Quantum Transitions, Detailed Balance, Black Holes and Nothingness

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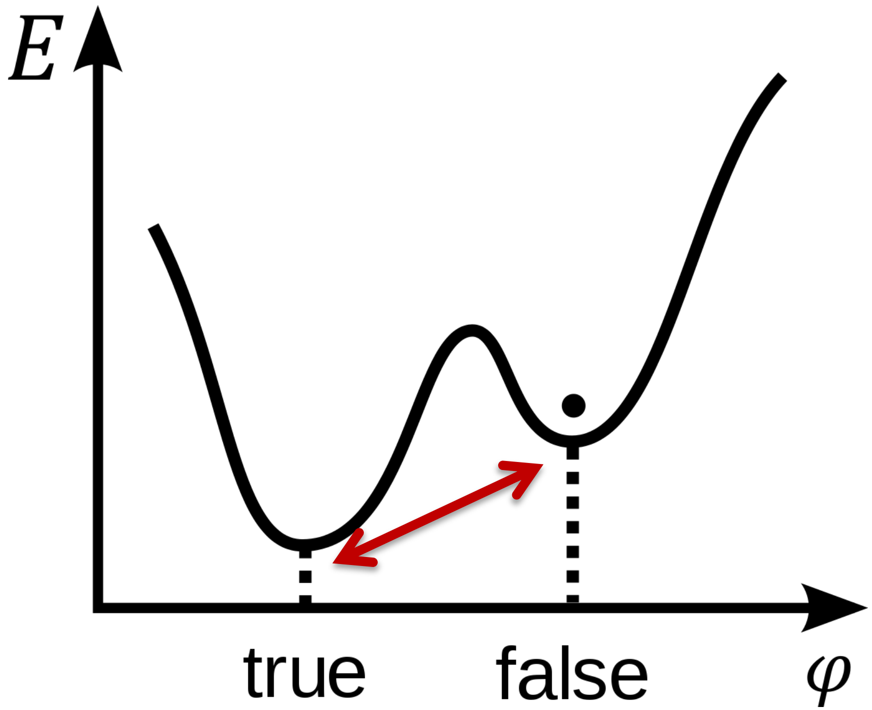
S. Céspedes, S. de Alwis, F. Muia, FQ (to appear)

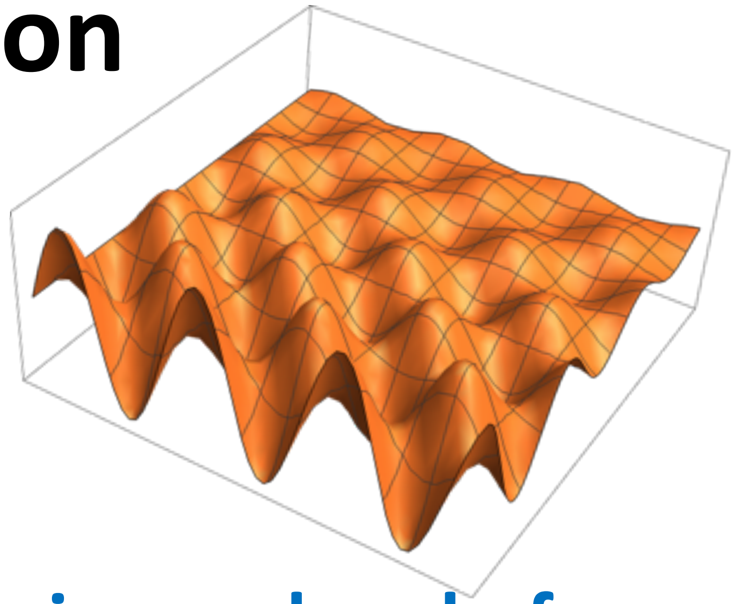
Also: S. de Alwis, F. Muia, V. Pasquarella, FQ 1909.01975
S. Céspedes, S. de Alwis, F. Muia, FQ 2011.13936, 2112.11650
V. Pasquarella, FQ 2211.07664

Old Question: Vacuum Transitions

Transitions among de Sitter, Minkowski and anti de Sitter spacetimes?



Motivation



- String landscape
- Vacuum transitions: beginning and end of our universe?
- Theoretical 'laboratory' to study quantum aspects of gravity

Early History

- **Coleman de Luccia (1980)**
- **Witten (1981)**
- **Vilenkin + Hartle-Hawking (1982-3)**
- **Brown-Teitelboim (1987)**
- **Farhi-Guth-Guven (1990)**
- **Fischler-Morgan-Polchinski (1990)**

Euclidean Approach

Wave functions of the universe

Mini-superspace

$$ds^2 = -N^2(t)dt^2 + a^2(t)(dr^2 + \sin^2 r d\Omega_2^2)$$

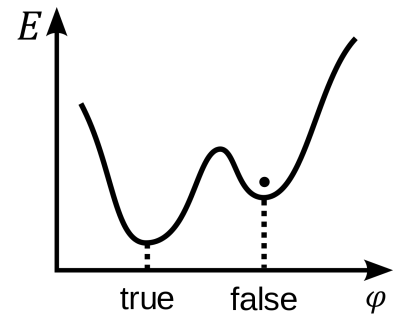
Hartle-Hawking vs Vilenkin (tunneling to dS from nothing)

entropy

$$\mathcal{P}_{\mathrm{HH}}(\mathrm{Nothing} \to \mathrm{dS}) = \|\Psi_{\mathrm{HH}}(\mathrm{H}_{\mathrm{dS}})\|^2 \propto e^{\frac{\pi}{GH_{\mathrm{dS}}^2}} = e^{+S_{\mathrm{dS}}}$$

$$\mathcal{P}_{\mathrm{T}}(\mathrm{Nothing} \to \mathrm{dS}) = \|\Psi_{\mathrm{T}}(\mathrm{H}_{\mathrm{dS}})\|^2 \propto \mathrm{e}^{-\frac{\pi}{\mathrm{GH}_{\mathrm{dS}}^2}} = e^{-S_{\mathrm{dS}}}$$

