Tackling the infamous g^6 term of the QCD pressure

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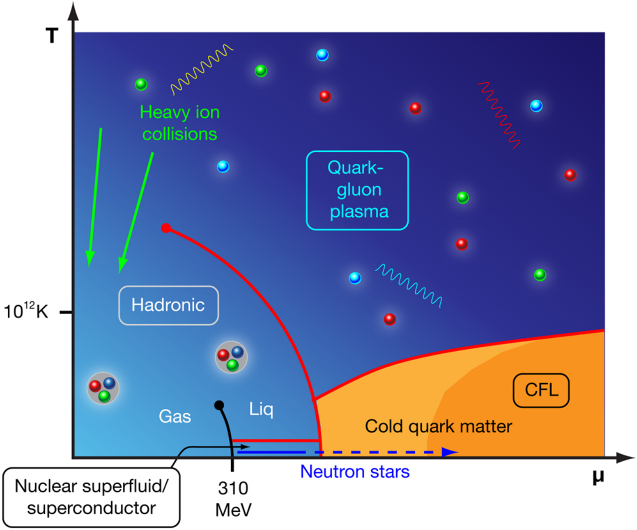
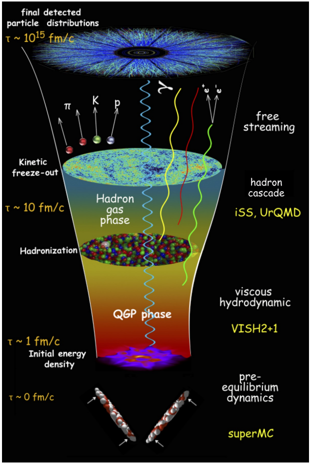
Motivation



T
Heavy ion
collisions
Quark-
gluon
plasma
10^{12}K
Hadronic
Gas
Liq
CFL
Cold quark matter
Nuclear superfluid/
superconductor
310
MeV
Neutron stars
μ

Motivation



Ongoing experiments:

- LHC: ALICE, ATLAS, CMS
- RHIC: Phenix, STAR

Interesting questions:

- Initial conditions
- Strongly interacting liquid-like QGP
- Thermal/Chemical freeze-out
- Relativistic hydrodynamic expansion

Hydrodynamics $\longrightarrow$ Pressure!

(among other things...)

Motivation

Matter in extreme physical conditions:

- Early Universe ($T \approx 10^{12} - 10^{15}$ K)
- Quark-gluon plasma in heavy ion collisions
- Inner cores of neutron stars ($n \approx 5 - 10\ n_s,\ n_s \approx 0.16\ \text{fm}^{-1}$)

$\longrightarrow$ Deconfined quarks and gluons at high enough temperature and/or density.

$$\boxed{\text{Asymptotic freedom} \Longrightarrow \alpha_s \ll 1 \Longrightarrow \text{Perturbation Theory}}$$

Equilibrium thermodynamics

The fundamental object to consider is the partition function ($\mu = 0$)

$$\mathcal{Z}(T) = \mathrm{Tr}[\exp(-\beta \hat{H})].$$

$\longrightarrow$ all thermodynamic information derived from $\mathcal{Z}$.

For QFT, (Euclidean) path integral representation:

$$\mathcal{Z}(T) = \int_{\mathrm{b.c}} \mathcal{D}[\mathrm{fields}] \exp\left\{-\frac{1}{\hbar}\int_0^{\beta\hbar} d\tau \int d^3x\, L\right\}.$$

- τ: compact dimension of length $\beta\hbar$
- Bosons: periodic in $\tau \Longrightarrow p_n = 2\pi n T$
- Fermions: anti-periodic in $\tau \Longrightarrow p_n = \pi T(2n+1)$

