LOG CORRECTIONS, ENTANGLEMENT, AND UV CUTOFFS IN DE SITTER

Felipe Díaz, in Collaboration with G. Arenas–Henriquez and P. Sundell

JHEP08(2022)261

Universidad Andrés Bello
Facultad de Ciencias Exactas

Outline

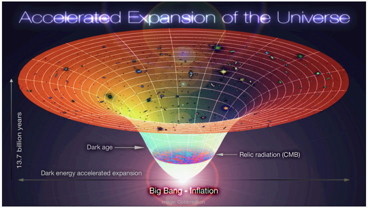
- Orbifolded dS spacetime
- Entropies and Free Energy
- Asymptotic Symmetry Algebra
- Log corrections & Renyi entropy
- UV Cutoff
- cod2 dS Holography
- Conclusions

De Sitter

Universe is expanding at an accelerating pace [Perlmutter et al 98, Riess et al 98]

If the expansion persist we will end up in a lonely cold world with the Milky Way only mergin with Andromeda [Nagamine, Loeb 02]

Other interesting era is the early stage of the universe that may be describe by an inflationary process [Guth 80]



De Sitter

This early/late stages of the universe may be described by dS spacetime, which is a maximally symmetric solution to Einstein gravity with positive cosmological constant

$$I_{\mathrm{EH}} = \frac{1}{16\pi G_d} \int d^d x \sqrt{g}\,(R - 2\Lambda) \ , \qquad \Lambda = +\frac{(d-1)(d-2)}{2\ell^2} \ ,$$

with field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0 \ ,$$

De Sitter

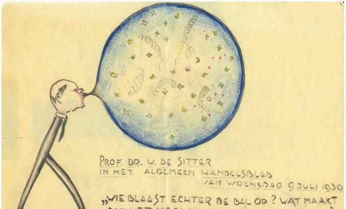
The solution can bee seen as timelike hyperboloid $X^A X_A = \ell^2$ embbeded into $\mathbb{R}^{1,d}$

$$ds^2 = -dX_0^2 + \sum_i^d dX_i^2 \ ,$$

with topology $\mathbb{R} \times S^d$ and manifest $O(1,d)$ symmetries.

It can be foliated by $(d-1)$-spheres along the global time coordinate

$$ds^2 = -dT^2 + \ell^2 \cosh^2(T/\ell) d\Omega_{d-1}^2$$



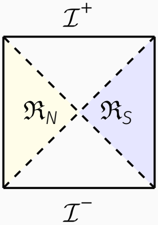
De Sitter

The exponential inflation prevents a static observer from communicating with the entire geometry.

In $d = 4$, observers can be described by the line element

$$ds^2 = \cos^2\theta\left[-\left(1-\frac{r^2}{\ell^2}\right)dt^2 + \frac{dr^2}{1-\frac{r^2}{\ell^2}}\right] + \ell^2 d\Omega_2^2$$

corresponding to an S^2 fibration along dS_2.



De Sitter

The solution has a Killing horizon $\mathcal{H}$ with an S^2 topology located at $r = \pm\ell$, described by the Killing vector field

$$\vec{\chi} = \vec{\partial}_t \,, \qquad \chi^2 = ds^2(\vec{\chi}, \vec{\chi}) = \frac{\cos^2\theta}{\ell^2}(r^2 - \ell^2) \,,$$

and has a surface gravity

$$\kappa^2 = -\lim_{\chi^2 \to 0} \frac{\nabla_\mu \chi^2 \nabla^2 \chi^2}{4\chi^2} = \frac{1}{\ell^2}$$

De Sitter

Treating dS path integral in the Euclidean regime by using Wick rotation methods, the Euclidean field theory has a Hawking temperature [Figari, Hoegh-Krohn, Nappi 75] with conjugated entropy [Gibbons, Hawking 77]

$$T_{\mathrm{H}} = \frac{\hbar}{2\pi\ell}\,, \qquad S_{\mathrm{GH}} = \frac{A_{\mathcal{H}}}{4\hbar G_d}\,.$$

associated to the Cosmological horizon $\mathcal{H}$

This is an observer dependant entropy!

De Sitter

Cosmological observations indicate that we are entering a phase dominated by a remarkably small, yet non-vanishing, positive cosmological constant. It's tempting to put it to zero in Einstein gravity.