Two types of vacuum transitions

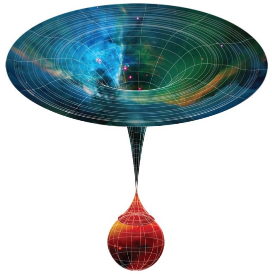


1. **Transition between two minima of scalar potential**

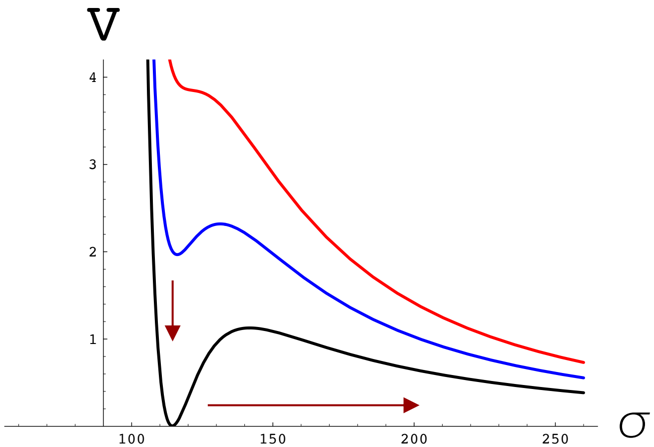
   Coleman-De Luccia 1980

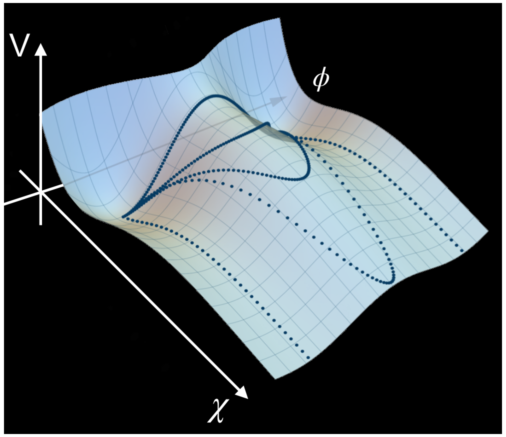
2. **No scalar field: M_1 to M_1+Wall+M_2**

   Brown-Teitelboim 87



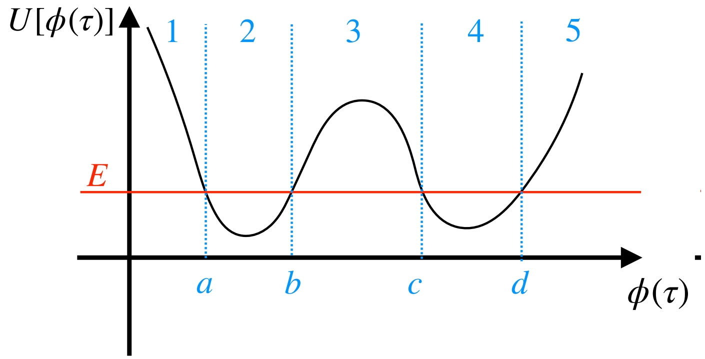
Both realised in string landscape





Approximate picture

WKB in Field Theory

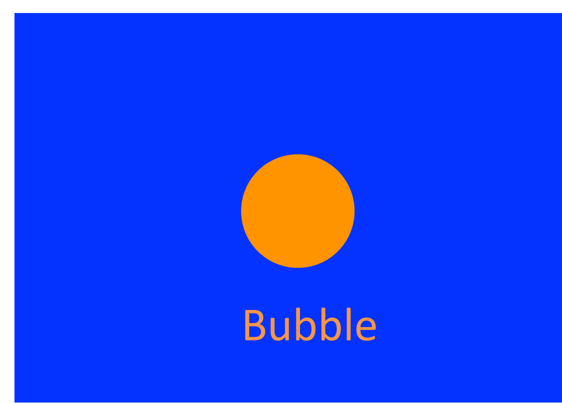


$U[\phi]$

Schrödinger equation

$$\int_X \left[-\frac{\hbar^2}{2} \frac{\delta^2}{\delta\phi(x)^2} + \underbrace{\frac{1}{2}\nabla_x^2 \phi + V(\phi)}_{U[\phi]} \right] \Psi[\phi] = E\Psi[\phi].$$

Infinite volume: Transition local



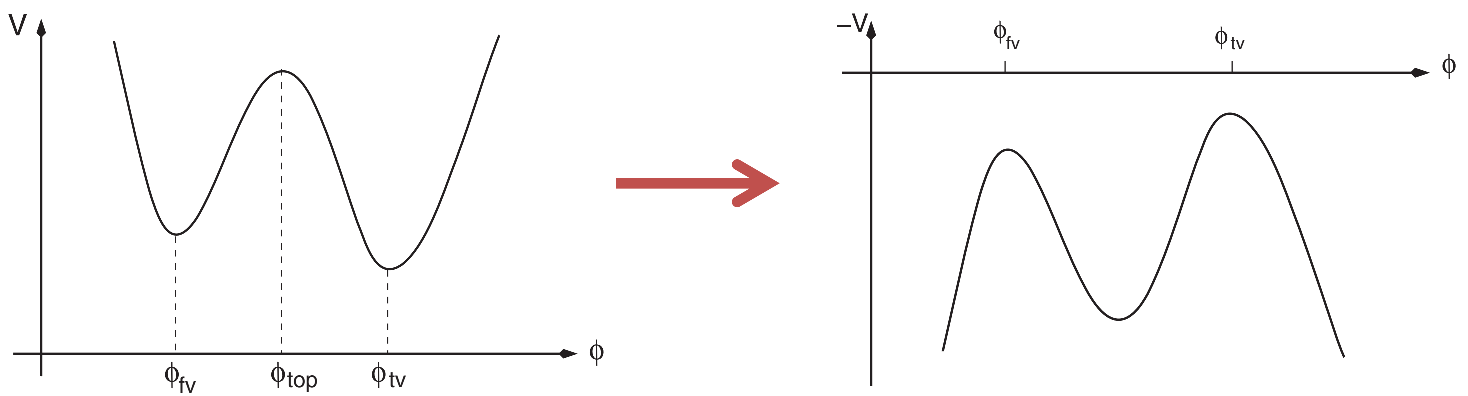
Decay rate

$$\Gamma \sim T^2 \sim \frac{1}{\lambda^2} \propto \exp\left(-2\int_b^c \kappa d\tau\right)$$

$$T^2 = \frac{|\Psi_4[\phi]|^2}{|\Psi_2[\phi]|^2}$$

Euclidean Approach

Coleman et al. (Field theory and Gravity)



O(4) Instanton (bounce)

$$\xi^2 = |x|^2 + \tau^2$$

$$ds^2 = d\xi^2 + \rho^2(\xi)(d\psi^2 + \sin^2\psi d\Omega_2^2)$$

Analytic continuation O(3,1)

$$\begin{aligned} ds^2 &= d\tilde{t}^{\,2} - \tilde{t}^{\,2}\left[\frac{d\tilde{r}^2}{1+\tilde{r}^2} + \tilde{r}^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right] \\ &\equiv d\tilde{t}^{\,2} - \tilde{t}^{\,2}\, d\Omega_T^2 \,. \end{aligned}$$

Open FLRW geometry!

