Basics of Higher-Spin Gauge Theory

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Symmetries

HS gauge theory: theory of maximal symmetries

Usual lower-spin symmetries

- **Relativistic theories: Poincaré symmetry:**

$\delta x^a = \varepsilon^a + \varepsilon^a{}_b x^b$ $\quad \varepsilon^a$: **translations;** ε^{ab} : **Lorentz rotations**

Lie algebra: $\delta x^a = [T, x^a]\,, \qquad T = \varepsilon^a P_a + \varepsilon^{ab} M_{ab}$

$$P_a = \frac{\partial}{\partial x^a}\,, \qquad M_{ab} = x_a \frac{\partial}{\partial x^b} - x_b \frac{\partial}{\partial x^a}$$

$$[M_{ab}, P_c] \;=\; P_a \eta_{bc} - P_b \eta_{ac}$$

$$[M_{ab},\, M_{cd}] = M_{ad}\eta_{bc} - M_{bd}\eta_{ac} - M_{ac}\eta_{bd} + M_{bc}\eta_{ad}$$

$$[P_a\,, P_b] = 0$$

Other Low-Energy Symmetries

$(A)dS$ **deformation**

$$[P_a\,, P_b] = \Lambda \mathbf{M_{ab}}$$

$\Lambda < 0$: AdS, $o(d-1,2)$

$\Lambda > 0$: dS, $o(d,1)$

$\Lambda = 0$: **Minkowski space,** $iso(d-1,1)$

- **SUSY**

$$P_a, M_{ab} \longrightarrow P_a, M_{ab}, \mathbf{Q}_\alpha\,, \qquad \alpha = 1,2,3,4$$

- **Inner symmetries: generators** T_i **are space-time invariant**

$$[T_i\,, (P_a, M_{ab})] = 0$$

Standard Model: $T_i \sim SU(3) \times SU(2) \times U(1)$

- **Conformal (super)symmetries**

Local Symmetries

Useful viewpoint: any global symmetry is the remnant of a local symmetry with parameters like $\varepsilon^a(x), \varepsilon^{ab}(x), \varepsilon^\alpha(x), \varepsilon^i(x)$ being arbitrary functions of space-time coordinates

Local symmetries: gauge fields A_a^n

$$\delta A_a^n = \partial_a \varepsilon^n + \ldots$$

$$S \longrightarrow S + \Delta S + \ldots\,, \qquad \Delta S = \int_{M^d} J_n^a(\varphi) A_a^n(x)$$

ΔS: Noether current interaction.

Subtlety

If $\varphi(x)$ were gauge fields with gauge parameters ε', $J_n^a(\varphi)$ may not be invariant under the ε' symmetry

Noether current interaction for several gauge fields may be obstructed

Why HS Theories?

Key question: is it possible to go to larger HS symmetries?

What are HS symmetries and HS counterparts of lower-spin theories including GR?

What are physical motivations for their study and possible outputs?

Fronsdal Fields

All $m = 0$ HS fields are gauge fields C.Fronsdal 1978

$\varphi_{a_1 \ldots a_s}$ **is a rank s symmetric tensor obeying** $\varphi^{c}{}_{c}{}^{b}{}_{b a_5 \ldots a_s} = 0$

Gauge transformation:

$$\delta \varphi_{a_1 \ldots a_s} = \partial_{(a_1} \varepsilon_{a_2 \ldots a_s)} , \qquad \varepsilon^{b}{}_{b a_3 \ldots a_{s-1}} = 0$$

Field equations: $G_{a_1 \ldots a_s}(x) = 0$ $\quad G_{a_1 \ldots a_s}(x)$: **Ricci-like tensor**

$$G_{a_1 \ldots a_s}(x) = \Box \varphi_{a_1 \ldots a_s}(x) - s \partial_{(a_1} \partial^{b} \varphi_{a_2 \ldots a_s b)}(x) + \frac{s(s-1)}{2} \partial_{(a_1} \partial_{a_2} \varphi^{b}{}_{a_3 \ldots a_s b)}(x)$$

