The HEFT, the SMEFT and Higgsing the On-Shell Way

Yael Shadmi, TECHNION

Reuven Balkin, Gauthier Durieux, Teppei Kitahara, YS, Yaniv Weiss '21
Hongkai Liu, Teng Ma, YS, Michael Waterbury '23

expanding on methods from:
YS Weiss '18
Durieux Kitahara YS Weiss '19
Durieux Kitahara Machado YS Weiss '20

PASCOS 2023, UCI

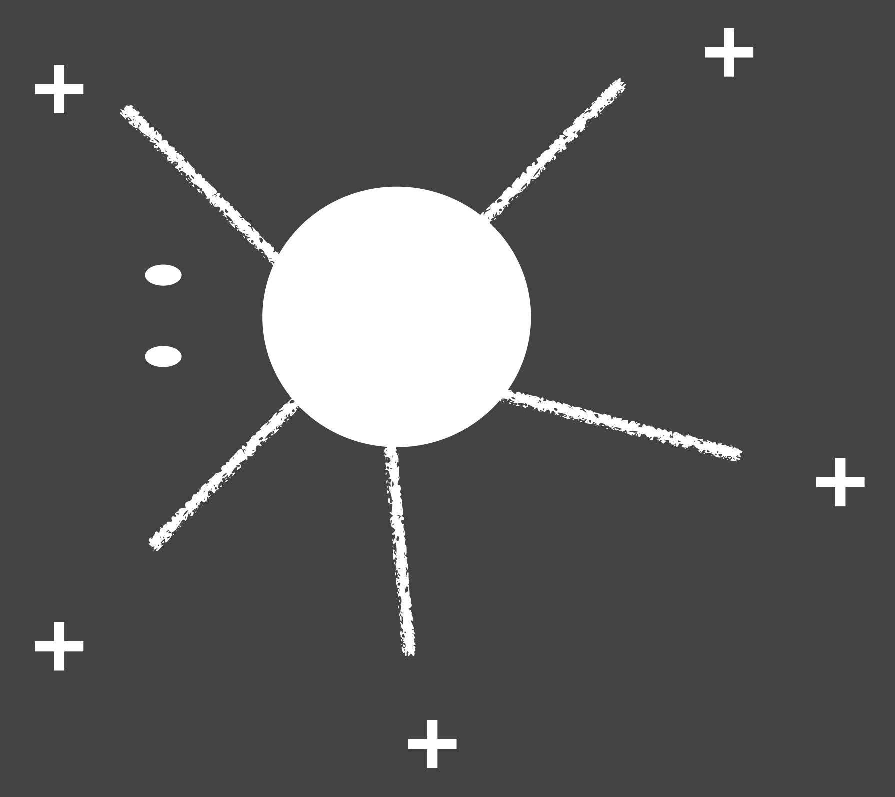
June 23

Why on-shell?

0) 1st clue: amplitudes: the whole is SMALLER than the sum of its parts:

gauge boson amplitudes: many Feynman diagrams   (~10 million for tree 10-gluon):

Mangano Parke review



= 0

Why on-shell?

describe massless *spin-1 particle* (2 dof’s) via *vector field* (4 dof’s)

➡ more efficient: focus on physical dof’s only

Why on-shell?

1) various ways developed for expressing amplitudes: make various properties/symmetries transparent

here:

massless & massive amplitudes in terms of **2-component spinor** products

- uniform description of amplitudes of different spins

- properties of amplitudes under Lorentz manifest: Little Group

    —> selection rules

- simple relations between massive <—> massless

Why on-shell?

2) bootstrapping amplitudes:

construct amplitudes based on their properties: little group; poles, cuts

$$\rightarrow \quad \mathcal{A}_{SM} + \mathcal{A}_{EFT}$$

rediscover SM
(more generally, gauge theory, Higgs mechanism)

- most general EFT amplitude
- model independent
- no issues of field redefinitions, basis dependence

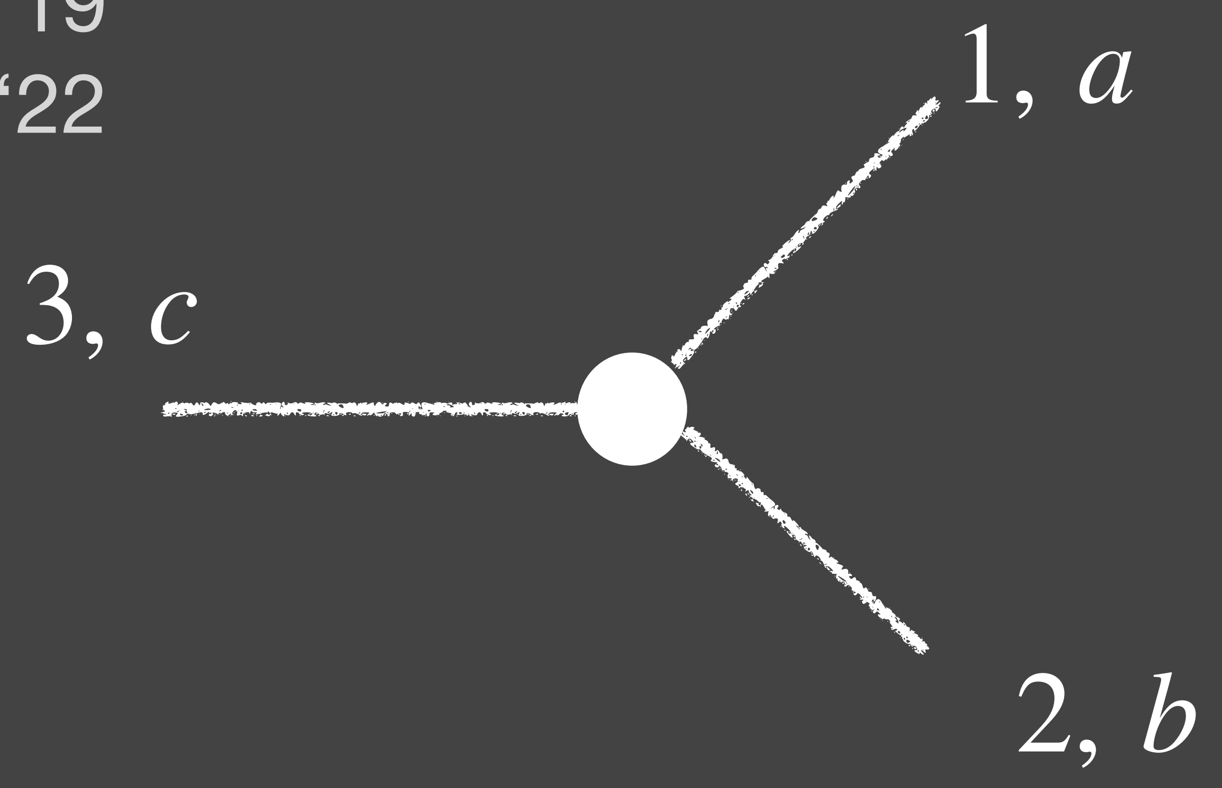
- natural approach as we try to go beyond SM rediscover SM

Lie groups (gauge symmetry) from amplitudes: (textbook example eg Schwartz)

the consistent interactions of spin-1 particles —> **LIE GROUPS**

3 massive degenerate spin-1 particles

Durieux Kitahara YS Weiss '19
Liu Yin '22



Lorentz (little group): most general amplitude:

$$C^{abc}\ \Big(\langle\mathbf{12}\rangle[\mathbf{23}]\langle\mathbf{31}\rangle + [\mathbf{12}]\langle\mathbf{23}\rangle[\mathbf{31}]\ +\ \text{perm}\Big)/M^2$$

$$+C'^{abc}\ \langle\mathbf{12}\rangle\langle\mathbf{23}\rangle\langle\mathbf{31}\rangle/\Lambda^2 + \mathbf{C}''^{\mathbf{abc}}\ [\mathbf{12}][\mathbf{23}][\mathbf{31}]/\Lambda^2$$

3 massive degenerate spin-1 particles

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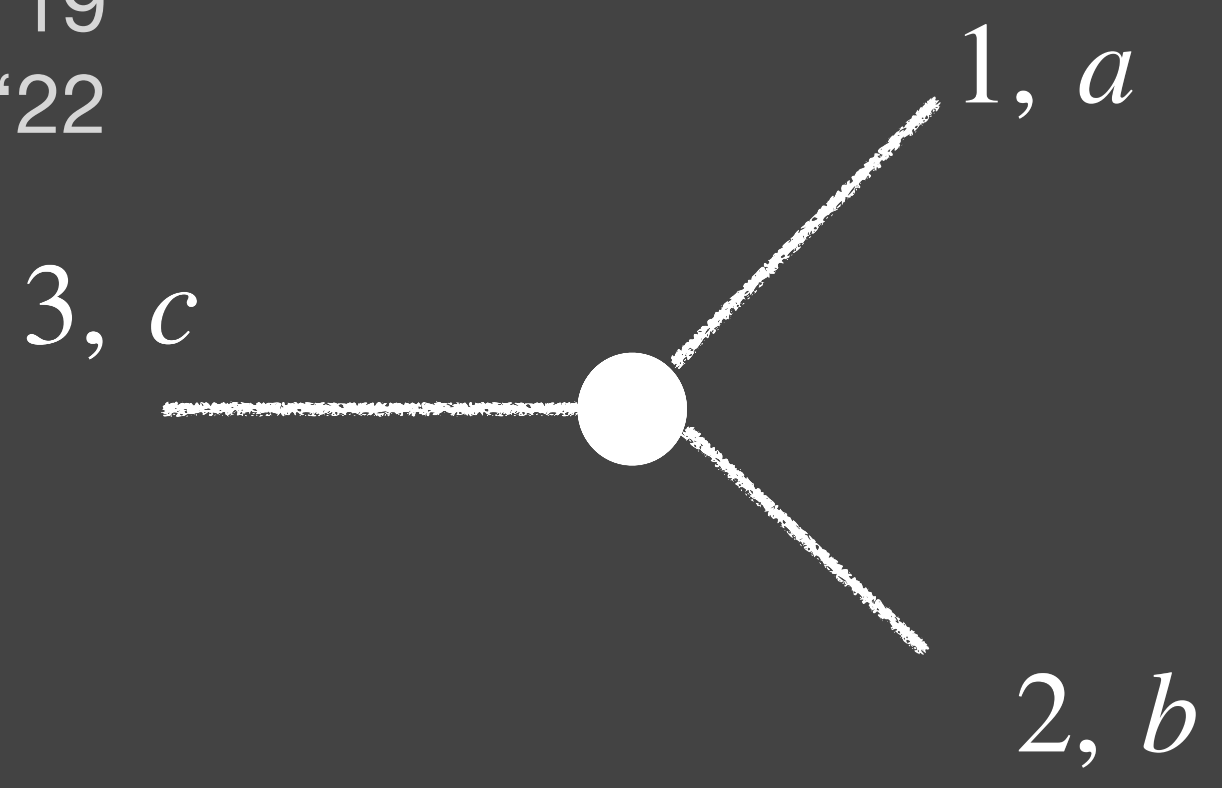

Lorentz (little group): most general amplitude:

$$C^{abc}\left(\langle 12\rangle[23]\langle 31\rangle + [12]\langle 23\rangle[31] \;+\; \text{perm}\right)/M^2$$

completely antisymmetric

$$+C'^{abc}\,\langle 12\rangle\langle 23\rangle\langle 31\rangle/\Lambda^2 + C''^{abc}\,[12][23][31]/\Lambda^2$$

—> C^{abc} completely antisymmetric

structure constants!

