Dark matter direct detection from single phonons to nuclear recoils

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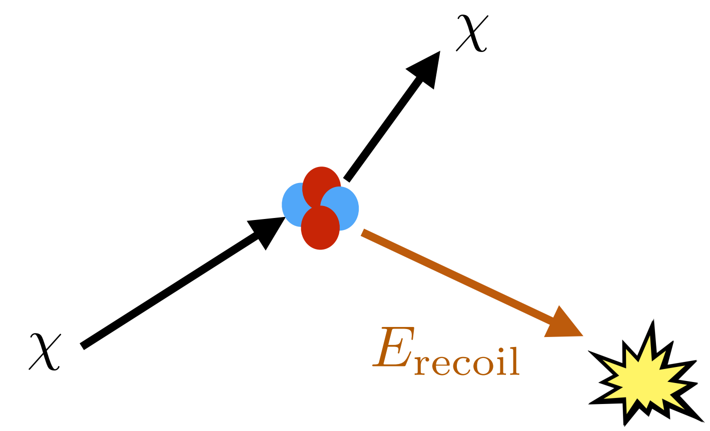
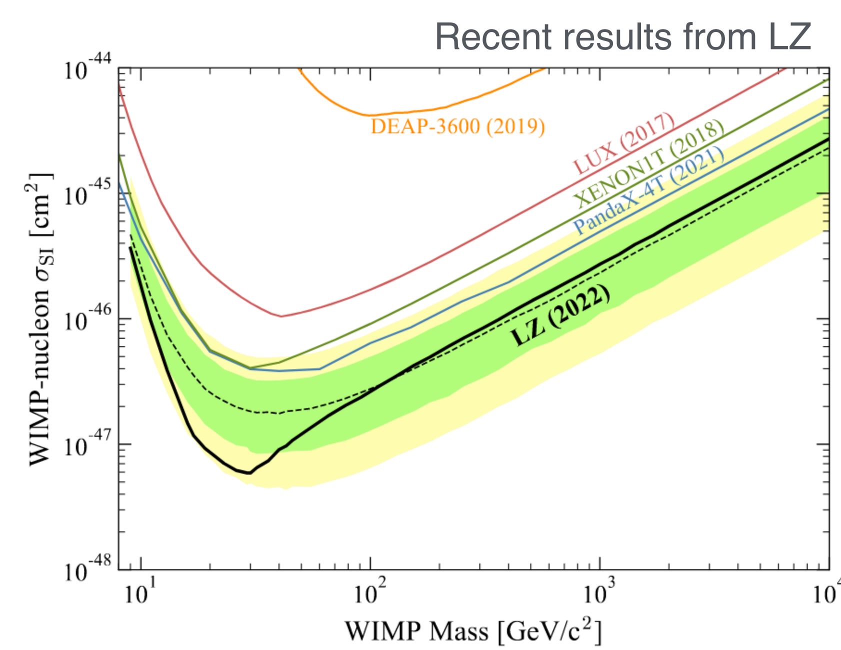
UCLA DM 2023

Dark matter mass

keV MeV GeV TeV



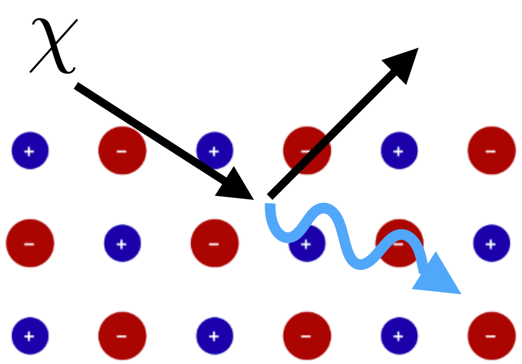
Nuclear recoils



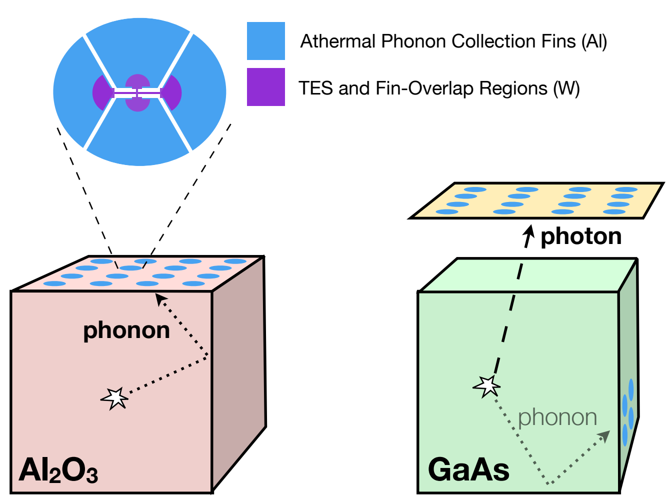
O(keV) thresholds

Dark matter mass

keV MeV GeV TeV

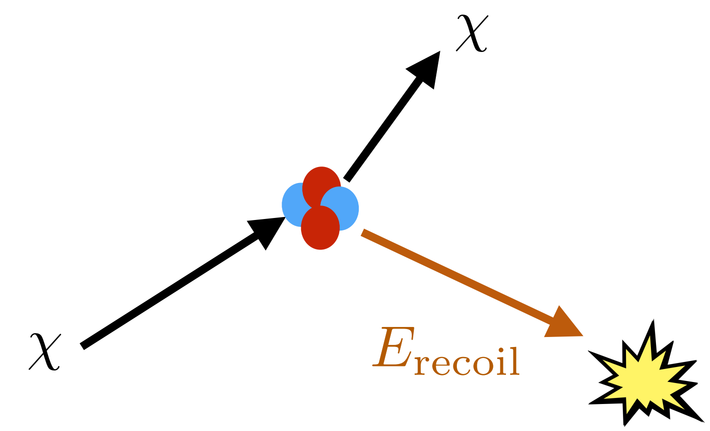


Single phonon excitation



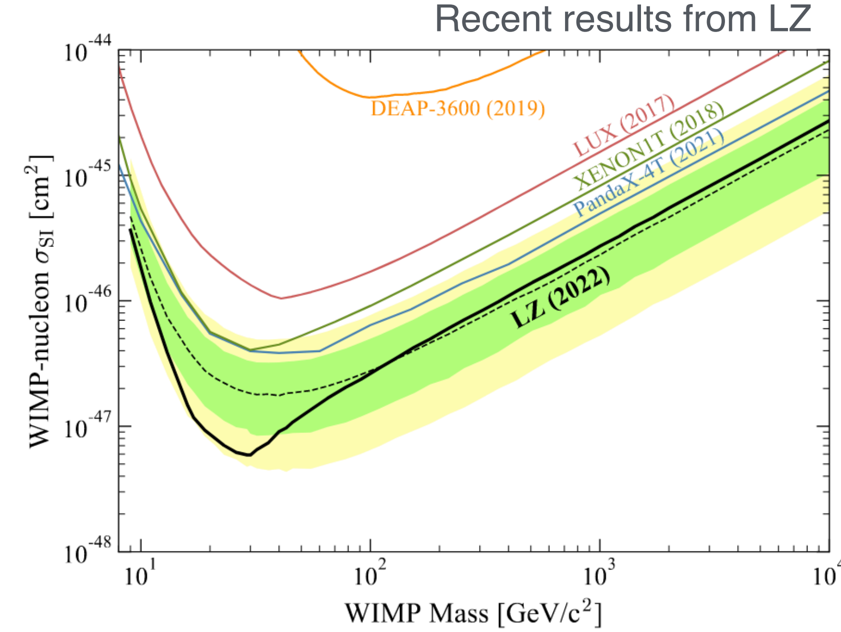
From TESSERACT white paper

Single phonons excitations with energy 1-100 meV (SPICE)