Including Gravity

Euclidean approach (Coleman-de Luccia, Lee-Weinberg, Brown-Teitelboim) :

$$\Gamma \sim e^{-B}, \qquad B = S[instanton] - S[background]$$

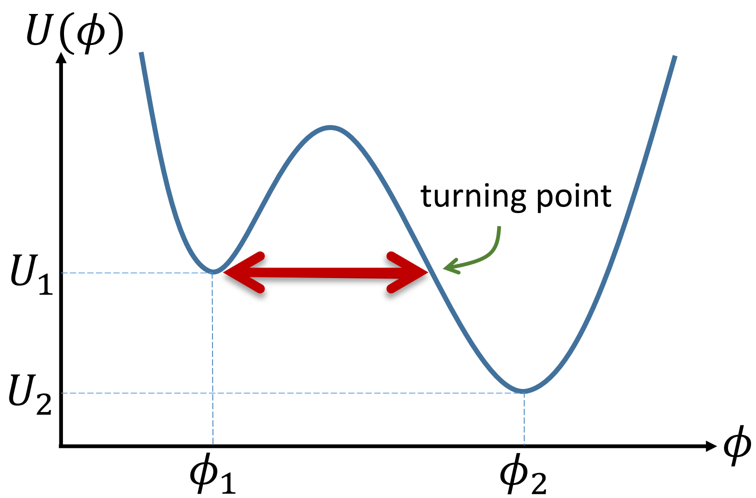
$$B = \frac{\pi}{2G}\left[\frac{\left[(H_{\rm O}^2 - H_{\rm I}^2)^2 + \kappa^2(H_{\rm O}^2 + H_{\rm I}^2)\right] R_{\rm o}}{4\kappa H_{\rm O}^2 H_{\rm I}^2} - \frac{1}{2}\left(H_{\rm I}^{-2} - H_{\rm O}^{-2}\right)\right]$$

$$R_{\rm o}^2 = \frac{4\kappa^2}{(H_{\rm O}^2 - H_{\rm I}^2)^2 + 2\kappa^2(H_{\rm O}^2 + H_{\rm I}^2) + \kappa^4}$$

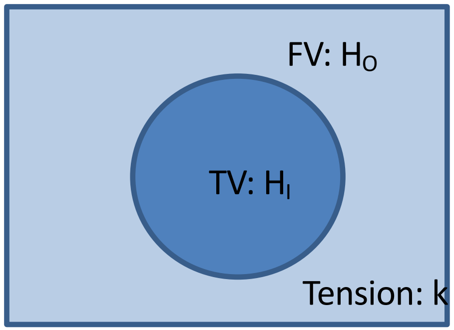
Vacuum transitions

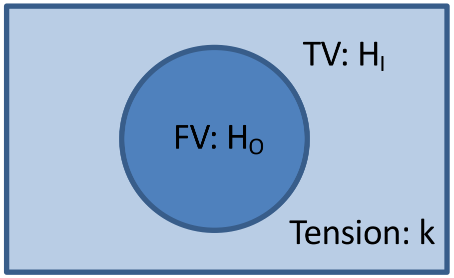
Gravity: Down and Up Tunneling

Lee+ Weinberg



Bubble nucleation





Euclidean CDL

- It reproduces the right decay rate Γ=e^{-B} as WKB and direct extension to field theory and gravity
- After bubble materialisation: Analytic continuation from Euclidean to Lorentzian. Implies open universe but just a "guess" (O(4) symmetry)
- Minkowski to de Sitter: (creating a universe in the lab), singular instanton?

Hamiltonian Approach

Hamiltonian Approach

Fischler, Morgan, Polchinski 1990

Metric $$ds^2 = -N_t^2(t,r)dt^2 + L^2(t,r)(dr + N_r dt)^2 + R^2(t,r)d\Omega_2^2\,,$$ **Spherically symmetric**

Action

$$S_{\rm tot} = \frac{1}{16\pi G}\int_{\mathcal{M}} d^4x\sqrt{g}\,\mathcal{R} + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^3y\sqrt{h}\,K + S_{\rm mat} + S_{\rm W}$$

$$S_{\rm W} = -4\pi\sigma\int dtdr\,\delta(r-\hat{r})[N_t^2 - L^2(N_r + \dot{\hat{r}})^2]^{1/2}$$

$$S_{\rm mat} = -4\pi\int dtdr\,LN_tR^2\,\rho(r)\,,$$

$$\rho = \Lambda_{\rm O}\,\theta(r-\hat{r}) + \Lambda_{\rm I}\,\theta(\hat{r}-r)$$

Conjugate variables

$$\pi_L = \frac{N_rR' - \dot{R}}{GN_t}R, \qquad \pi_R = \frac{(N_rLR)' - \partial_t(LR)}{GN_t}\,,$$

$$\mathcal{H}_g = \frac{GL\pi_L^2}{2R^2} - \frac{G}{R}\pi_L\pi_R + \frac{1}{2G}\left[\left(\frac{2RR'}{L}\right)' - \frac{R'^2}{L} - L\right]$$

$$P_g = R'\pi_R - L\pi_L'\,.$$

Constraints

$$\mathcal{H} = \mathcal{H}_g + 4\pi LR^2\rho(r) + \delta(r-\hat{r})E = 0\,,$$

$$P = P_a - \delta(r-\hat{r})\hat{p} = 0\,,$$

$$E = \sqrt{\frac{\hat{p}^2}{\hat{L}^2} + m^2}\,, \qquad m = 4\pi\sigma\hat{R}^2\,, \qquad \hat{p} = \partial\mathcal{L}/\partial\dot{\hat{r}}$$

Classical Trajectories

Solutions of constraints

$$\pi_L = \eta \frac{R}{G}\left[\frac{R'^2}{L^2} - A_\alpha\right]^{1/2}, \quad \alpha = \mathrm{O}, \mathrm{I}, \quad \eta = \pm 1,$$

$$A_\alpha = 1 - \frac{2GM_\alpha}{R} - H_\alpha^2 R^2, \qquad H_\alpha^2 = \frac{8\pi G}{3}\Lambda_\alpha,$$

$$ds_\alpha^2 = -A_\alpha(R)\,d\tau^2 + A_\alpha^{-1}(R)\,dR^2 + R^2\,d\Omega_2^2.$$