Action

$$S = \int_{M^d} \left(\frac{1}{2} \varphi^{a_1 \ldots a_s} G_{a_1 \ldots a_s}(\varphi) - \frac{1}{8} s(s-1) \varphi_{b}{}^{b\, a_3 \ldots a_s} G^{c}{}_{c\, a_3 \ldots a_s}(\varphi) \right)$$

No-go and the Role of $(A)dS$

In 60th it was argued (Weinberg, Coleman-Mandula) that
HS symmetries cannot be realized in a nontrivial local field theory in Minkowski space

In 70th it was shown by Aragone and Deser that HS gauge symmetries are incompatible with GR if expanding around Minkowski space

Green light: AdS background with $\Lambda \neq 0$ Fradkin, MV, 1987
In agreement with no-go statements the limit $\Lambda \to 0$ is singular

HS Symmetries Versus Riemann Geometry

HS symmetries do not commute with space-time symmetries

$$[T^a, T^{HS}] = T^{HS}, \qquad [T^{ab}, T^{HS}] = T^{HS}$$

HS transformations map gravitational fields (metric) to HS fields

Consequence:

Riemann geometry is not appropriate for HS theory:

concept of local event may become illusive!

Related feature: HS interactions contain higher derivatives:

How non-local HS gauge theory is?

HS Gauge Theory and Quantum Gravity

HS symmetry is in a certain sense maximal relativistic symmetry. Hence, it cannot result from spontaneous breakdown of a larger symmetry: HS symmetries are manifest at ultrahigh energies above any scale including Planck scale

- HS gauge theory should capture effects of Quantum Gravity: restrictive HS symmetry versus unavailable experimental tests
- Lower-spin theories as low-energy limits of HS theory: lower-spin symmetries: subalgebras of HS symmetry
- String Theory as spontaneously broken HS theory?! $(s > 2, m > 0)$

HS AdS/CFT Correspondence

AdS_4 HS theory is dual to $3d$ vectorial conformal models

Sezgin–Sundell (2002), Klebanov-Polyakov (2002); Giombi and Yin (2009)

AdS_3/CFT_2 correspondence Gaberdiel and Gopakumar (2010)

Analysis of HS holography helps to uncover the origin of AdS/CFT

Global HS Symmetry

HS symmetry in AdS_{d+1}:

Maximal symmetry of a d-dimensional free conformal field(s)=singletons

usually, scalar and/or spinor

What are symmetries of KG equation in Minkowski space?

$$\Box C(x) = 0\,, \qquad \Box = \eta^{ab}\frac{\partial^2}{\partial x^a \partial x^b}$$

Shaynkman, MV 2001 $3d$; Shapovalov, Shirokov 1992, Eastwood 2002 $\forall d$

i Poincaré

ii Scale transformation (dilatation)

$$\delta C(x) = \varepsilon D C(x)\,, \qquad D = x^a \frac{\partial}{\partial x^a} + \frac{d}{2} - 1$$

iii Special conformal transformations

$$\delta C(x) = \varepsilon_a K^a C(x)\,, \qquad K^a = (x^2\eta^{ab} - 2x^a x^b)\frac{\partial}{\partial x^b} + (2-d)x^a$$

Conformal HS Algebra

Algebraic construction simplifies in $d=3$ using spinor formalism most relevant in the context of AdS_4/CFT_3 HS holography Shaynkman, MV (2001)

$3d$ Lorentz algebra: $o(2,1)\sim sp(2,R)\sim sl_2(R)$. $3d$ spinors are real

$sp(2,R)$ invariant tensor $\epsilon^{\alpha\beta}=-\epsilon^{\beta\alpha}$ relates lower and upper indices

Unfolded massless equations take the form

$$\left(\frac{\partial}{\partial x^{\alpha\beta}}+\frac{\partial^2}{\partial y^\alpha\partial y^\beta}\right)C(y|x)=0\,,\qquad C(y|x)=\sum_{n=0}^{\infty}C^{\alpha_1\dots\alpha_{2n}}(x)y_{\alpha_1}\dots y_{\alpha_{2n}}$$