+ factorization of 4-points on 3-points: Jacobi identity

3 massive degenerate spin-1 particles

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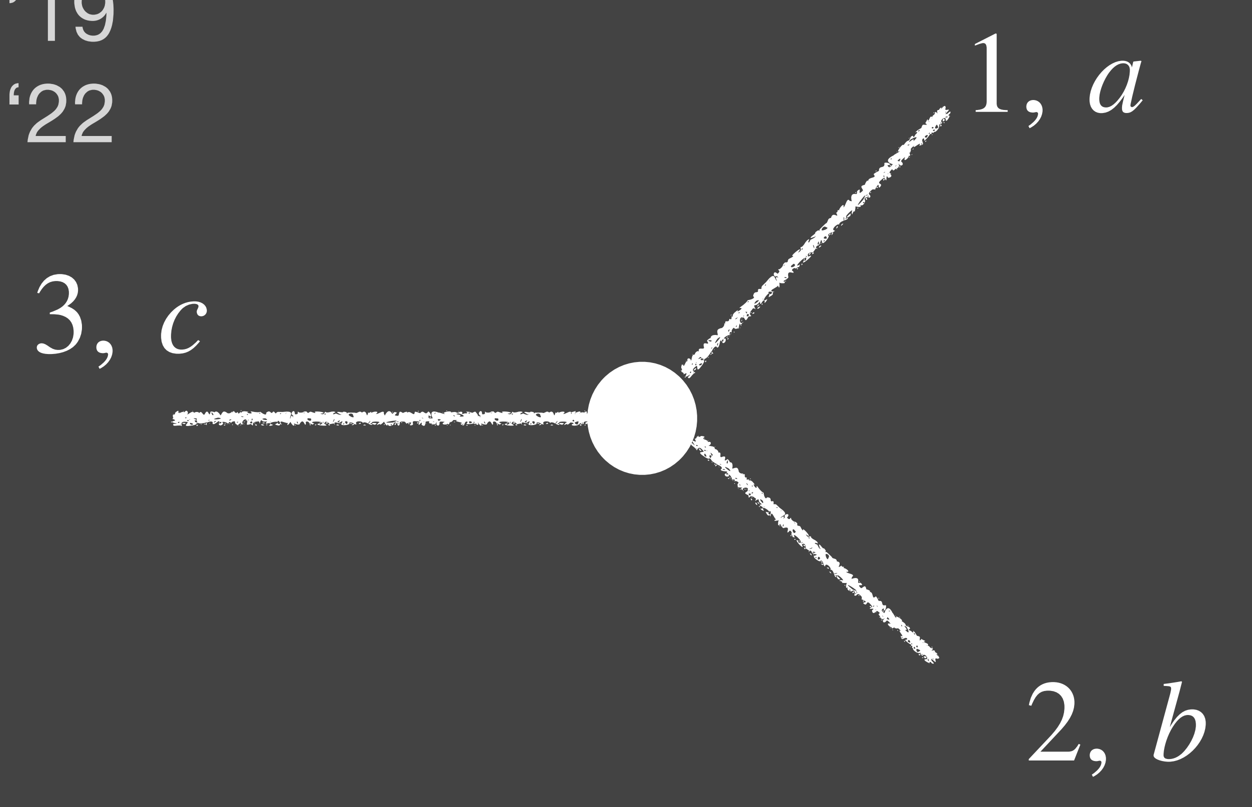

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$$+C'^{abc}\,\langle 12\rangle\langle 23\rangle\langle 31\rangle/\Lambda^2 + C''^{abc}\,[12][23][31]/\Lambda^2$$

power of Lorentz

—> C^{abc} completely antisymmetric

structure constants!

+ factorization of 4-points on 3-points: Jacobi identity natural to expect also general features of the **Higgs mechanism** to emerge from Lorentz

today:

- anatomy of the Higgs mechanism at the amplitude level
- application: on-shell derivation of SMEFT, HEFT amplitudes at ***low-energy***

2023:

1. EWSB ?? have only ad-hoc effective description: why is symmetry broken? what sets the scale? what stabilizes the scale?

2. know that we know nothing about the UV (*): motivates use of EFTs, on-shell construction of EFTs

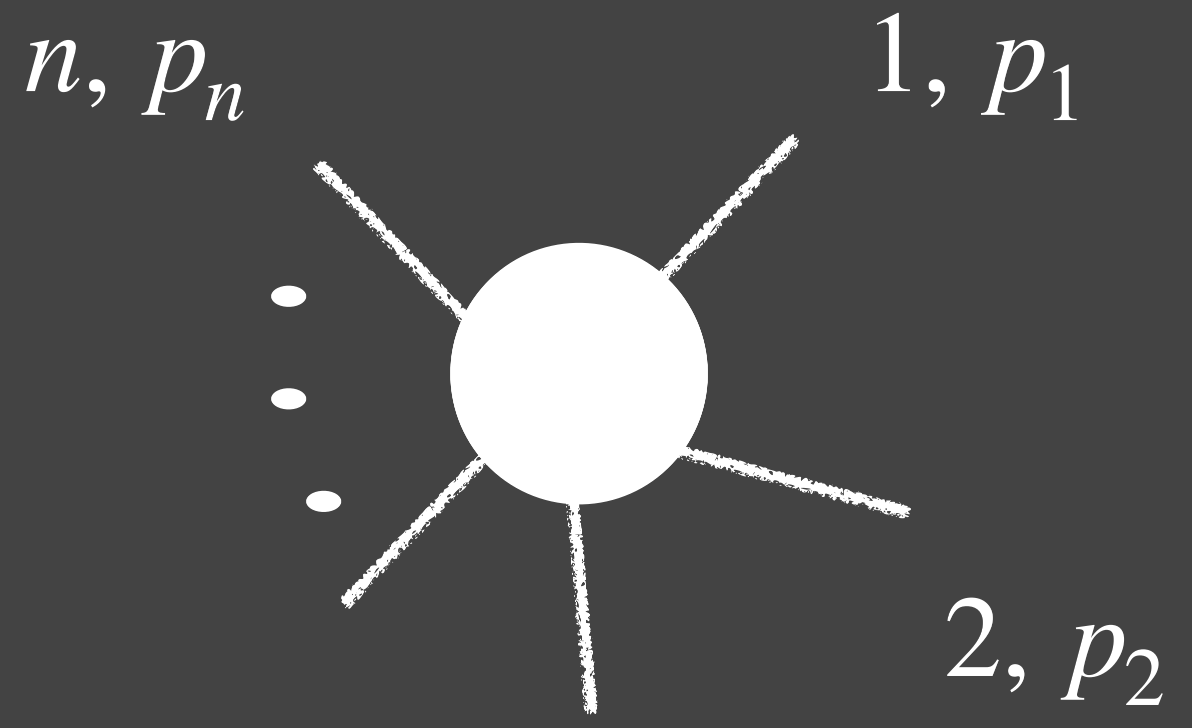
notations: spinor variables:

why suffer:

little group (LG) “charges” transparent —> selection rules

massless-massive relations transparent in LG covariant (“bolded”) massive spinor formalism

amplitude basics: spinor variables:



amplitude is function of momenta, polarizations ($s = 1/2, \quad s = 1$)

all can be written in terms of massless 2-component spinors:

$u_+(p) = p]$ or $u_-(p) = p\rangle$

$\bar{u}_+(p) = [p \qquad \bar{u}_-(p) = \langle p$

massless particle: one 3-vector/lightlike vector (momentum) —> one spinor

massive particle: two 3-vector/two lightlike vector (momentum+spin axis) —> two spinors

amplitude basics: spinor variables: massless

$p_i = i\rangle[i$: LG (U(1))= Lorentz transformations keeping p_i invariant:

$$i] \to e^{i\phi}\ i] : \quad charge + 1$$

$$i\rangle \to e^{-i\phi}\ i\rangle : \quad charge - 1$$

amplitude basics: spinor variables: massless

external leg i :

$i, h = 1/2 \qquad i]$

$i, h = -1/2 \qquad i\rangle$

$i, h = +1 \qquad i]i]$

$i, h = -1 \qquad i\rangle i\rangle$

amplitude basics: spinor variables: massive

Arkani-Hamed Huang Huang ‘17

$$p_i = p_i^{I=1} + p_i^{I=2}$$ lightlike vectors

$$p_i = i\rangle^I \, [i_I$$

LG (SU(2)) = Lorentz transformations keeping p_i invariant:

$$i\rangle^I \to W^I_J \; i\rangle^J \qquad [i_I \to (W^{-1})^J_I \, [\; i_J$$

amplitude basics: spinor variables:

external leg i :

massless		massive	
$i, h = 1/2$	$i]$	$i, \ s = 1/2$	$\mathbf{i}]$ or $\mathbf{i}\rangle$
$i, h = -1/2$	$i\rangle$	$i, \ s = +1$	$\mathbf{i}]\mathbf{i}]$ or $\mathbf{i}\rangle\mathbf{i}\rangle$ or $\mathbf{i}\rangle\mathbf{i}]$
$i, h = +1$	$i]i]$		
$i, h = -1$	$i\rangle i\rangle$		

$$\mathbf{i}]\mathbf{i}] \equiv i]^{\{I} i]^{J\}}$$

can construct any SU(2) rep from symm combinations of doublets

amplitude basics: spinor variables:

amplitude = function of spinor products $\langle ij\rangle, \;\; [ij]$, or $\langle \mathbf{ij}\rangle, \;\; [\mathbf{ij}]$

& Lorentz invariants $s_{ij} = (p_i + p_j)^2$

amplitude basics: more on LG covariant massive spinors

high-energy limit:

$$p = p^{I=1} + p^{I=2} \quad \equiv k + q$$

HE: $k = \mathcal{O}(E) \sim p \quad q = \mathcal{O}(m^2/E)$

eg, only $\mathbf{p}]^{I=1} \sim p]$ survives; $\mathbf{p}]^{I=2} = q]$ subleading