Nuclear recoils

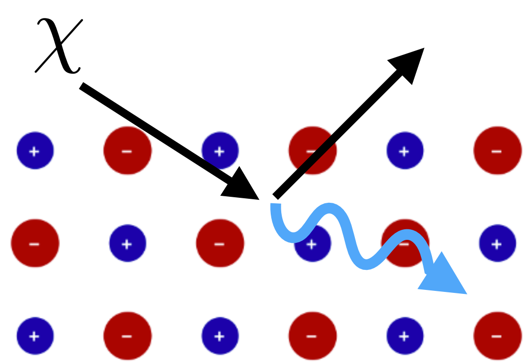
Recent results from LZ



O(keV) thresholds

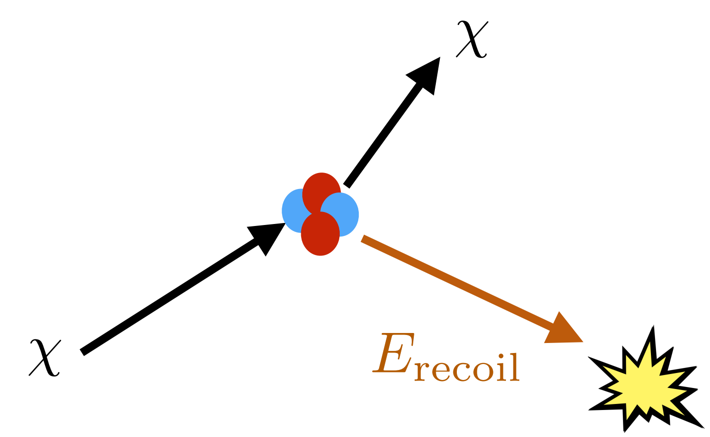
Dark matter mass

keV MeV GeV TeV

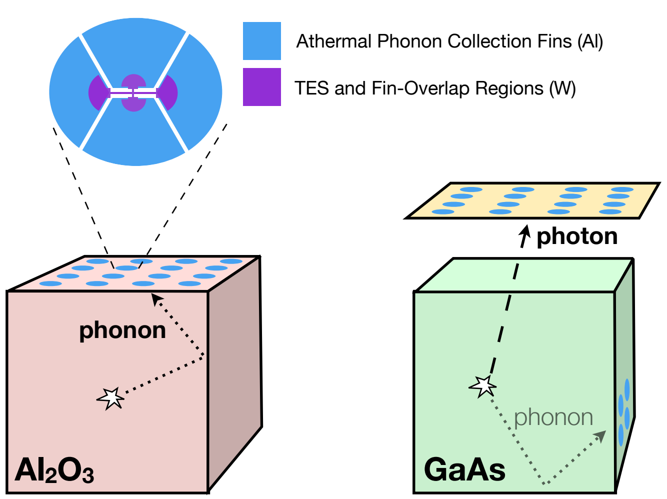


Single phonon excitation

?



Nuclear recoils



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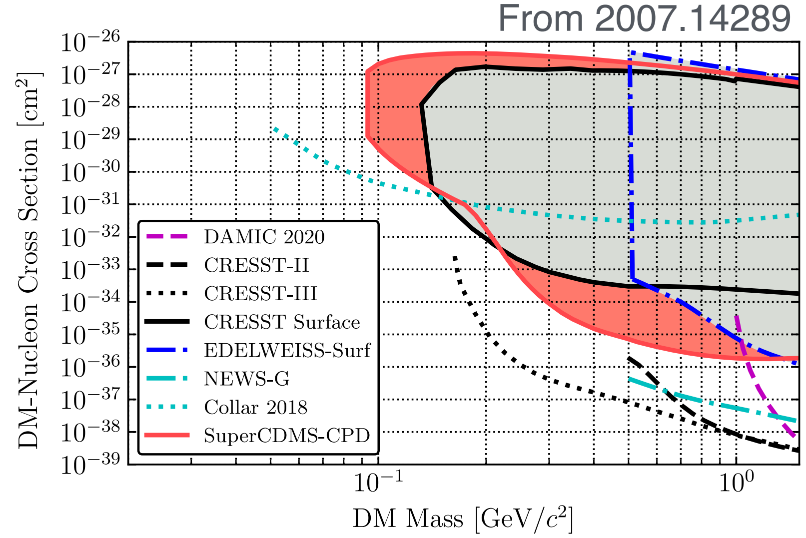
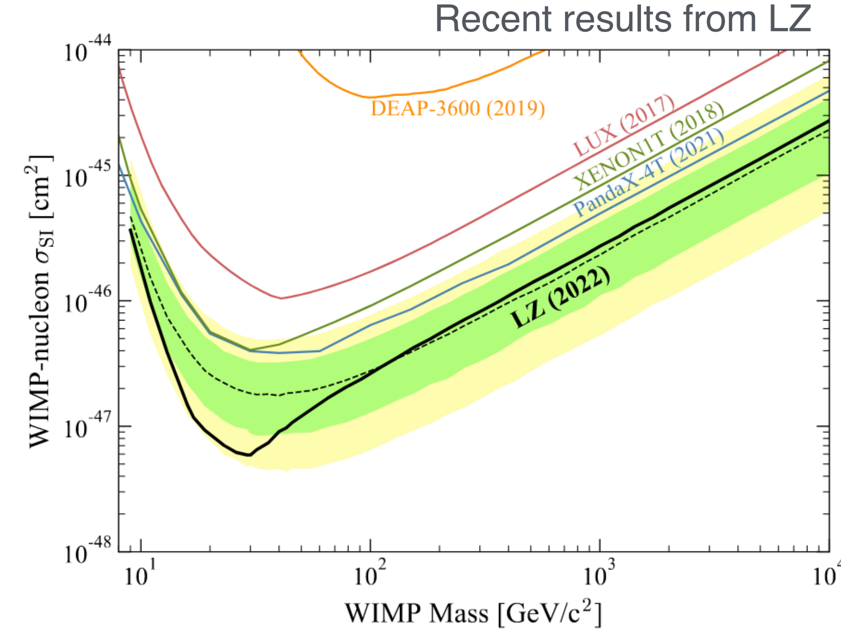
Single phonons excitations with energy 1-100 meV (SPICE)

