Asymptotic Symmetries in Einstein-Scalar Theory with an Exponential Potential

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Contents

Introduction

- Find B-H solutions that allow the presence of a Scalar Field (which naturally emerge in Super Gravity(SUGRA)).
- Holography.
- Asymptotic symmetries.
- Enhanced symmetry group.

The AdS story

- Henneaux-Teitelboim[1985]

$AdS_4 \qquad so(3,2)$

$Schw - AdS_4 \qquad \mathbb{R} \times so(3)$

$Schw\ AdS_4 \in asymptotically\ AdS_4\ metrics\ - so(3,2)$

- D=3 Brown-Henneaux[1986]

$AdS_3 \qquad so(2,2)$

$static\ BTZ \qquad \mathbb{R} \times S^1$

$$[L_n, L_m] = (m-n)L_{m+n}$$

$$\{Q_m, Q_n\} = (m-n)Q_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}$$

$$c = \frac{3l}{2G}$$

$static\ BTZ \in asymptotically\ AdS_3\ metrics$

Constructing a new flat story : Gravity coupled to a Scalar Field

With the objective of finding a realization of flat spacetime holography, we consider the E-H action with Λ=0 and a scalar matter field.

$$S = \int d^D x \sqrt{-g}\left[R - \frac{1}{2}\nabla_\mu \Phi \nabla^\mu \Phi - V[\Phi]\right]$$

$$V[\Phi] = \chi e^{-\sqrt{2\alpha}\Phi}$$

$$\Phi(r) = \sqrt{2\alpha}\, ln\left(\frac{r}{L}\right)$$

Potential of this form naturally appear in Supergravity and String Theory [Colgáin-Samtleben,2011][Freedman-Schwarz,1978]

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{2}T_{\mu\nu}$$

We will focus on solutions with a flat base manifold.

$$ds^2 = -f(r)dt^2 + \frac{1}{g(r)}dr^2 + r^2 d\Omega_{D-2}^2$$

We obtained solutions in D=2+1 and in D=3+1.

$$\Phi(r) = \sqrt{2\alpha}\, ln\left(\frac{r}{L}\right)$$

D=2+1

$$ds^2 = -\left(\frac{\chi}{2-\alpha}r^2 - \mu r^\alpha\right)dt^2 + \left(\frac{r}{L}\right)^{2\alpha}\frac{1}{\left(\frac{\chi}{2-\alpha}r^2 - \mu r^\alpha\right)}dr^2 + r^2 d\varphi^2$$

D=3+1

$$ds^2 = -\left(\frac{\chi}{6-4\alpha}r^2 - \mu r^{2\alpha-1}\right)dt^2 + \left(\frac{r}{L}\right)^{4\alpha}\frac{dr^2}{\left(\frac{\chi}{6-4\alpha}r^2 - \mu r^{2\alpha-1}\right)} + 2r^2 dz d\bar{z}$$

But what are we looking for? We are interested in solutions that are Asymptotically locally flat which is ${R^{\mu\nu}}_{\rho\sigma} \to 0$ as $r \to \infty$. Indeed, the Riemman tensor behaves like

D=2+1

$$ {R^{\mu\nu}}_{\rho\sigma} \sim C1\,\chi\, r^{-2\alpha} - C2\,\chi\, r^{-2-\alpha} $$

D=3+1

$$ {R^{\mu\nu}}_{\rho\sigma} \sim C1\,\chi\, r^{-4\alpha} - C2\; r^{-3-2\alpha} $$

In order to compute conserved charges, we'll use the covariant phase space [Compère,2019]
Which lead to the surface integrals

$$\delta\!\!\!/ H_\xi[\delta\Phi;\Phi] = \oint \mathbf{k}_\xi[\delta\Phi;\Phi].$$

In particular, our surface charge receives contributions both from the gravity and matter sectors.

$$k_G = 2\sqrt{-g}\left(\xi^{[\mu}\nabla_\alpha h^{\nu]\alpha} - \xi^{[\mu}\nabla^{\nu]}h - \frac{1}{2}h\,\nabla^{[\mu}\xi^{\nu]} - \xi_\alpha\nabla^{[\mu}h^{\nu]\alpha} + h^{\alpha[\mu}\nabla_\alpha\xi^{\nu]}\right)$$

$$k_\Phi = -2\sqrt{-g}(\xi^{[\mu}\nabla^{\nu]}\Phi\,\delta\Phi)$$

$$K = k_G + k_\Phi$$

We found that for the static solutions the conserved charges are

D=2+1

$$M = 2\pi\mu^{\frac{2}{2-\alpha}}$$

D=3+1

$$M = 2\pi^{\frac{1}{3}} \frac{(6-4\alpha)\mu}{\sqrt{\left(\frac{(6-4\alpha)\mu}{\chi}\right)^{-\frac{4\alpha}{3-2\alpha}}}}$$

Like in BTZ, in D=2+1 we apply a boost

$$dt \to \frac{dt + \omega l d\phi}{\sqrt{1-\omega^2}}$$

$$d\phi \to \frac{d\phi + \frac{\omega}{l} dt}{\sqrt{1-\omega^2}}$$

This way we found a new rotating solution

$$ds^2 = -\left(\frac{r^2}{l^2} - A\,r^\alpha\right)dt^2 + \frac{\left(\frac{r}{L}\right)^{2\alpha} dr^2}{\frac{r^2}{l^2} - A\left(1 - \frac{B^2}{l^2}\right)r^\alpha} + (r^2 + AB^2 r^\alpha)d\varphi^2 + 2\,ABr^\alpha d\varphi dt$$

