Quantum Hall effect inspired by quantum chromodynamics

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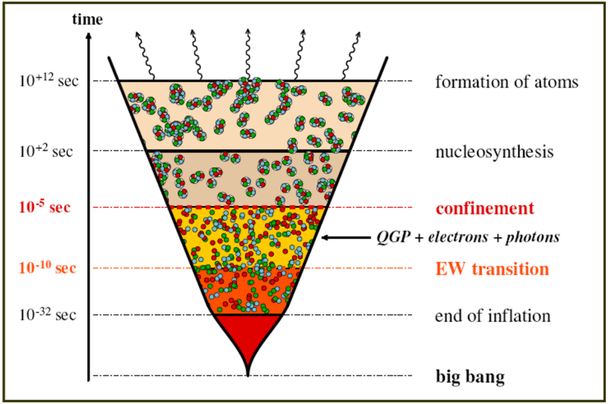
Huitzil collaboration with Cristian Villavicencio, Alfredo Raya, David Dudal, Alexandre Reily and Filipe Matusalém.

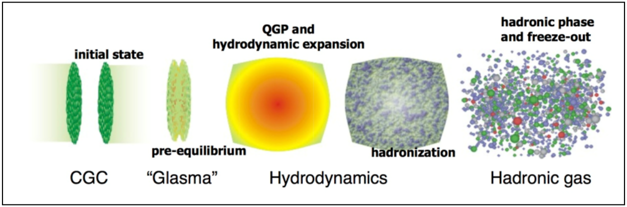
Overview

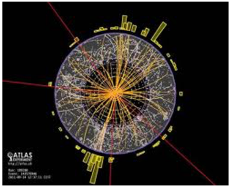
Heavy ion collisions: why?

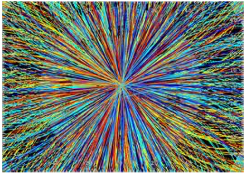


High temperature plasma – above 4 trillion Kelvin

Heavy Ion Collisions





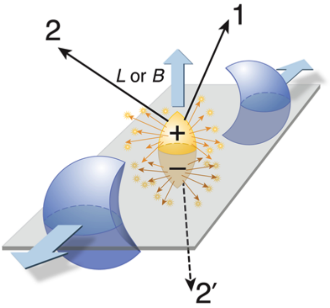


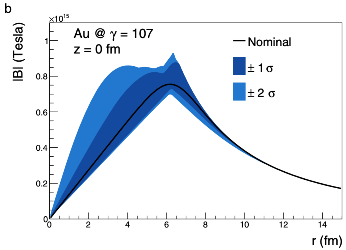
p-p collisions: $\sim$ 20 particles

Au-Au collisions: $\sim$ 4800 particles

Magnetic fields in HIC

Non-central collisions: the strongest magnetic fields observed in the laboratory (4 orders higher than the strongest magnetic field observed in nature - magnetars).





D. Brandenburg et al, Eur. Phys. J. A, 57, 299 (2021).

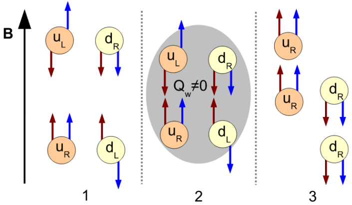
First experimental observation corroborates value predicted 12 years ago $eB \approx 10^{19}$G.

Overview

The chiral magnetic effect

The chiral anomaly generates a flip of chirality when chiral fermions interact with topologically non-trivial gauge fields.

$$(N_R - N_L) = 2N_f Q_w$$



[Kharzeev, McLerran and Warringa, NPA 803, 227 (2008).] Blue arrows denote spin and red arrows denote momentum.

- Effective description in terms of a chiral chemical potential:

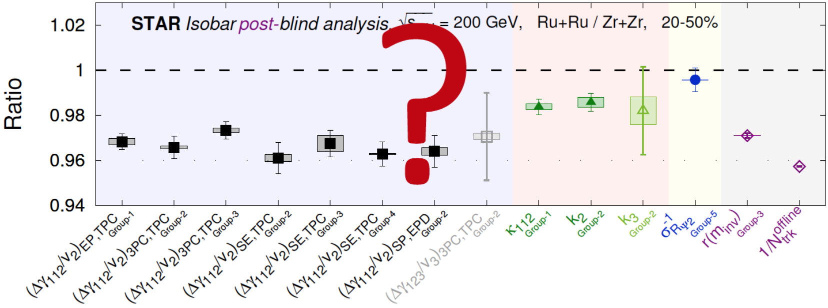
  Fukushima, Kharzeev and Warringa, PRD 85, 045104 (2008).

$$j_z \sim \sum_f q_f^2 B \mu_5 \quad \text{Independent of temperature and mass}$$

Why is it important?