—> HE limit: simply unbold spinor structures

Arkani-Hamed Huang Huang '17

massless <—> massive amplitudes from (un)bolding

anatomy of on-shell Higgsing

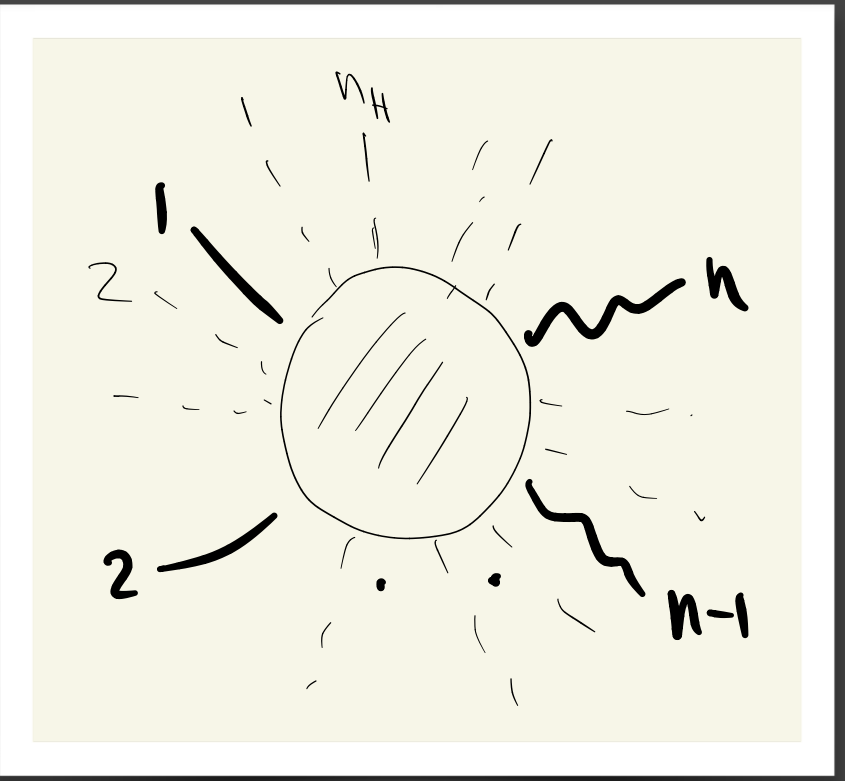
Balkin Durieux Kitahara YS Weiss ‘21

anatomy of on-shell Higgsing

Balkin Durieux Kitahara YS Weiss '21

start from massless amplitudes of unbroken theory and "Higgs" to get low-energy massive amplitudes

extra Higgs legs non-dynamical: soft: $H(q_i) \quad q_i \to 0$



probe field space

identify massless and massive amplitudes in high-energy/massless limit (where they coincide)

$$M_n(1,\ldots,n) \;=\; A_n(1,\ldots,n) + v \lim_{q\sim v\to 0} A_{n+1}(1,\ldots,n;H(q)) + \cdots$$

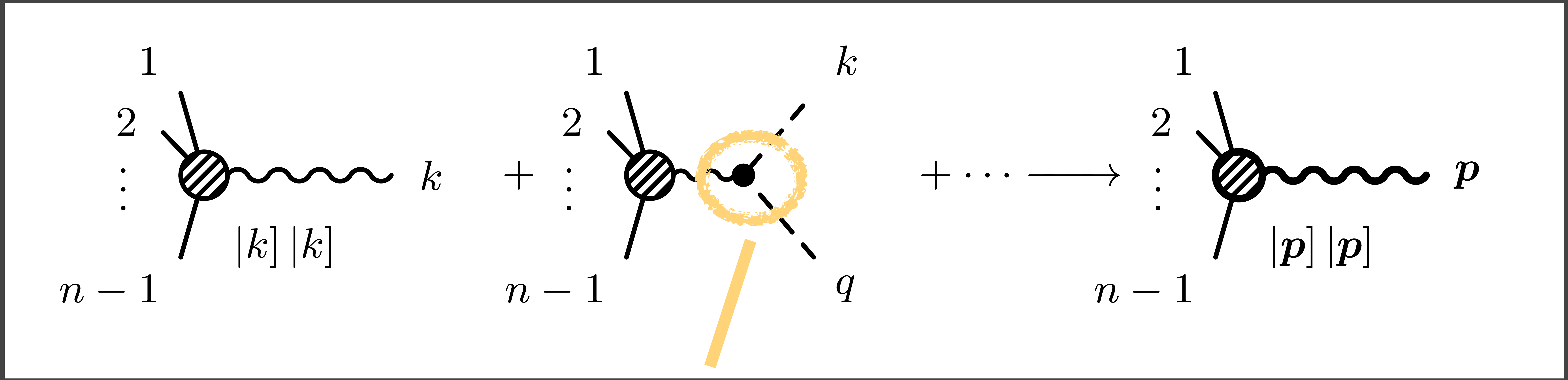
anatomy of on-shell Higgsing

Balkin Durieux Kitahara YS Weiss ‘21

- massless spinor structures get **bolded:**

n-pt amplitude with external vector n

(n+1)-pt amplitude with external Higgses n, (n+1)



n-pt amplitude with external massive vector n

known (universal) 3-pt amplitude $\propto g$

anatomy of on-shell Higgsing

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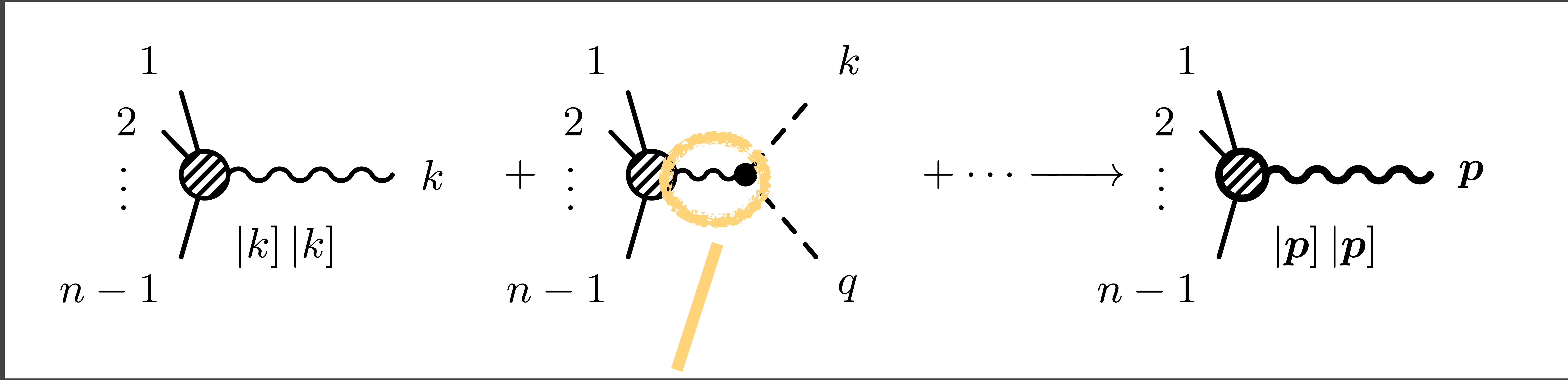

n-pt amplitude with external massive vector n

propagator $\propto 1/(k+q)^2 = 1/m^2$

anatomy of on-shell Higgsing

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n-pt amplitude with external vector n

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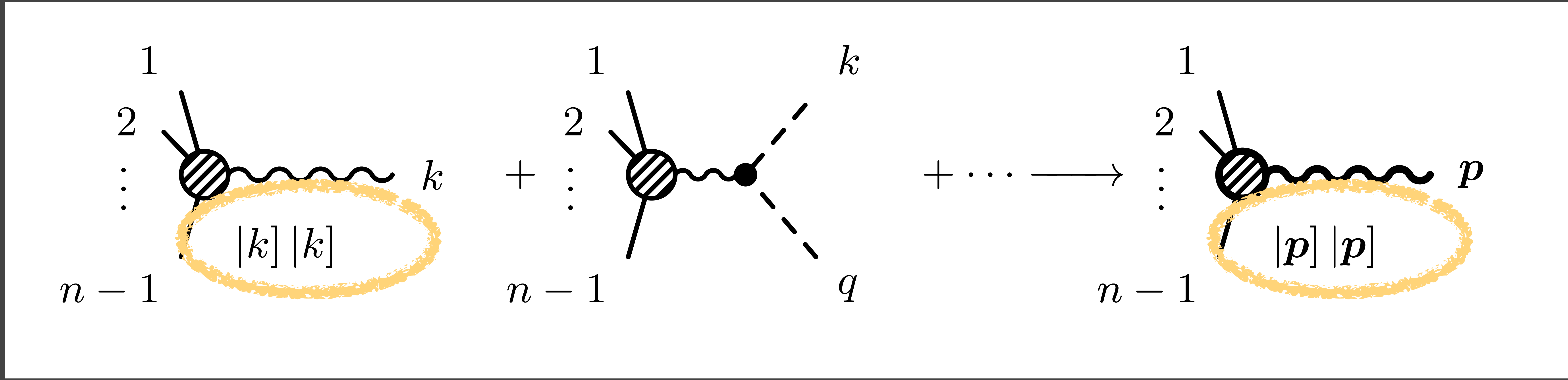

soft Higgs leg supplies second lightlike momentum to form massive momentum

$$\mathbf{p} = k + q$$

anatomy of on-shell Higgsing

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- massless spinor structures get **bolded:**

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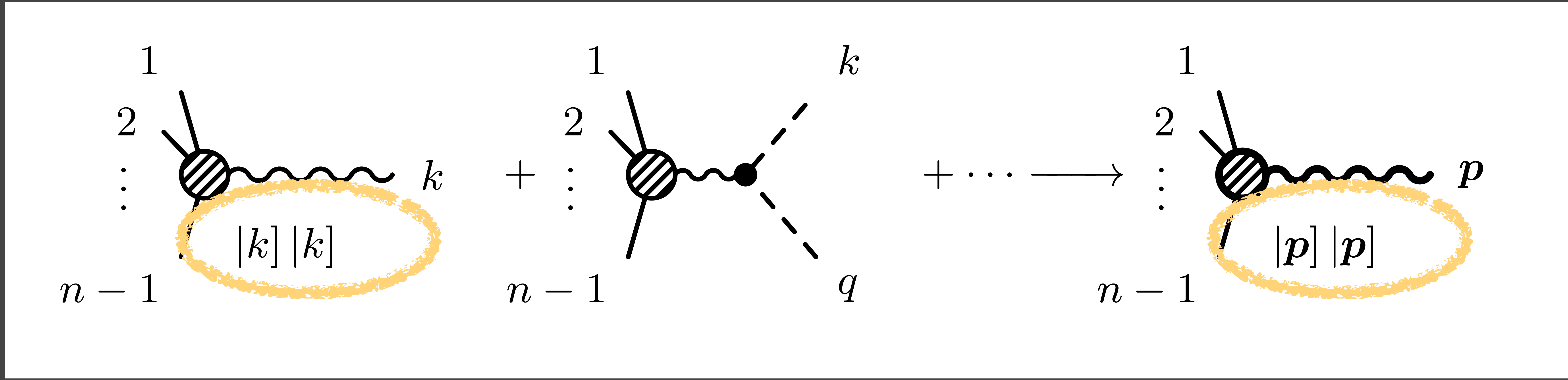


