Conformal Extensions of the Standard Model

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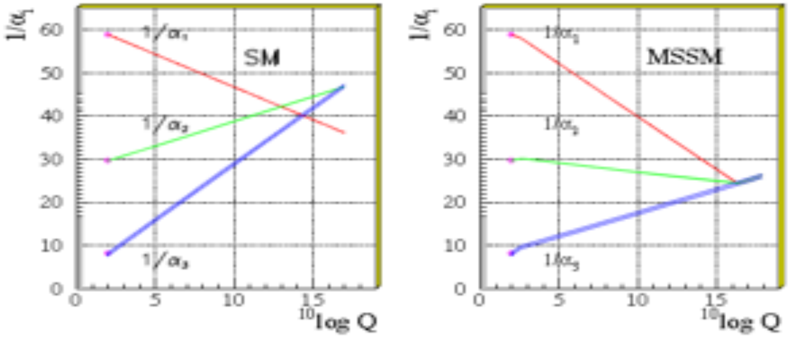
Reasons to go Beyond the Standard Model

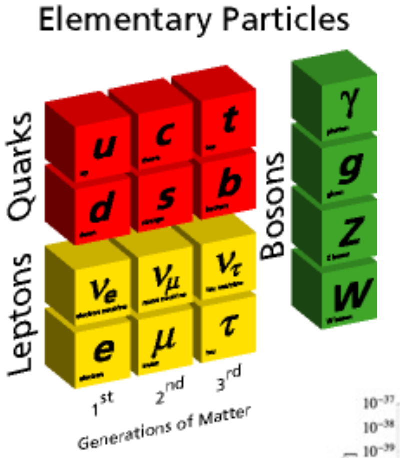
Theoretical arguments:

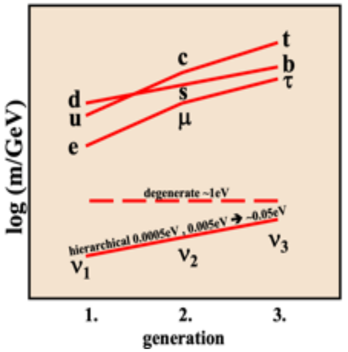
SM does not exist without cutoff
(triviality, vacuum stability)
Gauge hierarchy problem
Gauge unification ←→ charge quantization
Strong CP problem
Unification with gravity
Global symmetries & GR anomalies
Why: 3 generations, representations, d=4,
many parameters (flavour problem)

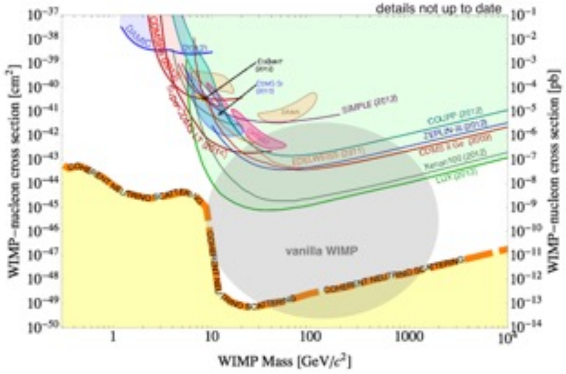
Facts, hints, problems:

- Electro-weak scale ≪ Planck scale
- Gauge couplings almost unify
- Neutrino masses & large mixings
- Flavour: Patterns of masses & mixings
- Baryon asymmetry of the Universe
- Dark Matter
- Inflation
- Dark Energy









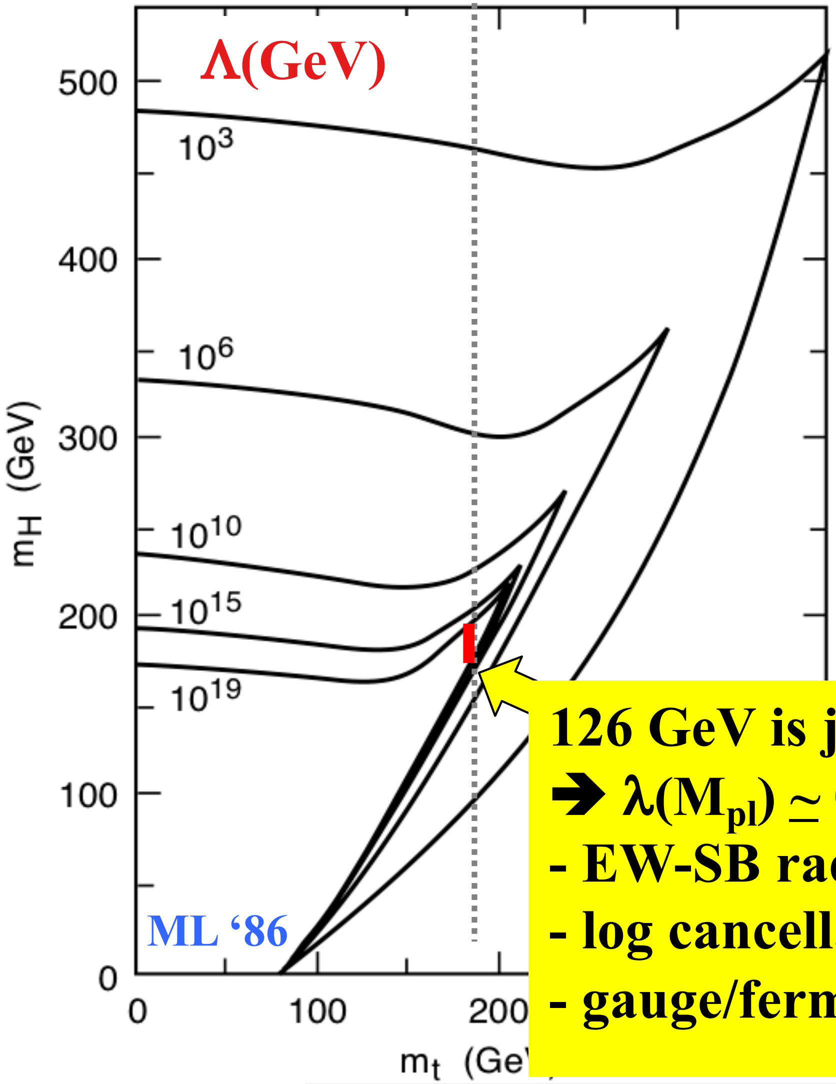
usualy:

- pick an idea, solve one problem
- be happy if it solves more probelms
- growing complexity...