- It sheds light on the quantum vacuum of the QCD: complex topological structure
- Sakharov conditions for baryogenesis:
  - Baryon number violation
  - C and CP violation
  - Dynamics out of thermal equilibrium
- These currents are non-dissipative. They are time reversal symmetric: $J_i = \sigma B_i$.

Isobar collisions



Isobar collisions: program dedicated to detect the CME did not find the signal. New analysis considering multiplicity difference between the isobars indicate a small signal.

We have a proposal to analyse the elements of the chiral magnetic effect through other observables, but this is subject for another talk.

Overview

Chiral magnetic effect observed in Dirac semimetals

LETTERS
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nature physics

Chiral magnetic effect in $ZrTe_5$

Qiang Li[1*], Dmitri E. Kharzeev[2,3*], Cheng Zhang[1], Yuan Huang[4], I. Pletikosić[1,5], A. V. Fedorov[6], R. D. Zhong[1], J. A. Schneeloch[1], G. D. Gu[1] and T. Valla[1*]

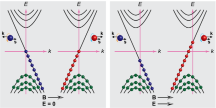
The chiral magnetic effect is the generation of an electric current induced by chirality imbalance in the presence of a magnetic field. It is a macroscopic manifestation of the quantum anomaly[1,2] in relativistic field theory of chiral fermions (massless spin 1/2 particles with a definite projection of spin on momentum)—a remarkable phenomenon arising from a collective motion of particles and antiparticles in the Dirac sea. The recent discovery[3–6] of Dirac semimetals with chiral quasiparticles opens a fascinating possibility to study this phenomenon in condensed matter experiments. Here we report on the measurement of magnetotransport in zirconium pentatelluride, $ZrTe_5$, that provides strong evidence for the chiral magnetic effect. Our angle-resolved photoemission spectroscopy experiments show that this material's electronic structure is consistent with a three-dimensional Dirac semimetal. We observe a large negative magnetoresistance when the magnetic field is parallel with the current. The mea-

causing the generation of primordial magnetic fields[13–17]. However, the interpretation in all these cases is under debate owing to lack of control over the chirality imbalance produced.

The most prominent signature of the CME in Dirac systems in parallel electric and magnetic fields is a positive contribution to the conductivity that has a quadratic dependence on magnetic field[8,18,19]. This is because the CME current is proportional to the product of the chirality imbalance and the magnetic field, and the chirality imbalance in Dirac systems is generated dynamically through the anomaly with a rate that is proportional to the product of electric and magnetic fields. As a result, the longitudinal magnetoresistance becomes negative[18,19].

Let us explain how this mechanism works in Dirac semimetals in more detail. In the absence of external fields, each Dirac point initially contains left- and right-handed fermions with equal chemical potentials, $\mu_L = \mu_R = 0$. If the energy degeneracy between the left- and right-handed fermions gets broken, we

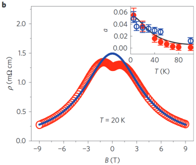
Chiral anomaly in 3D Dirac semi-metal $ZrTe_5$



[Kharzeev,Li Nuclear Physics A, 956, 107]

$$J_{CME} = \frac{e^2}{2\pi^2}\mu_5 B, \qquad \mu_5 \sim E \cdot B$$

$$J_{CME} \equiv \sigma^{ik}_{CME} E^k, \, \sigma^{zz}_{CME} \sim B^2$$



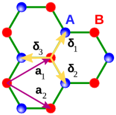
[Li et al, Nature Phys. 12, 550 (2016).]

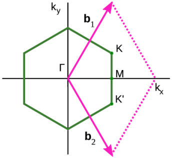
- The magnetoresistence in $ZrTe_5$ when a magnetic field is applied parallel to an electric field is in accordance with the predictions for the CME.