$3d$ conformal HS algebra is the algebra of various differential operators $\epsilon(y,\frac{\partial}{\partial y})$ obeying $\epsilon(-y,-\frac{\partial}{\partial y})=\epsilon(y,\frac{\partial}{\partial y})$

$$\delta C(y|x)=\epsilon(y,\frac{\partial}{\partial y}|x)C(y|x)$$

$$\epsilon(y,\frac{\partial}{\partial y}|x)=\exp\left[-x^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha\partial y^\beta}\right]\epsilon_{gl}(y,\frac{\partial}{\partial y})\exp\left[x^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha\partial y^\beta}\right]$$

$\epsilon_{gl}(y,\frac{\partial}{\partial y})$ describes global HS transformations

NonAbelian HS Algebra

$3d$ **Conformal HS symmetry =** AdS_4 **HS symmetry**

HS gauge fields: $\omega(Y|X)$

$Y_A = (y_\alpha, \bar{y}_{\dot\alpha})$, $\alpha, \dot\alpha = 1,2$ **two-component spinor indices**

$$\omega(Y|X) = \sum_{n,m=0}^{\infty} \frac{1}{2n!m!} \omega_{\alpha_1 \ldots \alpha_n, \dot\alpha_1 \ldots \dot\alpha_m}(X) y^{\alpha_1} \ldots y^{\alpha_n} \bar{y}^{\dot\alpha_1} \ldots \bar{y}^{\dot\alpha_m}$$

HS curvature and gauge transformation

$$R(Y|X) = d\omega(Y|X) + \omega(Y|X) * \wedge \omega(Y|X)$$

$$\delta\omega(Y|X) = D\epsilon(Y|X) = d\epsilon(Y|X) + [\omega(Y|X), \epsilon(Y|X)]_*$$

$$[y_\alpha, y_\beta]_* = 2i\varepsilon_{\alpha\beta}, \qquad [\bar{y}_{\dot\alpha}, \bar{y}_{\dot\beta}]_* = 2i\varepsilon_{\dot\alpha\dot\beta}$$

Star product is nonlocal in Y^A **!**

$$(f * g)(Y) = f(Y) \exp[i\overleftarrow{\partial_A}\overrightarrow{\partial_B} C^{AB}] g(Y)$$

Properties of HS Algebras

Global symmetry of symmetric vacuum of bosonic HS theory

Let T_s be a homogeneous polynomial of degree $2(s-1)$

$$[T_{s_1}\,,T_{s_2}] = T_{s_1+s_2-2} + T_{s_1+s_2-4} + \ldots + T_{|s_1-s_2|+2}\,.$$

Once spin $s>2$ appears, the HS algebra contains an infinite tower of higher spins: $[T_s,T_s]$ gives rise to T_{2s-2} as well as T_2 of $o(3,2)\sim sp(4)$.

Usual symmetries: spin-$s\leq 2$ $u(1)\oplus o(3,2)$: maximal finite-dimensional subalgebra of $hu(1,0|4)$. $u(1)$ is associated with the unit element.

Space-Time and Spin

Space-time M is where symmetry $G = O(d-1,2)$ acts

Spin s: different G-modules V_s where fields $\phi^A(x)$ are valued.

V_s contain ground (primary) fields $\phi^A(x)$ along with their derivatives $\partial_{n_1} \dots \partial_{n_k} \phi^A(x)$ (descendants)

HS vertices contain higher derivatives Bengtsson, Bengtsson, Brink (1983), Berends, Burgers and H. Van Dam (1984), (1985), Fradkin, MV; Metsaev,...

HS symmetries Fradkin, MV 1986 are infinite dimensional extesions of G

Infinite towers of spins $\Rightarrow$ infinite towers of derivatives.

How (non)local is HS gauge theory?

Locality and Non-Locality

arXiv: 2208.02004

Equations of motion in perturbatively local field theory $E_{A_0,s_0}(\partial,\phi)=0$

$$E_{A_0,s_0}(\partial,\phi)=\sum_{k=0,l=1}^{\infty} a^{n_1\dots n_k}_{A_0\,A_1\dots A_l}(s_0,s_1,\dots,s_l)\partial_{n_1}\dots\partial_{n_k}\phi^{A_1}_{s_1}\dots\phi^{A_l}_{s_l}$$

have a finite # of non-zero coefficients $a^{n_1\dots n_k}_{A_0\dots A_l}$ at any order l.

s_0 is the spin of the field on which the linearized equation is imposed

HS theory involves infinite towers of Fronsdal fields of all spins.

