Dark Matter Halo and Particle Properties From Mass and Energy Cascade

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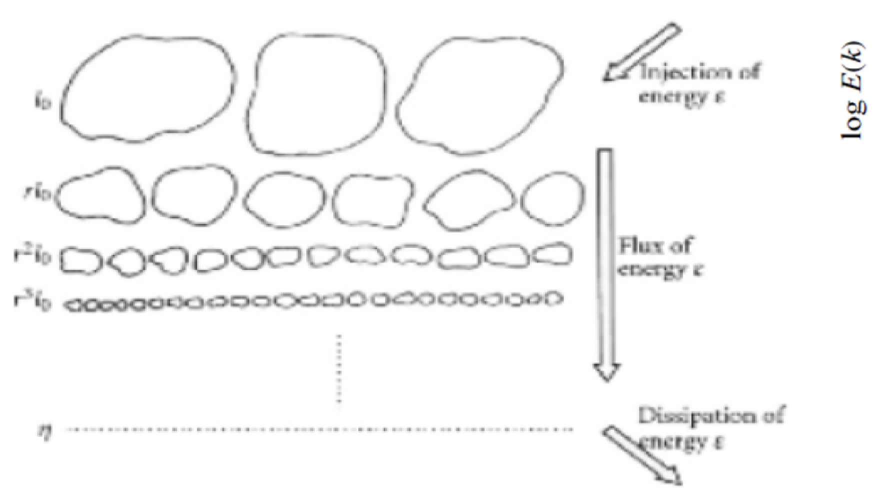
(a) Mass and Energy Cascade in Dark Matter

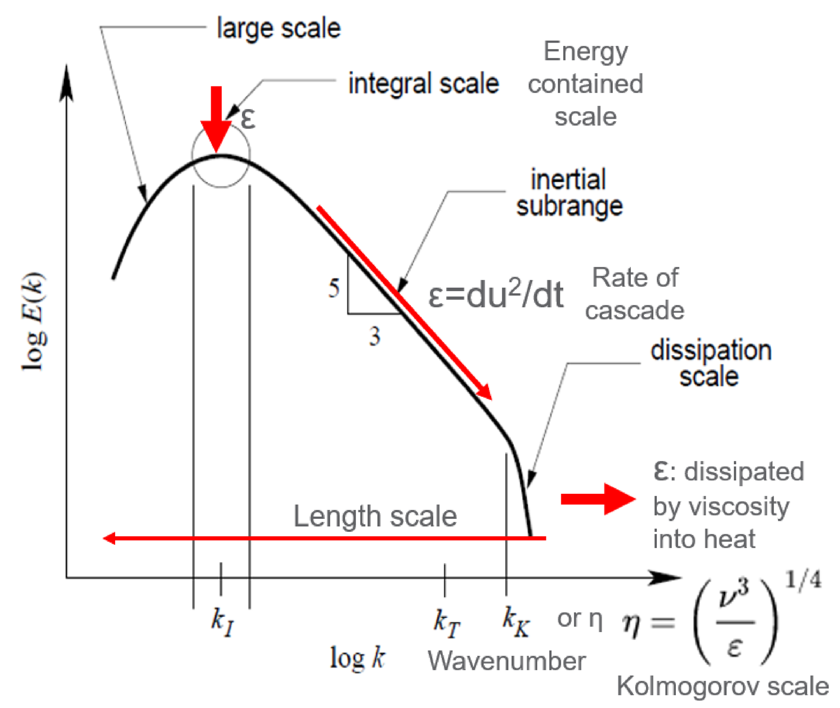
Key attributes of hydrodynamic turbulence	Key attributes of dark matter flow
Chaotic, random, Non-equilibrium	Chaotic, random, Non-equilibrium
Multiscale in length and time scales	Multiscale in mass/length/time scales
Dissipative and collisional	Dissipationless and collisionless
Short-range interaction	Long-range gravity
Velocity fluctuation	Velocity & acceleration fluctuation
Vortex as fundamental building block	Halos as fundamental building block
Direct energy cascade from large to small scales	Inverse mass and energy cascade from small to large mass scales

Eddy-mediated energy cascade in turbulence

- Inertial range: inertial >> viscous force (ε,r)
- Dissipation range: viscous dominant (ε,ν)

"Big whirls have little whirls, That feed on their velocity; And little whirls have lesser whirls, And so on to viscosity."