After the first observation, the CME was detected in several other 3D Dirac materials.

Could it replace superconductors in certain devices? Could it perform at a higher temperature?

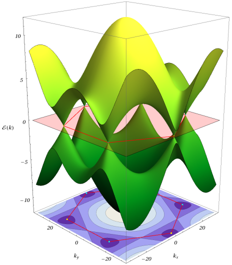
Honeycomb lattices: is it possible to reproduce the CME in two-dimensional materials?





- Represented in terms of two triangular sublattices.
- Hexagonal reciprocal lattice.

- Tight-binding approach: nearest neighboors.
- Hopping only between sublattices.



- Linear dispersion relation: $\mathcal{H} = \bar{\Psi}\, \hbar v_F \boldsymbol{\gamma} \cdot \boldsymbol{k}\, \psi$.
- Dirac points: valence and conduction band touch generating no gap.

Expanding around de K and K' points:

$$H_{K'}(\vec{q}) \approx \frac{3at}{2} \begin{pmatrix} 0 & \alpha(q_x + iq_y) \\ \alpha^*(q_x - iq_y) & 0 \end{pmatrix}, \qquad H_K = H^*_{K'}.$$

$$H_K = -i\hbar v_f \vec{\sigma}\nabla, \qquad H' = H_K^T.$$

Considering the 4-component spinor:

$$H = \begin{pmatrix} H_K & 0 \\ 0 & H'_K \end{pmatrix}, \qquad H_K = H^*_{K'}, \qquad H = -i\hbar v \tau_0 \otimes \vec{\sigma}\nabla.$$

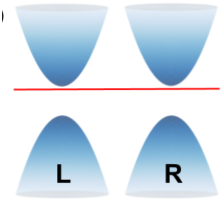
In the continuous limit:

$$\mathcal{L} = \sum_{\sigma=\pm} \bar{\psi}_\sigma(t,\mathbf{r}) \left[i\hbar\gamma^0\partial_t + i\hbar v_f \gamma^1 D_x + i\hbar v_f \gamma^2 D_y \right] \psi_\sigma(t,\mathbf{r}).$$

Chirality $\longleftrightarrow$ Dirac point (valley)

- Gaps between the conduction and valence band appear as a mass term $\hat{M}$ in the Lagrangian - interactions, deformations, substrates, doping, etc.
- $\hat{\mu}$ is a generalized chemical potential including spin interaction (Zeeman term), $\mu_\sigma = \mu - \frac{\sigma g}{2\mu_B} B$.
- QED3 fermion sector ($\hbar$=c=1):

$$\mathcal{L} = \bar{\psi}[\gamma_0(i\partial_0 + \hat{\mu}) - i(\boldsymbol{\gamma}_1 D_x + \gamma_2 D_y) - \hat{M}]\psi.$$



Overview

Pseudo/Reduced QED

[Marino (1993); Gonzalez, Guinea, Vozmediano (1994); Gorbar, Guysinin, Miranski (2001).]

- The gauge sector is not constrained to the plane.
- Coulomb rather than logarithmic interaction.
- Reduced QED: general (3+1) theory dimensionally reduced to a non-local effective (2+1) theory.
- $D = 4 \rightarrow$ Integrating over the gauge field and the third spatial dimension.
- Keeping $J^3 = 0$.
- Adding the fermion fields in (2+1)D.

$$S = \int d^3x \left[\bar{\psi} \left(i\not{D} + m \right) \psi + \frac{1}{2} F_{\mu\nu} \frac{1}{\sqrt{-\partial^2}} F^{\mu\nu} + \frac{1}{e^2 \xi} \partial_\mu A^\mu \frac{1}{\sqrt{-\partial^2}} \partial_\nu A^\nu \right].$$

Anomalous Quantum Hall Effect

[**AJM**, C. Villavicencio, D. Dudal, A. R. Rocha, F. Matusalém, Sci.Rep. 12 (2022) 1, 5439]

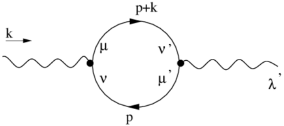
Linear response formalism: reaction of the system to external influences.

$$\delta S = \int d^4 x J_\mu(x) a_\mu(x) \tag{1}$$

The conductivity is given by

$$\sigma_\chi = -\lim_{\omega \to 0} \frac{1}{\hbar\omega} \tilde{\Pi}_R^{xy}$$

The polarization tensor is given by the diagram



We have shown that only 1-loop contributions are non-vanishing: Coleman-Hill theorem valid for RQED

[D. Dudal, AJM and P. Pais, PRD (2018)].