Equilibrium thermodynamics

Rest frame of heat bath $\Longrightarrow$ breaking of Lorentz invariance. Then, $(\hbar = c = k_B = 1)$

$$i \int d^4 p \longrightarrow T \sum_{n=-\infty}^{\infty} \int d^3 \vec{p}, \quad i \int d^4 x \longrightarrow \int_0^{\beta} d\tau \int d^3 \vec{x}.$$

Fourier analysis is carried out in the Matsubara formalism:

$$\phi(X) = \sum\!\!\!\!\!\!\int_P \tilde{\phi}(P) e^{iP\cdot X}, \quad \sum\!\!\!\!\!\!\int_P \equiv T \sum_{p_n} \int \frac{d^d \vec{p}}{(2\pi)^d},$$

with Euclidean four-momentum $P = (p_n, \vec{p})$.

QCD pressure

We are interested in the QCD pressure:

$$p(T) = \lim_{V\to\infty} \frac{T}{V} \log \int_{\text{periodic}} \mathcal{D}A_\mu^a \int_{\text{periodic}} \mathcal{D}\bar{c}^a \mathcal{D}c^a \int_{\text{anti-periodic}} \mathcal{D}\bar{\psi}\mathcal{D}\psi$$

$$\times \exp\left\{ -\int_0^{1/T} d\tau \int_{\vec{x}} \left[\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \sum_{i=1}^{N_f} \bar{\psi}_i \left(\gamma_\mu D_\mu + m\right) \psi_i + L_{gf} + L_{gh} \right] \right\} .$$

General strategy:

- Weak-coupling expansion in g
- $T \gg m$; N_f massless quarks and general $SU(N_c)$ gauge group
- Use general covariant R_ξ gauge; pressure is independent of ξ
- Generate diagrams $\rightarrow$ solve algebra $\rightarrow$ simplify $\rightarrow$ solve sum-ints

The Infrared Problem of hot QCD

Because of thermal interactions:

- Electric screening for $A_0^a \longrightarrow m_{\text{eff}} \sim gT$
- Magnetic screening for $A_i^a \longrightarrow m_{\text{eff}} \sim g^2 T$

Fermionic/bosonic largest loop expansion parameters (zero modes):

$$\epsilon_f \sim g^2, \quad \epsilon_b \sim \frac{g^2 T}{m_{\text{eff}}}.$$

Conclusions:

- Electrostatic contributions: $\epsilon_b \sim g \longrightarrow$ barely perturbative ✓
- Magnetostatic contributions: $\epsilon_b \sim 1 \longrightarrow$ perturbative χ
- Severe infrared divergences for the pressure at $\mathcal{O}(g^6)$ [Linde '80]

Electrostatic and Magnetostatic QCD

A hierarchy of energy scales shows up:

- hard scale T: energy of a typical particle
- soft scale gT: screening of electric fields
- ultrasoft scale g^2T: non-perturbative screening of magnetic fields

$\longrightarrow$ Define low energy (dimensionally reduced) effective field theories.

Electrostatic QCD (EQCD) for soft scale:

$$S_E = \frac{1}{T}\int d^3x \left\{ \frac{1}{4}\bar{F}^a_{ij}\bar{F}^a_{ij} + \frac{1}{2}(\mathcal{D}^{ab}_i \bar{A}^b_0)^2 + m_E^2\, Tr[\bar{A}_0^2] + \lambda_E\, Tr[\bar{A}_0^4] + \ldots \right\}.$$

Magnetostatic QCD (MQCD) for ultrasoft scale:

$$S_M = \frac{1}{T}\int d^3x \left\{ \frac{1}{4}\bar{\bar{F}}^a_{ij}\bar{\bar{F}}^a_{ij} + \ldots \right\}.$$

The QCD pressure

We can split the full QCD pressure as [Braaten/Nieto '96]

$$p_{QCD}(T) = p_h(T) + p_s(T) + p_u(T).$$

Here,

- $p_h \longrightarrow$ scale $T \longrightarrow$ weak-coupling expansion in g^2
- $p_s \longrightarrow$ scale $gT \longrightarrow$ weak-coupling expansion in g
- $p_u \longrightarrow$ scale $g^2 T \longrightarrow$ non-perturbative methods