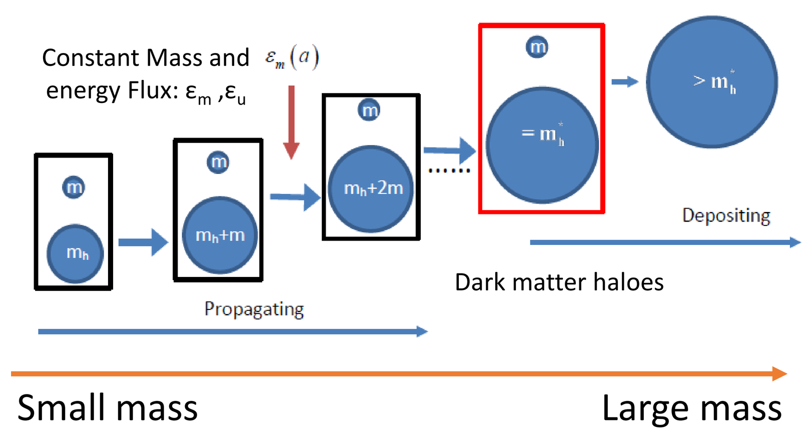


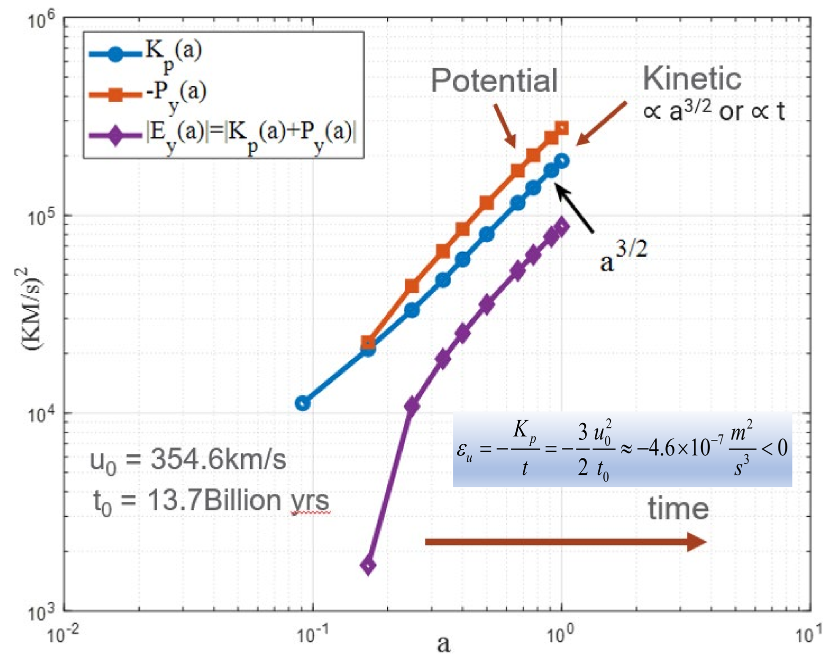


Halo-mediated mass/energy cascade in dark matter

- Propagation range: ε_u, G, r
- Deposition range: ε_u (m^2/s^3), u_0 (m/s)

"Little haloes have big haloes, That feed on their mass; And big haloes have greater haloes, And so on to growth."



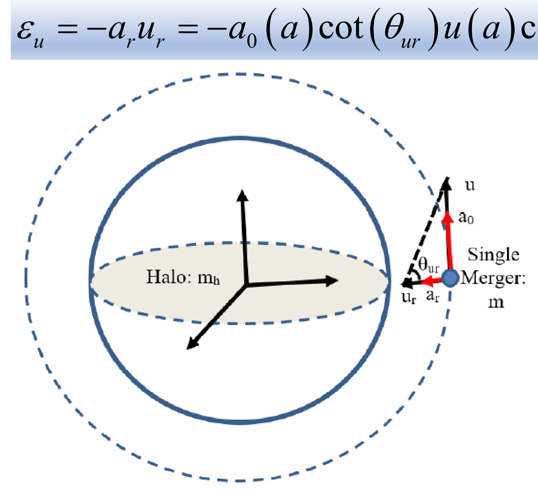


The time variation of specific kinetic and potential energies from N-body simulation.