From 2007.14289



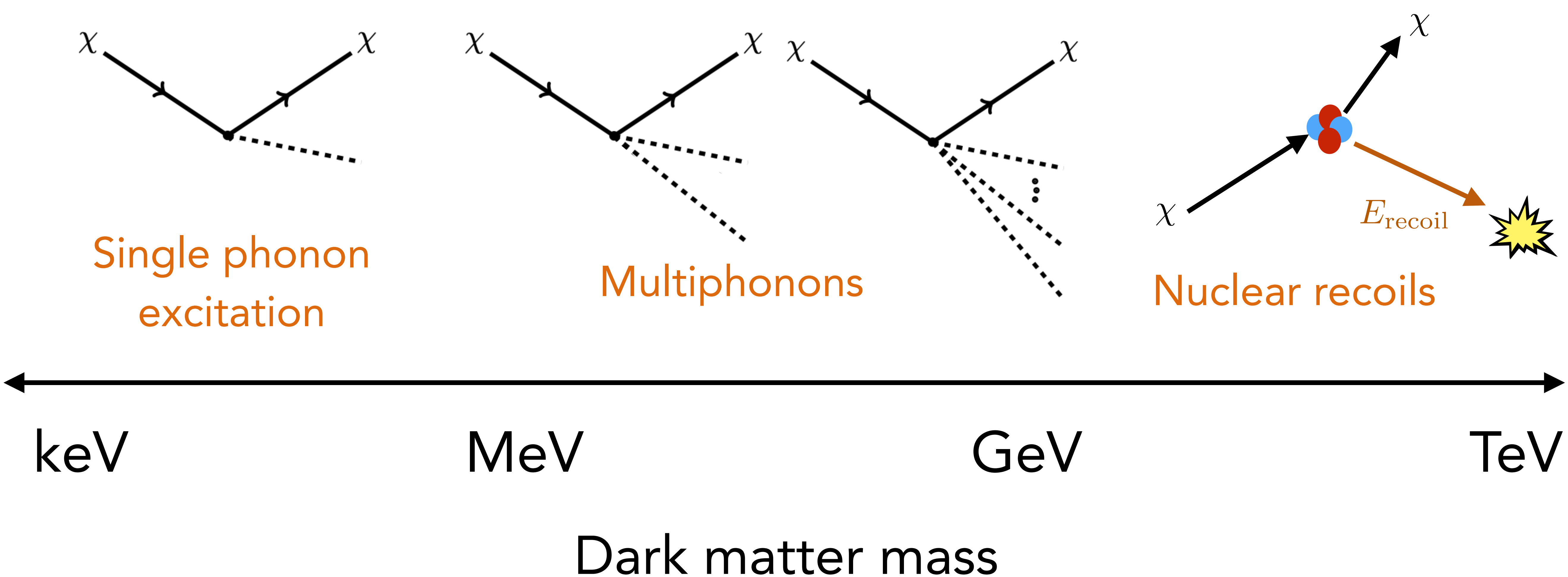
O(10) eV thresholds

Recent results from LZ



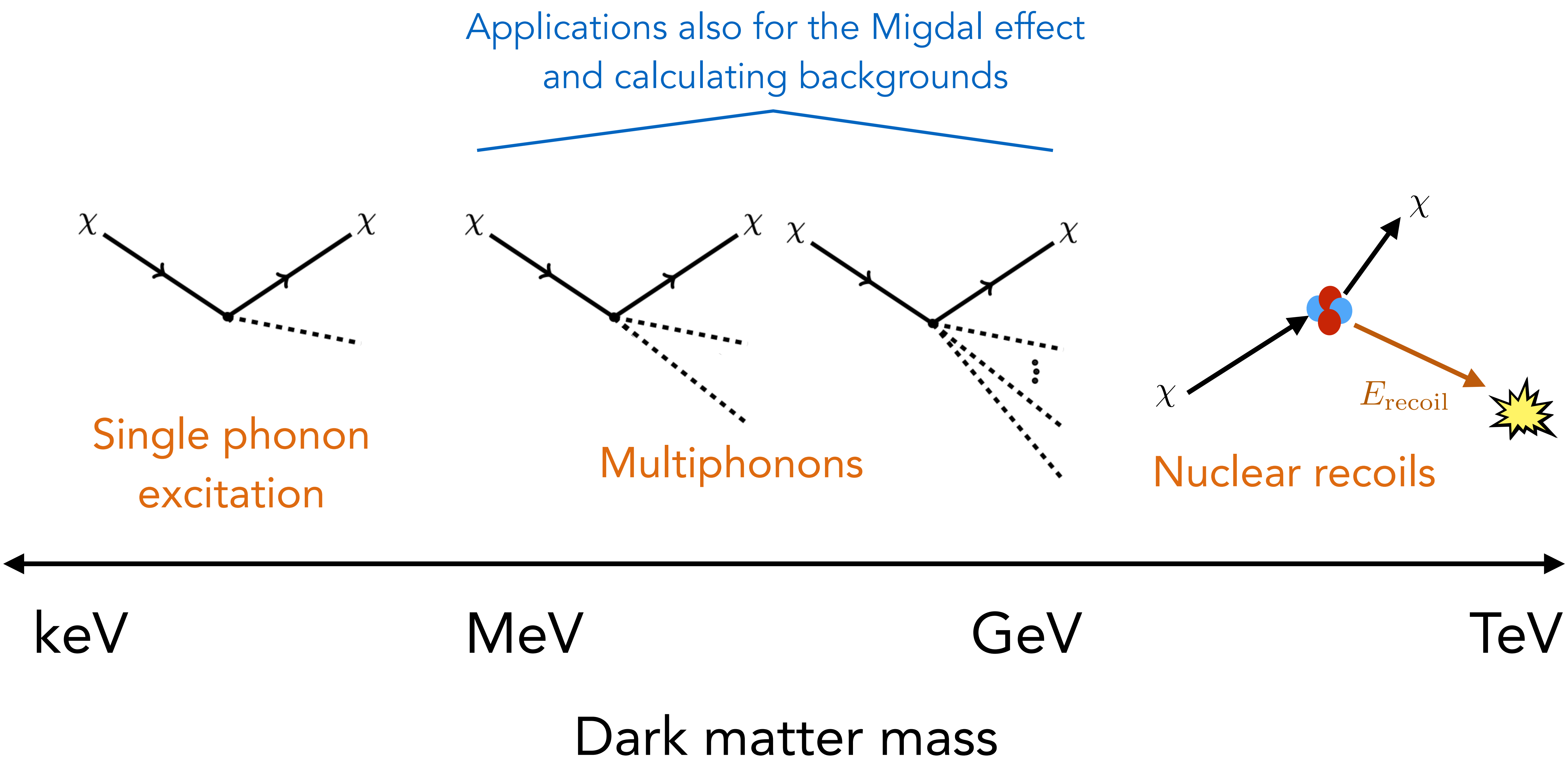
O(keV) thresholds

DM-nucleus scattering in crystals



Campbell-Deem, Cox, Knapen, TL, Melia 1911.03482
Knapen, Kozaczuk, TL 2011.09496
Campbell-Deem, Knapen, TL, Villarama 2205.02250
+ work in progress with Villarama, Shen, Sholapurkar

DM-nucleus scattering in crystals



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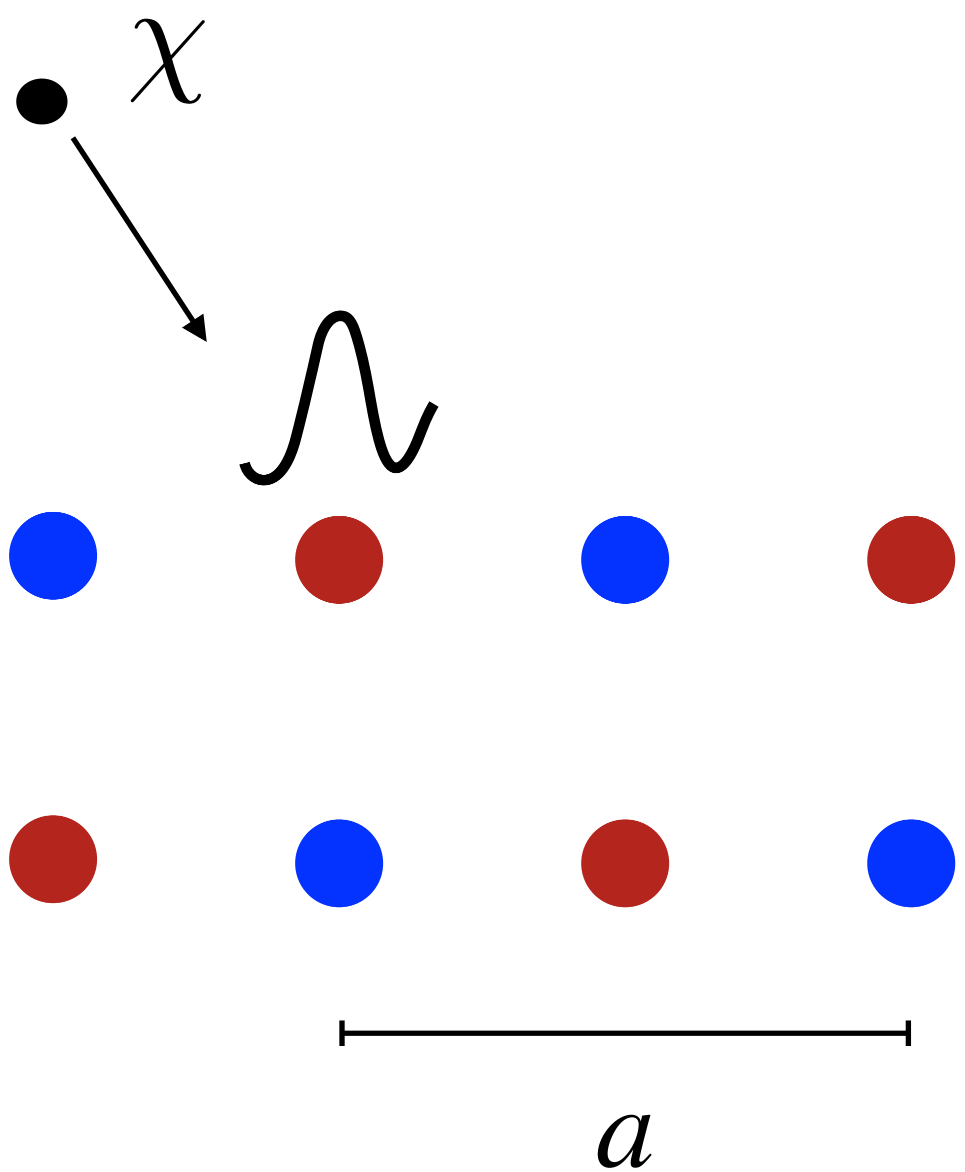
What does DM-nucleus scattering look like in a crystal?