soft Higgs leg supplies second lightlike momentum to form massive momentum

$$\mathbf{p} = k + q$$

symmetrization over LG indices: exchanging k, q in Higgs legs

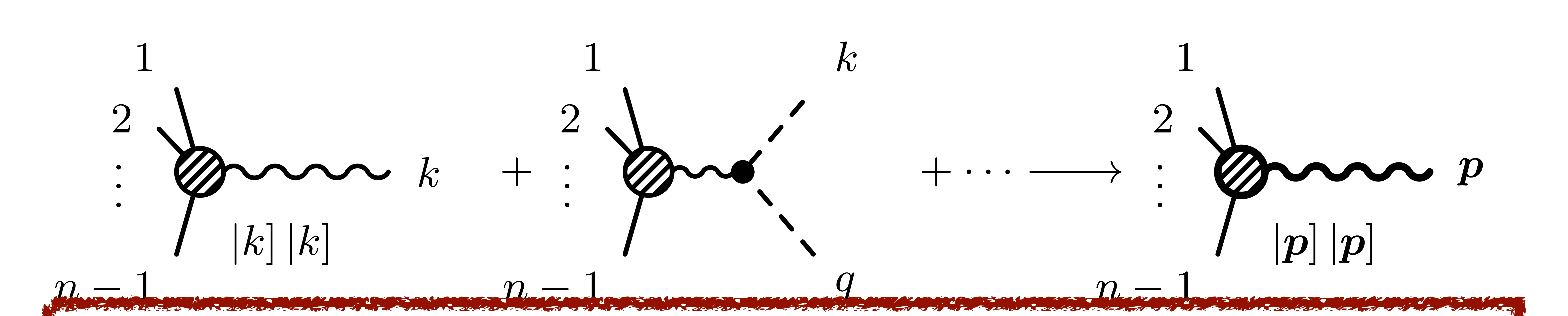
anatomy of on-shell Higgsing

Balkin Durieux Kitahara YS Weiss ‘21

- massless spinor structures get **bolded:**

n-pt amplitude with external vector n

(n+1)-pt amplitude with external Higgses n, (n+1)



massless spinor structure gets bolded $k]k] \to \mathbf{p]p]}$

anatomy of on-shell Higgsing

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massless fermion: $i] \rightarrow \mathbf{i}]$

massless vector $i]i] \rightarrow \mathbf{i}]\mathbf{i}]$

massless scalar amplitude with momentum insertion $p_i = i]\langle i$

—> 1. massive *scalar* amplitude with momentum insertion $\mathbf{p_i}$

—> 2. massive *vector amplitude* $p_i = i]\langle i \rightarrow \mathbf{i}]\langle\mathbf{i}$

(longitudinal vector from Goldstone boson)

anatomy of on-shell Higgsing

just as for gauge symmetry:

power of Lorentz

Higgs mechanism <—> Lorentz symmetry

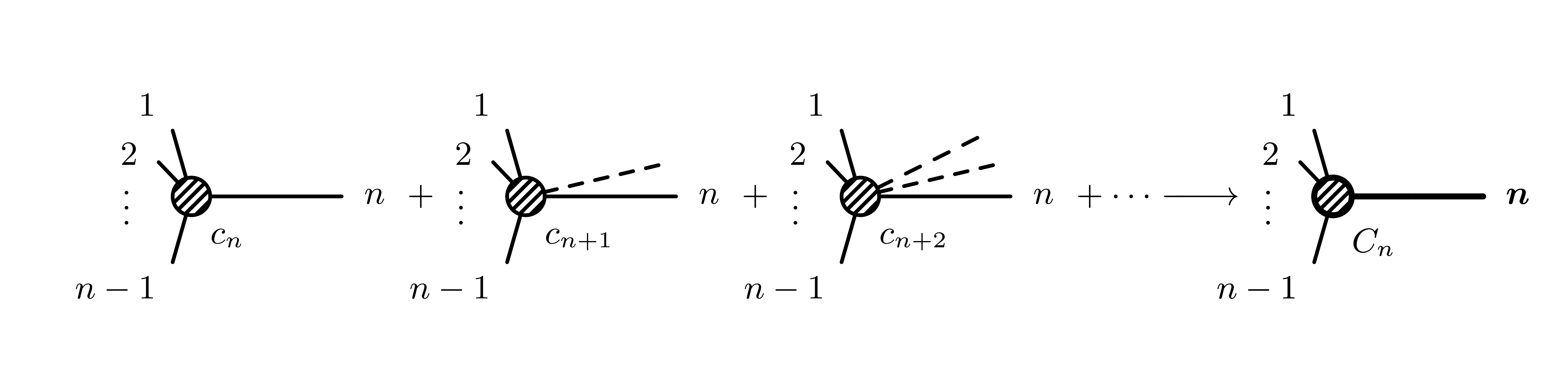
from Lorentz symmetry pov:

bolding the massless spinor structure = covariantizing wrt full massive LG

anatomy of on-shell Higgsing

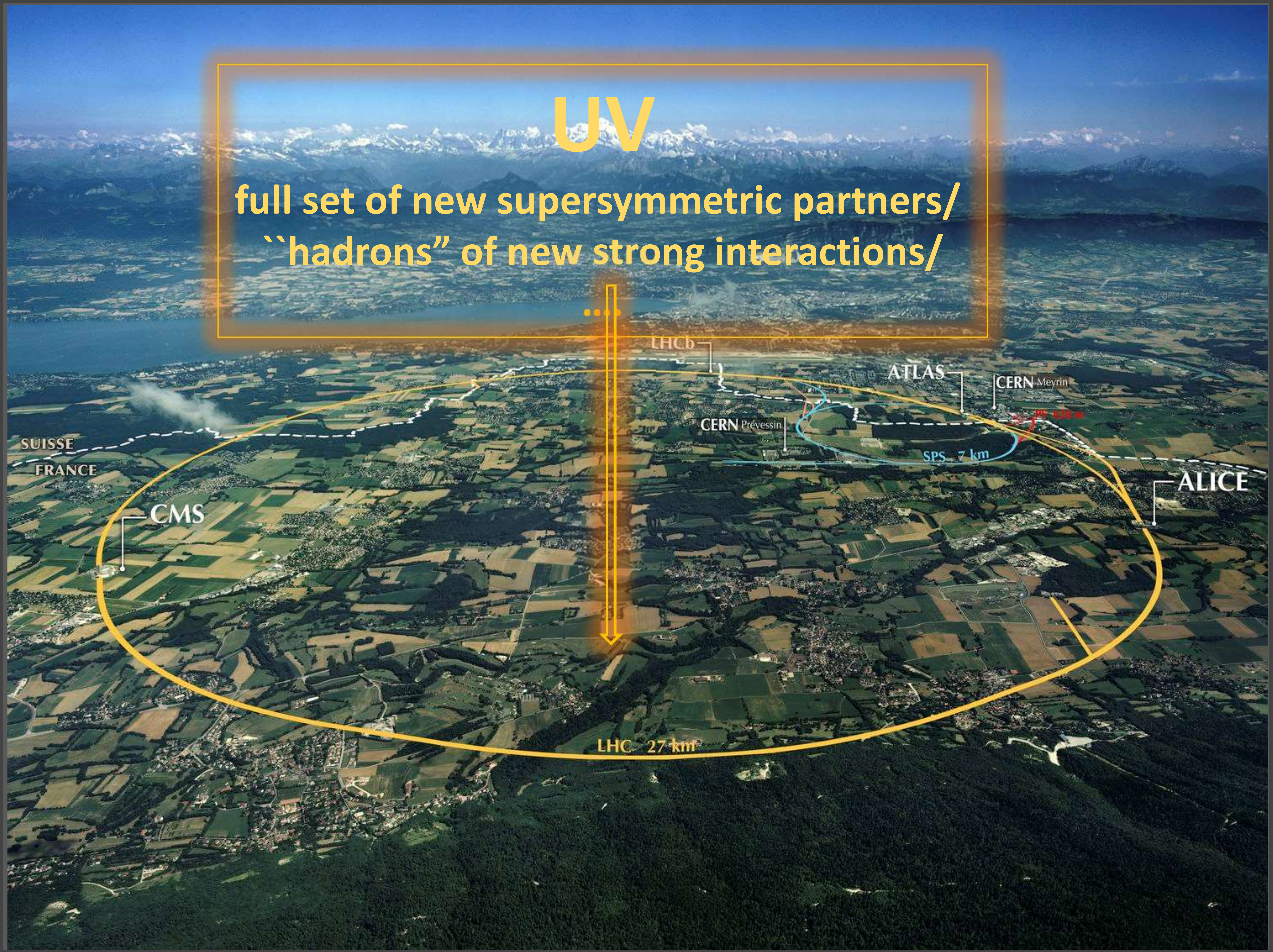
Balkin Durieux Kitahara YS Weiss '21

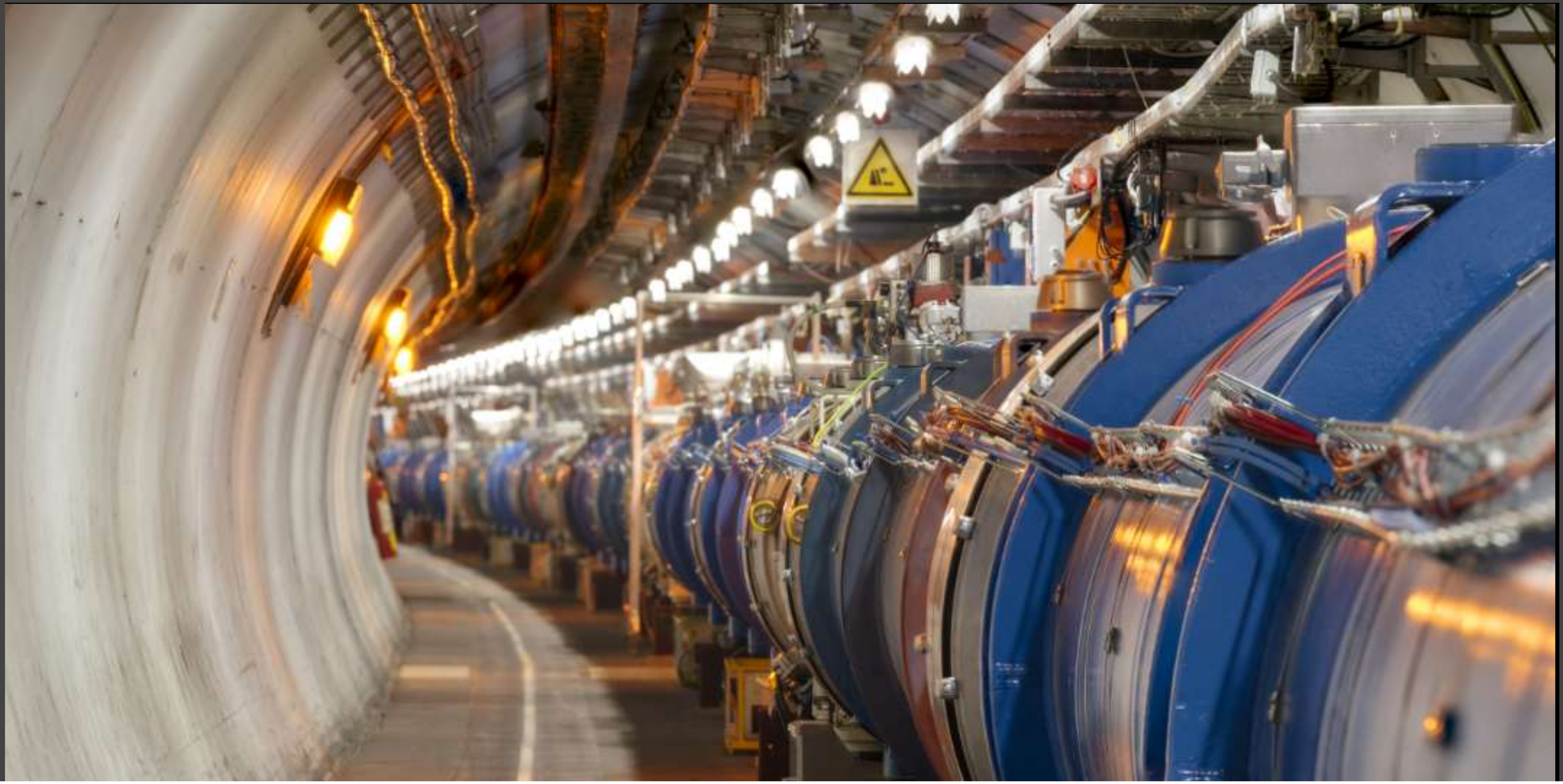
- couplings get $\mathcal{O}(v)$ corrections:



$$C_n = c_n + \#\, v c_{n+1} + \#\, v^2 c_{n+2} + \ldots$$

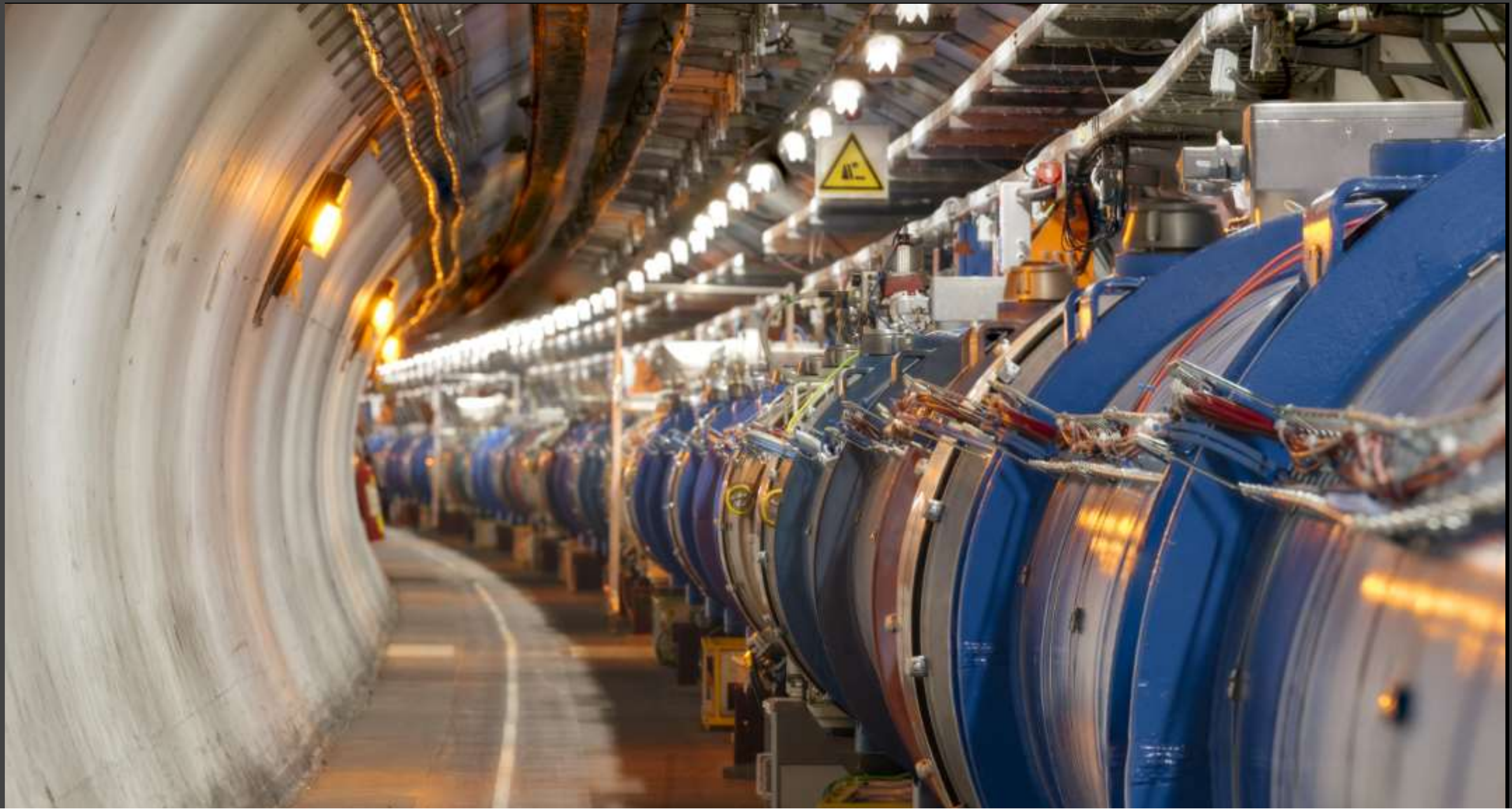
EFT applications


UV
full set of new supersymmetric partners/
``hadrons" of new strong interactions/
....
LHCb
ATLAS
CERN Meyrin
CERN Prévessin
SPS 7 km
SUISSE
FRANCE
ALICE
CMS
LHC 27 km



didn't quite work like this..

work our way up from the IR ~ -100m



EFTs: model independent parametrization of BSM

on-shell: focus on the physical DOFs

On-shell applications to EFTs (massless)

- selection rules: explain zeros in
  - matrix of anomalous dimensions of EFT operators (loop cuts & generalized cuts)

    Cheung Shen ’15
    Bern Parra-Martinez Sawyer ’20

  - interference of SM x EFT amplitudes (tree)

    Azatov Contino Machado Riva ‘16

- derive anomalous dimensions of EFT operators (loop cuts & generalized cuts)

Barratella Fernandez von Harling Pomarol ’20
Bern Parra-Martinez Sawyer ’20
Jiang Ma Shu ’20
De Angelis Accettulli-Huber ’21
Barratella ’22
…

On-shell applications to EFTs (massless + massive)

- count (& construct) bases of EFT operators:

YS Weiss ’18
Ma Shu Xiao ’19
Remmen Rodd ’19
Li Ren Shu Xiao Yu Zheng ’20
Durieux Machado ’20
…
also used in Henning Melia Murayama ‘15

in many of these:

amplitude

↓

$\mathscr{L}$

↓

amplitude ⟶ LHC

On-shell applications to EFTs (massless + massive) ★

work directly with amplitudes

bottom-up EFTs: parametrize our ignorance about the UV

bottom-up construction of amplitudes does just that

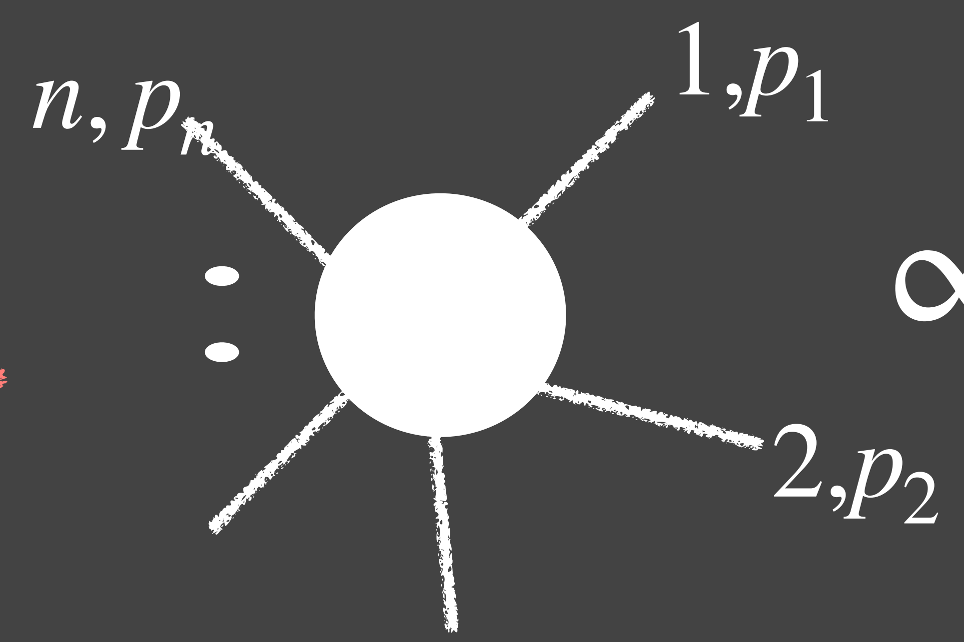
EFT via on-shell bootstrap

usually: start with SM fields: most general $\mathscr{L}$ consistent with symmetries (global, gauge)

on-shell: start with SM particles: most general $\mathscr{A}$ consistent with symmetries (global, gauge)

$$\mathscr{L} = \sum_i c_i \mathcal{O}_i(\phi_1, \ldots, \phi_n)$$

1-1 correspondence



$\propto c_i$

on-shell EFTs

bootstrapping amplitudes:

- most general 3-points (renormalizable + higher-dim): dictated by little group
- factorizable parts of higher-point amplitudes (determined by 3-pts)
- higher-point contact terms: dictated by little group

—> starting with the massive (and massless) particles we know:
construct **most general** amplitudes

contact-term (EFT) part of amplitude:

YS Weiss '18
Durieux Kitahara YS Weiss '19
...

$$\mathscr{A} = \frac{[\cdot\cdot]\cdot\cdot\langle\cdot\cdot\rangle}{\Lambda^{\#}}\; P\left(\frac{s_{ij}}{\Lambda^2}\right)$$

local: no poles

$$\mathscr{A} = \frac{[\cdot\cdot]\cdot\cdot\langle\cdot\cdot\rangle}{\Lambda^{\#}} P\left(\frac{s_{ij}}{\Lambda^2}\right)$$

carries LG weight; “stripped” off

all Lorentz invariants s_{ij}

“stripped contact term”   SCT

$$\mathscr{A} = \frac{[\cdot\cdot]\cdot\cdot\langle\cdot\cdot\rangle}{\Lambda^{\#}} P\left(\frac{s_{ij}}{\Lambda^2}\right)$$

carries LG weight; “stripped” of all Lorentz invariants s_{ij}

“stripped contact term” SCT

polynomial in Lorentz invariants s_{ij}

subject to kinematical constraints,

eg, $s_{12} + s_{13} + s_{23} = \sum m^2$

derivative expansion

easy part!