Dynamics

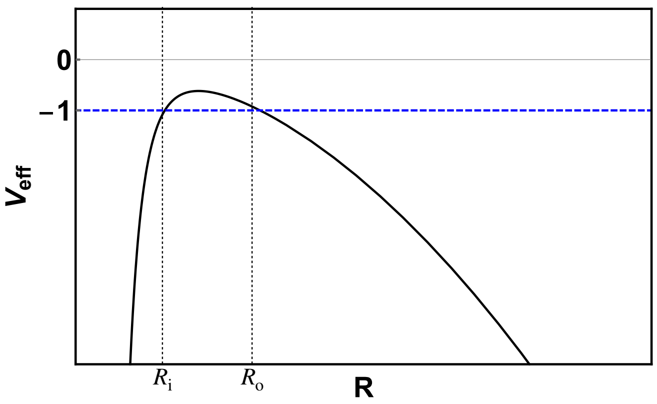
$$\dot{\hat{R}}^2 + V = -1$$

$$V = -\frac{1}{(2\kappa\hat{R})^2}\left((\hat{A}_\mathrm{I} - \hat{A}_\mathrm{O}) - \kappa^2\hat{R}^2\right)^2 + (\hat{A}_\mathrm{O} - 1)$$

Matching conditions

$$\frac{\hat{R}}{\hat{L}}(R'(\hat{r}+\epsilon) - R'(\hat{r}-\epsilon)) = -GE,$$

$$\pi_L(\hat{r}+\epsilon) - \pi_L(\hat{r}-\epsilon) = \frac{\hat{p}}{\hat{L}} = 0,$$



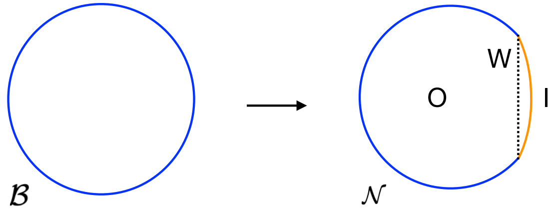
Tunneling Probability and WDW

Wheeler DeWitt Equation $\mathcal{H}\Psi = 0$ **(no time evolution)**

WKB $\Psi = ae^{I} + be^{-I}$ $iS = I$

Transition Probability

$$\mathcal{P}(\mathcal{B} \to \mathcal{N}) = \left| \frac{\Psi_{\mathcal{N}}}{\Psi_{\mathcal{B}}} \right|^2 \qquad \mathcal{P}(\mathcal{B} \to \mathcal{N}) \equiv \Gamma_{\mathcal{B}\to\mathcal{N}}$$



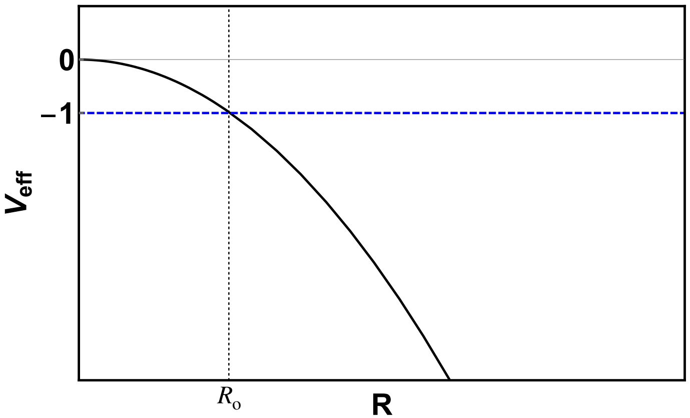
$$\mathcal{P}(\mathcal{B} \to \mathcal{N}) \simeq \exp\left[2\,\mathrm{Re}\left(I_{\mathrm{tot}}(\mathcal{N}) - I(\mathcal{B})\right)\right]$$

$$\mathcal{P}(A \to A/B \oplus \mathrm{W}) = \frac{|\Psi(A/B \oplus \mathrm{W})|^2}{|\Psi(A)|^2}$$

De Sitter to de Sitter

(M=0)

$$\mathcal{P}(\mathrm{dS} \to \mathrm{dS}/\mathrm{dS} \oplus \mathrm{W}) = \frac{|\Psi(\mathrm{dS}/\mathrm{dS} \oplus \mathrm{W})|^2}{|\Psi(\mathrm{dS})|^2}$$



$$V = -\frac{1}{4\kappa^2}\hat{R}^2 \left[(H_\mathrm{O}^2 - H_\mathrm{I}^2)^2 + 2\kappa^2(H_\mathrm{O}^2 + H_\mathrm{I}^2) + \kappa^4\right]$$

$$A_\mathrm{O} = 1 - H_\mathrm{O}^2 R^2\,, \qquad A_\mathrm{I} = 1 - H_\mathrm{I}^2 R^2$$