A remarkable Coincidence of the SM

➔ **SM is a renormalizable QFT like QED w/o hierarchy problem**

➔ **Cutoff "Λ" has no meaning ➔ triviality, vacuum stability**



126 GeV is just here!

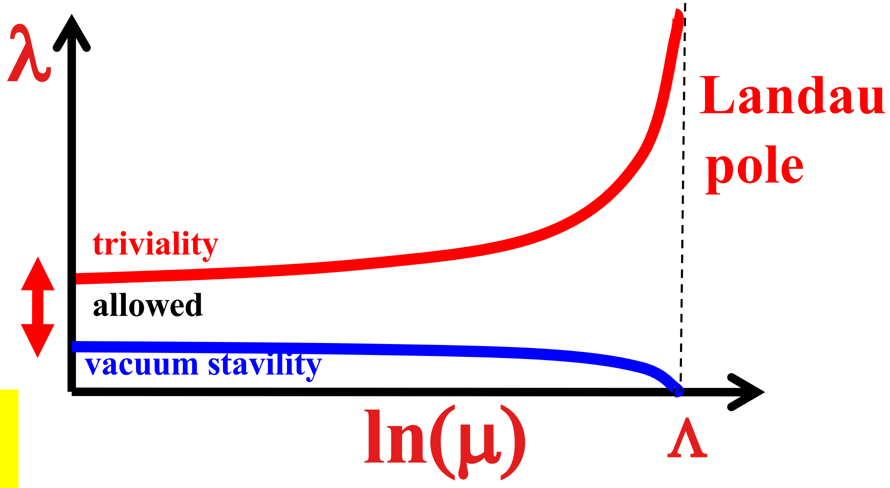
➔ $\lambda(M_{pl}) \simeq 0$

- **EW-SB radiative**
- **log cancellations**
- **gauge/fermion/scalar**

126 GeV < m_H < 174 GeV

SM does not exist w/o embedding

- U(1) coupling , Higgs self-coupling



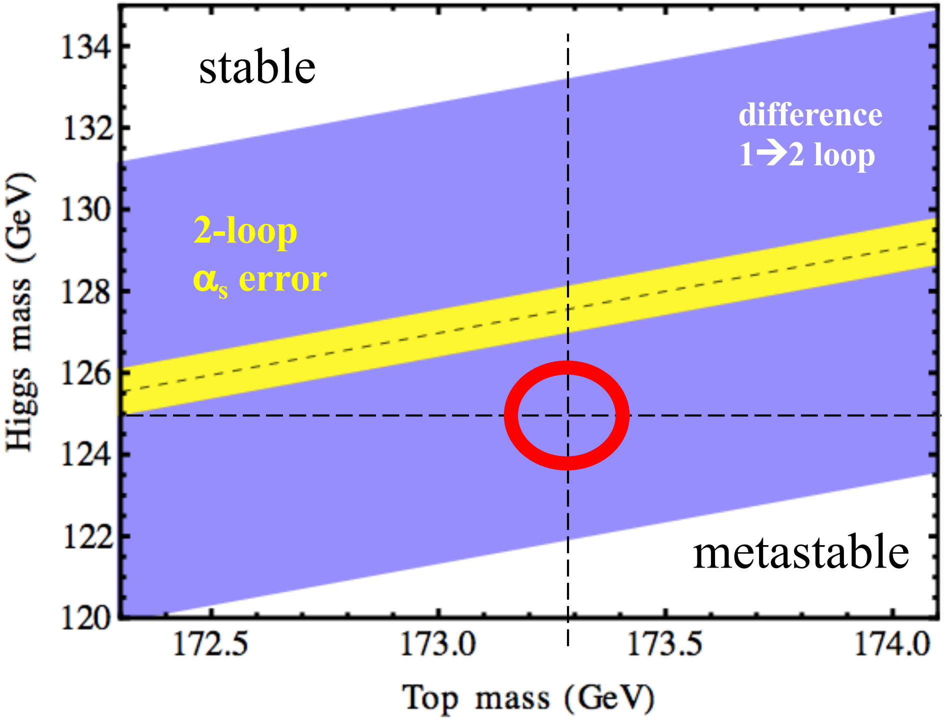
➔ RGE arguments seem to work

➔ but we need some embedding

Is the Higgs Potential at M_{Planck} flat?

Holthausen, ML, Lim

12 Dec 2011



Elias-Miro, Espinosa, Giudice, Isidori, Riotto, Strumia

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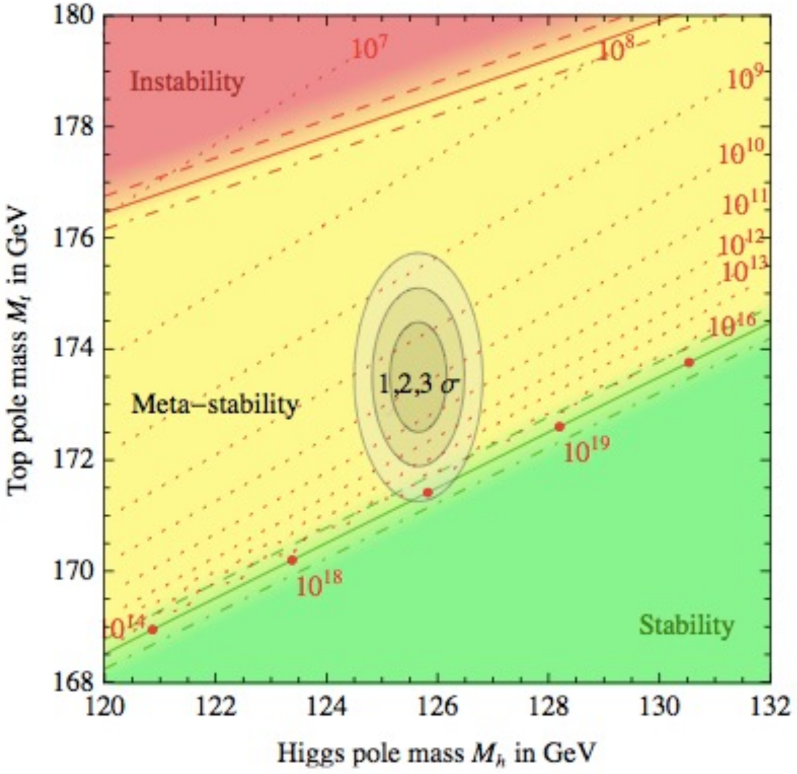


