The University of
Nottingham

How to calculate Feynman rules in scalar-tensor theories

and not collapse in the process

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FeynMG

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FeynMG: a FeynRules extension for scalar-tensor theories of gravity

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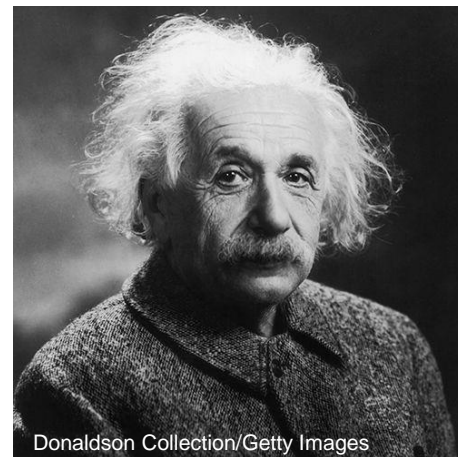[b] Department of Physics and Astronomy, University of Manchester,
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Based on arXiv:2211.14300

Sevillano

What are Scalar-Tensor theories

We all know and love Einstein's General Relativity, which
Is usually described by the Einstein-Hilbert action

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[-\frac{M_{\mathrm{pl}}^2}{2} R + \mathcal{L}_{\mathrm{m}}\{\psi_i, \phi_i, g_{\mu\nu}, ...\} \right]$$

Curvature

Matter action

Donaldson Collection/Getty Images
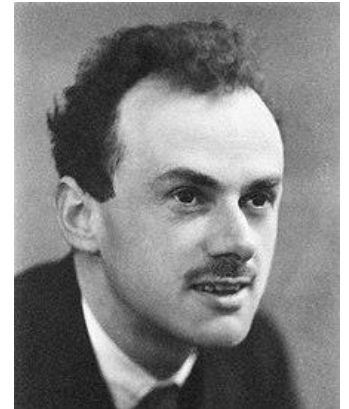
However, this might not be the whole story…

Some history

One of the first motivations for modified gravity started with Dirac, and its **large number hypothesis**

He noticed that both dimensionless ratios were of the same order

$$\frac{c\,t}{r_{\text{e}}} \approx 3.47 \cdot 10^{41} \approx 10^{42} \qquad \frac{e^2}{4\pi\epsilon_0 G m_{\text{p}} m_{\text{e}}} \approx 10^{40}$$

From which he hypothesised that the strength of gravity might be changing with time!

$$G \sim \frac{1}{t}$$

Some history

Time-dependent gravitational constant? Quite crazy... $\quad G \sim \dfrac{1}{t}$

However, of course, we cannot just insert some function of time as:

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[-\frac{M_{\mathrm{pl}}(t)^2}{2} R + \mathcal{L}_{\mathrm{m}} \{\psi_i, \phi_i, g_{\mu\nu}, ...\} \right]$$

Some history

However, Brans, Dicke and Jordan had already been thinking about something similar, but using a scalar field

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[-\frac{F(X)}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu X \partial_\nu X - U(X) + \mathcal{L}_{\mathrm{m}} \{ \psi_i, \phi_i, g_{\mu\nu}, ... \} \right]$$

Such that at late times, when the scalar stabilizes

$$F(v_X) \equiv M_{\mathrm{pl}}^2$$

This is a consistent theory with changing gravitational force!

What are Scalar-Tensor theories

Either from an **effective field theory** standpoint or more **fundamental theories** of gravity (such as compactifications of extra dimensions) it is natural to extend the gravitational action:

Different functions of gravity

$$S_{\mathrm{grav}} = \int \mathrm{d}^4 x \sqrt{-g} \left[-aR - F(X)R - bR^2 - cR_{\mu\nu}R^{\mu\nu} + \frac{1}{2} g^{\mu\nu} \partial_\mu X \partial_\nu X + ... \right]$$

Non-minimal couplings to a scalar field

These extensions of gravity are called **Scalar-Tensor theories**

What are Scalar-Tensor theories

$$S_{\text{grav}} = \int \mathrm{d}^4 x \sqrt{-g} \left[-aR - F(X)R - bR^2 - cR_{\mu\nu}R^{\mu\nu} + \frac{1}{2}g^{\mu\nu}\partial_\mu X \partial_\nu X + ... \right]$$

In most cases, they introduce new dynamics into the matter sector (**fifth forces**)