2 to 2 with massless initial state particles:

$$\mathscr{A} = \frac{[\cdot\cdot]\cdot\cdot\langle\cdot\cdot\rangle}{\Lambda^{\#}} \quad P\left(\frac{s}{\Lambda^2}, \frac{t}{\Lambda^2}\right)$$

SCT

scattering angle
and
decay angles

scattering angle

2 to 2 with massless initial state particles:

$\mathscr{A} = \frac{[\cdot\cdot}{}$

scatte

and

decay angles

bottom up construction; input: physical particles

SU(3)xU(1)

higgs = gauge singlet

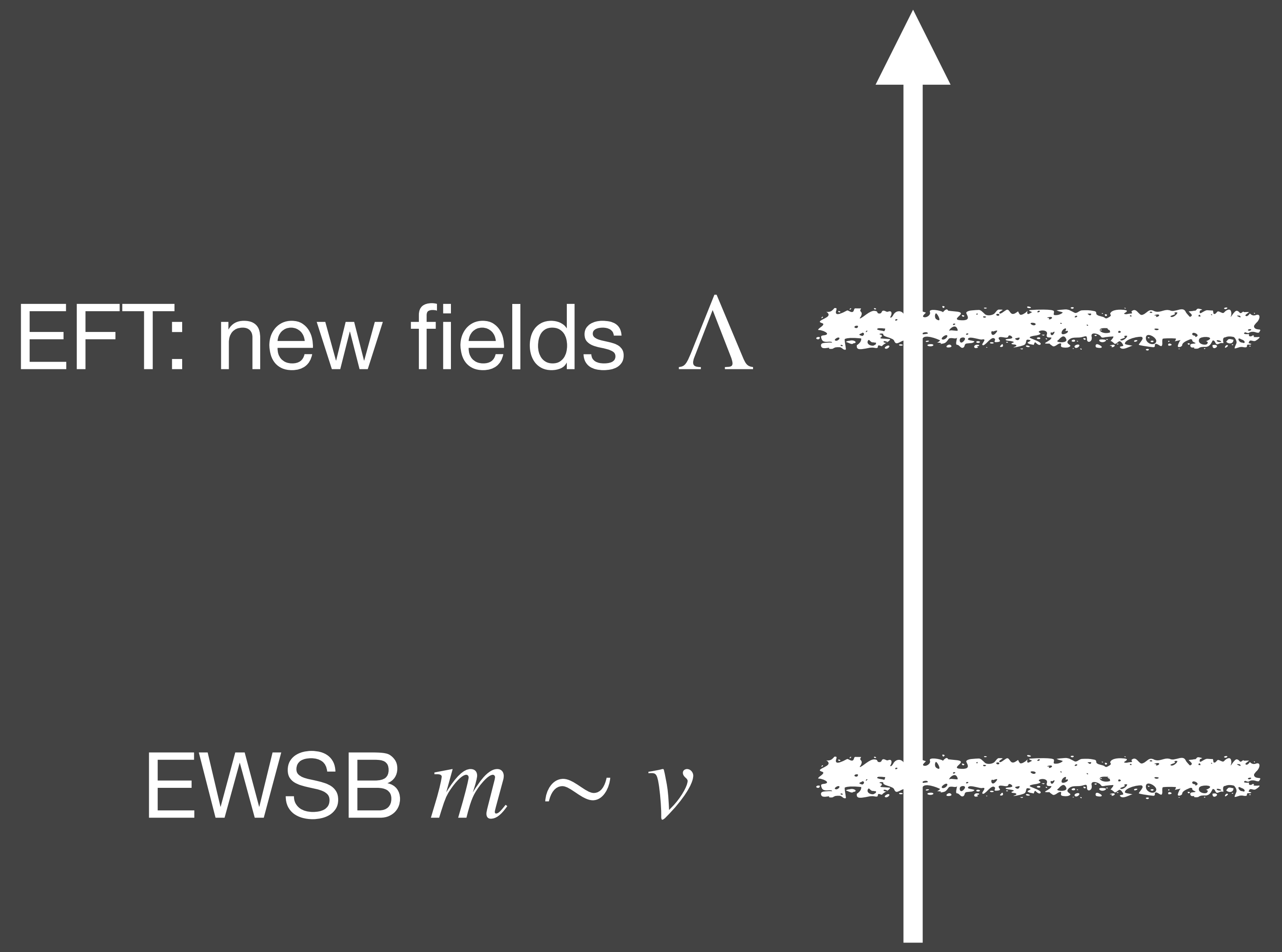
gives **HEFT** amplitudes

What about **(low-energy) SMEFT** amplitudes?

use on-shell Higgsing

construct amplitudes of unbroken theory & “Higgs” them to get massive amplitudes

Balkin Durieux Kitahara YS Weiss ‘21



massless $\mathscr{A}$ (impose full SU(3)xSU(2)xU(1))

↓

derive massive $\mathscr{M}$

(contact term part only)

[another way: start with most general amplitudes and require perturbative unitarity] Durieux Kitahara YS Weiss ‘19

results: HEFT, SMEFT

HEFT inventory *(**observables**; many more results on operators, anomalous dim's via on-shell)*

- *all HEFT 3-points (+matching to SMEFT)* — *Durieux Kitahara YS Weiss '19*
- *[all generic 3-points for spins up to 3*
- *all generic 4-pt SCTs for spins 0, 1/2, 1]* — *Durieux Kitahara Machado YS Weiss'20*
- *HEFT 4-points: hggg, Zggg, ffVh, WWhh* — *Shadmi et al '18, Durieux et al '19, Balkin et al '21*

+ some full amplitudes (factorizable + contact terms): ffWh, ffZh, WWhh

- *5V (4W+Z etc)* — *De Angelis '21*
- *Higgs, top 4pts in terms of momenta+polarizations* — *Chang et al '22, '23*
- *all HEFT 4pts up to d=8* ← — *Liu Ma YS Waterbury '23*

Full set of EFT contact terms featuring E^2 growth: (mostly dim-6 operators)

Massive amplitudes	E^2 contact terms
$\mathcal{M}(WWhh)$	$C^{00}_{WWhh}\langle\mathbf{12}\rangle[\mathbf{12}]$, $C^{\pm\pm}_{WWhh}(\mathbf{12})^2$
$\mathcal{M}(ZZhh)$	$C^{00}_{ZZhh}\langle\mathbf{12}\rangle[\mathbf{12}]$, $C^{\pm\pm}_{ZZhh}(\mathbf{12})^2$
$\mathcal{M}(gghh)$	$C^{\pm\pm}_{gghh}(12)^2$
$\mathcal{M}(\gamma\gamma hh)$	$C^{\pm\pm}_{\gamma\gamma hh}(12)^2$
$\mathcal{M}(\gamma Zhh)$	$C^{\pm}_{\gamma Zhh}(1\mathbf{2})^2$
$\mathcal{M}(hhhh)$	C_{hhhh}
$\mathcal{M}(f^c fhh)$	$C^{\pm\pm}_{ffhh}(\mathbf{12})$
$\mathcal{M}(f^c fWh)$	$C^{+-0}_{ffWh}[\mathbf{13}]\langle\mathbf{23}\rangle$, $C^{-+0}_{ffWh}\langle\mathbf{13}\rangle[\mathbf{23}]$, $C^{\pm\pm\pm}_{ffWh}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(f^c fZh)$	$C^{+-0}_{ffZh}[\mathbf{13}]\langle\mathbf{23}\rangle$, $C^{-+0}_{ffZh}\langle\mathbf{13}\rangle[\mathbf{23}]$, $C^{\pm\pm\pm}_{ffZh}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(f^c f\gamma h)$	$C^{\pm\pm\pm}_{ff\gamma h}(\mathbf{1}3)(\mathbf{2}3)$
$\mathcal{M}(q^c qgh)$	$C^{\pm\pm\pm}_{qqgh}(\mathbf{1}3)(\mathbf{2}3)$
$\mathcal{M}(f^c ff^c f)$	$C^{\pm\pm\pm\pm,1}_{ffff}(\mathbf{12})(\mathbf{34})$, $C^{--++}_{ffff}\langle\mathbf{12}\rangle[\mathbf{34}]$, $C^{-+-+}_{ffff}\langle\mathbf{13}\rangle[\mathbf{24}]$, $C^{-++-}_{ffff}\langle\mathbf{14}\rangle[\mathbf{23}]$ $C^{\pm\pm\pm\pm,2}_{ffff}(\mathbf{13})(\mathbf{24})$, $C^{++--}_{ffff}[\mathbf{12}]\langle\mathbf{34}\rangle$, $C^{+-+-}_{ffff}[\mathbf{13}]\langle\mathbf{24}\rangle$, $C^{+--+}_{ffff}[\mathbf{14}]\langle\mathbf{23}\rangle$

$(12) = [12]$ or $\langle 12\rangle$

C's: Wilson coefficients

most suppressed by $\bar{\Lambda}^2$
(amplitude dim-less)

Ma Liu YS Waterbury 2301.11349

Full set of EFT contact terms featuring E^2 growth: (mostly dim-6 operators)

Massive amplitudes	E^2 contact terms
$\mathcal{M}(WWhh)$	$C^{00}_{WWhh}\langle\mathbf{12}\rangle[\mathbf{12}]$, $C^{\pm\pm}_{WWhh}(\mathbf{12})^2$
$\mathcal{M}(ZZhh)$	$C^{00}_{ZZhh}\langle\mathbf{12}\rangle[\mathbf{12}]$, $C^{\pm\pm}_{ZZhh}(\mathbf{12})^2$
$\mathcal{M}(gghh)$	$C^{\pm\pm}_{gghh}(\mathbf{12})^2$
$\mathcal{M}(\gamma\gamma hh)$	[illegible]
$\mathcal{M}(\gamma Zhh)$	[illegible]
$\mathcal{M}(hhhh)$	[illegible]
$\mathcal{M}(f^cfhh)$	[illegible]
$\mathcal{M}(f^cfWh)$	[illegible] $_{ffWh}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(f^cfZh)$	[illegible] $_{ffZh}[\mathbf{13}]\langle\mathbf{23}\rangle$, $C^{-+0}_{ffZh}\langle\mathbf{13}\rangle[\mathbf{23}]$, $C^{\pm\pm\pm}_{ffZh}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(f^cf\gamma h)$	$C^{\pm\pm\pm}_{ff\gamma h}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(q^cqgh)$	$C^{\pm\pm\pm}_{qqgh}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(f^cff^cf)$	$C^{\pm\pm\pm\pm,1}_{ffff}(\mathbf{12})(\mathbf{34})$, $C^{--++}_{ffff}\langle\mathbf{12}\rangle[\mathbf{34}]$, $C^{-+-+}_{ffff}\langle\mathbf{13}\rangle[\mathbf{24}]$, $C^{-++-}_{ffff}\langle\mathbf{14}\rangle[\mathbf{23}]$ $C^{\pm\pm\pm\pm,2}_{ffff}(\mathbf{13})(\mathbf{24})$, $C^{++--}_{ffff}[\mathbf{12}]\langle\mathbf{34}\rangle$, $C^{+-+-}_{ffff}[\mathbf{13}]\langle\mathbf{24}\rangle$, $C^{+--+}_{ffff}[\mathbf{14}]\langle\mathbf{23}\rangle$

LOW ENERGY AMPLITUDES (physical particles)

$(12) = [12]$ or $\langle 12\rangle$

C's: Wilson coefficients

most suppressed by $\bar{\Lambda}^2$ (amplitude dim-less)

Ma Liu YS Waterbury 2301.11349

similarly: full set of $d \leq 8$ HEFT amplitudes
(E^3, E^4 growth)

see backup slides

some of these already derived in:

YS Weiss '18
Durieux Kitahara YS Weiss '19 (which also has all 3 points)
Balkin Durieux Kitahara YS Weiss '21

What about **SMEFT** amplitudes?

use on-shell Higgsing

start with massless SU(2)xU(1) symm amplitudes

and Higgs these to get massive amplitudes

for completeness provide full mapping
of 4-pt $d \leq 6$ EFT amplitudes
to Warsaw basis