$a^{n_1\dots n_k}_{A_0\dots A_l}$ may take an infinite # of values.

It makes sense to distinguish between   Gelfond, MV 2018

local: finite number of derivatives at any order

$$a^{n_1\dots n_k}_{A_0\dots A_l}(s_0,s_1,\dots s_l)=0 \quad \text{at } k>k_{max}(l)$$

spin-local: finite number of derivatives for any finite subset of fields

$$a^{n_1\dots n_k}_{A_0\dots A_l}(s_0,s_1,s_2,\dots s_l)=0 \quad \text{at } k>k_{max}(s_0,s_1,s_2,\dots s_l)$$

non-local: infinite number of derivatives for a finite subset of fields at some order.

Compact Spin-Locality

The simplest option: replacement of the class of local field theories with the finite # of fields by spin-local models with infinite # of fields.

Spin-local-compact vertices in addition obey

$$a^{n_1 \dots n_k}_{A_0\, A_1 \dots A_l}(s_0, s_1, \dots , s_k + t_k , \dots , s_l) = 0 \quad t_k > t_k^0 \quad \forall k$$

non-compact otherwise.

Compactness is in the space of spins, not in space-time

Both types of vertices in HS theory:

Cubic HS vertices $\omega * \omega$ built from HS gauge potentials are spin-local-compact: spins s_0, s_1, s_2 obey the triangle inequalities $s_0 \le s_1 + s_2$ etc.

Vertices associated with the conserved currents built from gauge invariant field strength are spin-local non-compact. These include conserved currents of any integer s_0 built from two spin-zero fields $(s_1 = s_2 = 0)$.

Field Redefinitions

A local theory remains local under perturbatively local field redefinitions

$$\phi^B_{s_0} \to \phi^B_{s_0} + \delta\phi^B_{s_0}\,, \qquad \delta\phi^B_{s_0} = \sum_{k=0,l=1}^{\infty} b^{Bn_1\dots n_k}{}_{A_1\dots A_l}(s_0, s_1, \dots, s_l)\partial_{n_1}\dots\partial_{n_k}\phi^{A_1}_{s_1}\dots\phi^{A_l}_{s_l}$$

with a finite # of non-zero coefficients at any order.

Which field redefinitions leave vertices spin-local?

General spin-local field redefinitions do not work since contributions of all spin s_p redefined fields may develop non-locality

$$\delta E_{A_0,s_0}(\partial,\phi) = \sum_{s_p=0}^{\infty} \sum_{p,k,k'=0,l,l'=1}^{\infty} a^{n_1\dots n_k}_{A_0\,A_1\dots A_l}(s_0, s_1, s_2, \dots, s_p, \dots, s_l)$$

$$\partial_{n_1}\dots\partial_{n_k}\phi^{A_1}_{s_1}\dots\phi^{A_{p-1}}_{s_{p-1}}\phi^{A_{p+1}}_{s_{p+1}}\dots\phi^{A_l}_{s_l} b^{A_p m_1\dots m_{k'}}{}_{B_1\dots B_{l'}}(s_p, t_1, \dots, t_{l'})\partial_{m_1}\dots\partial_{m_k}\phi^{B_1}_{t_1}\dots\phi^{B_{l'}}_{t_{l'}}$$

Spin-local-compact field redefinitions in spin-local theories: proper substitute since summation over s_p is finite.

One of the central problems in HS theory is to find a field frame making it (spin-)local. Given non-locally looking field theory, the essential question is whether or not it is spin-local in some other variables.

HS Multiplets

Infinite set of spins $s = 0, 1/2, 1, 3/2, 2 \dots$

$\omega_{\alpha_1 \dots \alpha_n, \dot\beta_1 \dots \dot\beta_m}$ **and** $C_{\alpha_1 \dots \alpha_n, \dot\beta_1 \dots \dot\beta_m}$ **with all** $n \geq 0$ **and** $m \geq 0$.