-Cosmological scales (Gravitational waves)   Bertotti et al. 2003

-Solar system scales (Cassini spacecraft)   Avilez et al. 2013

-Atomic scales (Atom interferometry)   YT talk by Clare Burrage
in "Cosmology Talks"

Intro to Scalar–Tensor theories

However, we can also study them on the subatomic scale using particle theory, unveiling the effect that the modification of gravity has in the Standard Model

For that, instead of working with modified gravity, we will make the following transformation

Our plan:

Modified theory of gravity + Standard Model	Using Field theory →	General Relativity + Beyond the Standard Model
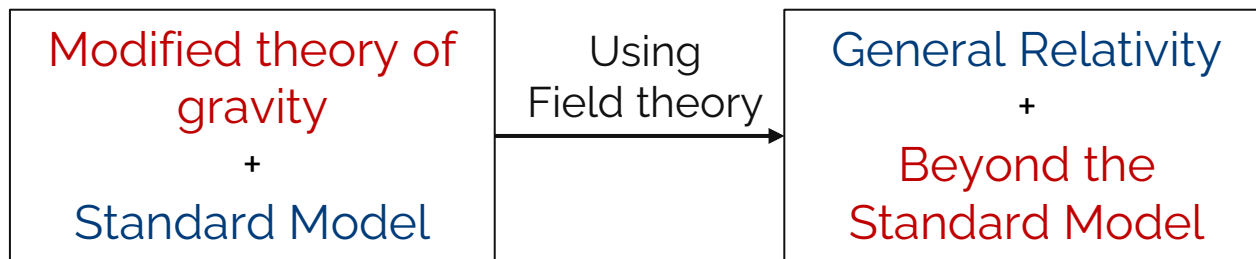
Intro to Scalar–Tensor theories

As an example, let's study the simplest modified theory of gravity

Brans-Dicke Action:

Matter action

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[-\frac{F(X)}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu X \partial_\nu X - U(X) + \mathcal{L}_{\mathrm{m}} \{\psi_i, \phi_i, g_{\mu\nu}, \ldots\} \right]$$

To satisfy basic constraints, we need at late times: $F(v_X) \equiv M_{\mathrm{pl}}^2$

Intro to Scalar–Tensor theories

The common way of solving this called Transforming to the Einstein frame

Make a conformal transformation such that gravity becomes canonical

$$g_{\mu\nu} \rightarrow \frac{\tilde{M}_{\rm Pl}^2}{F(X)} \tilde{g}_{\mu\nu}, \qquad\qquad g^{\mu\nu} \rightarrow \frac{F(X)}{\tilde{M}_{\rm Pl}^2} \tilde{g}^{\mu\nu},$$

$$S_{\rm EF} = \int {\rm d}^4 x \sqrt{-\tilde{g}} \left[-\frac{\tilde{M}_{\rm pl}^2}{2} \tilde{R} + \frac{\tilde{M}_{\rm pl}^2}{2} \left[\frac{1}{F(X)} + \frac{3 F'(X)^2}{2 F(X)^2} \right] \tilde{g}^{\mu\nu} \partial_\mu X \partial_\nu X \right.$$

$$\left. - \frac{\tilde{M}_{\rm pl}^4}{F(X)^2} U(X) + \mathcal{L}_{\rm m} \{ \psi_i, \phi_i, X, \tilde{g}_{\mu\nu}, ... \} \right]$$

Jordan frame

If the gravitational action is very complicated, there is no transformation of the metric that takes us to the Einstein frame, then we are stuck in the <u>Jordan frame</u>

For the simplest Brans-Dicke case:

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[-\frac{F(X)}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu X \partial_\nu X - U(X) + \mathcal{L}_{\mathrm{m}} \{ \psi_i, \phi_i, g_{\mu\nu}, ... \} \right]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + \cdots$$
$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu},$$

graviton

Jordan frame

Fifth forces leak into the system via a kinetic mixing with gravity!