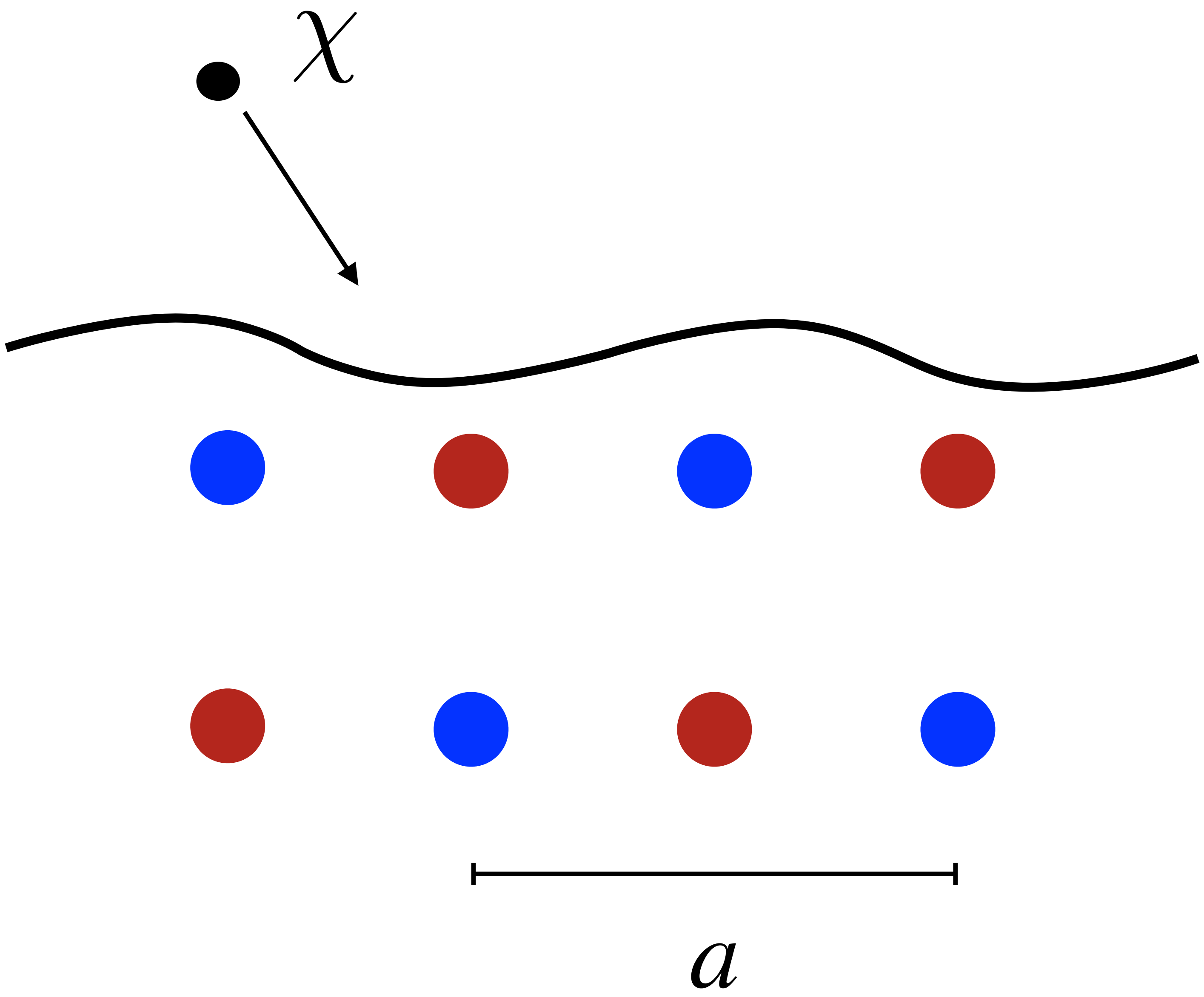
When momentum transfer

$$q \gg q_{\rm BZ} = \frac{2\pi}{a} \sim \text{few keV}$$

and $\omega \gg \bar{\omega}_{\rm phonon} \sim 10\text{-}100\ \text{meV}$

DM scatters off an individual nucleus

What does DM-nucleus scattering look like in a crystal?



When momentum transfer

$$q \ll q_{\mathrm{BZ}} = \frac{2\pi}{a}$$

and $\omega \sim \bar{\omega}_{\mathrm{phonon}}$

DM excites collective excitations = phonons

DM scattering rate

DM-mediator form factor

$$\frac{d\sigma}{d^3\mathbf{q}\,d\omega} \propto \sigma_{\chi p}\,|\tilde{F}_{\rm med}(q)|^2\,S(\mathbf{q},\omega)\,\delta(\omega - \mathbf{q}\cdot\mathbf{v} + \tfrac{q^2}{2m_\chi})$$

Dynamic structure factor:

Target response for momentum transfer $\mathbf{q}$

and energy deposited ω

DM scattering rate

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For free nuclei and spin-independent interactions:

$$S(\mathbf{q},\omega) \propto A_N^2\, \delta\left(\omega - \frac{q^2}{2m_N}\right)$$

DM scattering rate

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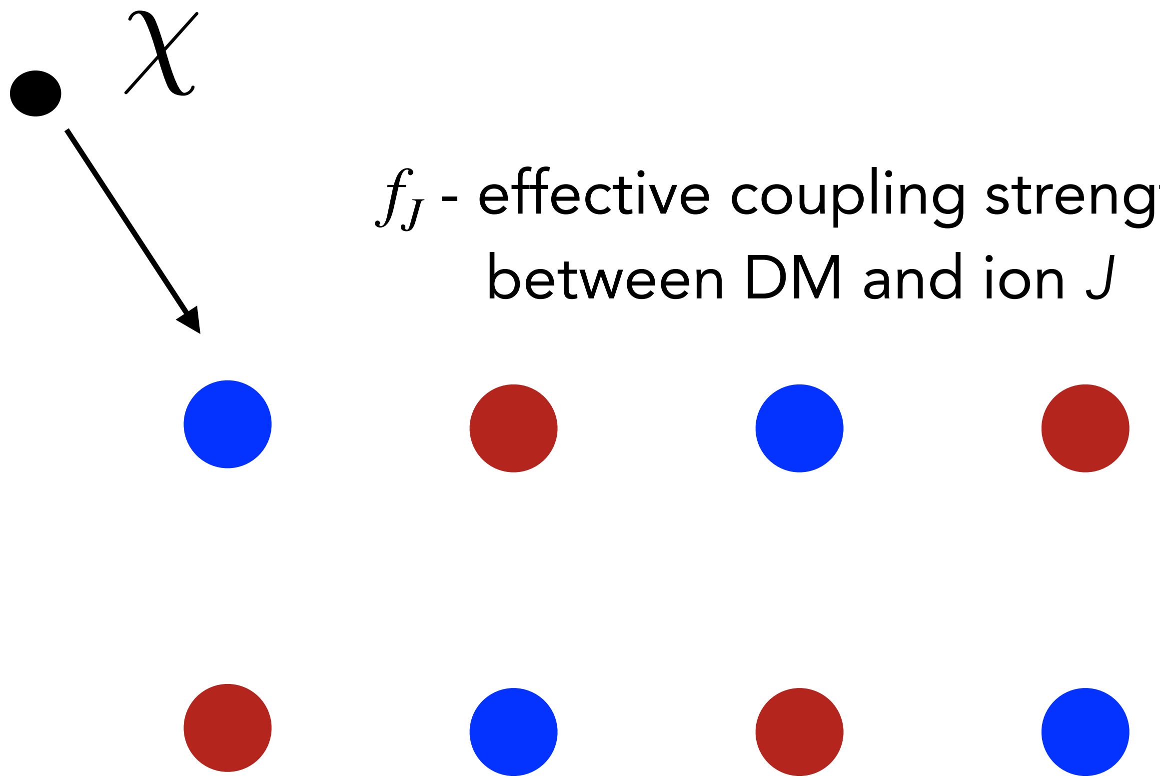
Goal: understand $S(\mathbf{q},\omega)$ from the single phonon to the nuclear recoil regime