Generating functions $\omega(Y|x)$ **and** $C(Y|x)$: **unrestricted functions of commuting spinor variables** $Y = (y_\alpha, \bar y_{\dot\alpha})$

$$A(Y|x) = \sum_{n,m=0}^{\infty} \frac{1}{2n!m!} A_{\alpha_1 \dots \alpha_n, \dot\alpha_1 \dots \dot\alpha_m}(x) y^{\alpha_1} \dots y^{\alpha_n} \bar y^{\dot\alpha_1} \dots \bar y^{\dot\alpha_m}$$

Gauge one-forms $\qquad \omega_{\alpha_1 \dots \alpha_n, \dot\beta_1 \dots \dot\beta_m}, \qquad n + m = 2(s-1)$

$s = 1: \quad \omega(x) = dx^\nu \omega_\nu(x)$

$s = 2: \quad \omega_{\alpha\dot\beta}(x), \qquad \omega_{\alpha\beta}(x), \qquad \bar\omega_{\dot\alpha\dot\beta}(x)$

$s = 3/2: \quad \omega_\alpha(x), \qquad \bar\omega_{\dot\alpha}(x)$

Frame-like fields: $|n - m| = 0$ **(bosons) or** $|n - m| = 1$ **fermions**

Auxiliary Lorentz-like fields: $|n - m| = 2$ **(bosons)**

Extra fields: $|n - m| > 2$ **and zero-forms** $C(Y|x)$: **higher derivatives**

Free Field Unfolded Massless Equations

The full unfolded system for free massless bosonic fields is 1989

$$\star \qquad R_1(y,\overline{y}\mid x) = \frac{i}{4}\Big(\eta \overline{H}^{\dot\alpha\dot\beta}\frac{\partial^2}{\partial \overline{y}^{\dot\alpha}\partial \overline{y}^{\dot\beta}}\, C(0,\overline{y}\mid x) + \bar\eta H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\, C(y,0\mid x)\Big)$$

$$\star\star \qquad \tilde{\mathrm{D}}_0 C(y,\overline{y}\mid x) = 0$$

$$R_1(y,\bar y\mid x) := D_0^{ad}\omega(y,\bar y\mid x) \qquad D_0^{ad} := D^L - e^{\alpha\dot\beta}\Big(y_\alpha\frac{\partial}{\partial \bar y^{\dot\beta}} + \frac{\partial}{\partial y^\alpha}\bar y_{\dot\beta}\Big)$$

$$\tilde{\mathrm{D}}_0 = D^L + e^{\alpha\dot\beta}\Big(y_\alpha \bar y_{\dot\beta} + \frac{\partial^2}{\partial y^\alpha \partial \bar y^{\dot\beta}}\Big) \qquad D^L := \mathrm{d}_x - \Big(\omega^{\alpha\beta}y_\alpha\frac{\partial}{\partial y^\beta} + \bar\omega^{\dot\alpha\dot\beta}\bar y_{\dot\alpha}\frac{\partial}{\partial \bar y^{\dot\beta}}\Big)$$

$$H^{\alpha\beta} := e^{\alpha}{}_{\dot\alpha}e^{\beta\dot\alpha}, \qquad \overline{H}^{\dot\alpha\dot\beta} := e_\alpha{}^{\dot\alpha}e^{\alpha\dot\beta}$$

$\star\star$ implies that higher-order terms in y and $\bar y$ describe higher-derivative descendants of the primary HS fields

Zero-Form Sector

Equations on the gauge invariant zero-forms C

$$C(Y;K|x) = \sum_{n,m=0}^{\infty} \frac{1}{2n!m!} C_{\alpha_1\dots\alpha_n\,,\dot\alpha_1\dots\dot\alpha_m}(x) y^{\alpha_1}\dots y^{\alpha_n}\bar{y}^{\dot\alpha_1}\dots\bar{y}^{\dot\alpha_m}$$

decompose into independent subsystems associated with different spins

Spin-s zero-forms are $C_{\alpha_1\dots\alpha_n\,,\dot\alpha_1\dots\dot\alpha_m}(x)$ with

$$n-m=\pm 2s$$

Perturbative unfolded equations

$$\mathrm{d}_x C = \sigma_- C + \text{lower-derivative and nonlinear terms}$$

$$\sigma_- := \mathbf{e}^{\alpha\dot\beta}\frac{\partial^2}{\partial \mathbf{y}^\alpha \partial \bar{\mathbf{y}}^{\dot\beta}}\,, \qquad \sigma_-^2 = 0$$