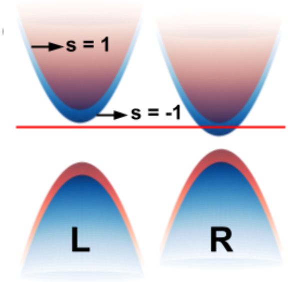
The limit can only be taken if we consider a configuration of the magnetic field that implies an electric field when the limit $\omega \to 0$ is taken.

Considering a chemical potential, we obtain for the net current

$$\sigma_\chi = \sum_s \frac{e^2}{4\pi}\left[\frac{m_{s,k}}{|m_{s,k}|}\theta(m^2_{s,k}-\mu^2) - \frac{m_{s,k'}}{|m_{s,k'}|}\theta(m^2_{s,k'}-\mu^2)\right]$$

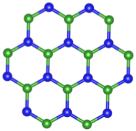
Quantum Hall Effect, with fractional Chern-number. **TOPOLOGICALLY PROTECTED!**

Spin flip causes a flip in the mass sign: we need a **lift of spin degeneracy**



Physical meaning of the interactions

- Center symmetry breaking "mass": $M = m_3\gamma_3$
- This can be obtained if **sublattice symmetry is broken.**

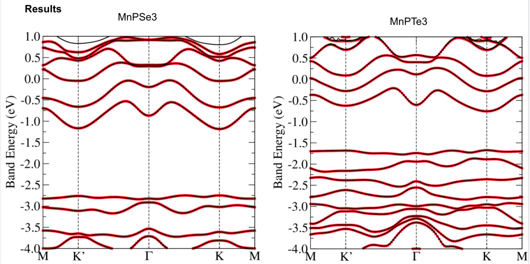


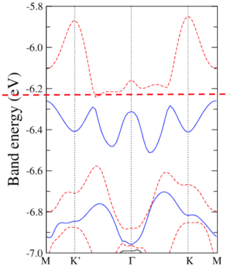
- Broken T symmetry: complex next to nearest neighbors term. In the Lagrangian: $M = m_3\gamma_3\gamma_5$. **Spin orbit?**
- Similar to the (3+1)D case, external electric and magnetic fields (represented by a Chern-Simons term) can dynamically generate Haldane mass in RQED. [J. Casimiro, L. Albino, AJM, A. Raya, Phys.Rev.D 102 (2020) 9, 096023]

Ab Initio simulations (Filipe Matusalém)

We look for materials that can present an intrinsic effect.

Band structure of $MnPX_3$, $X = Se, Te$





Doped with 1 Cu atom in order to generate a Zeeman effect and lift spin degeneracy.

Other promising materials:

- Heterostructures: MnPSe3/CrBr3, MnPSe3/MoS2 and WS2/h-VN
- Dichalcogenides: NbSe2 and WS2

Overview

Final remarks

- The chiral magnetic effect is an important anomalous transport effect that may take place in the quark-gluon plasma and, besides sheding light on the QCD vacuum structure, has deep implications for the early universe.
- It has been observed in condensed matter systems: three-dimensional Dirac materials.
- Although the chiral magnetic effect is not allowed in two-dimensional materials, an analogue effect based on the parity anomaly is possible.
- Analogy depends on considering honeycomb lattice strictly 2D or quasi-2D.
- First principle simulations are on the way: band structure, conductivity, stability.
- Simulations to motivate experimental search.

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South American Institute
for Fundamental Research

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Research Foundation
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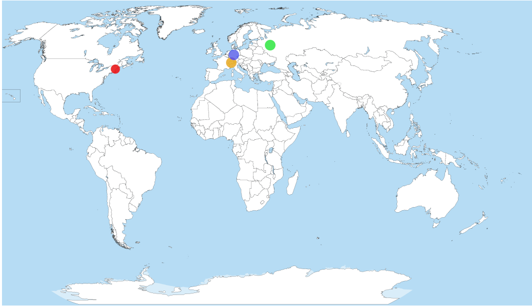
Opening new horizons

CONICYT
Ministerio de Educación
Gobierno de Chile

Backup

Heavy ion collisions: where?