DM-nucleus interaction



Short range SI interaction

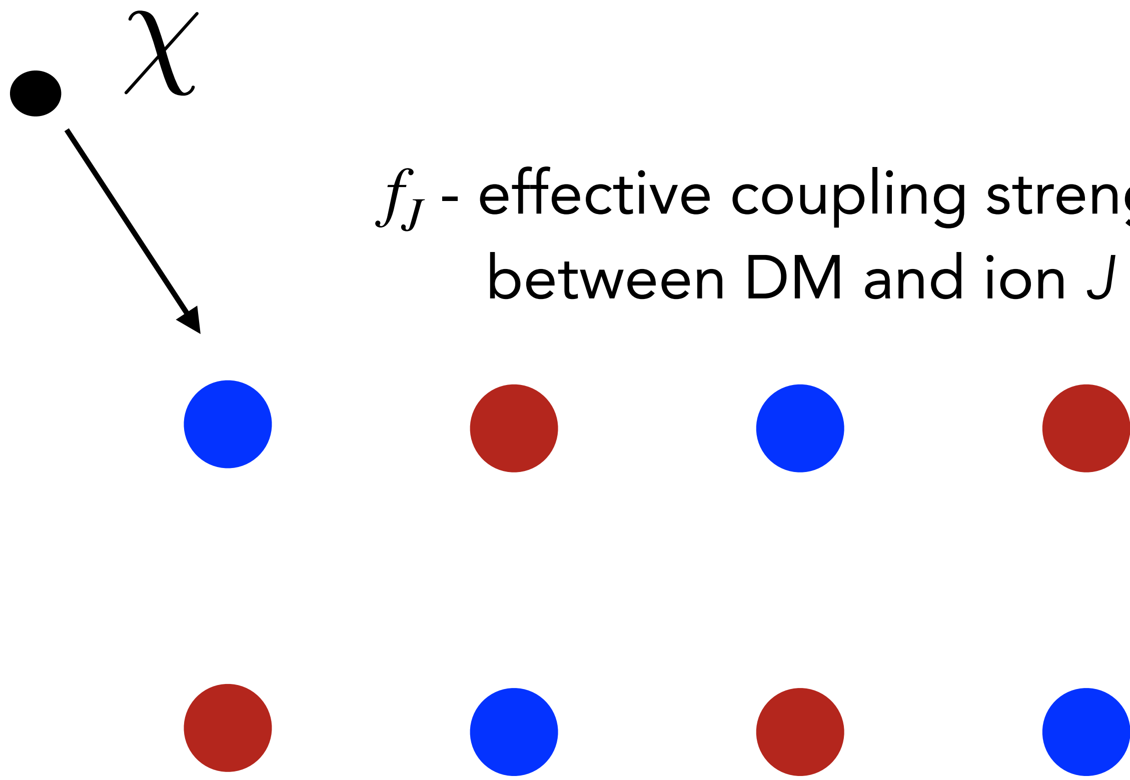
$$\sigma_{\chi p} = 4\pi b_p^2$$

Scattering potential in Fourier space

$$V(\mathbf{q}) \propto b_p \sum_J f_J \, e^{i\mathbf{q}\cdot\mathbf{r}_J}$$

Theory of neutron scattering:
Squires 1996, Schober 2014

DM-nucleus interaction



f_J - effective coupling strength between DM and ion J

Short range SI interaction

$$\sigma_{\chi p} = 4\pi b_p^2$$

Scattering potential in Fourier space

$$V(\mathbf{q}) \propto b_p \sum_J f_J \, e^{i\mathbf{q}\cdot\mathbf{r}_J}$$

$$S(\mathbf{q},\omega) \equiv \frac{2\pi}{V} \sum_f \left| \sum_J \langle \Phi_f | f_J e^{i\mathbf{q}\cdot\mathbf{r}_J} | 0 \rangle \right|^2 \delta\left(E_f - \omega\right)$$

$$= \frac{1}{V} \sum_{J,\, J'}^{N} f_J f_{J'}^* \int_{-\infty}^{\infty} dt \, \langle e^{-i\mathbf{q}\cdot\mathbf{r}_{J'}(0)} e^{i\mathbf{q}\cdot\mathbf{r}_J(t)} \rangle e^{-i\omega t}$$

Contains interference terms between different atoms → single phonon excitations

Theory of neutron scattering: Squires 1996, Schober 2014

Dynamic structure factor

$$S(\mathbf{q},\omega) = \frac{1}{V}\sum_{J,\,J'}^{N} f_J f_{J'}^{*} \int_{-\infty}^{\infty} dt\,\langle e^{-i\mathbf{q}\cdot\mathbf{r}_{J'}(0)} e^{i\mathbf{q}\cdot\mathbf{r}_J(t)}\rangle e^{-i\omega t}$$

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Phonons appear through positions of ions:

$$\mathbf{r}_J(t) = \mathbf{r}_J^0 + \mathbf{u}_J(t)$$

↑

Quantized phonon field given in terms of phonon dispersions $\omega_{\nu\mathbf{k}}$ and eigenvectors $\mathbf{e}_{\nu\mathbf{k}}$

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Quantized phonon field given in terms of phonon dispersions $\omega_{\nu\mathbf{k}}$ and eigenvectors $\mathbf{e}_{\nu\mathbf{k}}$

Single phonon contribution has been studied extensively in literature

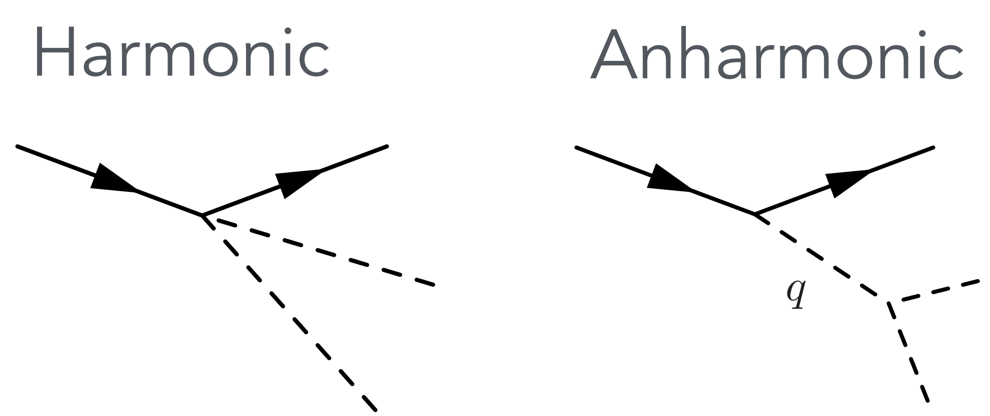
$$S^{n=1}(\mathbf{q}, \omega) \sim \sum_{J,J'} f_J f_{J'} \int dt \, \langle \mathbf{q} \cdot \mathbf{u}_J(0) \; \mathbf{q} \cdot \mathbf{u}_{J'}(t) \rangle e^{-i\omega t}$$

Griffin, Knapen, TL, Zurek 1807.10291; Griffin, Inzani, Trickle, Zhang, Zurek 1910.10716
Griffin, Hochberg, Inzani, Kurinsky, TL, Yu 2020; Coskuner, Tickle, Zhang, Zurek 2102.09567

Many phonons

Expansion in $q^2/(M_N\omega)$ (and anharmonic interactions):

$$S(\mathbf{q},\omega) = \text{(0-phonon)} + \text{(1-phonon)} + \text{(2-phonon)} + \cdots$$



Campbell-Deem, Cox, Knapen, TL, Melia 1911.03482

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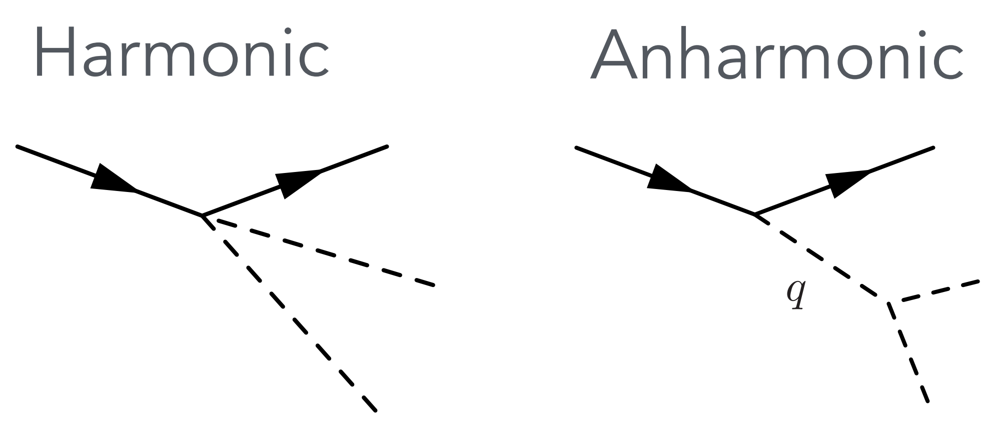