$C_{\alpha_1\dots\alpha_n\,,\dot\alpha_1\dots\dot\alpha_m}(x)$ contain $\frac{n+m}{2}-\{s\}$ space-time derivatives of the spin-s dynamical fields. Presence of zero-forms C in the HS vertices may induce infinite towers of derivatives and, hence, non-locality.

HS Vertices

The problem: consistent non-linear corrections 1988 in the local frame

$$\mathsf{d}_x\omega = -\omega * \omega + \Upsilon(\omega,\omega,C) + \Upsilon(\omega,\omega,C,C) + \dots\,,$$

$$\mathsf{d}_x C = -[\omega, C]_* + \Upsilon(\omega,C,C) + \dots$$

The vertices can be put into the form

$$\Upsilon(\Phi,\Phi,\dots) = F(Q^i, P^{nm}; \bar{Q}^j, \bar{P}^{kl})\Phi(Y_1)\dots\Phi(Y_n)|_{Y_i=0}$$

with $\Phi = \omega$, C and some non-polynomial functions $F(Q^i, P^{nm}; \bar{Q}^j, \bar{P}^{kl})$ of the Lorentz-covariant combinations

$$Q^i := y^\alpha \frac{\partial}{\partial {y_i}^\alpha}\,, \qquad P^{ij} := \frac{\partial}{\partial y_i^\alpha}\frac{\partial}{\partial y_{j\alpha}}\,, \qquad \bar{Q}^i := \bar{y}^{\dot\alpha}\frac{\partial}{\partial {\bar{y}_i}^{\dot\alpha}}\,, \qquad \bar{P}^{ij} := \frac{\partial}{\partial {\bar{y}_i}^{\dot\alpha}}\frac{\partial}{\partial \bar{y}_{j\dot\alpha}}$$

The fundamental problem: find a proper class of functions $F(Q^i, P^{nm}; \bar{Q}^j, \bar{P}$ guaranteeing spin-locality (minimal non-locality) of the HS theory

Spinor Spin-Locality

Polynomiality of $F(Q^i, P^{ij}, \bar{Q}^j, \bar{P}^{kl})$ in either P^{ij} or $\bar{P}^{ij}$ $\forall i, j$ associated with C

Restriction to the fixed spin relates the degrees in P^{ij} and $\bar{P}^{kl}$ since

$$n - m = \pm 2s$$

Non-linear corrections can affect the relation between spinor and space-time spin-locality making obscure the space-time interpretation of the locality analysis in the spinor space.

This does not happen for projectively-compact spin-local vertices with

$$F(Q^i, P^{ij}, \bar{Q}^j, \bar{P}^{kl}) = Q_\omega G(Q^i, P^{ij}, \bar{Q}^j, \bar{P}^{kl}) + \bar{Q}_\omega \bar{G}(Q^i, P^{ij}, \bar{Q}^j, \bar{P}^{kl})$$

Q_ω and $\bar{Q}_\omega$ being associated with the one-forms ω among Φ.

Projectiely-Compact Spin-Local Vertices

Using background frame $e^{\alpha\dot\beta}$ HS equations can be represented as

$$D^L C(y,\bar y) = e^{\alpha\dot\alpha}\Big(\partial_\alpha\bar\partial_{\dot\alpha}F^{++}(y,\bar y)+y_\alpha\bar\partial_{\dot\alpha}F^{-+}(y,\bar y)+\bar y_{\dot\alpha}\partial_\alpha F^{+-}(y,\bar y)+y_\alpha\bar y_{\dot\alpha}F^{--}(y,\bar y$$

Generally, nonlinear corrections can contribute to any of F^{ab}.