Ma Shu Xiao '19

Amplitude	Contact term	Warsaw basis operator	Coefficient
$\mathcal{A}(H_i^c H_j^c H_k^c H^l H^m H^n)$	$T^{+\,lmn}_{ijk}$	$\mathcal{O}_H/6$	$c_{(H^\dagger H)^3}$
$\mathcal{A}(H_i^c H_j^c H^k H^l)$	$s_{12} T^{+\,kl}_{ij}$	$\mathcal{O}_{HD}/2 + \mathcal{O}_{H\Box}/4$	$c^{(+)}_{(H^\dagger H)^2}$
$\mathcal{A}(H_i^c H_j^c H^k H^l)$	$(s_{13} - s_{23}) T^{-\,kl}_{ij}$	$\mathcal{O}_{HD}/2 - \mathcal{O}_{H\Box}/4$	$c^{(-)}_{(H^\dagger H)^2}$
$\mathcal{A}(B^\pm B^\pm H_i^c H^j)$	$(12)^2 \delta_i^j$	$(\mathcal{O}_{HB} \pm i\mathcal{O}_{H\tilde{B}})/2$	$c^{\pm\pm}_{BBHH}$
$\mathcal{A}(B^\pm W^{I\pm} H_i^c H^j)$	$(12)^2 (\sigma^I)_i^j$	$\mathcal{O}_{HWB} \pm i\mathcal{O}_{H\tilde{W}B}$	$c^{\pm\pm}_{BWHH}$
$\mathcal{A}(W^{I+} W^{J+} H_i^c H^j)$	$(12)^2 \delta^{IJ} \delta_i^j$	$(\mathcal{O}_{HW} \pm i\mathcal{O}_{H\tilde{W}})/2$	$c^{\pm\pm}_{WWHH}$
$\mathcal{A}(g^{A\pm} g^{B\pm} H_i^c H^j)$	$(12)^2 \delta^{AB} \delta_i^j$	$(\mathcal{O}_{HG} \pm i\mathcal{O}_{H\tilde{G}})/2$	$c^{\pm\pm}_{GGHH}$
$\mathcal{A}(L_i^c e H_j^c H^k H^l)$	$[12] T^{+\,kl}_{ij}$	$\mathcal{O}_{eH}/2$	c^{++}_{LeHHH}
$\mathcal{A}(Q^c_{a,i} d^b H_j^c H^k H^l)$	$[12] T^{+\,kl}_{ij} \delta_a^b$	$\mathcal{O}_{dH}/2$	c^{++}_{QdHHH}
$\mathcal{A}(Q^c_{a,i} u^b H_j^c H_k^c H^l)$	$[12] \varepsilon_{im} T^{+\,ml}_{jk} \delta_a^b$	$\mathcal{O}_{uH}/2$	c^{++}_{QuHHH}
$\mathcal{A}(e^c e H_i^c H^j)$	$\langle 142] \delta_i^j$	$\mathcal{O}_{He}/2$	c^{-+}_{eeHH}
$\mathcal{A}(u_a^c u^b H_i^c H^j)$	$\langle 142] \delta_i^j \delta_a^b$	$\mathcal{O}_{Hu}/2$	c^{-+}_{uuHH}
$\mathcal{A}(d_a^c d^b H_i^c H^j)$	$\langle 142] \delta_i^j \delta_a^b$	$\mathcal{O}_{Hd}/2$	c^{-+}_{ddHH}
$\mathcal{A}(u_a^c d^b H^i H^j)$	$\langle 142] \epsilon^{ij} \delta_a^b$	$\mathcal{O}_{Hud}/2$	c^{-+}_{udHH}
$\mathcal{A}(L_i^c L^j H_k^c H^l)$	$[142\rangle T^{+\,jl}_{ik}$	$\left(\mathcal{O}^{(1)}_{HL} + \mathcal{O}^{(3)}_{HL}\right)/8$	$c^{+-,(+)}_{LLHH}$
$\mathcal{A}(L_i^c L^j H_k^c H^l)$	$[142\rangle T^{-\,jl}_{ik}$	$\left(\mathcal{O}^{(1)}_{HL} - \mathcal{O}^{(3)}_{HL}\right)/8$	$c^{+-,(-)}_{LLHH}$
$\mathcal{A}(Q^c_{a,i} Q^{b,j} H_k^c H^l)$	$[142\rangle T^{+\,jl}_{ik} \delta_a^b$	$\left(3\mathcal{O}^{(1)}_{HQ} + \mathcal{O}^{(3)}_{HQ}\right)/8$	$c^{+-,(+)}_{QQHH}$
$\mathcal{A}(Q^c_{a,i} Q^{b,j} H_k^c H^l)$	$[142\rangle T^{-\,jl}_{ik} \delta_a^b$	$(\mathcal{O}^{(1)}_{HQ} - \mathcal{O}^{(3)}_{HQ})/8$	$c^{+-,(-)}_{QQHH}$
$\mathcal{A}(L_i^c e B^+ H^j)$	$[13][23] \delta_i^j$	$-i\mathcal{O}_{eB}/(2\sqrt{2})$	c^{+++}_{LeBH}
$\mathcal{A}(Q^c_{a,i} d^b B^+ H^j)$	$[13][23] \delta_i^j \delta_a^b$	$-i\mathcal{O}_{dB}/(2\sqrt{2})$	c^{+++}_{QdBH}
$\mathcal{A}(Q^c_{a,i} u^b B^+ H_j^c)$	$[13][23] \epsilon_{ij} \delta_a^b$	$-i\mathcal{O}_{uB}/(2\sqrt{2})$	c^{+++}_{QuBH}
$\mathcal{A}(L_i^c e W^{I+} H^j)$	$[13][23] (\sigma^I)_i^j$	$-i\mathcal{O}_{eW}/(2\sqrt{2})$	c^{+++}_{LeWH}
$\mathcal{A}(Q^c_{a,i} d^b W^{I+} H^j)$	$[13][23] (\sigma^I)_i^j \delta_a^b$	$-i\mathcal{O}_{dW}/(2\sqrt{2})$	c^{+++}_{QdWH}
$\mathcal{A}(Q^c_{a,i} u^b W^{I+} H_j^c)$	$[13][23] (\sigma^I)_{ik} \epsilon_j^k \delta_a^b$	$-i\mathcal{O}_{uW}/(2\sqrt{2})$	c^{+++}_{QuWH}
$\mathcal{A}(Q^c_{a,i} d^b g^{A+} H^j)$	$[13][23] \delta_i^j (\lambda^A)_a^b$	$-i\mathcal{O}_{dG}/(2\sqrt{2})$	c^{+++}_{QdGH}
$\mathcal{A}(Q^c_{a,i} u^b g^{A+} H_j^c)$	$[13][23] \epsilon_{ij} (\lambda^A)_a^b$	$-i\mathcal{O}_{uG}/(2\sqrt{2})$	c^{+++}_{QuGH}
$\mathcal{A}(W^{I\pm} W^{J\pm} W^{K\pm})$	$(12)(23)(31) \epsilon^{IJK}$	$(\mathcal{O}_W \pm i\mathcal{O}_{\tilde{W}})/6$	$c^{\pm\pm\pm}_{WWW}$
$\mathcal{A}(g^{A\pm} g^{B\pm} g^{C\pm})$	$(12)(23)(31) f^{ABC}$	$(\mathcal{O}_G \pm i\mathcal{O}_{\tilde{G}})/6$	$c^{\pm\pm\pm}_{GGG}$

Table 2: Massless $d = 6$ SMEFT contact terms [34] and their relations to Warsaw basis operators [3]. For each operator (or operator combination) $\mathcal{O}$ in the third column, $c\,\mathcal{O}$ generates the structure in the second column with the coefficient c given in the fourth column. c-superscripts denote charge conjugation.

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get SMEFT low-energy contact terms

here: up to $d \leq 6$

d=8: Goldberg Liu YS in progress

Massive $d=6$ amplitudes	SMEFT Wilson coefficients
$\mathcal{M}(W_L^+W_L^-hh) = C^{00}_{WWhh}\langle\mathbf{12}\rangle[\mathbf{12}]$	$C^{00}_{WWhh} = (c^{(+)}_{(H^\dagger H)^2} - 3c^{(-)}_{(H^\dagger H)^2})/2$
$\mathcal{M}(W_\pm^+W_\pm^-hh) = C^{\pm\pm}_{WWhh}(\mathbf{12})^2$	$C^{\pm\pm}_{WWhh} = 2c^{\pm\pm}_{WWHH}$
$\mathcal{M}(Z_LZ_Lhh) = C^{00}_{ZZhh}\langle\mathbf{12}\rangle[\mathbf{12}]$	$C^{00}_{ZZhh} = -2c^{(+)}_{(H^\dagger H)^2}$
$\mathcal{M}(Z_\pm Z_\pm hh) = C^{\pm\pm}_{ZZhh}(\mathbf{12})^2$	$C^{\pm\pm}_{ZZhh} = c_W^2 c^{\pm\pm}_{WWHH} + s_W^2 c^{\pm\pm}_{BBHH} + c_W s_W c^{\pm\pm}_{BWHH}$
$\mathcal{M}(g_\pm g_\pm hh) = C^{\pm\pm}_{gghh}(12)^2$	[illegible]
$\mathcal{M}(\gamma_\pm\gamma_\pm hh) = C^{\pm\pm}_{\gamma\gamma hh}(12)^2$	[illegible] $c^{\pm\pm}_{BWHH}$
$\mathcal{M}(\gamma_\pm Zh$ [illegible]	[illegible] $c_W^2)c^{\pm\pm}_{BWHH}$
$\mathcal{M}(hh$ [illegible]	[illegible]
$\mathcal{M}(f^c_\pm f_\pm h$ [illegible]	[illegible]
$\mathcal{M}(f^c_+ f'_- W_L h)$ [illegible]	[illegible]
$\mathcal{M}(f^c_- f'_+ W_L h)$ [illegible]	[illegible]
$\mathcal{M}(f^c_\pm f'_\pm W_\pm h)$ [illegible]	$C^{\pm\pm\pm}_{ffWh} = c^{\pm\pm\pm}_{\Psi\psi WH}/2$
$\mathcal{M}(f^c_+ f_- Z_L h)$ [illegible] $_{ffZh}[\mathbf{13}]\langle\mathbf{23}\rangle$	$C^{+-0}_{e_Le_LZh} = -i\sqrt{2}c^{+-,(+)}_{\Psi\Psi HH},\ C^{+-0}_{\nu_L\nu_LZh} = -i(c^{+-,(+)}_{\Psi\Psi HH} + c^{+-,(-)}_{\Psi\Psi HH})/\sqrt{2}$
$\mathcal{M}(f^c_- f_+ Z_L h) = C^{-+0}_{ffZh}\langle\mathbf{13}\rangle[\mathbf{23}]$	$C^{-+0,\mathrm{CT}}_{ffZh} = -i\sqrt{2}c^{-+}_{\psi\psi HH}$
$\mathcal{M}(f^c_\pm f_\pm Z_\pm h) = C^{\pm\pm\pm}_{ffZh}(\mathbf{13})(\mathbf{23})$	$C^{\pm\pm\pm}_{ffZh} = -(s_W c^{\pm\pm\pm}_{\Psi\psi BH} + c_W c^{\pm\pm\pm}_{\Psi\psi WH})/\sqrt{2}$
$\mathcal{M}(f^c_\pm f_\pm \gamma_\pm h) = C^{\pm\pm\pm}_{ff\gamma h}(\mathbf{13})(\mathbf{23})$	$C^{\pm\pm\pm}_{ff\gamma h} = (-s_W c^{\pm\pm\pm}_{\Psi\psi WH} + c_W c^{\pm\pm\pm}_{\Psi\psi BH})/\sqrt{2}$
$\mathcal{M}(q^c_\pm q_\pm g^A_\pm h) = C^{\pm\pm\pm}_{qqgh}\lambda^A(\mathbf{13})(\mathbf{23})$	$C^{\pm\pm\pm}_{qqgh} = c^{\pm\pm\pm}_{\Psi\psi GH}/\sqrt{2}$

LOW ENERGY AMPLITUDES (Physical particles)

Table 3: The low-energy E^2 contact terms (left column) and their $d=6$ coefficients in the SMEFT (right column). $c_{(H^\dagger H)^2}$ without a superscript is the renormalizable four-Higgs coupling. The mapping for four fermion contact terms is trivial, so we do not include them here.