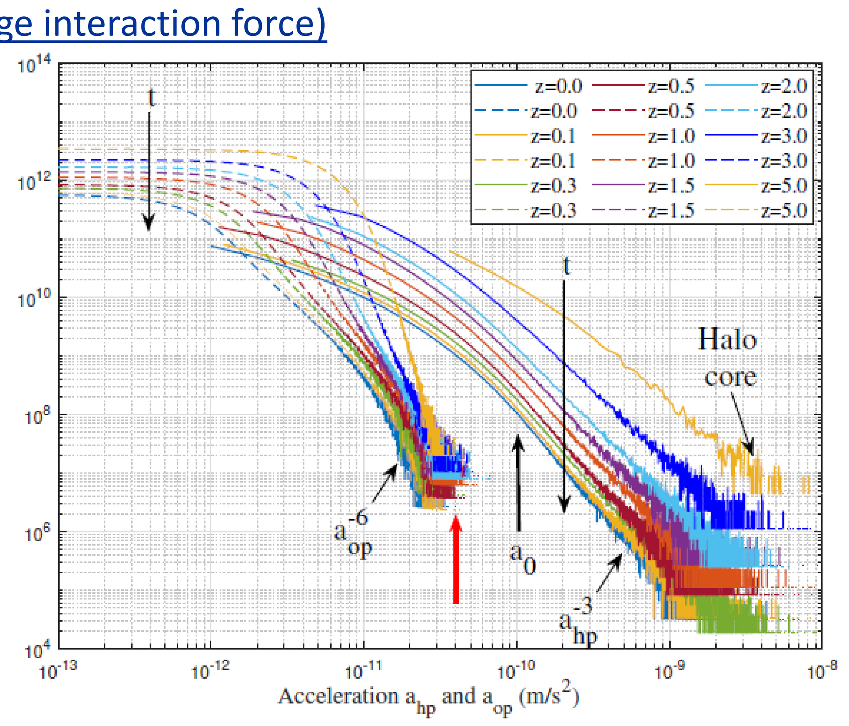
Velocity & acceleration fluctuation (due to long-range interaction force)

- Velocity fluctuation with a velocity scale u_0
- Acceleration fluctuation with an acc. scale a_0 (MOND?)
- Two fluctuations related by energy cascade ε_u

$$\varepsilon_u = -a_r u_r = -a_0(a)\cot(\theta_{ur})u(a)\cot(\theta_{ur})$$



The schematic plot for inverse mass/energy cascade from a series of merging between halo and single mergers.

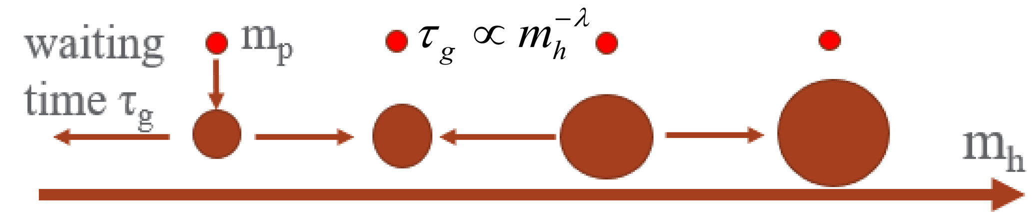


$$a_0(a) = -(3\pi)^2\frac{\varepsilon_u}{u} = 1.2\times10^{-10}a^{-3/4}\,m/s^2 \propto t^{-1/2}$$

The distribution (fluctuation) of halo particle acceleration and out-of-halo particle acceleration