Quickly becomes more complicated to evaluate for more than 1 phonon

Campbell-Deem, Cox, Knapen, TL, Melia 1911.03482

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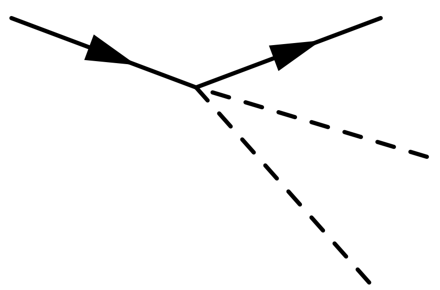
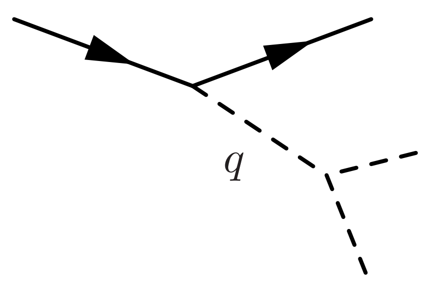
Harmonic



Anharmonic



Quickly becomes more complicated to evaluate for more than 1 phonon

Our approach: use harmonic & incoherent approximations

Campbell-Deem, Cox, Knapen, TL, Melia 1911.03482

Incoherent approximation for $q > q_{\mathrm{BZ}}$

Neglect interference terms entirely:

$$S(\mathbf{q},\omega) = \frac{1}{V}\sum_{J,\,J'}^{N} f_J f_{J'}^* \int_{-\infty}^{\infty} dt\, \langle e^{-i\mathbf{q}\cdot\mathbf{r}_{J'}(0)} e^{i\mathbf{q}\cdot\mathbf{r}_J(t)}\rangle e^{-i\omega t}$$

$$\approx \frac{1}{V}\sum_{J}^{N} (f_J)^2 \int_{-\infty}^{\infty} dt\, \langle e^{-i\mathbf{q}\cdot\mathbf{u}_J(0)} e^{i\mathbf{q}\cdot\mathbf{u}_J(t)}\rangle e^{-i\omega t}$$

Motivation: scatter off individual nuclei at large q compared to inverse lattice spacing

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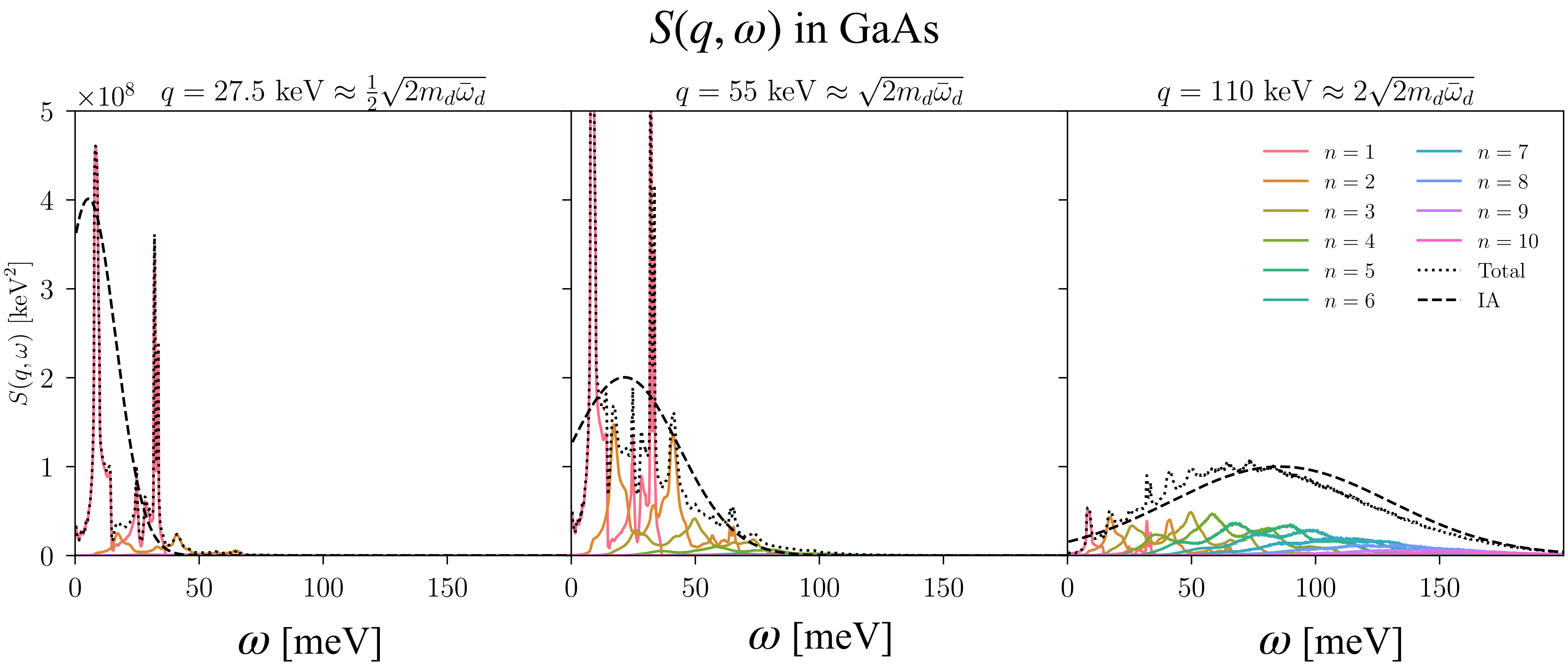
Analytic result for phonon expansion in terms of phonon density of states $D(\omega)$:

$$S(q,\omega) \propto \sum_J e^{-2W_J(q)} (f_J)^2 \sum_n \frac{1}{n!}\left(\frac{q^2}{2m_N}\right)^n \left(\prod_{i=1}^{n} \int d\omega_i \frac{D(\omega_i)}{\omega_i}\right) \delta\left(\sum_j \omega_j - \omega\right)$$

n = number of phonons

Multiphonons become important around $q = \sqrt{2 m_N \bar{\omega}_{\rm ph}}$



$q = \frac{1}{2}\sqrt{2 m_N \bar{\omega}_{\rm ph}}$:
dominated by
n=1 phonon

$q = \sqrt{2 m_N \bar{\omega}_{\rm ph}}$:
contributions from
n=1,2,3,4…

$q = 2\sqrt{2 m_N \bar{\omega}_{\rm ph}}$:
can be approximated
by Gaussian envelope

Nuclear recoil limit

When $q \gg \sqrt{2m_N \bar{\omega}_{\rm ph}}$, “re-sum” the n-phonon contributions:

$$S^{\rm IA}(q,\omega) \propto \sum_J f_J^2 \sqrt{\frac{2\pi}{\Delta^2}} \exp\left(-\frac{(\omega - \frac{q^2}{2m_N})^2}{2\Delta^2}\right), \quad \Delta^2 = \frac{q^2 \bar{\omega}_{\rm ph}}{2m_N}$$