Modified theory of gravity
+
Standard Model
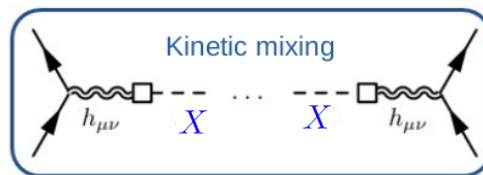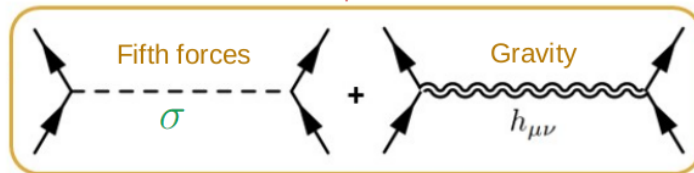
General Relativity
+
Beyond the Standard Model

Beyond Standard Model

Once we have the Beyond Standard Model description, we can calculate from quantum corrections to scattering amplitudes!

However, these calculations are very tedious and model-dependent

Simplest Jordan frame calculation

Expansion of gravity
Canonical normalization
Expansion around non-trivial vevs
Kinetic mixings to graviton
Mass mixings

(even for the Einstein frame)

Beyond Standard Model

Once we have the Beyond Standard Model description, we can calculate from quantum corrections to scattering amplitudes!

However, these calculations are very tedious and model-dependent:

<u>Simplest Jordan frame calculation</u>

(Graphic representation
of your collapse)
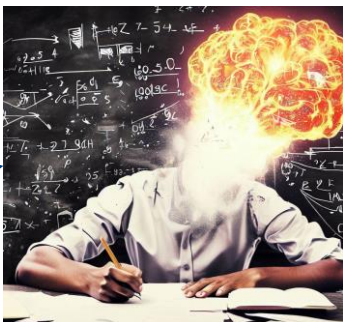
Stable Diffusion

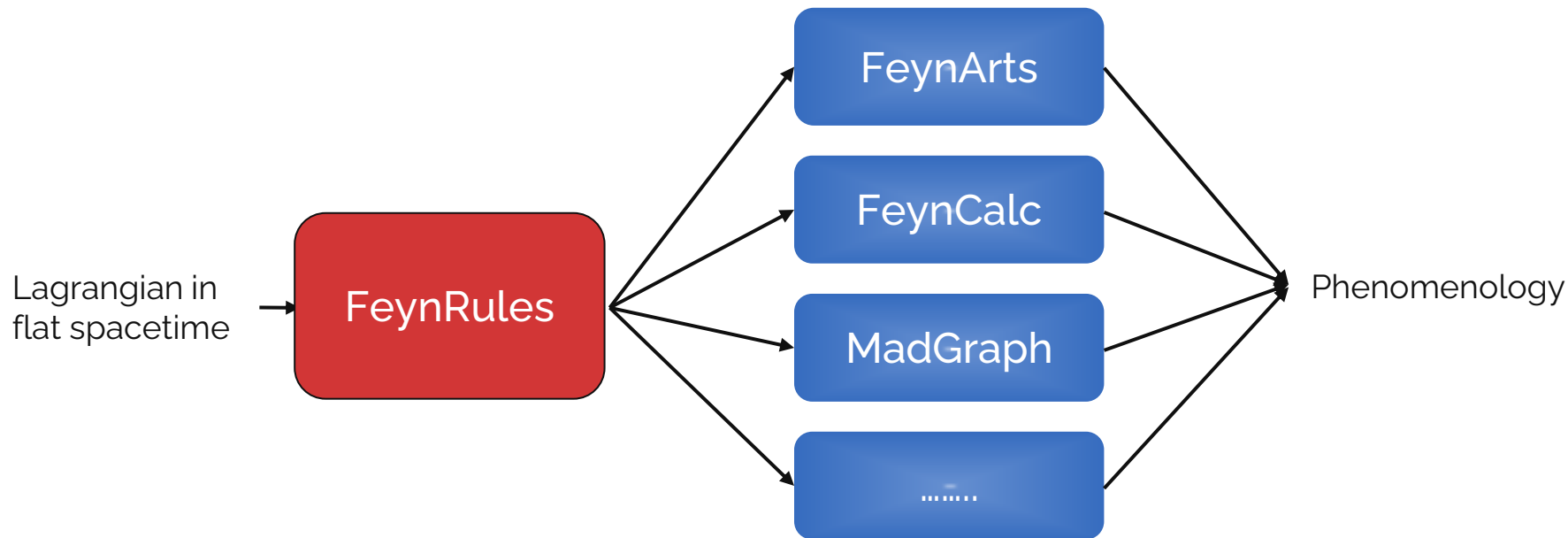Almost impossible to study scalar-tensor theories consistently!!!