Experimental values indicate metastability, but,

→ **we need to include DM, neutrino masses, …? are all errors (EX+TH) fully included?**

→ **be cautious about claiming that metastability is established**

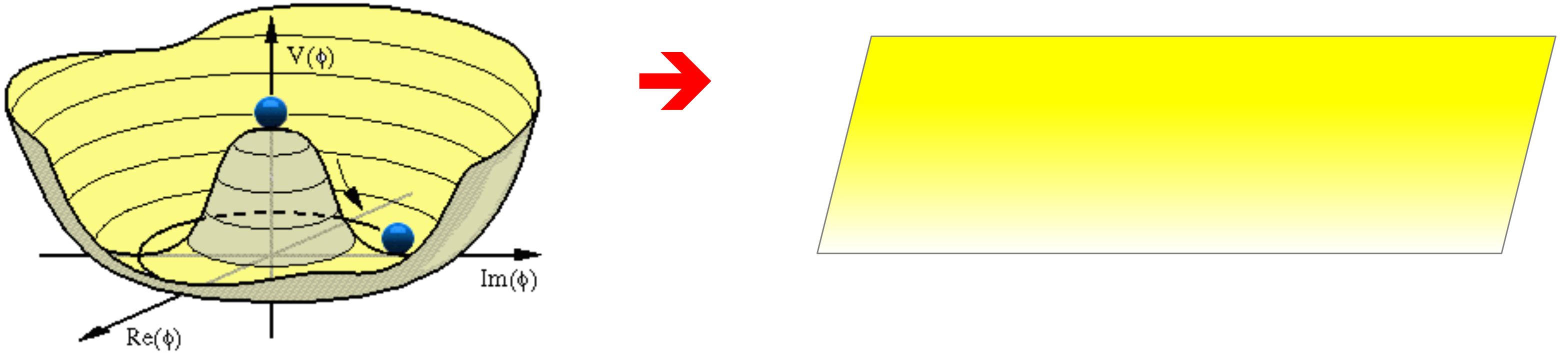
→ Important observation:

- **remarkable relation between weak scale, m_t, couplings and M_{Planck} ←→ precision**
- **interplay between gauge, Higgs and top loops: log divergences – not quadartic div.**

Is there a Message?

- $\lambda(M_{Planck}) \simeq 0$? ➔remarkable log cancellations ⇔ CA~ β-fcts.
  M_{planck}, M_{weak}, gauge, Higgs & Yukawa couplings are unrelated

- remember: μ is the only single scale of the SM ➔ special role
  ➔ if in addition $\mu^2 = 0$ ➔ $V(M_{Planck}) \simeq 0$
  ➔ flat Mexican hat (<1%) at the Planck scale!



➔ **conformal (or shift) symmetry as solution to the HP?**

➔ **combined conformal & EW symmetry breaking**

- conceptual issues
- minimal realizations ⇔ SM seems to know about high scales → bottom-up
  ⇔ many new d.o.f. (fields, big reps.) ~ UV-instabilities

Reminder: Scales and Hierarchy Problems

1) Why are (tree level) scales vastly different or couplings tiny?

2) Stability of vastly different scales under quantum corrections?

- SM with an embedding at Λ (new physics, not a SM regulator)

$$\delta M_H^2 = \frac{\Lambda^2}{32\pi^2 V^2}\left(6M_W^2 + 3M_Z^2 + 3M_H^2 - 12M_t^2\right)$$ **$\sim\Lambda^2 \gg M^2_H$**

- SM + Dirac neutrino masses: no problem – just like SM
- SM + Majorana neutrino masses ➔ more scales M_i

➔ generates another HP problem for large M even if y_ν is tiny

➔ $\delta m_H^2 \simeq \frac{y_\nu^2}{16\pi^2} M^2$; $y_\nu^2 = M m_\nu / v^2$

$M \lesssim 10^7 - 10^8$ GeV ⬅➔ **see-saw, leptogenesis, ...**

The Problem: Two or more EXPLICIT Scales

- Renormalizable QFT with two scalars φ , Φ with masses m, M and a hierarchy $m \ll M$
- $\varphi^+\varphi$ and $\Phi^+\Phi$ are singlets ➔ must interact via portal term ➔ $\lambda_{mix}(\varphi^+\varphi)(\Phi^+\Phi)$ in addition to φ^4 and Φ^4
- Quantum corrections $\sim M^2$ drives m to the (heavy) scale M ➔ vastly different explicit scalar scales are generically unstable

- **Since SM Higgs exists ➔ problem: embedding with a 2nd scalar**
  - **gauge extensions: LR, PS, GUTs → must be broken…**
  - **even for SUSY GUTS → doublet-triplet splitting...**
  - **also for fashionable Higgs-portal scenarios...**
- **Ways out:**
  - **No Higgs ...**
  - **Symmetry: SUSY, ... ; ➔ here: conformal symmetry = no explicit scales!**
  - **Question: Is one physical scale ($\mu^2 \neq 0$) of the SM an issue?**

Scale / Conformal Symmetry & SM

➔ **scale or conformal invariance:** Lagrangian without any dimensionful parameter

Conformal symmetry is an old topic:

- Scale invariance is hardly broken by scale anomaly: Callan, ’70; Symanzik,’70
- The scale anomaly cannot directly generate a mass gap
- To generate a mass gap, scale invariance has to be spontaneously broken

What about the Standard Model:

- It is a one-scale theory (←→ more scales for Majorana masses?)
- $\mu^2 \neq 0$ required by SSB ➔ not scale invariant!
- For $\mu^2 = 0$ increased symmetry → makes classical scale invariance exact
  ➔ can $\mu^2 \neq 0$ be a quantum effect?
- Loops: log. running coupling constants break scale invariance ➔ β-functions

$$\partial^\mu J_\mu = T^\mu{}_\mu = \sum_i \beta_i \cdot \hat{O}_i + \mathcal{C}$$

$\hat{O}_i$ = dim. 4 operators $\mathcal{C}$ = Weyl anomaly ←→ curved backgrounds

- log running and quadratic divergences are different breakings of scale invariance

Explaining Masses without Mass

SM: Quadratic divergences?←→ 2nd scale (cutoff, heavy particle...)