The contribution to F^{++} can be singled out by the projector

$$\Pi^{des} := N_y^{-1}\bar N_{\bar y}^{-1} y^\alpha \bar y^{\dot\alpha}\frac{\partial}{\partial e^{\alpha\dot\alpha}}\,, \qquad N_y := y^\alpha\partial_\alpha\,, \quad N_{\bar y} := \bar y^{\dot\alpha}\bar\partial_{\dot\alpha}$$

A spin-local vertex Υ is called projectively compact if $\Pi^{des}\Upsilon$ is spin-local-compact. In particular, if $\Pi^{des}\Upsilon = 0$.

The contribution of the projectively-compact spin-local vertices can affect the expressions of the descendants in terms of derivatives of the ground fields only by spin-local-compact terms that preserve space-time locality of the vertex associated with the spin-local spinor vertex.

Projectiely-Compact Spin-Local Vertices in $d_x C$

The $d_x C$ vertex is 2017

$$\Upsilon = \Upsilon_\eta(e, C) + \Upsilon_{\bar\eta}(e, C)$$

$$\Upsilon_\eta(e, C) = \frac{1}{2}\eta \exp\left(i\bar{P}^{1,2}\right)\int_0^1 d\tau e(y, (1-\tau)\bar{p}_1 - \tau\bar{p}_2) C(\tau y, \bar{y}; K) C(-(1-\tau)y, \bar{y}; K)\,,$$

$$\Upsilon_{\bar\eta}(e, C) = \frac{1}{2}\bar\eta \exp i(P^{1,2})\int_0^1 d\tau e((1-\tau)p_1 - \tau p_2, \bar{y}) C(y, \tau\bar{y}; K) C(y, -(1-\tau)\bar{y}; K)\,,$$

where $e(a, \bar{a}) := e^{\alpha\dot\alpha} a_\alpha \bar{a}_{\dot\alpha}\,.$

Being non-polynomial either in P^{12} or in $\bar{P}^{12}$, Υ is spin-local

Since Υ contains either $e^{\alpha\dot\alpha} y_\alpha$ or $e^{\alpha\dot\alpha} \bar{y}_{\dot\alpha}$,

$$\Pi^{des}\Upsilon = 0 \quad \Rightarrow \Upsilon \quad \text{is projectively-compact spin-local}$$

PCSL vertices contain the minimal possible number of derivatives.

Holographic Higher Spins

Sezgin-Sunell-Klebanov-Polyakov conjecture: HS theory in AdS_4 is holographically dual to $3d$ vector model of scalar fields ϕ^i $(i = 1 \dots N)$.

Sleight and Taronna argued 2017 that a HS theory resulting from holographic analysis based on the is essentially non-local

Since HS holography is a weak-weak duality, it should be possible to test it.

No locality analysis of the full HS theory in AdS_4 has been done except for that of the Lebedev group Didenko, Gelfond, Korybut, MV 2017-2022
What has been shown so far indicates that HS theory is spin-local?!

Suggests gauged version of the SSKP conjecture with conformal HS boundary theory MV 2012

Conclusion

HS gauge theories contain gravity along with infinite towers of other fields with various spins including ordinary matter fields: singlet scalar!

Infinite-dimensional HS symmetry

HS theories exist in various dimensions.

Unbroken HS symmetries demand AdS background

HS vertices contain higher derivatives.

Customary concepts of Riemann geometry are not applicable: study of exact solutions is very instructive:

BH-like solutions Didenko, MV 2009, Iazeola, Sundell 2010

One of the central problems is the mechanism of spontaneous breakdown of HS symmetries

HS holography is closely related with the locality properties of HS theory

Concepts of compact and projectively-compact vertices are introduced. These apply to various versions of HS theories.

For projectively-compact vertices spin-locality in the spinor space and space-time are equivalent.

PCSL vertices are conjectured to form a proper class of solutions of the non-linear HS equations that guarantee spin-locality of the HS theory at higher orders.

Analysis of HS gauge theory has a potential to affect the paradigm of holographic corresondence replacing the gauge-gravity correspondence by the conformal gravity - gravity correspondence.