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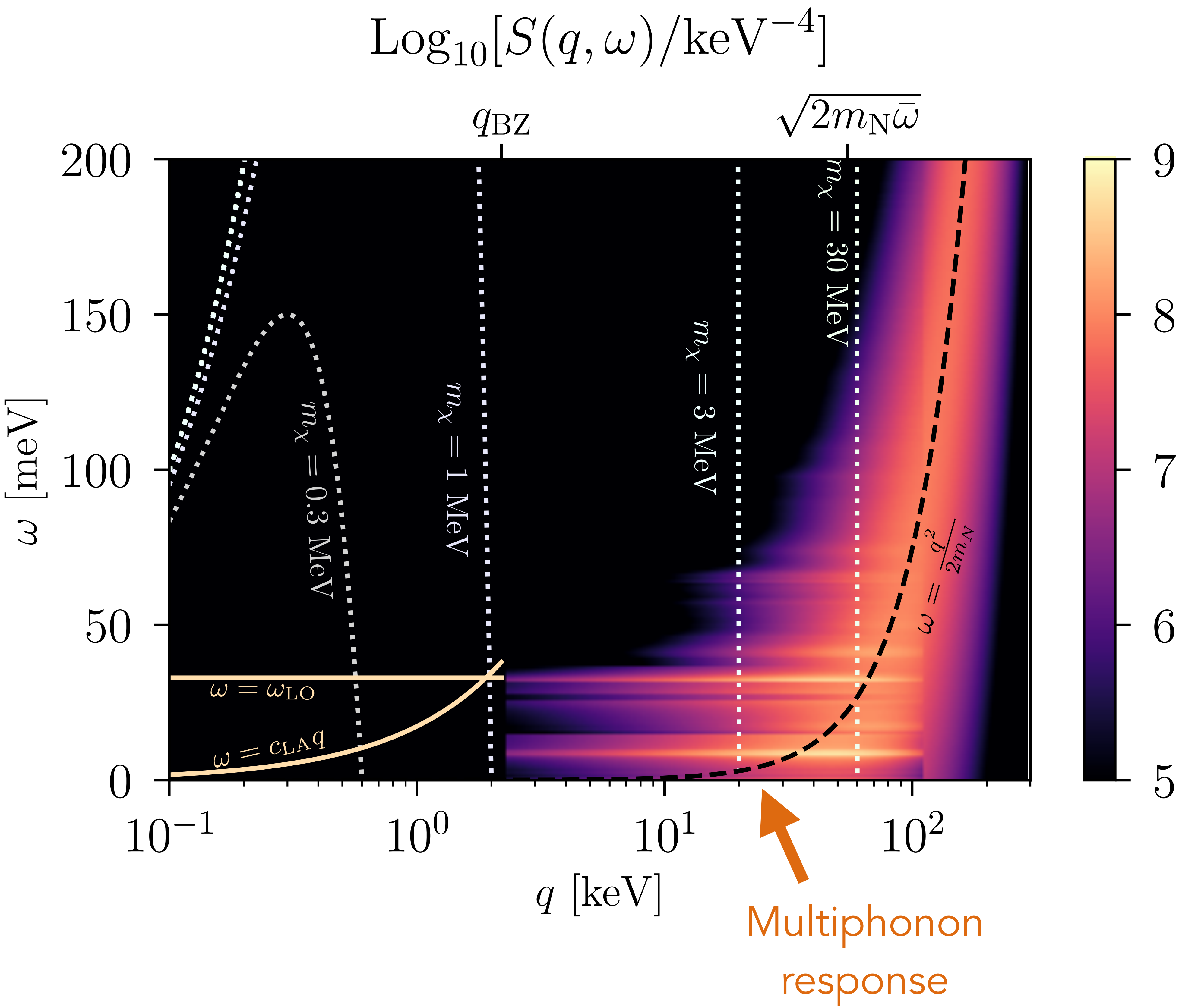
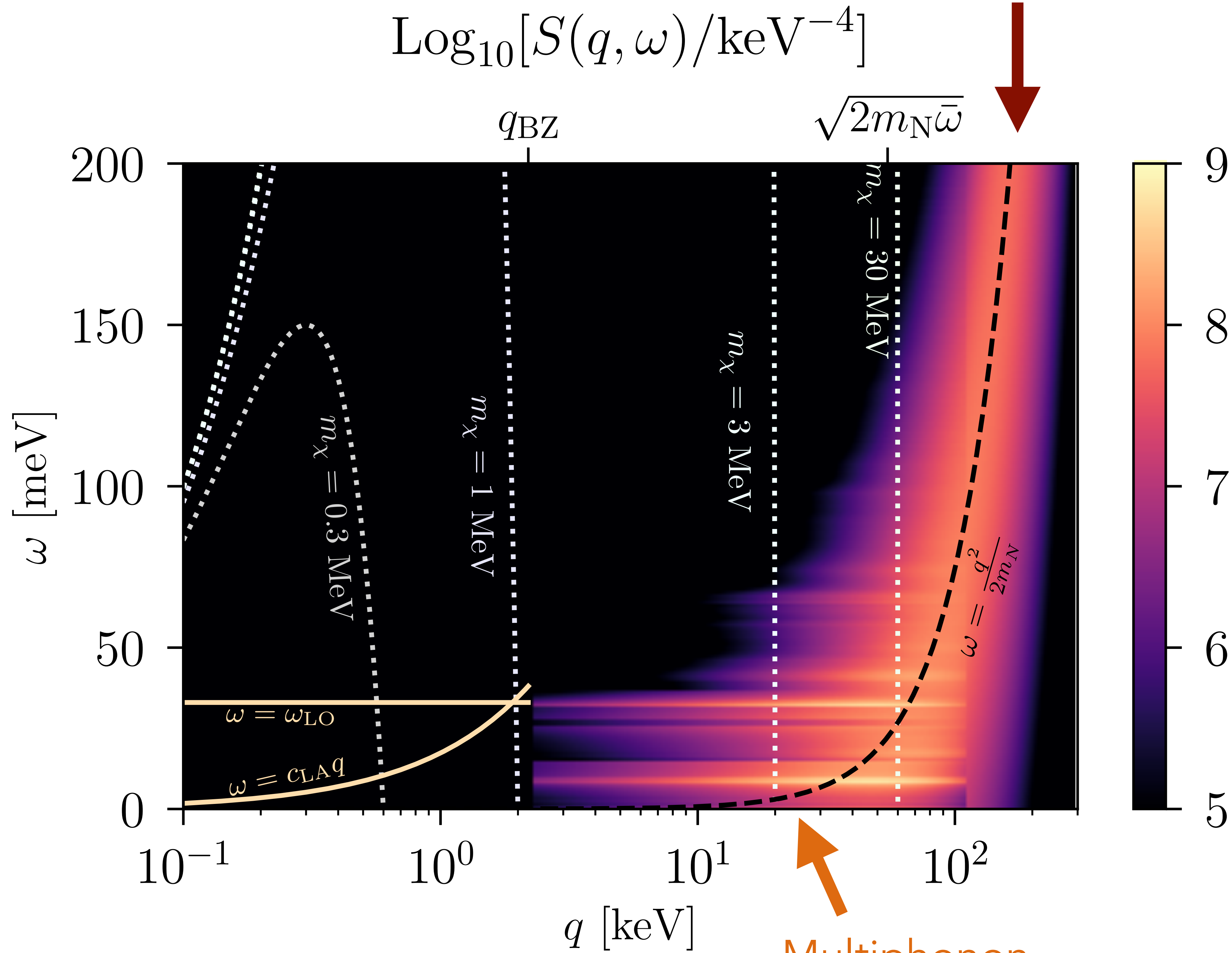
As $\omega \gg \bar{\omega}_{\text{ph}}$, $\Delta/\omega \to 0$, take narrow-width limit:

$$S(q,\omega) \propto \sum_J f_J^2 \; \delta\left(\omega - \frac{q^2}{2m_N}\right)$$

reproducing free nuclear recoils!



Campbell-Deem, Knapen, TL, Villarama 2205.02250



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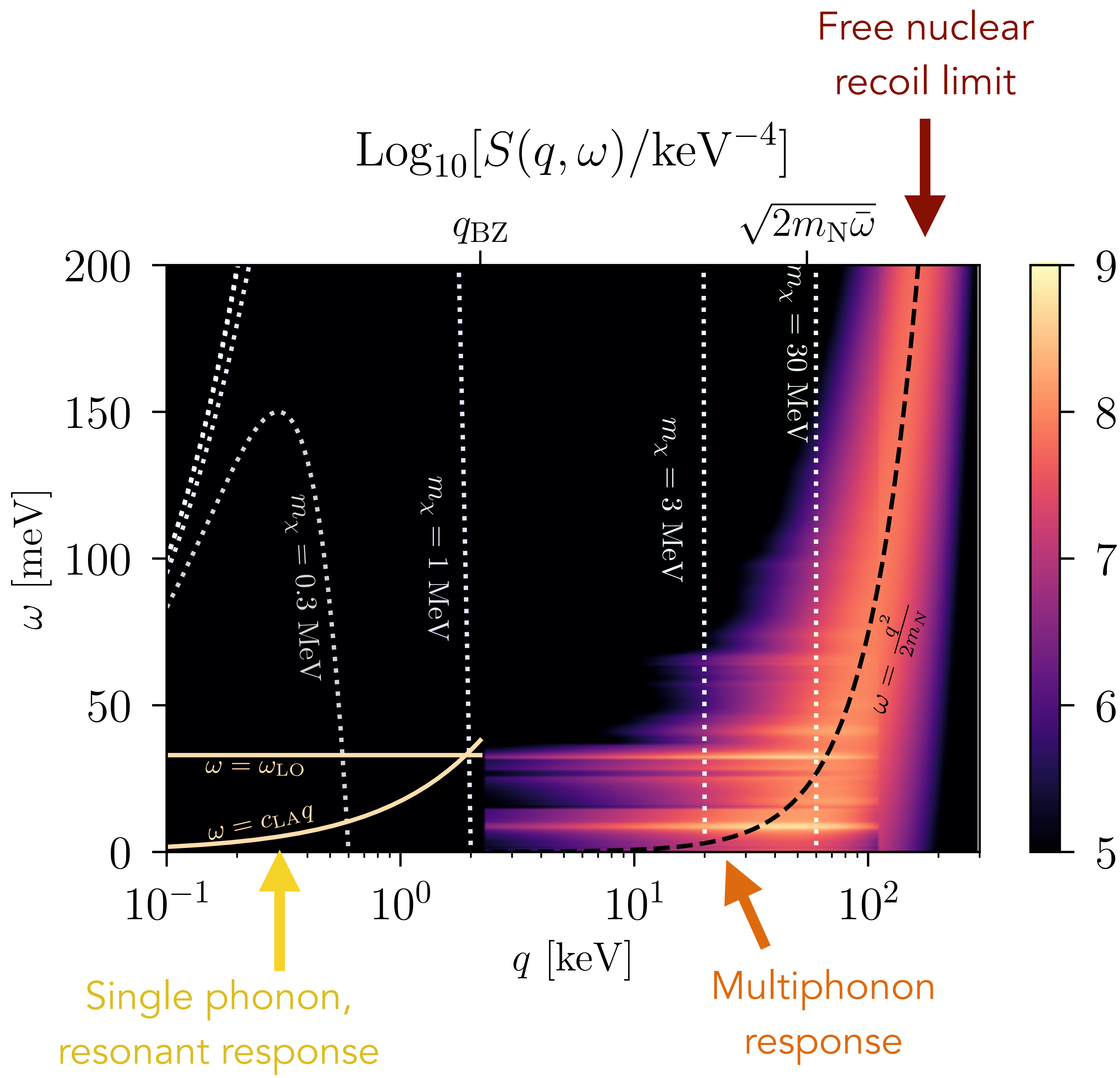