- Relativistic Heavy Ion Collider - BNL (USA)
- Large Hadron Collider - CERN (Switzerland)
- Facility for Antiproton and Ion Research - GSI (Germany)
- Nuclotron-based Ion Collider fAcility - JINR (Russia)

The chiral magnetic effect

Chirality: an intrinsic quantum number related to parity transformation - mirror image. For massless particles chirality can be identified with helicity.

How does it happen? Axial anomaly: imbalance of chirality

Topological invariant:

$$Q_w = \frac{g^2}{32\pi^2}\int d^4x F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a.$$

Non-conservation of axial current:

$$\partial_\mu J^5_\mu = 2\sum_f m_f \langle \bar{\psi}_f i\gamma_5 \psi_f \rangle_A - \frac{N_f g^2}{16\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a.$$

The anomaly will affect the Ward identities of a quark interacting with a gauge field and it is possible to relate the winding number with the eigenvalues of the equation of motion.

Summing over the eigenvalues of the chiral operator:

$$\boxed{(N_R - N_L) = 2N_f Q_w.}$$

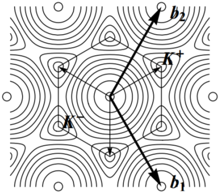
Pseudo-chirality: a physical quantity or an elegant theoretical modeling?

- Chirality in odd dimensions: impossible to define a γ^5 matrix composed by the other matrices of the relevant group that anti-commute with all of them.
- Mecklenburg and Regan: what if graphene is not so $2+1$D? (PRL 106, 116803 (2011)).
  - Consider a break of sublattice symmetry: the system is aware of the **z** axis:

$$H_{K'}(\vec{q}) \approx \frac{3at}{2}\begin{pmatrix} \Delta & \alpha(q_x+iq_y) \\ \alpha^*(q_x-iq_y) & -\Delta \end{pmatrix},$$

    where $\Delta = \varepsilon_A - \varepsilon_B$.

For this case an "extra" angular momentum is necessary in order to commute with the Hamiltonian $[H, \mathbf{L}+\mathbf{S}] = \mathbf{0}$.



Rotational invariance around the K points.

- Missing angular momentum: $[H, \mathbf{L}] \neq \mathbf{0}$ while the Hamiltonian has rotational symmetry around the axis perpendicular to the graphene plane.
- Possible to define a vector S analogous to spin collectively generated by the background lattice such that $[H, \mathbf{L} + \mathbf{S}] = \mathbf{0}$

Pseudo-chirality: a physical quantity or an elegant theoretical modeling?

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Photonic graphene with broken sublattice symmetry exhibits vorticities that can be associated to an angular momentum of the pseudo-spin. **"Unveiling pseudospin and angular momentum in photonic graphene"** [Nature comm. 6, 6272 (2015)].

Two possible algebras for the pseudo-spin σ:

- Rotations in 3 spatial dimensions:
  $[S_i, S_j] = i\hbar\varepsilon_{ijk} S_k, \qquad \{i,j,k\} \in \{1,2,3\}$

- 2 boosts and 1 rotation:
  $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \qquad \{\mu,\nu\} \in \{0,1,2\}$

Honeycomb lattice as a strictly two dimensional structure, or as a quasi-two dimensional structure embedded in three dimensional space?

Honeycomb lattice as a (3+1)D system

AJM, S. Hernández-Ortiz, A. Raya and C. Villavicencio, Eur.Phys.J.C 78, 11 (2018).

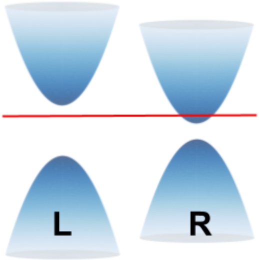
- No dynamics in the third space coordinate.
- However there is interaction with the third component of the gauge field.
- Interactions are chosen in a way to mimic chiral imbalance (this will be clear in a moment.)

$$\mathcal{L} = \bar{\psi}[i\partial\!\!\!/ + \mu\gamma^0 + (eA_3^{\text{ext}} - m_3)\gamma^3 - m_o\gamma^3\gamma^5]\psi.$$

- This configuration preserves iso-flavor chiral symmetry.