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Massive amplitudes	E^2 contact terms
$\mathcal{M}(WWhh)$	$C^{00}_{WWhh}\langle\mathbf{12}\rangle[\mathbf{12}]$, $C^{\pm\pm}_{WWhh}(\mathbf{12})^2$
$\mathcal{M}(ZZhh)$	$C^{00}_{ZZhh}\langle\mathbf{12}\rangle[\mathbf{12}]$, $C^{\pm\pm}_{ZZhh}(\mathbf{12})^2$
$\mathcal{M}(gghh)$	$C^{\pm\pm}_{gghh}(12)^2$
$\mathcal{M}(f^c f f^c f)$	$C^{\pm\pm\pm\pm,2}_{ffff}(\mathbf{13})(\mathbf{24})$, $C^{++--}_{ffff}[\mathbf{12}]\langle\mathbf{34}\rangle$, $C^{+-+-}_{ffff}[\mathbf{13}]\langle\mathbf{24}\rangle$, $C^{+--+}_{ffff}[\mathbf{14}]\langle\mathbf{23}\rangle$

simple: each term: complex number (scattering angle; W/Z/h/t spin polarization direction)

SMEFT relations or lack thereof reflected directly in coefficients of specific observables
(obviously after adding in factorizable part of amplitude and squaring)

good starting point for isolating specific contributions

in progress: De Angelis Durieux Grojean YS compare to standard approach: to derive SMEFT predictions:

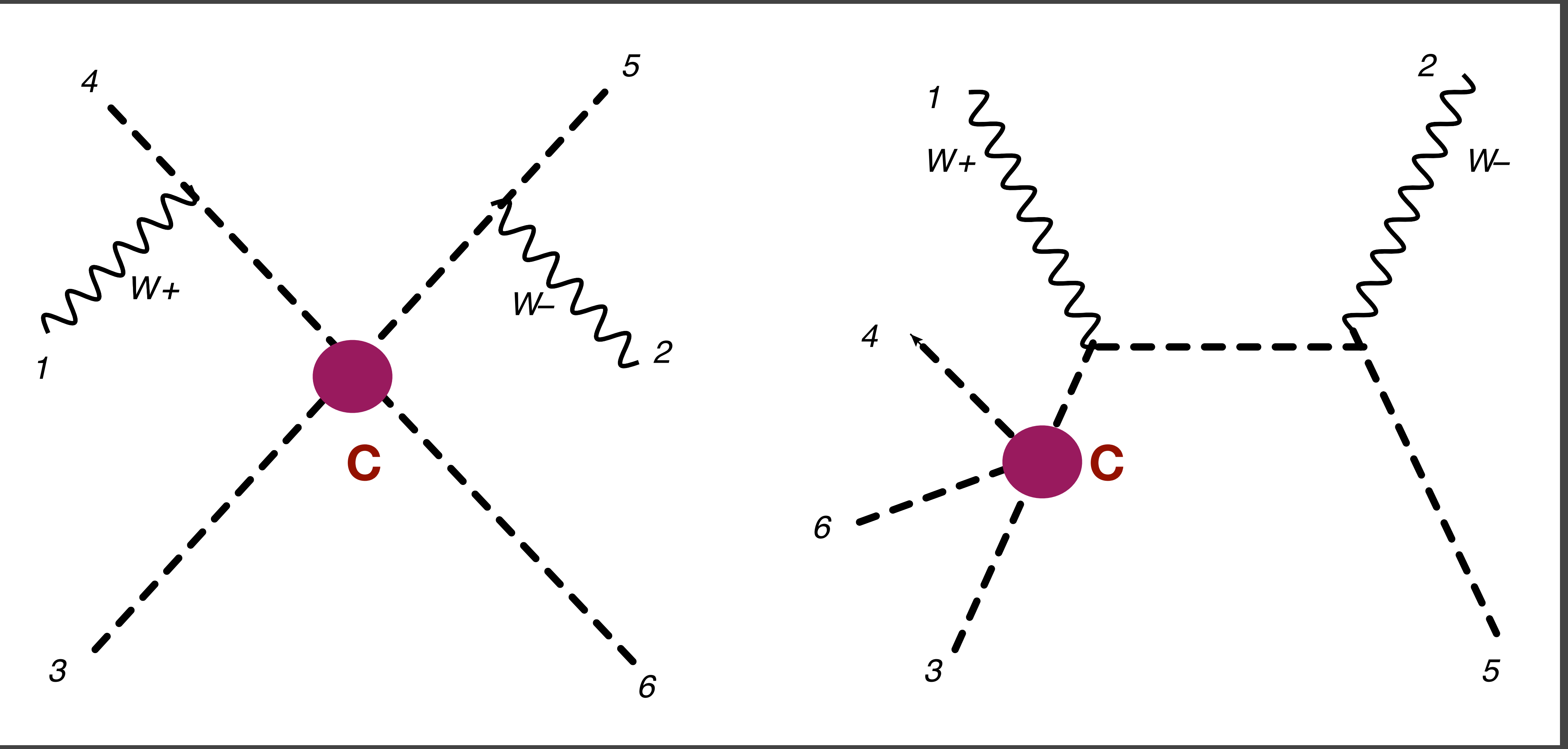
- basis of operators in unbroken theory
- turn on Higgs VEV —> Lagrangian in broken theory, SM couplings shift
- derive Feynman rules of broken theory in some gauge
- redefine parameters from physical masses, couplings

here: directly get physical parameters, working with on-shell dof’s only example: shifts of SM couplings from d=6 operators

WWh coupling shift from $2H-2H^\dagger$ d=6 contact term

6-point $(H^\dagger H)^2WW$ amplitude with this contact term

taking three Higgs momenta to be soft



—> $\mathcal{M}^m_{d=6}(h(W^+)^+(W^-)^-)=g(1+v^2C)\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{M_W}$

to conclude:

- mature(ing) methods for on-shell derivations of low-energy EFT amplitudes:
- clear distinction between HEFT, SMEFT
- all HEFT 4-pts up to d=8; all SMEFT 4-pts up to d=6
    - directly in terms of **physical particles, couplings**
    - *amplitudes are what we need to compare with experiment*
- start to develop an understanding of field space — Higgs mechanism

Backup

2 to 2:

$$\mathscr{A} = \frac{[\cdot\cdot]\cdot\cdot\langle\cdot\cdot\rangle}{\Lambda^{\#}} \quad P\left(\frac{s}{\Lambda^2}, \frac{t}{\Lambda^2}\right)$$

SCT

scattering angle

scattering angle
and
decay angles

4.1.7 W^+W^-ZZ

$$
\begin{array}{rcll}
0000: & [\mathbf{12}][\mathbf{34}]\langle\mathbf{12}\rangle\langle\mathbf{34}\rangle, [\mathbf{13}][\mathbf{24}]\langle\mathbf{13}\rangle\langle\mathbf{24}\rangle + (3\leftrightarrow 4) & (4;8) & \# = 2 \\
++00: & [\mathbf{12}]^2[\mathbf{34}]\langle\mathbf{34}\rangle;\ \text{PF} & (6;8) & \# = 2 \\
+0+0: & \{[\mathbf{12}][\mathbf{34}][\mathbf{13}]\langle\mathbf{24}\rangle, [\mathbf{14}][\mathbf{23}][\mathbf{13}]\langle\mathbf{24}\rangle\} + (3\leftrightarrow 4); (1\leftrightarrow 2);\ \text{PF} & (6;8) & \# = 8 \\
00++: & [\mathbf{34}]^2[\mathbf{12}]\langle\mathbf{12}\rangle;\ \text{PF} & (6;8) & \# = 2 \\
+-00: & [\mathbf{13}][\mathbf{14}]\langle\mathbf{23}\rangle\langle\mathbf{24}\rangle;\ \text{PF} & (6;8) & \# = 2 \\
+0-0: & \{[\mathbf{12}][\mathbf{14}]\langle\mathbf{23}\rangle\langle\mathbf{34}\rangle + (3\leftrightarrow 4), (1\leftrightarrow 2)\};\ \text{PF} & (6;8) & \# = 4 \\
00+-: & [\mathbf{13}][\mathbf{23}]\langle\mathbf{14}\rangle\langle\mathbf{24}\rangle + (3\leftrightarrow 4) & (6;8) & \# = 1 \\
++++: & \{[\mathbf{12}]^2[\mathbf{34}]^2, [\mathbf{13}]^2[\mathbf{24}]^2 + (3\leftrightarrow 4)\};\ \text{PF} & (8;8) & \# = 4 \\
++--: & [\mathbf{12}]^2\langle\mathbf{34}\rangle^2;\ \text{PF} & (8;8) & \# = 2 \\
-+-+: & [\mathbf{14}]^2\langle\mathbf{23}\rangle^2 + (3\leftrightarrow 4);\ \text{PF} & (8;8) & \# = 2
\end{array}
\tag{9}
$$

At order E^5 several new $vvvv$ SCTs become independent in the $(+000)$, $(+++0)$, and $(++-0)$ helicity categories.

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HEFT, naive SMEFT dim's

# of indep structures/couplings

PF = parity flip angle <—> square

do new SCTs appear at higher dim's and where example: higgs + 3 gluons:

YS Weiss ‘18

$$\mathcal{M}\left(h; g^{a+}(p_1)\, g^{b+}(p_2)\, g^{c+}(p_3)\right) = \frac{[12][13][23]}{\Lambda}\Bigg[f^{abc}\left(-i\,\frac{m^4\, g_s\, c_5^{hgg}}{s_{12}s_{13}s_{23}} + \frac{c_7}{\Lambda^2} + \frac{c_{11}}{\Lambda^6}\left(s_{12}s_{23}+s_{13}s_{23}+s_{12}s_{13}\right) + \frac{c_{13}}{\Lambda^8} s_{12}s_{13}s_{23}\right) + d^{abc}\frac{c'_{13}}{\Lambda^8}\left(s_{12}-s_{13}\right)\left(s_{12}-s_{23}\right)\left(s_{13}-s_{23}\right)\Bigg] + \cdots$$

derivative expansion

- factorizable + EFT (most general)
- full kinematic behavior of amplitude
- going to dim-13: academic exercise: here see that nothing important beyond dim-7
- by-product: counting & classifying basis of EFT operators