Can we get a computer to do it?

Learn from particle phenomenologists (thank you):

They use FeynRules: a Mathematica Package that from a Lagrangian defined in flat spacetime gives the list of Feynman Rules
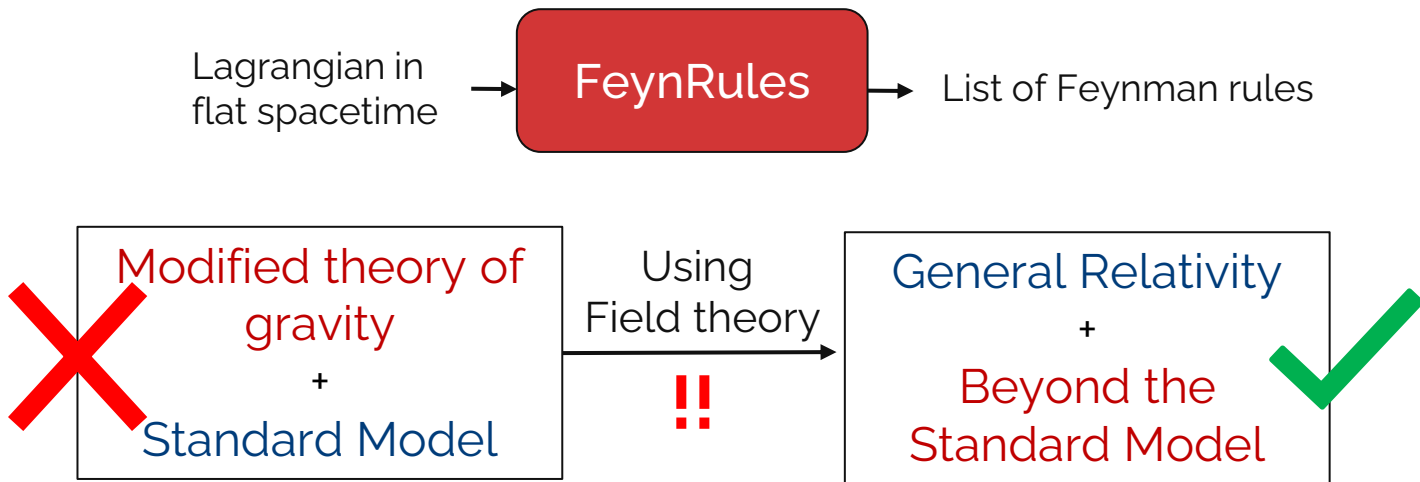
-Defined Fields
-Defined Parameters
-Lagrangian in flat spacetime → FeynRules → List of Feynman rules
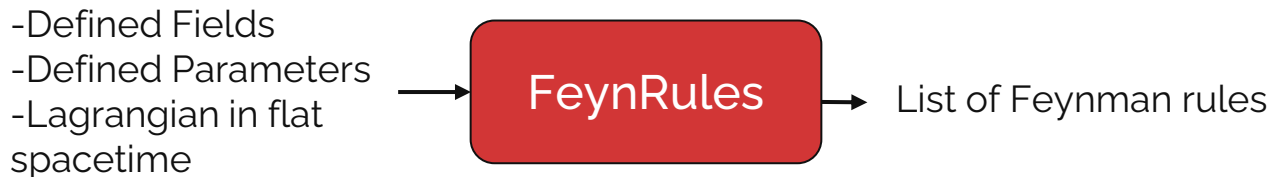
Can we get a computer to do it?

Can we get a computer to do it?

However, the difficult thing is getting to the BSM formalism

Lagrangian in flat spacetime → **FeynRules** → List of Feynman rules

❌ Modified theory of gravity + Standard Model → Using Field theory ‼️ → General Relativity + Beyond the Standard Model ✅

Can we get a computer to do it?