- In the chiral basis, we define: $\psi_\pm \equiv \frac{1}{2}(1 \pm \gamma^5)\psi$ and $m_\pm = m_3 \pm m_0$:

$$\mathcal{L} = \sum_{\chi=\pm} \bar{\psi}_\chi \left[i\partial\!\!\!/ + \mu\gamma^0 + \left(eA_3^{ext} - m_\chi \right) \right] \psi_\chi .$$



- Using the Schwinger proper time method:

$$\tilde{G}(k;M_\pm) = -i\int_0^\infty ds\, e^{-sK_\parallel{}^2-\left[k^2+M_\pm\right]\tanh(eBs)/eB}$$
$$\left\{\not{K}_\parallel\left[1-i\gamma^2\gamma^3\tanh(eBs)\right]-\left[k_2\gamma^2+M_\pm\gamma^3\right]\mathrm{sech}^2(\mathrm{eBs})\right\},$$

$$K_\parallel = (k_0+\mu, k_1, 0)$$

- We calculate the currents at finite temperature:

$$J_\mu(x) = -e\langle\bar{\psi}\gamma_\mu\psi\rangle, \qquad J_{\mu 5}(x) = -e\langle\bar{\psi}\gamma_\mu\gamma_5\psi\rangle$$

$$j(\eta) = 2\frac{e^2BT}{2\pi}\sum_n\int_{-\infty}^{\infty} ds\, r_s(\omega_n,\mu)\,(\omega_n-i\mu)\left[\frac{tan(eBs)}{eBs}\right]^{1/2} e^{-s(\omega-i\mu)^2-eB\eta^2}.$$

- Only $i = 1$ component is non-vanishing.

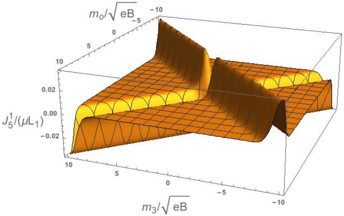


Figure: $L_2\sqrt{eB} = 0.2$

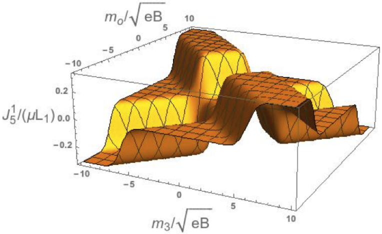
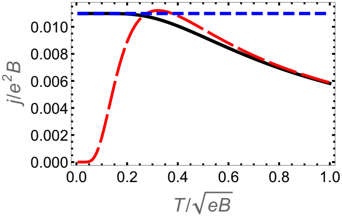


Figure: $L_2\sqrt{eB} = 8$

Electric current



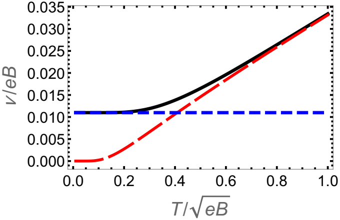
For $|eB| \ll (\pi T)^2 - \mu^2$:

$$j(\eta) = \frac{e^2B}{2\pi}[n_F(eB\eta - \mu) - n_F(eB\eta + \mu)], \qquad \text{where } n_F = (1 + e^{x/T})^{-1}.$$

For $|eB| \gg (\pi T)^2 - \mu^2$:

$$j(\eta) = \frac{e^2B}{\sqrt{|eB|}} \frac{\mu}{\pi^{3/2}} e^{-|eB|\eta^2}.$$

Density number



For $|eB| \ll (\pi T)^2 - \mu^2$:

$$\nu(\eta) = \frac{eB\eta}{\pi} T^2 \left[\frac{|eB\eta|}{T} ln \left(\frac{1+e^{(|eB\eta|-\mu)/T}}{1+e^{(|eB\eta|+\mu)/T}} \right) + Li_2 \left(-e^{(|eB\eta|-\mu)/T} \right) + Li_2 \left(-e^{(|eB\eta|+\mu)/T} \right) \right].$$