Bardeen '95 quadratic divergences may be an artefact (like a regulator)

➔ Λ^2 not surprising - regulator is 2nd explicit scale!

not a problem in SM only - renormalizable like QED

➔ problems left: triviality, vacuum stability...

SM with $\mu^2 = 0$ ➔ no explicit scale ➔ more symmetry

➔ quadratic divergences may be an artefact

Conformal anomaly = breaking of CS by quantum effects (loops)

anomaly $\simeq$ trace of energy momentum tensor

←→ β-functions ←→ log running

←→ UV fixed points ←→ anomaly matching

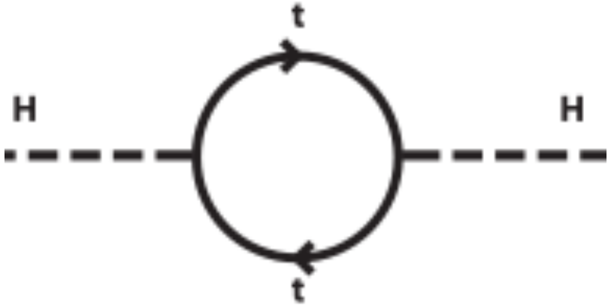
no Λ^2 divergences if the theory has no scale

Wheeler '62: Starting with a theory with mass scales one has no chance to explain its origin

The program:

1. **Start with conformally symmetric theories (CS)**
2. **CS broken by quantum effects**
   ➔ **conformal anomaly ~ β-functions**
3. **CS is maintained if the anomaly is matched**
   ➔ **UV fixedpoints** (later? selecting representations or automatically?
4. **no quadratic divergences if anomaly is matched**

Implications:



naïve power counting would be wrong
since after SSB CS is non-linearly realized
➔ no Λ^2 divergence

➔ dimensional transmutation of conformal theories
by log running of couplings like in chiral QCD

Warning: The Lagrangian in the broken phase explicitely violates conformal symmetry
➔ naive power counting would be misleading

Bottom-up realizations

Why the minimalistic SM does not work

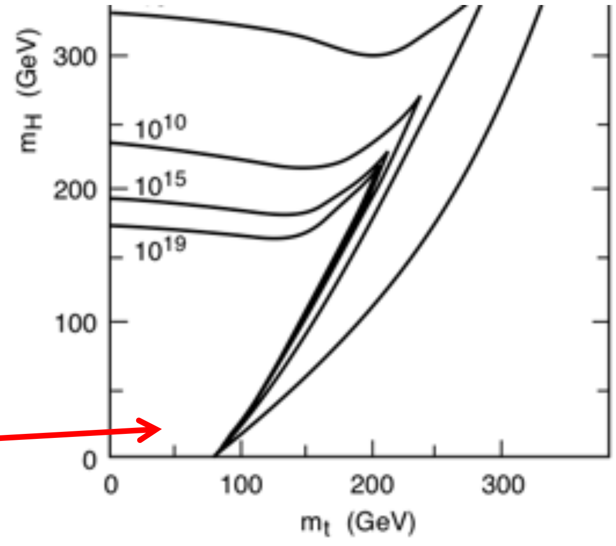
Minimalistic version: → "SM-"

SM + with μ= 0 ←→ CS

Coleman Weinberg: effective potential

→ CS breaking (dimensional transmutation)

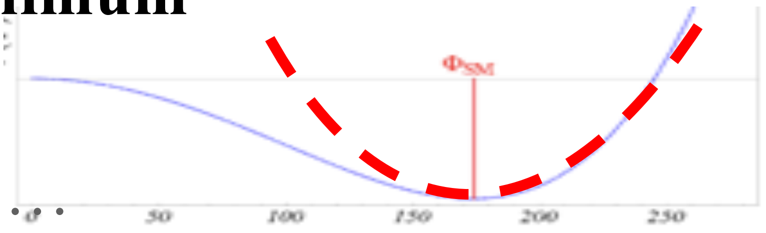
→ induces for $m_t < 79$ GeV
a Higgs mass $m_H = 8.9$ GeV



- **Success: no-scale SM → broken SM but: Higgs and top do not fit**

- **DSB for weak coupling ←→ CS= phase boundary**
  → scale set by log-running couplings ←→ gap eqn: hierarchical!

- **Reason for $m_H << v$: V_{eff} flat around minimum**
  ←→ m_H ~ loop factor ~ $1/16\pi^2$



AND: We need neutrino masses, dark matter, …

Realizing the Idea via Higgs Portals

- **SM scalar Φ plus some new scalar φ (or more scalars)**
- **CS → no scalar mass terms**
- **the scalar portal $\lambda_{mix}(\varphi^+\varphi)(\Phi^+\Phi)$ must exist**

→ a condensate of $<\varphi^+\varphi>$ produces $\lambda_{mix}<\varphi^+\varphi>(\Phi^+\Phi) = \mu^2(\Phi^+\Phi)$
→ effective mass term for Φ

- **no CA… → breaking only ln(Λ)**

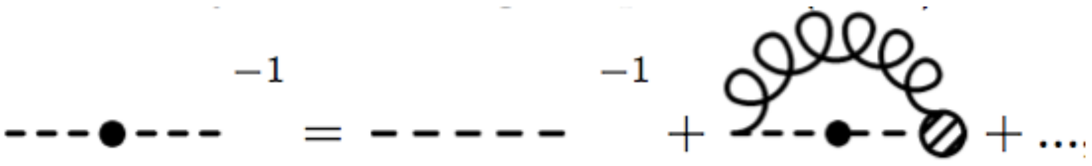
→ implies a TeV-ish condensate for φ to obtain <Φ> = 246 GeV

- **Many model building possibilities / phenomenological aspects:**
  - **- φ could be an effective field of some hidden sector DSB**
  - **- further particles could exist in hidden sector; e.g. confining…**
  - **- extra hidden U(1) potentially problematic ←→ U(1) mixing**
  - **- avoid Yukawas which couple visible and hidden sector**

→phenomenology safe due to Higgs portal →suppressed TeV-ish BSM physics!