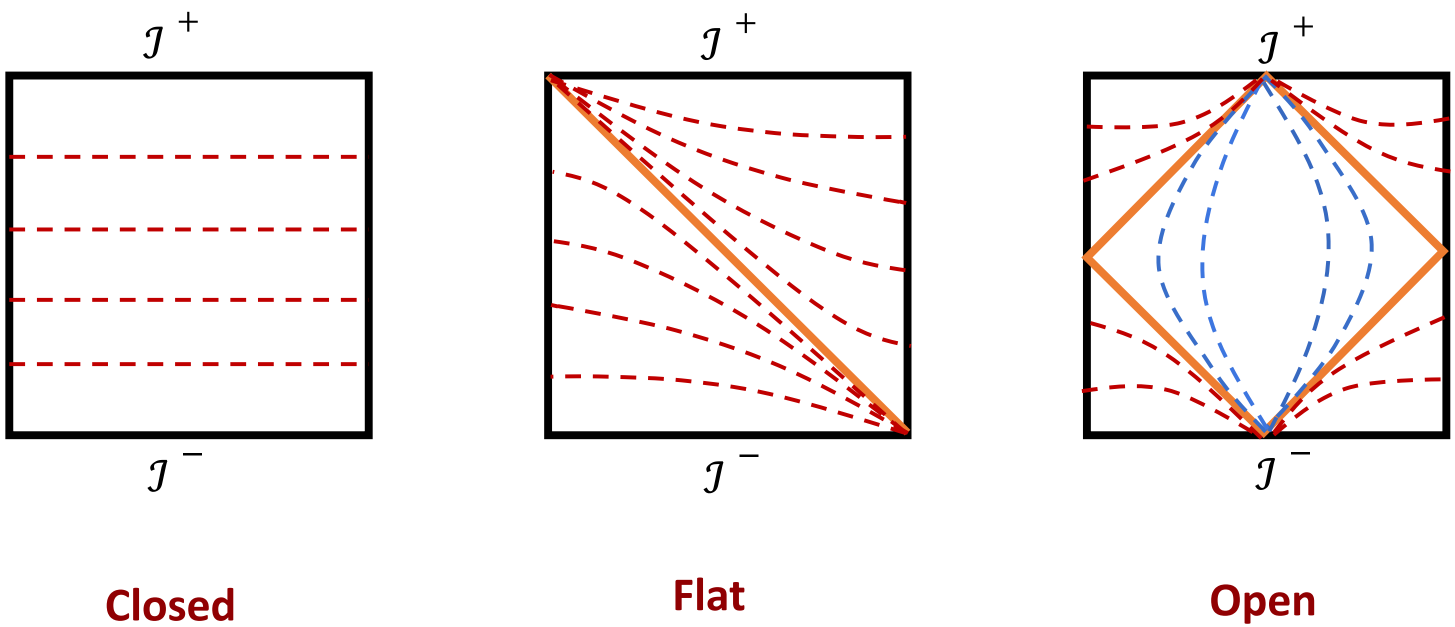
$$I_\mathrm{tot}\Big|_\mathrm{tp} - \bar{I} = -\frac{\eta\pi}{G}\left[\frac{\left[(H_\mathrm{O}^2 - H_\mathrm{I}^2)^2 + \kappa^2(H_\mathrm{O}^2 + H_\mathrm{I}^2)\right] R_\mathrm{o}}{8\kappa H_\mathrm{O}^2 H_\mathrm{I}^2} - \frac{1}{4}\left(H_\mathrm{I}^{-2} - H_\mathrm{O}^{-2}\right)\right]$$

Same result as Euclidean approach

$\eta = +1$ **Background Hartle-Hawking**

$\eta = -1$ **Background Vilenkin**

De Sitter Slicings



From Hamiltonian approach: O(3) symmetry, closed slicing.
Universe inside the bubble is closed for global slicing.

Up-Tunneling and Minkowski limit

Detailed balance

entropies

$$\Gamma_{\rm up} = \Gamma_{\rm down} \exp\left[\frac{\pi}{G}\left(\frac{1}{H_{\rm I}^2} - \frac{1}{H_{\rm O}^2}\right)\right] = \Gamma_{\rm CDL} \exp\left(S_{\rm I} - S_{\rm O}\right)$$

For HH sign only!

De Sitter to Minkowski ?

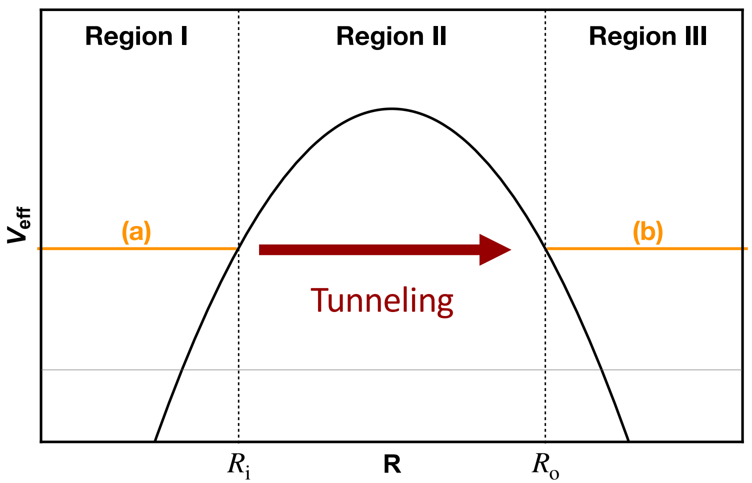
$$H_I \to 0, \qquad \Gamma_{\rm down} \to \exp\left[-\frac{\pi}{2G}\frac{\kappa^4}{H_O^2\left(H_O^2+\kappa^2\right)^2}\right]$$

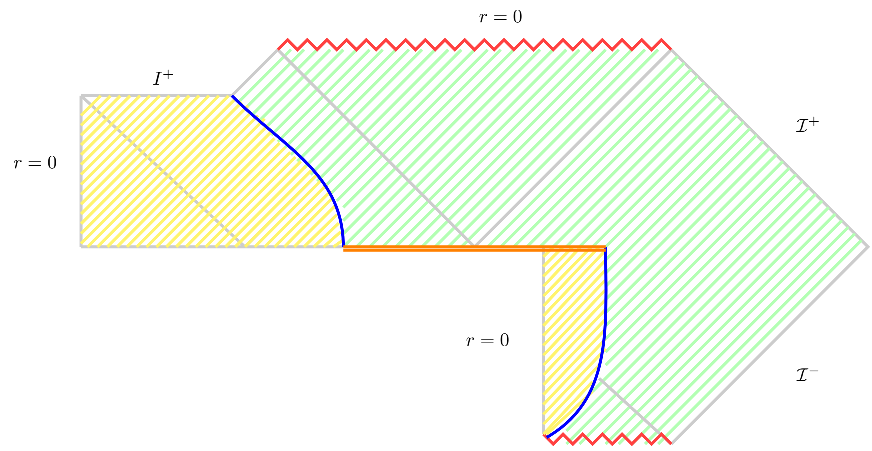
$$H_O \to 0, \qquad \Gamma_{\rm up} \to 0$$

Schwarzschild to de Sitter

(H_O=0)

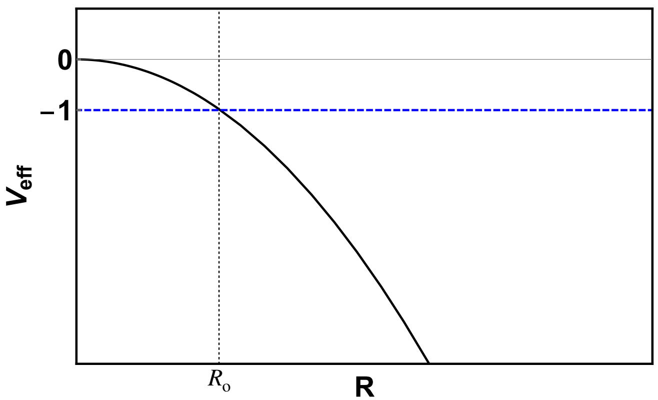
Farhi,Guth, Guven (Euclidean) + Fischler, Morgan, Polchinski (Hamiltonian)