Where $A = \frac{\mu}{1-\omega^2}$ and $B = \omega l$, ω is the rotating parameter. Here, the finiteness behaves the same as Henneaux[2004] for scalars with slow fall-offs

$$M = -\frac{(\alpha - 2)(l^2 - B^2)^{\frac{\alpha}{2-\alpha}}(l^2 + B^2)\,A^{\frac{2}{2-\alpha}}}{16Gl^2}$$

$$J = \frac{(2-\alpha)AB}{8G\left(A(l^2-B^2)\right)^{\frac{\alpha}{\alpha-2}}}$$

Thermodynamics

It is necessary to study the thermodynamic of these solutions, in particular we'll see that the most interesting quantities are the entropy, that it is calculated by the Ward formula, $S = \frac{A}{4G}$ and the temperature, which in the future will be useful to see the first law $dM = T\,dS$.

D=2+1

$$T = \frac{r_0 \chi}{4\pi} \quad \rightarrow \quad S = \frac{2\pi r_0}{4G}$$

D=3+1

$$T = \frac{r_0 \chi}{8\pi} \quad \rightarrow \quad S = \frac{r_0^2}{4G} l_x l_y$$

CFT

$d = 2 \quad S \sim T$

$d = 3 \quad S \sim T^2$

$d = n \quad S \sim T^{n-1}$

Asymptotic symmetries

Now, our objective is to find the Killing vector in the Asymptotic region $L_{\xi} g_{\mu\nu} = O(r^{\sigma})$

We found that particular asymptotic Killing vector is [Detournay,2018]

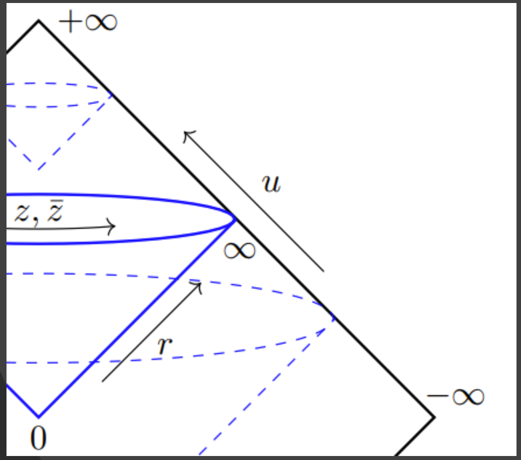
$$\vec{\xi} = \{f(u), -r\partial_{\phi} Y(u,\phi), Y(u,\phi)\}$$

Where the fall-offs conditions with $\alpha = 1$ are

$$\begin{aligned} h_{uu} &= O(r^2) & h_{ur} &= O(r) \\ h_{u\phi} &= O(r^2) & h_{rr} &= O(1) \\ h_{r\phi} &= O(1) & h_{\phi\phi} &= O(r) \end{aligned}$$

$$\Phi(u,r,\phi) = \sqrt{2\alpha}\, ln\left(\frac{r}{L}\right) + f_s(u,\phi)$$

Solutions with $V[\Phi] = \chi e^{-\sqrt{2}\Phi}$



The solutions found in D= 2+1 are

$$g_{uu} = f_{uu}^{(2)}(u,\phi)r^2 + f_{uu}^{(1)}(u,\phi)r \qquad g_{ur} = f_{ur}^{(1)}(u,\phi)r$$
$$g_{u\phi} = f_{u\phi}^{(2)}(u,\phi)r^2 + w(u,\phi)r \qquad g_{\phi\phi} = r^2 + f_{\phi\phi}^{(1)}(u,\phi)r$$

And thanks to the Einstein equations, the following relations are obtained

$$f_S(u,\phi) = \theta(\phi) \qquad f_{uu}^{(2)}(u,\phi) = C_2 e^{\sqrt{2}\theta(\phi)}$$
$$f_{ur}^{(1)}(u,\phi) = 2e^{\sqrt{2}\,\theta(\phi)} \qquad f_{u\phi}^{(2)}(u,\phi) = -4\left(\frac{L}{l}\right)^2 \partial_\phi e^{\sqrt{2}\theta(\phi)}$$

Solutions with $V[\Phi] = \chi e^{-\sqrt{2}\Phi}$

With this, we can now calculate the conserved charge

$$Q[Y(u,\phi)] = \int_0^{2\pi} d\phi(-e^{-\sqrt{2}\theta(\phi)}\, w(\phi)\, Y(u,\phi)\,)$$

And with this result, we can use the relation $\delta_1 Q_2 = \{Q_1, Q_2\}$

$$\{Q(Y_1), Q(Y_2)\} = Q([Y_2, Y_1])$$

And if we choose the rotation base $Y = \sum Y_n e^{in\phi}$ we also find Virasoro without central charge.

$$\{Y_n, Y_m\} = i(n-m)Y_{n+m}$$

Conclusion

- Holography is well understood for asymptotically AdS spaces.
- Realization of Asymptotically flat holography.
- Entropy behaves the same as some conformal theories.
- Asymptotic behavior leads to Witt algebra that connects with the possible conformal symmetry.
- We are trying to extend these results for arbitrary values of $V[\Phi] = e^{-\sqrt{2\alpha}\Phi}$

¡Thanks!

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