Rather minimalistic: SM + QCD Scalar S

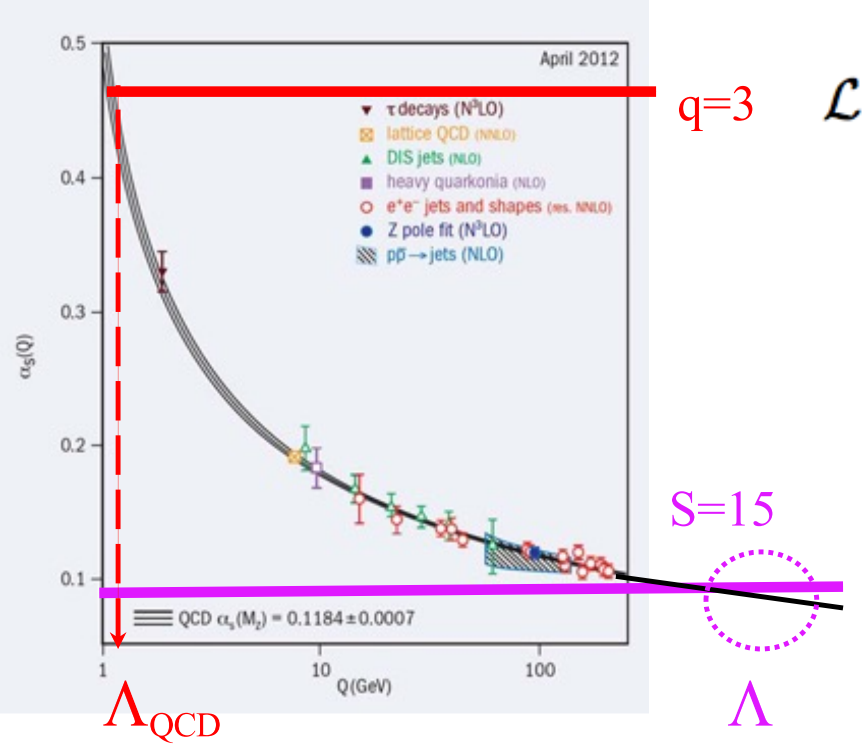
J. Kubo, K.S. Lim, ML New scalar representation S → QCD gap equation:



→ $\boxed{C_2(S)\alpha(\Lambda) \gtrsim X}$

$C_2(\Lambda)$ increases with larger representations

←→ condensation for smaller values of running α



$$\mathcal{L} = \mathcal{L}_{\mathrm{SM},m^2\to 0} + (D_{\mu,ij}S_j)^\dagger (D^\mu_{ik}S_k) + \lambda_{HS}H^\dagger H S^\dagger S - \lambda_{\mathbf{1}_i}\left[\bar{\boldsymbol{S}}\times\boldsymbol{S}\times\bar{\boldsymbol{S}}\times\boldsymbol{S}\right]_{\mathbf{1}_i}$$

$$\lambda_{HS}\langle S^\dagger S\rangle H^\dagger H \to \lambda_{HS}\Lambda^2 H^\dagger H$$

$$m_h^2 = 2\lambda_{HS}\Lambda^2 \qquad \frac{\lambda_h}{\lambda_{HS}} = \frac{\Lambda^2}{v^2}$$

SM ⊗ hidden $SU(3)_H$ Gauge Sector

Holthausen, Kubo, Lim, ML

- hidden $SU(3)_H$:

$$\mathcal{L}_{\rm H} = -\frac{1}{2}{\rm Tr}\ F^2 + {\rm Tr}\ \bar{\psi}(i\gamma^\mu D_\mu - yS)\psi$$

gauge fields ; $\psi = 3_H$ with $SU(3)_F$; S = real singlet scalar

- SM coupled by S via a Higgs portal:

$$V_{{\rm SM}+S} = \lambda_H (H^\dagger H)^2 + \frac{1}{4}\lambda_S S^4 - \frac{1}{2}\lambda_{HS} S^2 (H^\dagger H)$$

- no scalar mass terms
- use similarity to QCD, use NJL approximation, …
- χ–ral symmetry breaking in hidden sector: $SU(3)_L x SU(3)_R$ → $SU(3)_V$ ➔ generation of TeV scale

➔ transferred into the SM sector through the singlet S

➔ dark pions are PGBs: naturally stable ➔ DM

Realizing the Idea: Many more Models

SM + extra singlet or doublet: Φ, φ
Nicolai, Meissner Farzinnia, He, Ren, Foot, Kobakhidze, Volkas, Hill, …

Minimal B-L extension if SM: $SU(3)c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$
Iso, Okada, Orikasa

Minimal LR-model: $SU(3)c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ Holthausen, ML, Schmidt

SM $\otimes$ $SU(N)_H$ with new N-plet in a hidden sector
Ko, Carone, Ramos, Holthausen, Kubo, Lim, ML, Hambye, Strumia , …

SM + QCD colored scalar which condenses at TeV scale Kubo, Lim, ML

SM $\otimes$ $[SU(2)_X \otimes U(1)_X]$
Altmannshofer, Bardeen, Bauer, Carena, Lykken

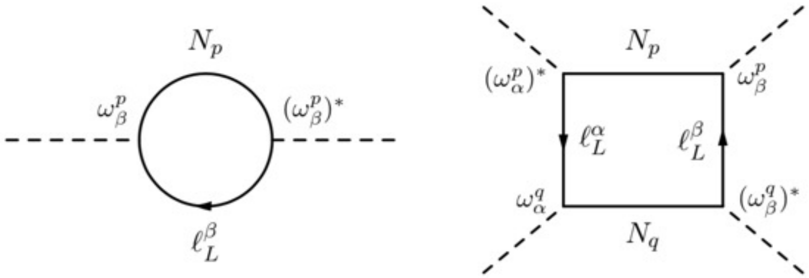
… more ...

Since the SM-only version does not work ➔ observable effects:

- Higgs coupling to other scalars (singlet, hidden sector, …)
- dark matter candidates ←→ hidden sectors & Higgs portals
- consequences for neutrino masses

The Neutrino Option

Connection between EWSB and neutrinos ⇔ ν-hierarchy problem



Neutrino option: Brivio, Trott

➔ symmetry breaking V_{eff} induced by neutrino loops

Conformal Realization of the Neutrino Option: Brdar, Emonds, Helmboldt, ML

➔ conformal symmetry + V_{eff} from neutrino loops (not from Higgs portal)