For $|eB| \gg (\pi T)^2 - \mu^2$:

$$\nu(\eta) = \sqrt{|eB| \frac{\mu}{\pi^3/2}} e^{-|eB|\eta^2}.$$

$$J_1 = |e| sign(B) N_5$$

Linear response theory

The expectation value of the current to leading order in the gauge field

$$\langle j(t)\rangle = \frac{i}{\hbar}\langle 0|^{t}_{-infty} d\tau \left[\Delta H(\tau), \vec{j}(t)\right]|0\rangle$$

$$\langle j_x(t)\rangle = \frac{iB}{\hbar\omega}\langle 0|\, i\int_{-\infty}^{t} d\tau [j_y(\tau), j_x(t)]\,|0\rangle \cos(\omega\tau)$$

$$= \mathrm{Im}\left\{\frac{B}{\hbar\omega}\langle 0|\, i\int_{-\infty}^{t} d\tau [j_y(\tau), j_x(t)]\,|0\rangle\, e^{-i\omega\tau}\right\}.$$

After using time translational invariance, we get

$$\langle j_x(t)\rangle = \frac{B}{\hbar\omega}\mathrm{Im}\left\{ i\int_{0}^{\infty} d\tau \left(\langle 0|\,[j_y(0), j_x(\tau)]\,|0\rangle\, e^{i\omega\tau}\right) e^{-i\omega t}\right\},$$

from which the Kubo relation for the DC PME anomalous conductivity follows as

$$\sigma = \lim_{\omega\to 0}\frac{1}{\hbar\omega}\mathrm{Im}\left\{ i\int_{0}^{\infty} d\tau e^{i\omega\tau}\langle 0|\,[j_y(0), j_x(\tau)]\,|0\rangle\right\}.$$

$$\tilde{\Pi}^{ij}(\vec{p}) = -\frac{m}{|m|}\frac{e^2}{4\pi}\theta(m^2-\mu^2)\varepsilon^{ijk}p_k + \mathcal{O}(p^2),$$

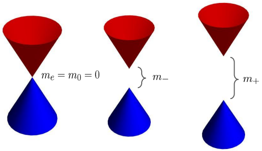
The limits $p_0 = 0$, $\vec{p} \to 0$ commute, and we get that

$$\begin{aligned}
\sigma &= -\lim_{\omega\to 0}\frac{1}{\hbar\omega}\Pi^{yx} \\
&= \lim_{\omega\to 0}\frac{1}{\hbar\omega}\frac{m}{|m|}\frac{e^2}{4\pi}\theta(m^2-\mu^2)\varepsilon_{210}\omega \\
&= -\frac{e^2}{4\pi\hbar}\frac{m}{|m|}\theta(m^2-\mu^2).
\end{aligned}$$

Chiral chemical potential (Fermi liquid model)

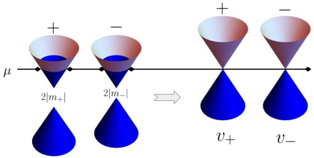
[AJM, A. Raya and C. Villavicencio, hep/ph:1803.05794.]

$$\mathcal{L} = \bar{\psi}[i\partial\!\!\!/ + \mu\gamma^0 + (eA_3^{\text{ext}} - m_3)\gamma^3 - m_o\gamma^3\gamma^5]\psi.$$



Expanding around $p_\chi = \sqrt{\mu^2 - m_\chi^2}$,

$$\mathcal{L} = \sum_{\chi=\pm} \bar{\psi}'_\chi \left[i\gamma^0\partial_0 - v_\chi \gamma\cdot\nabla \right] \psi'_\chi.$$

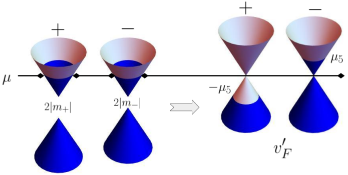


Expanding around $p_\chi = \frac{m_\chi v'_F}{\sqrt{1-v'^2_F}}$,

$$\mathcal{L} = \sum_{\chi=\pm} \bar{\Psi}' [i\gamma^0\partial_0 - v'_F \boldsymbol{\gamma}\cdot\boldsymbol{\nabla} + \mu_5\gamma_0\gamma_5]\Psi'_\chi$$

$$\mu' = \mu - \frac{|m_-| + |m_+|}{2\sqrt{1 - v'^2_F}}$$

$$\mu_5 = \frac{|m_-| - |m_+|}{2\sqrt{1 - v'^2_F}},$$



Description in terms of the chiral chemical potential μ_5.