(b) Double-λ Halo Mass Function from Mass Cascade

Reference	Mass Function $f(\sigma_\delta, z)$	Mass Range of Fit	Redshift range of Fit
PS, Press & Schechter	$\sqrt{\frac{2}{\pi}}\frac{\delta_c}{\sigma_\delta}\exp\left[-\frac{\delta_c^2}{2\sigma_\delta^2}\right]$	unspecified	unspecified
ST, Sheth & Tormen	$A\sqrt{\frac{2q}{\pi}}\frac{\delta_c}{\sigma_\delta}\exp\left[-\frac{q\delta_c^2}{2\sigma_\delta^2}\right]\left[1+\left(\frac{\sigma_\delta^2}{q\delta_c^2}\right)^p\right]$	unspecified	unspecified
JK, Jenkins et al.	$0.315\exp\left[-\lvert\ln\sigma_\delta^{-1}+0.61\rvert^{3.8}\right]$	$-1.2\leqslant\ln\sigma_\delta^{-1}\leqslant1.05$	$z=0-5$
WR, Warren et al.	$0.7234\left(\sigma_\delta^{-1.625}+0.2538\right)\exp\left[-\frac{1.1982}{\sigma_\delta^2}\right]$	$(10^{10}-10^{15})h^{-1}M_\odot$	$z=0$
Double-λ, This work Eq. (20)	$\frac{2p(2\sqrt{\eta_0})^{-q}}{\Gamma(q/2)}\left(\frac{\delta_c}{\sigma_\delta}\right)^{pq}\exp\left[-\frac{1}{4\eta_0}\left(\frac{\delta_c}{\sigma_\delta}\right)^{2p}\right]$	unspecified	unspecified



1D Random walk equation in mass space (similar to diffusion):

$$\frac{\partial m_h(t)}{\partial t} = \frac{m_p\xi(t)}{\tau_g(m_h)} = \sqrt{2D_p(m_h)}\varsigma(t)$$

kg^2/s Noise $s^{-1/2}$

Position-dependent diffusivity: $D_p(m_h)\propto m_h^{2\lambda}$

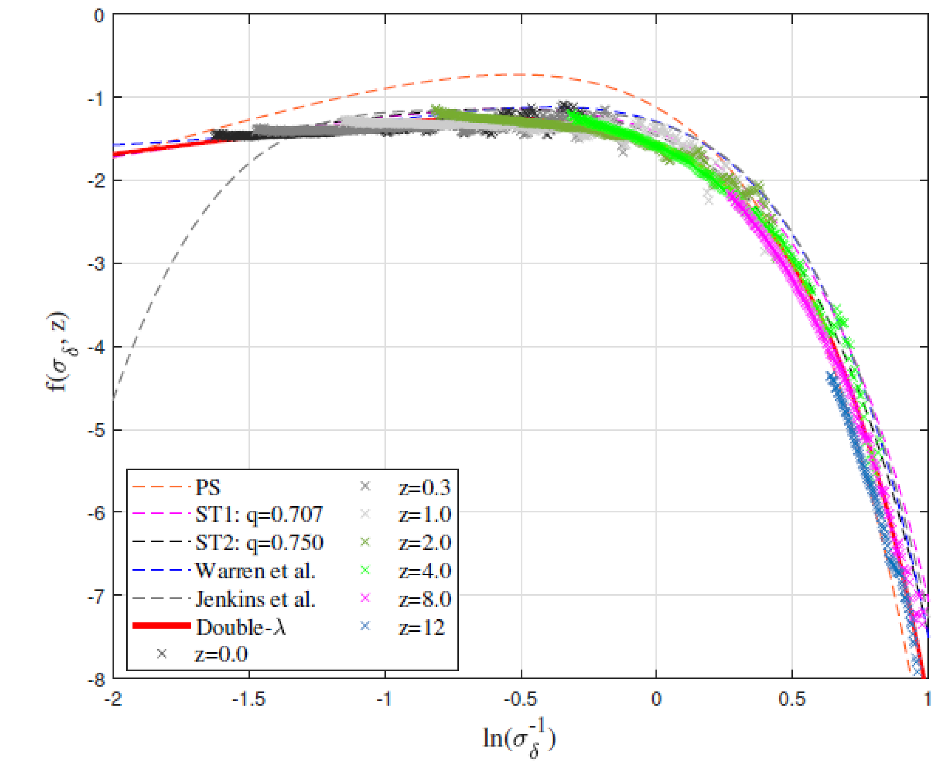
Fokker-Planck equation for distribution function:

$$\frac{\partial P_h}{\partial t} = \frac{\partial}{\partial m_h}\left[\sqrt{D_p}\frac{\partial}{\partial m_h}\left(\sqrt{D_p}P_h\right)\right] = D_{p0}\frac{\partial}{\partial m_h}\left[m_h^\lambda\frac{\partial}{\partial m_h}\left(m_h^\lambda P_h\right)\right]$$

Solving Fokker-Planck Eq. leads to Halo mass function:

$$f_M(m_h,a) = \frac{(1-\lambda)}{\sqrt{\pi\eta_0}}\left(\frac{m_h^*}{m_h}\right)^{\lambda}\frac{1}{m_h^*}\exp\left[-\frac{1}{4\eta_0}\left(\frac{m_h}{m_h^*}\right)^{2-2\lambda}\right]$$

Reduce to Press-Schechter (PS) if λ=2/3 !