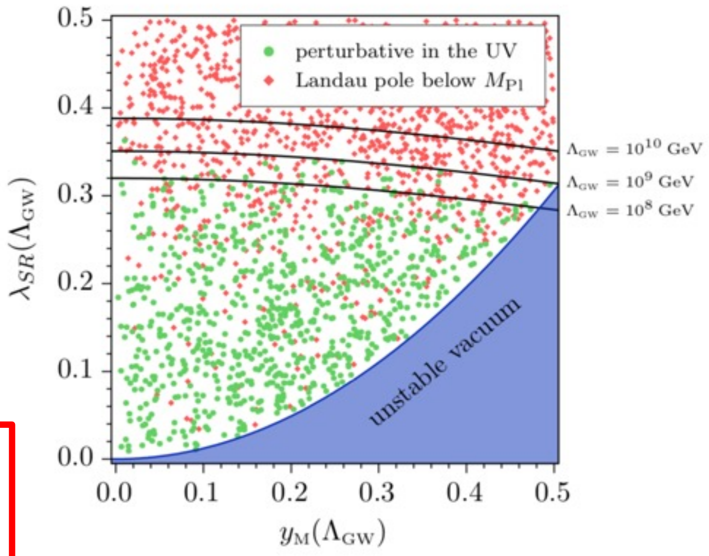
Fields: SM + 3x NR + 2x scalar SM singlets: S, R

$$\begin{aligned}\mathcal{L} \supseteq\ & \tfrac{1}{2}\partial_\mu S \partial^\mu S + \tfrac{1}{2}\partial_\mu R \partial^\mu R + \mathrm{i}\bar{N}_R \not{\partial} N_R \\ & - V(H,S,R) - \left(\tfrac{1}{2} y_{\mathrm{M}} S \bar{N}_R N_R^c + y_\nu \bar{L}\tilde{H} N_R + \text{h.c.}\right)\end{aligned}$$

➔ consistent UV-complete realization of the idea

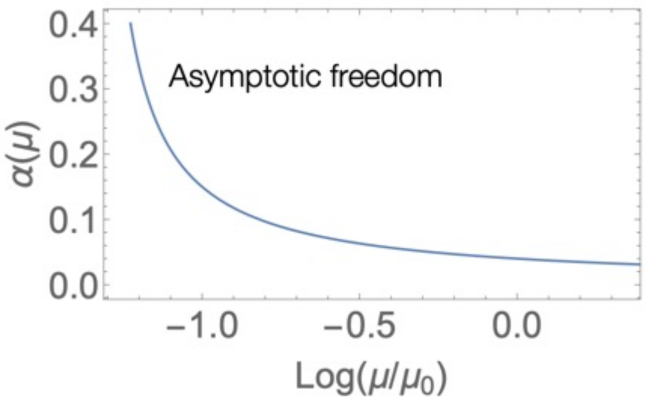
➔ very nice feature:

$$\boxed{\lambda_{HS} \ll \frac{3}{16\pi^2} y_\nu^2(\Lambda_{\mathrm{GW}}) \cdot y_{\mathrm{M}}^2(\Lambda_{\mathrm{GW}}) \simeq \mathrm{O}(10^{-12})}$$

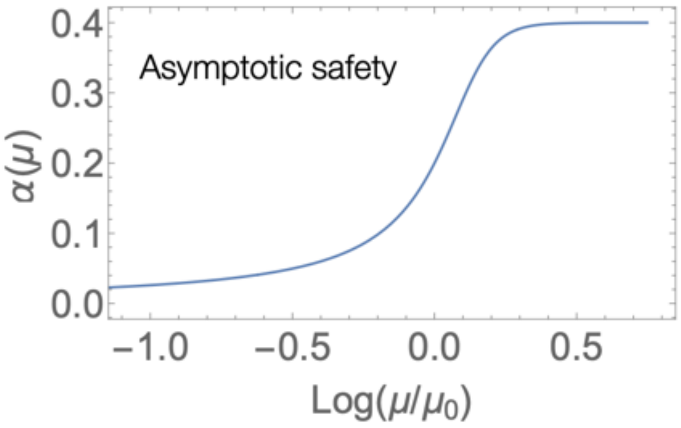


UV-Completion

Successful theories should have a meaningful UV-completion
➔ vanishing β-functions for all couplings (UV fixedpoints) ←→ restored scale symmetry



Interacting UV-fixedpoint ➔
← trivial fixedpoint



Interacting UV-fixedpoints:
- requires carefully selected particle content ➔ explanation?
- scalar self-couplings and Yukawa couplings tend to have Landau poles...

Better trivial fixedpoints:
- no fundamental scalars
- no Yukawa couplings
  ➔ all sclars composite
  ➔ automatically safe models

Conformal Little Higgs

Aqeel Ahmed,* Manfred Lindner,† and Philipp Saake‡

Max-Planck-Institut für Kernphysik (MPIK),
Saupfercheckweg 1, 69117 Heidelberg, Germany

to appear

Conformal Symmetry & Neutrino Masses

ML, S. Schmidt and J.Smirnov

- No explicit scale ➔ no explicit (Dirac or Majorana) mass term
  ➔ only Yukawa couplings ⊗ generic scales
- Enlarge the Standard Model field spectrum
  like in 0706.1829 - R. Foot, A. Kobakhidze, K.L. McDonald, R. Volkas
- Consider direct product groups: SM ⊗ HS
- Two scales: CS breaking scale at O(TeV) + induced EW scale

Important consequence for fermion mass terms:

➔ spectrum of Yukawa couplings ⊗TeV or ⊗EW scale
➔ interesting consequences ←→ Majorana mass terms are no longer expected at the generic L-breaking scale → anywhere

Examples

Yukawa seesaw:

SM + ν_R + singlet

$$\langle\phi\rangle \approx \mathrm{TeV}$$

$$\langle H\rangle \approx 1/4\,\mathrm{TeV}$$

$$\mathcal{M} = \begin{pmatrix} 0 & y_D\langle H\rangle \\ y_D^T\langle H\rangle & y_M\langle\phi\rangle \end{pmatrix}$$

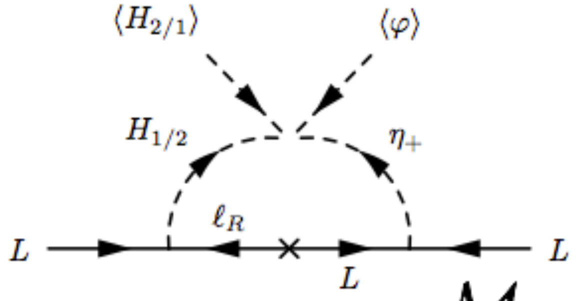
➔ generically expect a TeV seesaw

BUT: y_M can be tiny

➔ wide range of sterile masses ➔ including pseudo-Dirac case

➔ suppressed 0νββ

Radiative masses



$$\mathcal{M} = m_L \quad \text{or}$$

$$\mathcal{M} = \begin{pmatrix} \mu_1 & y_D\langle H\rangle \\ y_D^T\langle H\rangle & \mu_2 \end{pmatrix}$$