The structure is the following:

$$p_h = T^4 \left[a_0 + a_1 g^2 + a_2 g^4 + a_3 g^6 + \dots\right] \qquad \text{4d thermal QCD}$$

$$p_s = m_E^3 T \left[b_0 + \dots + b_3 (g_E^2/m_E)^3 + \dots\right] \qquad \text{3d YM + adjoint Higgs}$$

$$p_u = g_M^6 T \left[c_0 + \dots\right] \qquad \text{3d pure YM}$$

The g^6 term of the QCD pressure

Each contributions starts at:

- p_h at order g^0 (Non-interacting gas)
- p_s at order g^3
- p_u at order g^6

Order $\mathcal{O}(g^6)$ pressure: physical leading order.

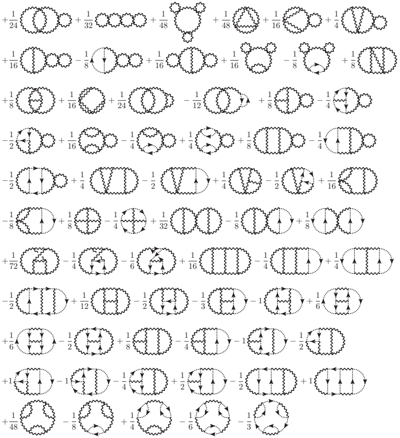
The contribution of order g^6 to p_h is not known:

$$p_h\Big|_{\mathcal{O}(g^6)} = \sum \text{(connected 4-loop vacuum diagrams in 4d thermal QCD)}\,.$$

Status of the computation:

1. Generate diagrams and solve algebra ✓
2. Simplify (reduce) calculation ✓ (?)
3. Solve remaining master sum-integrals !!

Results: Bosonic sector



- 65 bosonic Feynman diagrams
- Hardest: $2^9\, 6^6 \approx 24M$ terms
- Polynomial up to ξ^6
- Sum-ints to solve: 176119
- After shifts → 25047
- After symmetries → 945
- Sum all diagrams → 21
- Explicit gauge invariance in d dimensions is obtained [PN/Schröder in preparation]
- Finally: solve 21 sum-ints (IBP improvement?)

Status of the calculation

After simplification: solve 21 remaining 4-loop bosonic sum-integrals:

- 2 factorized as $(\text{1-loop})^4$: all known
- 3 factorized as $(\text{1-loop})^2\times(\text{2-loop})$: all known [see Andrei's talk]
- 6 factorized as $(\text{1-loop})\times(\text{3-loop})$: all known [PN/Schröder '22]
- 10 genuine 4-loops: 9 unknown

Now extending 3-loop methods to 4 loops: [Ghişoiu/Schröder '12]

$$U^{s_{8\ldots11}}_{s_{1\ldots7}} = \sum\!\!\!\!\!\!\!\int_P \frac{(P_0)^{s_8}}{(P^2)^{s_1}} \Pi^{s_9}_{s_2 s_5} \Pi^{s_{10}}_{s_3 s_6} \Pi^{s_{11}}_{s_4 s_7}, \quad \Pi^c_{ab} \equiv \sum\!\!\!\!\!\!\!\int_Q \frac{(Q_0)^c}{(Q^2)^a[(P-Q)^2]^b}$$

General strategy:

$$U = U^{\text{finite}} + U^{\text{divergent}} + U^{\text{zero modes}}.$$

Conclusions

- Thermal QCD is the framework for addressing a wide range of extreme phenomena, from cosmology to particle physics and astrophysics
- An interesting infrared structure emerges in non-Abelian gauge theories, making an interplay between perturbative and non-perturbative methods necessary
- Calculation methods at finite temperature are far less automated, as, for example, zero-temperature particle physics
- For the g^6 term, probably need to tackle 4-loop sum-integral structures never met before in the literature