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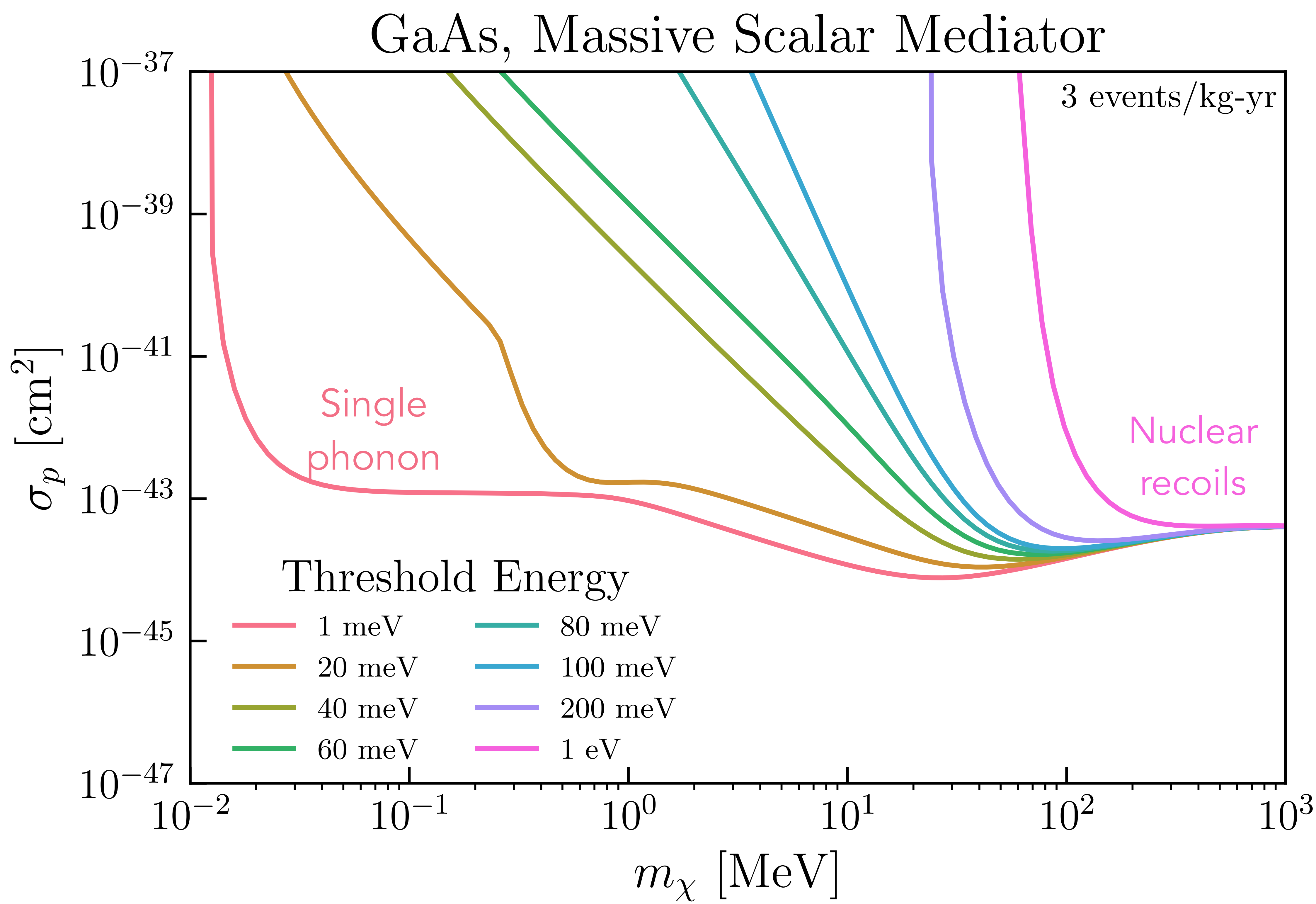
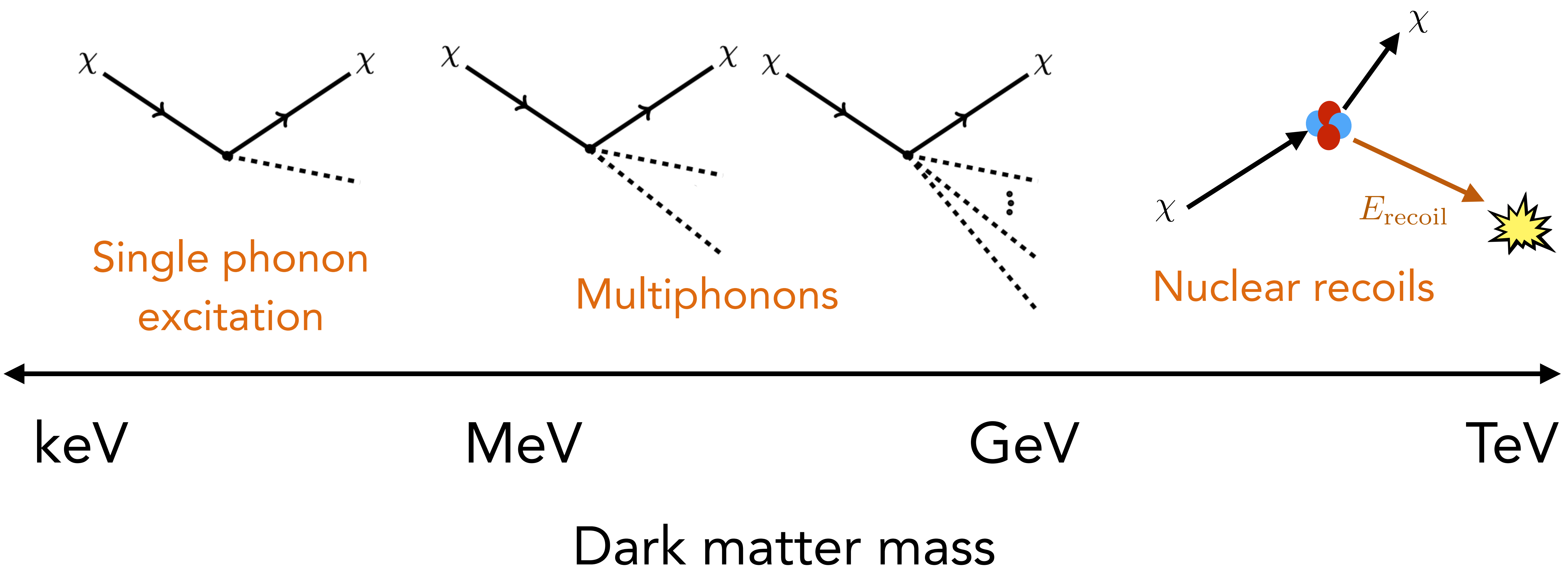
DM scattering rate



Campbell-Deem, Knapen, TL, Villarama 2205.02250
Harmonic oscillator model: Kahn, Krnjaic, Mandava 2011.09477

DM scattering in crystals



Future steps: quantifying theory uncertainties and improved calculations, a closer look at 2-3 phonon processes