We developed **FeynMG** to help through the process
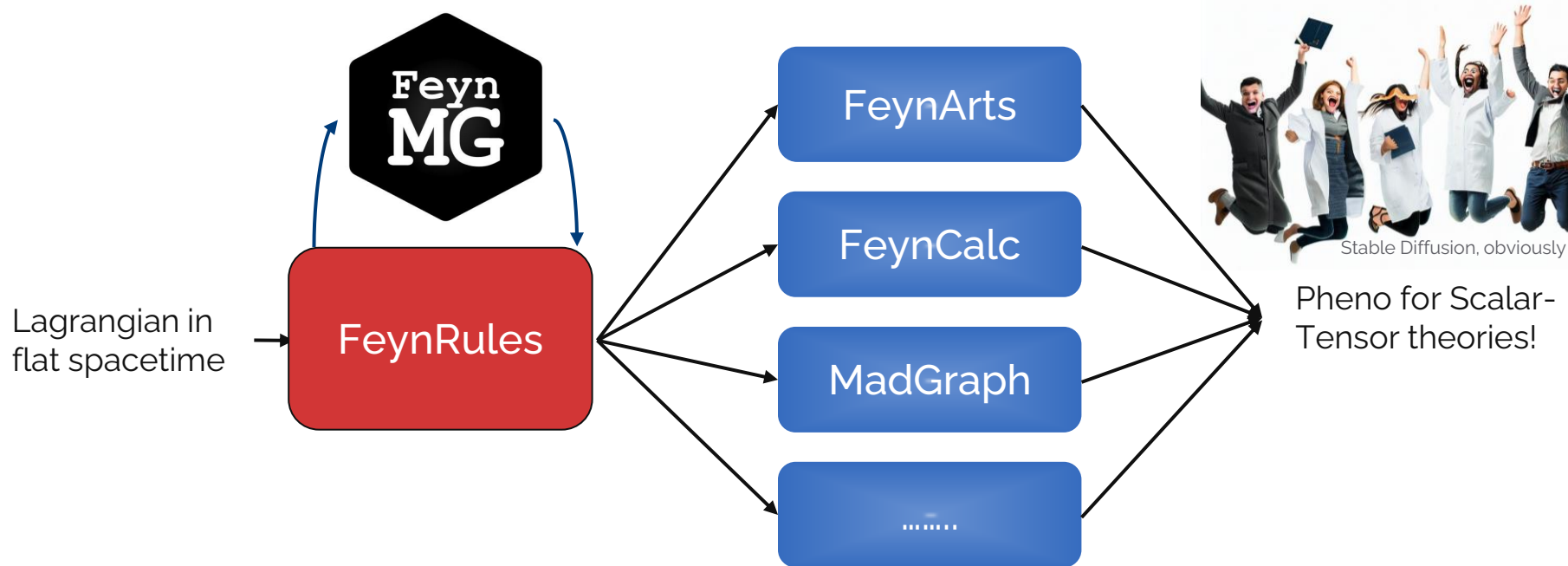
Subpackage of FeynRules

What can FeynMG do?

-Allows the user to insert new scalar degrees of freedom and any gravitational theory

-Perform all the necessary operations to calculate the Beyond the Standard Model description

Simplest Jordan frame calculation

Expansion of gravity   `LinearizeGravity`

`CanonScalar`   Canonical normalization

`VevExpand`   Expansion around non-trivial vevs

Kinetic mixings to graviton   `GravKinMixing`

Mass mixings   `MassDiagMG`
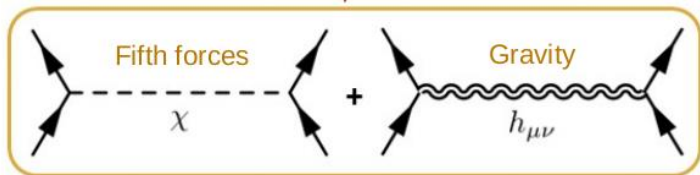
Test Scalar-Tensor theories in colliders!

Based on arXiv:2211.14300

FeynMG!

Lagrangian in flat spacetime → **FeynRules**

FeynRules → FeynArts

FeynRules → FeynCalc

FeynRules → MadGraph

FeynRules → ....

Stable Diffusion, obviously

Pheno for Scalar-Tensor theories!