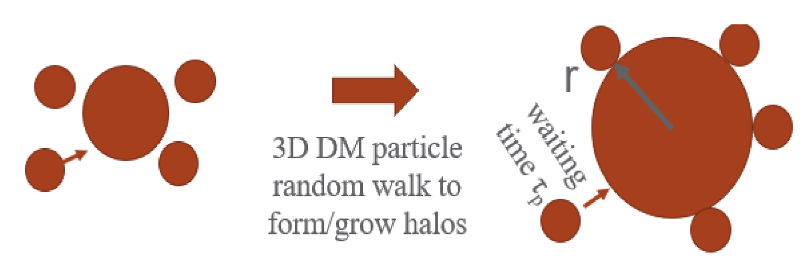


Comparison between different halo mass functions $f(\sigma_\delta, z)$ and Illustris simulation at different redshift z.

(c) Universal Scaling Laws and Double-γ Halo Density Profile

In propagation range, all relevant quantities are determined by G, ε_u, and scale r. This predicts:

Mass:	$m_r = \alpha_r\varepsilon_u^{2/3}G^{-1}r^{5/3}$	5/3 law
Density:	$\rho_r = \beta_r\varepsilon_u^{2/3}G^{-1}r^{-4/3}$	-4/3 law
Kinetic energy:	$v_r^2 = (\gamma_s\varepsilon_u)^{2/3}r^{2/3}$	2/3 law



Waiting time dependent on halo size r (position-dependent):

$$\tau_P(r)\propto m_r(r)^{-\lambda}\propto r^{-\gamma}$$

The larger halo, the shorter waiting time

3D Random walk equation: $\frac{dX_t}{dt} = \sqrt{2D_P(X_t)}\xi(t)$.

$$D_P(X_t) = D_0(t)r^{2\gamma}$$

Fokker-Planck equation for distribution function:

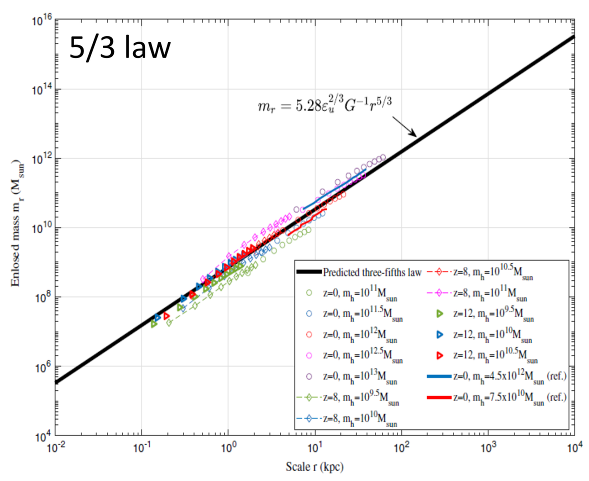
$$\frac{\partial P_r(X,t)}{\partial t} = D_0\frac{\partial}{\partial X_i}\left[r^\gamma\frac{\partial}{\partial X_i}\left(r^\gamma P_r(X,t)\right)\right]$$

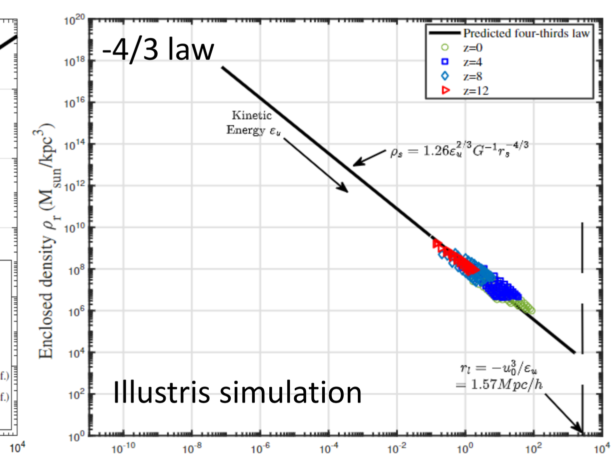
$$\alpha = 2-2\gamma_2 \qquad \beta = \frac{2-2\gamma_2}{2-\gamma_1}$$