Zero Schwarzschild mass limit

(Minkowski ≈ Schwarzschild in the M=0 limit)



$$V = -\frac{\left(H^2+\kappa^2\right)^2}{4\kappa^2}\,\hat{R}^2$$

$$\mathcal{P}(\mathcal{M} \to \mathcal{M}/\mathrm{dS} \oplus \mathrm{W}) = \exp\left[\frac{\eta\pi}{GH^2}\left(1 - \frac{\kappa^4}{(H^2+\kappa^2)^2}\right)\right]$$

Up-tunneling

$$\mathcal{P}(\mathrm{dS} \to \mathrm{dS}/\mathcal{M} \oplus \mathrm{W}) = \exp\left[\frac{\eta\pi}{GH^2}\left(-\frac{\kappa^4}{(H^2+\kappa^2)^2}\right)\right]$$

Down-tunneling

Detailed Balance

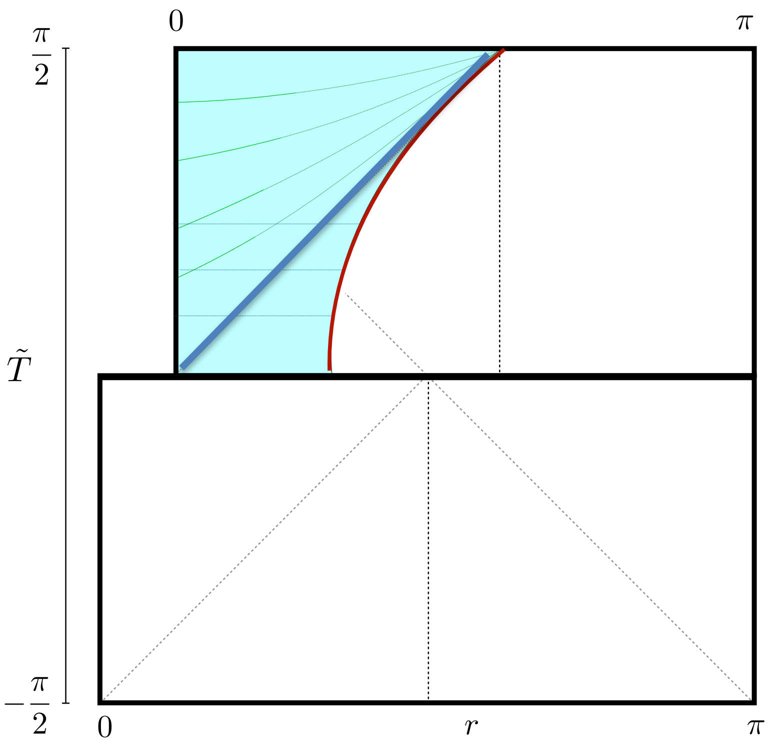
$$\frac{\mathcal{P}(\mathcal{M} \to \mathcal{M}/\mathrm{dS} \oplus \mathrm{W})}{\mathcal{P}(\mathrm{dS} \to \mathrm{dS}/\mathcal{M} \oplus \mathrm{W})} = \exp\left[\eta\frac{\pi}{G}\frac{1}{H^2}\right]$$

Entropy

M=0 Schwarzschild ≠ H=0 de Sitter

(Difference on background wave function)

Bubble Trajectory



$$\dot{\hat{R}}^2 + V = -1; \qquad V = -\frac{\hat{R}^2}{R_0^2},$$

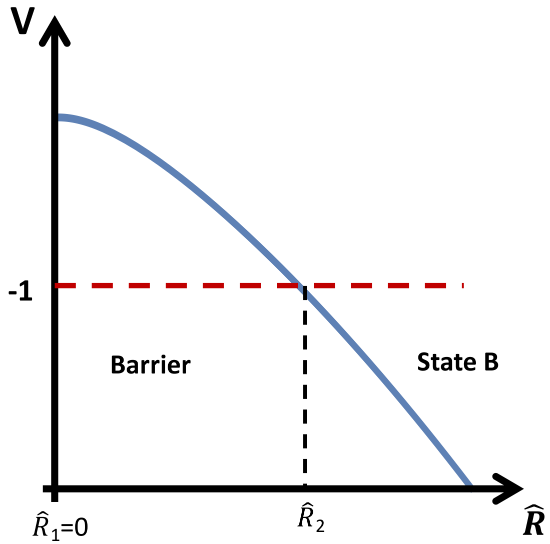
$$R(t) = R_0 \cosh \frac{t}{R_0}$$

$$\cos(\rho) = \sqrt{1 - H^2 R_0^2} \cos T$$

Asymptotic speed= speed of light – $(M/M_P)^2$<c!

Even though calculation done in global slicing, trajectories follow geodesics of open slicing

AdS to AdS



$$\kappa < \left|\sqrt{|H_I^2|} - \sqrt{|H_O^2|}\right|, \qquad \text{or} \qquad \kappa > \left|\sqrt{|H_I^2|} + \sqrt{|H_O^2|}\right|,$$

$$B = -\frac{\eta\pi}{2G}\left[\frac{\left(\left|H_I^2\right| - \left|H_O^2\right|\right)^2 - \kappa^2\left(\left|H_I^2\right| + \left|H_O^2\right|\right)}{2\kappa\left|H_I^2\right|\left|H_O^2\right|} R_0 - \left(\frac{1}{\left|H_O^2\right|} - \frac{1}{\left|H_I^2\right|}\right)\right]$$

$$\mathcal{P}_{\text{up}}^{\text{AdS}\to\text{AdS}} = \mathcal{P}_{\text{down}}^{\text{AdS}\to\text{AdS}},$$

Detailed balance if Entropy of AdS = 0 !