What can FeynMG do?
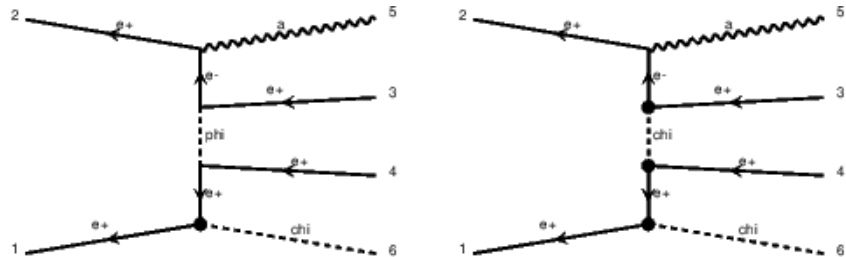
A quick example to express my excitement:

Calculating by hand
fifth forces for an electron

vs

Using MadGraph:
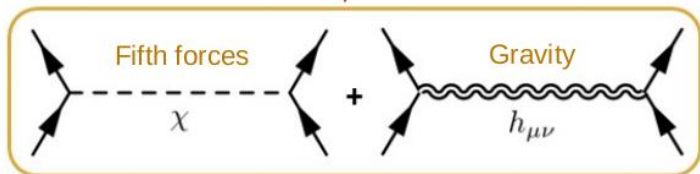

3-4 months of learning and mistakes
in the process

It took **X min.** to generate the
possible 344 diagrams

It can work with any scalar-tensor theory

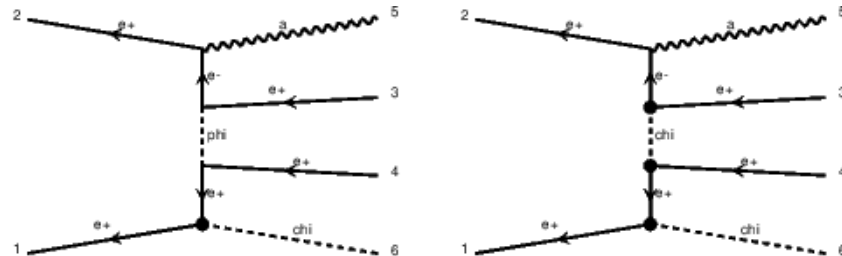# What can FeynMG do?

A quick example to express my excitement:

Calculating by hand
fifth forces for an electron

vs

Using MadGraph:

3-4 months of learning and mistakes
in the process

It took **1.2 s** to generate the
possible **344** diagrams

It can work with any scalar-tensor theory

Conclusion

-Scalar-tensor theories can be studied as a particle theory through a Beyond Standard Model description

-FeynMG help us through the calculation, making it possible to work with the whole Standard Model

-Being inside FeynRules, we can use all the compatible packages to do pheno studies!

FeynMG allows to consistently test Scalar-tensor theories of

gravity at colliders

Thank you for listening

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https://gitlab.com/feynmg/FeynMG

EXTRA!

$$S_{\text{grav}} = \int \mathrm{d}^4 x \sqrt{-g} \left[-aR - F(X)R - bR^2 - cR_{\mu\nu}R^{\mu\nu} + \frac{1}{2}g^{\mu\nu}\partial_\mu X \partial_\nu X + ... \right]$$

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EXTRA!

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In most cases, they introduce new dynamics into the matter sector (fifth forces)
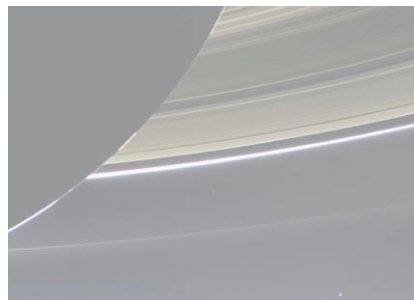
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in "Cosmology Talks"

They give an **effective mass** to the new scalar around **high-density environments**, so any local measurement is **exponentially suppressed**
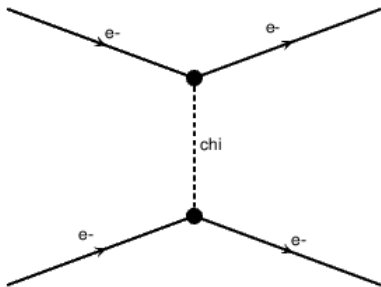
Screening mechanisms

EXTRA!

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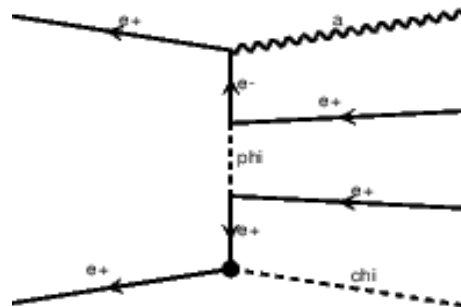
Two cases can occur:

Modification as an internal propagator

Missing energy due to external state:

Screened due to effective mass

Not screened!