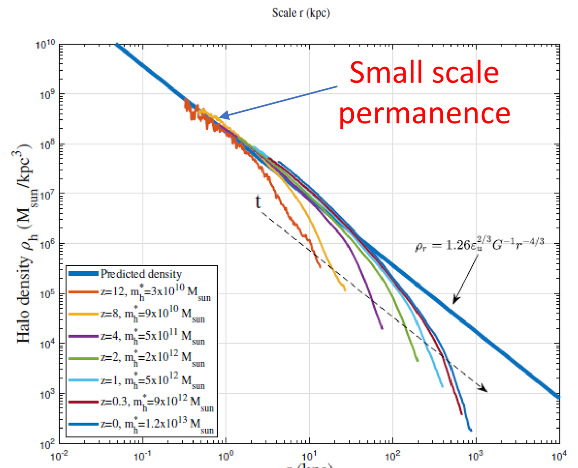
Double-γ halo density profile: $x = r/r_s(t)$

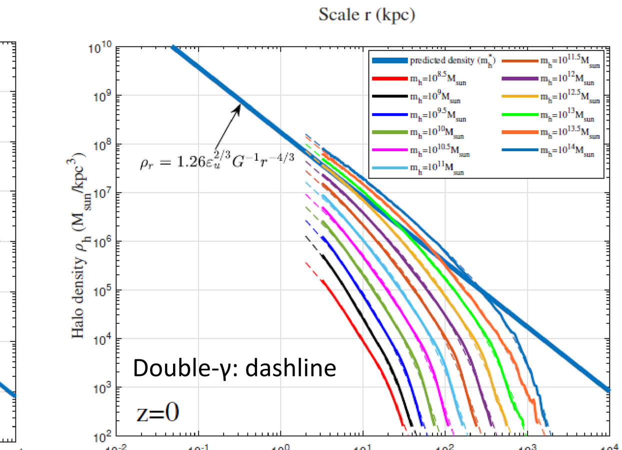
$$\rho_{D\gamma}\left(x=\frac{r}{r_s(t)}\right) = \frac{\alpha\beta^{-(1/\alpha+1/\beta)}}{4\pi\Gamma(1/\alpha+1/\beta)}x^{\frac{\alpha}{\beta}-2}\exp\left(-\frac{x^\alpha}{\beta}\right)$$

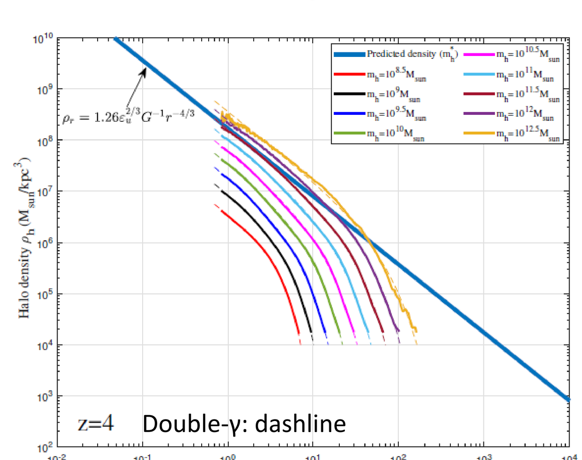
Reduce to Einasto if α=2β !

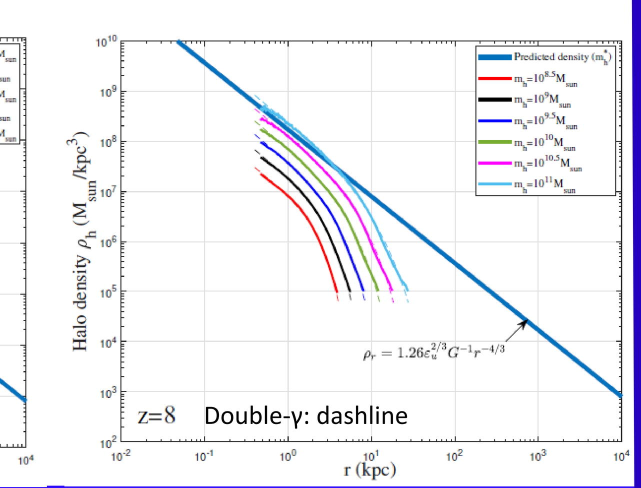












(d) Dark Matter Particle Properties

For collisionless dark matter:

- Dark matter is fully collisionless
- Gravity is the only interaction

All relevant quantities determined by G, Planck constant h and ε_u :

On the smallest scale:

$m_X v_X \cdot l_X/2 = \hbar$	Uncertainty principle
$v_X^2 = Gm_X/l_X$	Virial theorem
$(-\varepsilon_u) = v_X^3/l_X$	Constant energy cascade

Mass: $m_X \propto \left(-\varepsilon_u\hbar^5/G^4\right)^{1/9}\approx10^{12}GeV$

Length: $l_X\propto\left(-G\hbar/\varepsilon_u\right)^{1/3}\approx10^{-13}m$

Velocity: $V_X\propto\left(G\hbar\varepsilon_u^2\right)^{1/9}\approx4\times10^{-7}m/s$

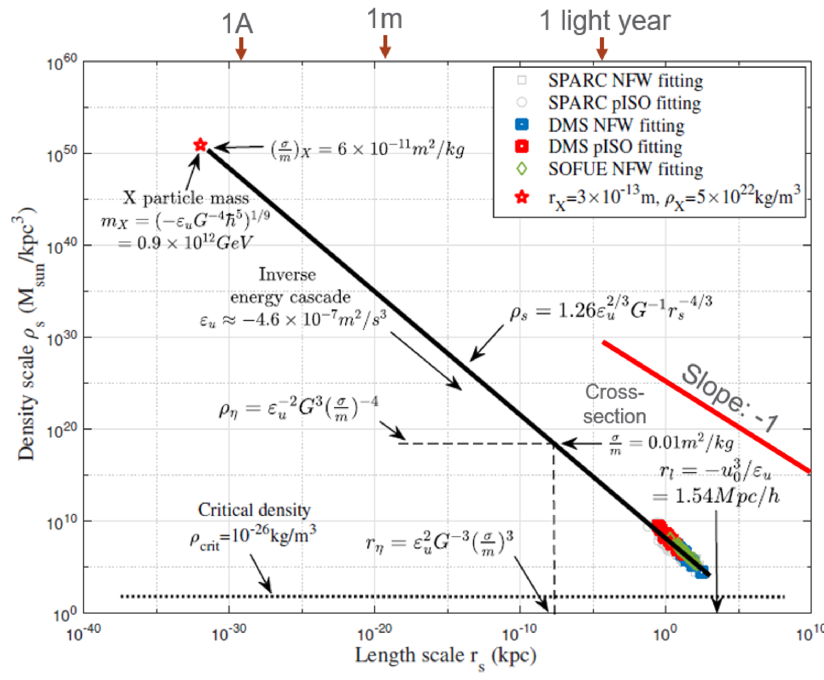
Density: $\rho = m_X/l_X^3\approx5.33\times10^{22}kg/m^3$

Energy: $m_X v_X^2 = 0.87\times10^{-9}eV$ km wave

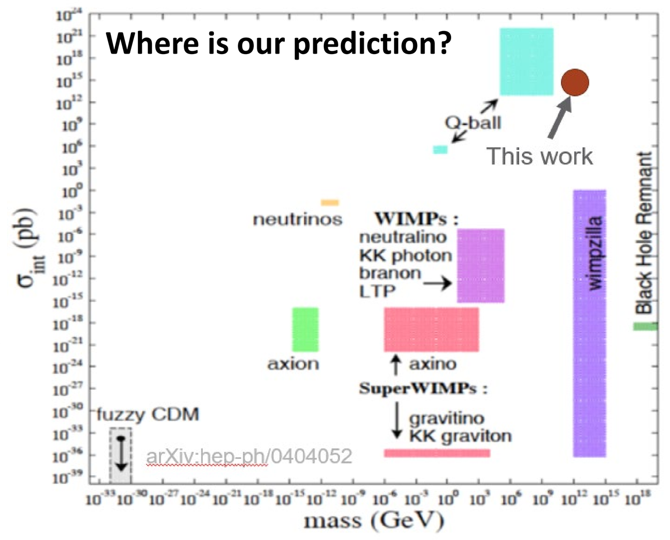
Power: $\mu_X = m_X a_X\cdot v_X = -m_X\varepsilon_u = 0.0046eV/s$

Particle lifetime: $\tau_X = -c^2/\varepsilon_u = 6.2\times10^{15}yr$

Cross section: $l_X^2 V_X = 4\times10^{-32}m^3/s$



Extend the predicted – 4/3 law to the smallest scale where quantum effect can be important (red star).