Minkowski to AdS

$$H_0 \to 0 \qquad B = 2\left(I_{\rm tot}|_{\rm tp} - \bar{I}\right) = -\frac{\eta\pi}{2G|H_I|^2}\left[\frac{2\kappa^4}{(|H_I|^2-\kappa^2)^2}\right],$$

As in CDL

AdS to dS

$$B^{\rm AdS->dS} = \frac{\eta\pi}{G}\left\{\frac{\left\{(|H_B^2|+H_A^2)^2+\kappa^2(-|H_B^2|+H_A^2)\right\}R_{\rm o}}{4\kappa|H_B^2|H_{\rm A}^2} + \frac{1}{2}\left(\frac{1}{H_A^2}-\frac{1}{|H_B^2|}\right)\right\},$$

$$\frac{P^{\rm AdS->dS}}{P^{\rm dS->AdS}} = \frac{e^{B^{\rm AdS->dS}}}{e^{B^{\rm dS->AdS}}} = \frac{\exp\left(\frac{\eta\pi}{2G}\frac{1}{H_A^2}\right)}{\exp\left(-\frac{\eta\pi}{2G}\frac{1}{H_A^2}\right)} = e^{\eta(S_{\rm dS}-(S_{\rm AdS}=0))},$$

Detailed balance if AdS entropy=0!

Minkowski limit from dS blows-up but from AdS is finite!?

To Nothingness and Back?

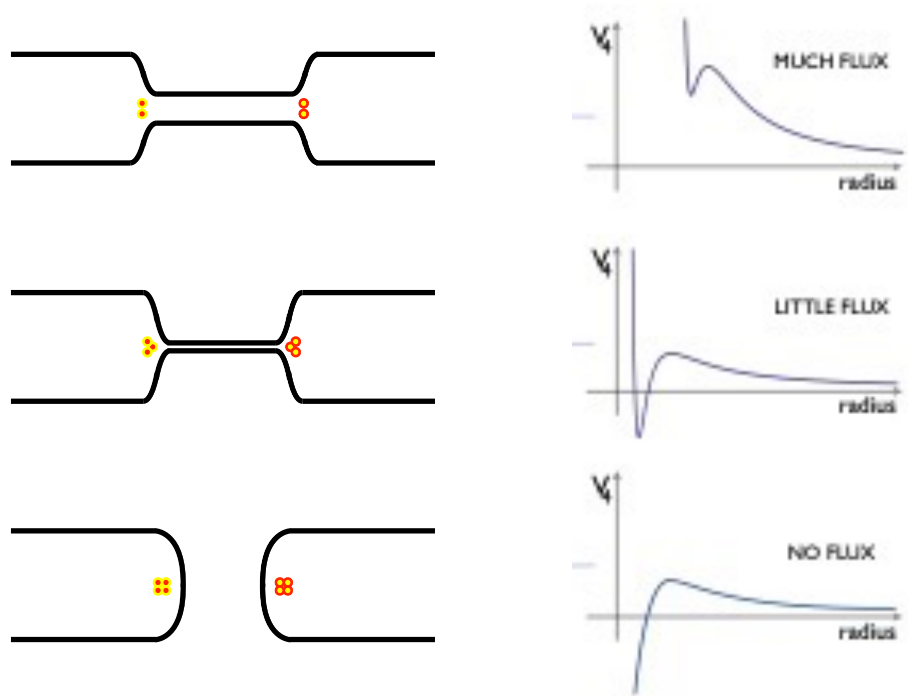
For SAdS to dS $H_{\rm O} \gg H_{\rm I}, M, \kappa$

$$B^{\rm AdS->dS} \to \frac{\eta\pi}{G}\left\{\frac{\left\{(|H_B^2|)^2\right\}2\kappa/|H_B^2|}{4\kappa|H_{\rm B}^2|H_A^2} + \frac{1}{2}\left(\frac{1}{H_A^2}+0\right)\right\} = \frac{\eta\pi}{2G}\frac{1}{H_A^2}.$$

The same as Vilenkin, Hartle-Hawking wave functions!

≈ Brown-Dahlen:
Nothing as AdS

$H_{\rm O} \to \infty$



General: S(A)dS to S(A)dS

Total Action

$$I_{\rm tot} = I_{\rm B} + I_{\rm W}$$

$$I_{\rm B} = \frac{\eta}{G}\int_0^{\hat r-\epsilon} dr R\left[\sqrt{A_{\rm I}L^2 - R'^2} - R'\cos^{-1}\left(\frac{R'}{L\sqrt{A_{\rm I}}}\right)\right] + \int_{\hat r+\epsilon}^{\pi} dr\ [{\rm I}\to{\rm O}]\ ,$$

$$I_{\rm W} = \frac{\eta}{G}\int \delta\hat R\,\hat R\cos^{-1}\left(\frac{R'}{L\sqrt{\hat A}}\right)\Bigg|_{\hat r-\epsilon}^{\hat r+\epsilon},$$

$$I_{\rm W} = -\frac{\eta}{G}\int dR R\cos^{-1}\left(\frac{\frac{2G}{R}(M_{\rm O}-M_{\rm I}) + R^2(\pm H_{\rm O}^2 \mp H_{\rm I}^2 - \kappa^2)}{2\kappa R\sqrt{1-\frac{2GM_{\rm O}}{R} \mp H_{\rm O}^2R^2}}\right)$$
$$+\frac{\eta}{G}\int dR R\cos^{-1}\left(\frac{\frac{2G}{R}(M_{\rm O}-M_{\rm I}) + R^2(\pm H_{\rm O}^2 \mp H_{\rm I}^2 + \kappa^2)}{2\kappa R\sqrt{1-\frac{2GM_{\rm I}}{R} \mp H_{\rm I}^2R^2}}\right).$$

Wall integrals cannot be done analytically but symmetric under

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Bulk Contributions

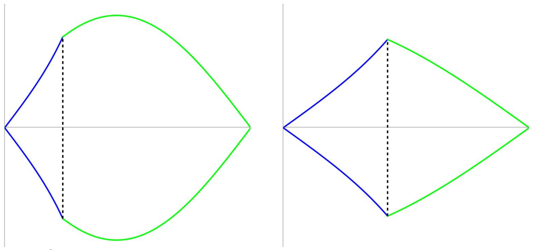
SdS to SdS

Turning point geometry $\pi_L = 0$

$$\frac{R'^2}{L^2} = A(R) = 1 - \frac{2MG}{R} - H^2 R^2$$

$$I_B \propto \int R dR \Theta(-R')$$

$$I_B \left[\hat{R}\right] = \frac{\eta\pi}{2G}\left[\theta(-\hat{R}'_-)\left(R^2_{I,c} - \hat{R}^2\right) + \theta(-\hat{R}'_+)\left(\hat{R}^2 - R^2_{O,s}\right)\right] + \frac{\eta\pi}{2G}\left[\theta(\hat{R}'_+)\left(R^2_{O,c} - \hat{R}^2\right) + \theta(\hat{R}'_-)\left(\hat{R}^2 - R^2_{I,s}\right)\right].$$



$$I_B\,[R_2] = \frac{\eta\pi}{2G}\left[\left(R^2_{I,c} - R^2_{O,s}\right)\right],\; I_B\,[R_1] = \frac{\eta\pi}{2G}\left[\left(R^2_{O,c} - R^2_{I,s}\right)\right]$$