➔pseudo-Dirac case

The punch line:

all usual neutrino mass terms can be generated

- ➔ suitable scalars required
- ➔ no explicit masses: all via Yukawa couplings
- ➔ different numerical expectations ←→ could easily explain keV masses

Conformal Symmetry & Dark Matter

Different natural and viable options:

1) eV, **keV = DM**, TeV, … sterile neutrino mass easily possible ←→ not so easy in standard see-saw's

2) New particles which are fundamental or composite DM candidates:
   - hidden sector pseudo-Goldstone-bosons
   - stable color neutral bound states from new QCD representations

➔ some look like WIMPs
➔ others are extremely weakly coupled (via Higgs portal)
➔ or even coupled to QCD (threshold suppressed…)

Including the Planck Scale

The Planck Scale from CS Breaking

Conformal Gravity (CG):

- more symmetry ➔ claimed to be power counting renormalizable
- CG may have a ghost... ➔ see later
- Spontaneous generation (SG) of M_{Pl} = SG of Einstein-Hilbert theory
- most economic and simple way:

$$\frac{\xi_S}{2} S^2 R \rightarrow \frac{\xi_S}{2} \langle S \rangle^2 R \rightarrow \frac{M_{\rm Pl}^2}{2} R$$

$$M_{\rm Pl} = \sqrt{\xi_S} \langle S \rangle$$

Brans+Dicke,'61; Fujii,'74; Englert+Truffin+Grastmans,'76; Minkowsky,'77;......

Idea: Generate M_{Planck} from conformal gravity ⊗ SU(N)

➔ gauge assisted condensate via SU(N) field ➔ M_{Planck} = effective scale

Kubo, ML, Schmitz, Yamada similar ideas: **Donoghue, Menezes, ...**

$$S_C = \int d^4x \sqrt{-g} \Big[-\hat{\beta}\, S^\dagger S\, R + \hat{\gamma}\, R^2 - \frac{1}{2} \mathrm{Tr}\, F^2 + \\ + g^{\mu\nu} (D_\mu S)^\dagger D_\nu S - \hat{\lambda} (S^\dagger S)^2 + a\, R_{\mu\nu} R^{\mu\nu} + b\, R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \Big]$$

R = Ricci curvature scalar, $R_{\mu\nu}$ = Ricci tensor, $R_{\mu\nu\alpha\beta}$ = Riemann tensor

F = field-strength tensor of the SU(N_c) gauge theory, **S = complex scalar in fund. rep. ➔ N_c**

➔ most general diffeomorphism invariance, gauge invariance, and global scale invariance

Condensation in SU(N_c) gauge sector

➔ dimensional transmutation: $\langle S^+ S \rangle$ ➔ effective Planck mass

$$M_{\mathrm{planck}} = 2\,\beta\, f_0 = \frac{N_c \beta}{16\pi^2} (2\,\lambda\, f_0) \left(1 + 2\,\ln \frac{2\,\lambda\, f_0}{\Lambda^2} \right) \quad \text{with} \quad f_0 = <S^+ S>$$

➔ Effectively normal gravity with a dynamically generated M_{Planck}

Dilaton-Scalaron Inflation

Effective Jordan-frame Lagrangian:

$$\frac{\mathcal{L}^J_{\text{eff}}}{\sqrt{-g_J}} = -\frac{1}{2}\, B\left(\chi\right) M^2_{\text{Pl}}\, R_J + G\left(\chi\right) R^2_J + \frac{1}{2}\, g^{\mu\nu}_J\, \partial_\mu\chi\, \partial_\nu\chi - U\left(\chi\right)$$ ➔ auxiliary field Ψ ➔

$$\frac{\mathcal{L}^J_{\text{eff}}}{\sqrt{-g_J}} = -\left[\frac{1}{2}\, B\left(\chi\right) M^2_{\text{Pl}} - 2\, G\left(\chi\right) \psi\right] R_J + \frac{1}{2}\, g^{\mu\nu}_J\, \partial_\mu\chi\, \partial_\nu\chi - U\left(\chi\right) - G\left(\chi\right) \psi^2$$

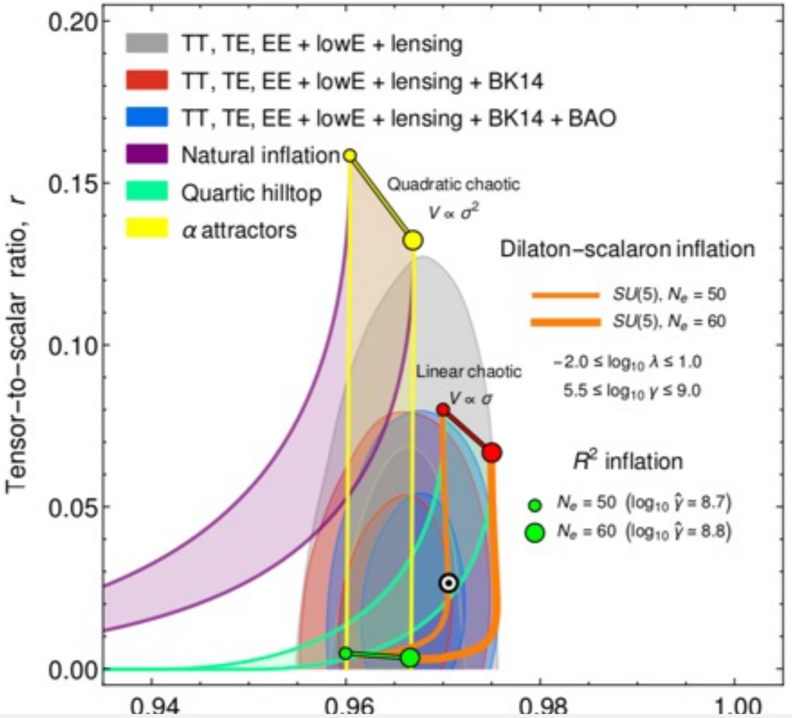
Weyl rescaling: $g_{\mu\nu} = \Omega^2\, g^J_{\mu\nu}$ $\qquad \Omega^2 = e^{\Phi(\phi)}\,, \qquad \Phi\left(\phi\right) = \frac{\sqrt{2}\,\phi}{\sqrt{3}\, M_{\text{Pl}}}$

Einstein-frame scalar potential:

$$V\left(\chi, \phi\right) = e^{-2\,\Phi(\phi)} \left[U\left(\chi\right) + \frac{M^4_{\text{Pl}}}{16\, G\left(\chi\right)} \left(B\left(\chi\right) - e^{\Phi(\phi)}\right)^2\right]$$

➔ Slow role inflation

➔ fits data very well!