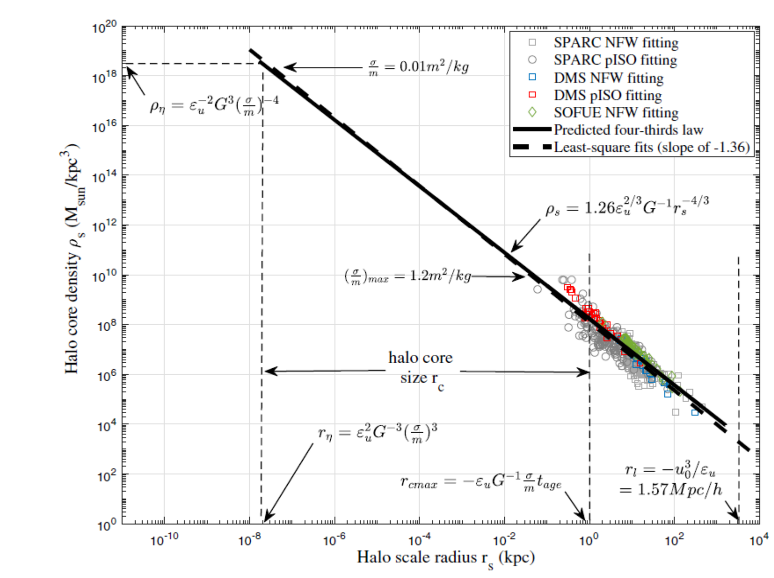
For self-interacting dark matter:

All relevant quantities determined by G, cross-section σ/m and ε_u :

On the smallest length scale:

$\rho_r(\sigma/m)v_r t_r = 1$	Elastic scatter
$v_s^2 = Gm_r(r_s)/r_s$	Virial theorem
$-\varepsilon_u = v_s^3/\gamma_s r_s$	Constant energy cascade

Minimum halo core size:	$r_\eta = \varepsilon_u^2G^{-3}(\sigma/m)^3$
Minimum mass scale:	$m_\eta = \varepsilon_u^4G^{-6}(\sigma/m)^5$
Maximum density scale:	$\rho_\eta = \varepsilon_u^{-2}G^3(\sigma/m)^{-4}$



The predicted – 4/3 law tested against actual data from galaxy rotation curves.

Maximum halo core size r_{cmax} :

$\rho_r\frac{\sigma}{m}v_r t_{age} = 1$ t_{age}: age of Universe

$$\frac{r_{cmax}}{(\sigma/m)} = -\varepsilon_u G^{-1}t_{age}\approx10kpc\frac{g}{cm^2}$$

(e) Conclusions

- Inverse mass/energy cascade from small to large scales (rate: ε_m -- kg/s; ε_u – m^2/s^3)
- Mass cascade leads to the random walk of halos in mass space with a position-dependent waiting time
- Random walk of halos in mass space leads to halo mass function and density profile (just like diffusion)
- Inverse energy cascade predicts scaling laws: mass, size, etc. for collisionless and self-interacting dark matter

Data availability:

A Comparative Study of Dark Matter Flow & Hydrodynamic Turbulence. 10. 5281/ zenodo.6569901 (2022).

References:

[1] Xu, Z. A Unified Theory for Dark Matter Halo Mass Function and Density Profile. 10. 48550/ ARXIV. 2210. 01200 (2022).
[2] Xu, Z. Dark matter particle mass and properties from rotation curves and energy cascade. 10. 48550/ ARXIV. 2202. 07240 (2022).
[3] Xu, Z. Universal scaling laws and density slopes for dark matter haloes. Scientific Reports 13:4165 (2023).

Acknowledgments

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