B → A vs A → B

$$P_\uparrow\left(\hat{R}\right) = \frac{|a|^2 e^{2I^{AB}_{\rm Bu}(\hat{R}) + 2I^{AB}_{\rm W}(\hat{R})} + \ldots}{|a|^2 e^{2I^{AB}_{\rm Bu}(R_1) + 2I^{AB}_{\rm W}(R_1)} + \ldots},\quad P_\downarrow\left(\hat{R}\right) = \frac{|a|^2 e^{2I^{BA}_{\rm Bu}(\hat{R}) + 2I^{BA}_{\rm W}(\hat{R})} + \ldots}{|a|^2 e^{2I^{BA}_{\rm Bu}(R_1) + 2I^{BA}_{\rm W}(R_1)} + \ldots}$$

$$\frac{P_\uparrow}{P_\downarrow} = e^{\frac{\pi}{G}\left[\left(R^2_{B,c} - R^2_{A,s}\right) - \left(R^2_{A,c} - R^2_{B,s}\right)\right]} = e^{S_B - S_A}$$

SAdS to dS

$$I_{\rm B}\Big|_{\rm tp} \equiv I_{\rm B}\Big|_{R_{\rm I}}^{R_{\rm O}} = \begin{cases} \frac{\eta\pi}{2G}(R_{\rm O}^2 - R_{\rm I}^2)\,, & M > M_{\rm S}\,, \\ \frac{\eta\pi}{2G}(R_{\rm O}^2 - R_{\mathcal{S}}^2)\,, & M_{\rm S} > M > M_{\rm D} \\ \frac{\eta\pi}{2G}(R_{\rm dS}^2 - R_{\mathcal{S}}^2)\,, & M_{\rm D} > M\,. \end{cases}$$

$$M_{\rm S} = \frac{H_{\rm O}^2 + H_{\rm I}^2 + \kappa^2}{2G\left(H_{\rm I}^2 + \kappa^2\right)^{3/2}}, \qquad M_{\rm D} = \frac{H_{\rm O}^2 + H_{\rm I}^2 - \kappa^2}{2GH_{\rm I}^3}.$$

Need numerical estimates for wall contribution but the transition is allowed however detailed balance is OK only for M_D>M (?)

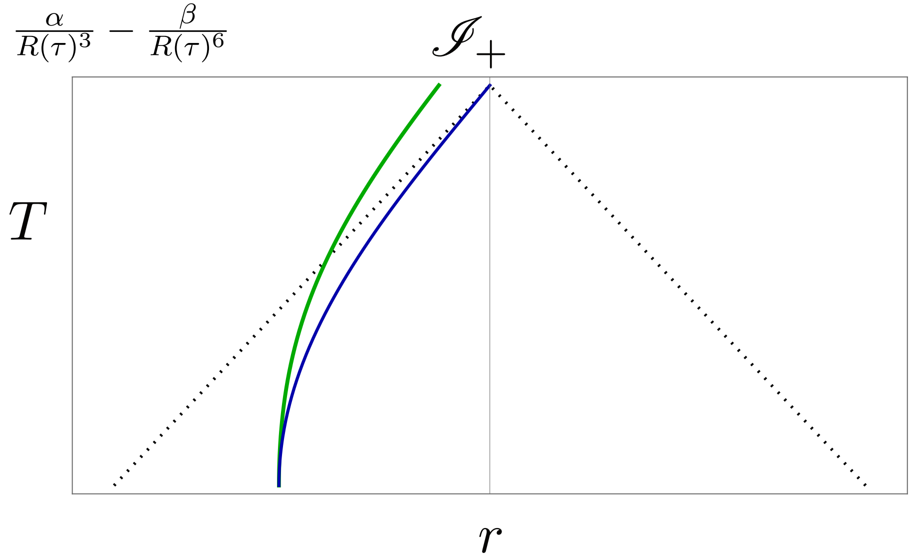
$$\frac{P^{\rm AdS->dS}}{P^{\rm dS->AdS}} = \frac{e^{B^{\rm AdS->dS}}}{e^{B^{\rm dS->AdS}}} = \frac{\exp\left(\frac{\eta\pi}{2G}\frac{1}{H_A^2}\right)}{\exp\left(-\frac{\eta\pi}{2G}\frac{1}{H_A^2}\right)} = e^{\eta(S_{\rm dS}-(S_{\rm AdS}=0))},$$

Wall Trajectory

$$ds^2 = -dt^2 + \hat{R}^2(t)d\Omega^2$$

Acceleration:

$$-K_{\tau\tau} = \frac{\ddot{R} - H^2 R}{\sqrt{1 - H^2R^2 + \dot{R}^2}} = \frac{\frac{(1-H_0^2R_*^2)}{R_0^2} - \frac{\alpha}{2R(\tau)^3} - \frac{2\beta}{R(\tau)^6}}{\sqrt{\frac{(1-H_0^2R_*^2)}{R_0^2} - \frac{\alpha}{R(\tau)^3} - \frac{\beta}{R(\tau)^6}}}$$



Non open universe geodesics !
Observable implications?

Vacuum Transitions

Standard	**Non-Standard**
• Euclidean	• Hamiltonian
• No Minkowski to dS	• BH, Minkowski/AdS to dS
• Open Universe	• Closed Universe
• Unrelated to V,HH	• Related to V, HH

* **Hamiltonian approach only available in minisuperspace or transitions without scalar potential**

Conclusions

- Hamiltonian approach to quantum transitions
- Minkowski BH and AdS BH to de Sitter not forbidden (no O(4) symmetry)
- Minkowski entropy from M→0 BH or |H|→0 AdS and no H→0 dS!
- Consistent with a closed universe after bubble nucleation (predictions?).
- Wall trajectory not an open universe geodesic
- Up-tunneling from AdS, Minkowski if their entropies vanish!
- Up-tunneling from AdS limit H→∞ = Hartle-Hawking/Vilenkin from nothing!
- Hartle-Hawking vs Vilenkin (detailed balance)
- Many open questions (very few closed)

e.g Transition with scalar potentials beyond mini-superspace

THANK YOU !