The Ghost Problem in quadratic Gravity

Unlike GR, **quadratic gravity is renormalizable** thanks to four derivatives of the metric

$$\mathcal{L}_{\rm EH} = \sqrt{-g}\,\underset{\text{dimensionful}}{M_{\rm pl}^2}\,R \qquad \mathcal{L}_{\rm QG} = \sqrt{-g}\Big(-\beta\phi^2 R + \gamma R^2 - \kappa C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\Big)$$

dimensionless

Problem: Double pole ➔ **classical Ostrogradsky instability**

$$\Delta_{hh} \sim \frac{1}{p^2} - \frac{1}{(p^2 - m_{\rm gh}^2)} \quad \Rightarrow \quad \mathcal{H} \sim c_+\pi_+^2 - c_-\pi_-^2 + \cdots$$

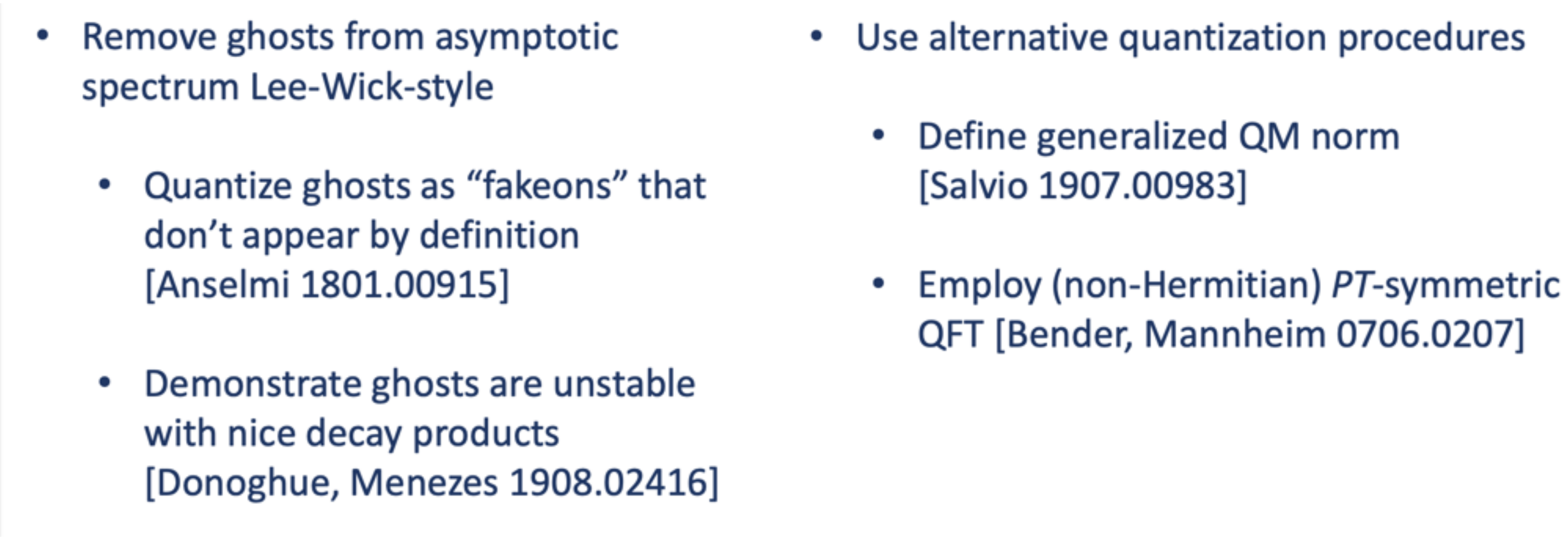
unbounded Hamiltonian

Leads after quantization to negative norm states ➔ **unitarity violation**

$$\begin{aligned}\big[\hat{a}_h(\mathbf{p}),\, \hat{a}_h^\dagger(\mathbf{q})\big] &= \delta^3(\mathbf{p}-\mathbf{q})\\ \big[\hat{a}_H(\mathbf{p}),\, \hat{a}_H^\dagger(\mathbf{q})\big] &= -\delta^3(\mathbf{p}-\mathbf{q})\end{aligned} \quad \Rightarrow \quad \sum_n \big|\langle n|S|\alpha\rangle\big|^2 \neq 1$$

breakdown of probability interpretation

Potential Solutions



- Remove ghosts from asymptotic spectrum Lee-Wick-style
  - Quantize ghosts as “fakeons” that don’t appear by definition [Anselmi 1801.00915]
  - Demonstrate ghosts are unstable with nice decay products [Donoghue, Menezes 1908.02416]

- Use alternative quantization procedures
  - Define generalized QM norm [Salvio 1907.00983]
  - Employ (non-Hermitian) *PT*-symmetric QFT [Bender, Mannheim 0706.0207]

- **Unitarity OK if interaction energies are below the ghost mass**

  ➔ conformal theories OK if ghost becomes massive after SSB

  $M_{ghost} \simeq M_{Planck}$ ➔ no unitarity violation except in the early (pre-inflation) universe

  Kubo, Kuntz 2202.08298, 2208.12832

Conclusions

- **Explaining masses without masses ➔ conformal symmetry**
  - → inspiring SM features...
  - → close, but does not work → non-minimal versions + DM, ν's, ..., GR
- **SM embedings into QFTs with conformal symmetry**
  - → combined conformal & electro-weak symmetry breaking
  - → implications for BSM phenomenology
  - → implications for Higgs couplings, neutrino physics, dark matter, …
  - **➔ testable consequences: @LHC, dark matter, neutrinos**
- **Planck scale generation by gauge induced breaking of conformal GR**
  - → very nice phenomenology: inflation…
  - → consistent quantum gravity: renormalizability!, ghosts?
    - ←→ normal GR from a theory with more symmetry