Nonetheless, is it hard to understand how such small CC could originate this enormous entropy $S_{\mathrm{GH}} \sim 10^{10^{120}}$.

Positive CC has many consequences, such as finite bound on entropy of the observable universe [Dyson, Kleban, Susskind 02] or quantization of gravity in this background [Witten 01]

De Sitter

It has been also argued by Banks using M-theory methods that the Hilbert space of quantum gravity theory on a asymptotically dS background would be finite dimensional

$$S_{\mathrm{GH}} = \log \dim H \ ,$$

which would implies problems in quantization of EH action.

We will try to see this property by considering a more general geometry

Orbifolding the Horizon

By considering a $\mathbb{Z}_q$ action on dS_4 we [Arias, D, Sundell 20; Arias, D, Olea, Sundell 20] obtained that the Renyi entropy associated to a single observer at the semiclassical level

$$S_q = \frac{\log \mathrm{Tr} \rho^q}{q-1} = 2 S_{\mathrm{GH}} \ , \quad \forall q$$

being a maximally mixed entangled state, in agreement with [Dong, Silverstien, Torroba 19] (and later by [Chandrasekaran, Longo, Pennington, Witten 22]), up to factors.

Orbifolding the horizon

The $\mathbb{Z}_q$ action makes a conically singular geometry and the Einstein tensor contain a term of the form [Fursave, Solodukhin 95]

$$\left(1-\frac{1}{q}\right)\nabla^2 \log\rho \sim \left(1-\frac{1}{q}\right)\delta(\rho)\,, \qquad \rho = \ell\cos\theta$$

such that for consistency of the variational principle to hold we need

$$T^{(q)}_{ij}(y) = \frac{1}{4G_4}\left(1-\frac{1}{q}\right)h^{(q)}_{ij}(0,y)$$

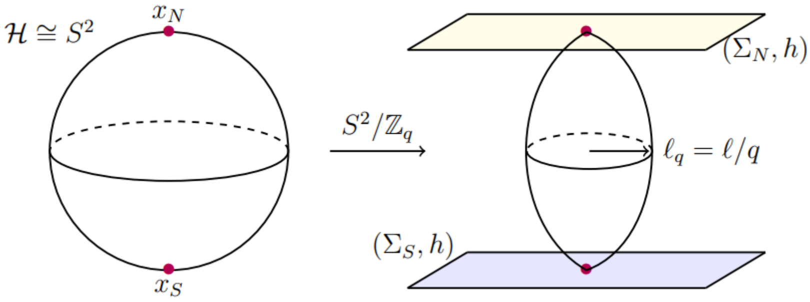
which can be achieved by coupling a pair of Nambu–Goto actions to Einstein gravity

$$I = \frac{1}{16\pi G_4}\int_M d^4x\sqrt{g}\left(R-6\ell^{-2}\right) - \frac{1}{4G}\sum_{i=N,S}\left(1-\frac{1}{q_i}\right)\int_{\Sigma_i}\sqrt{h}$$

Orbifolding the Horizon

Where we take $q_N = q_S = q$, and $\Sigma_{N,S}$ are cod-2 antipodal defects located at $\theta = 0, \pi$ with geometry of dS_2.

The thermodynamics properties and original dS space is recovered in the tensionless limit $q \to 1$.



Orbifolding the Horizon

Taking the modular Hamiltonian of a single observer $\rho = \exp(-H)$, and using replica tricks we can compute $\rho^q = \exp(-qH)$, and interpret q as the inverse of the temperature in a modular partition function [Calabrese, Cardy 04]

$$Z = \mathrm{Tr}\rho^q = \mathrm{Tr}\exp(-qH) ,$$

with moduar Free energy

$$F_q = -\frac{1}{q}\log Z = -\frac{1}{q}\log \mathrm{Tr}\rho^q$$

which can be computed in the orbifold geometry [Arias, D, Olea, Sundell 20]

Orbifolding the Horizon

Using the Replica trick on a single observer [Arias, D, Sundell 20]

$$F_q = -q \log \mathrm{Tr}\rho^q = -q \log \frac{Z[S^4_q]}{Z^q[S^4]} = 2\left(1-\frac{1}{q}\right)\frac{\pi\ell^2}{\hbar G_4} ,$$

We can use the Baez relation [Baez 11]

$$S_q = \left(1-\frac{1}{q}\right)^{-1} F_q = 2S_{\mathrm{GH}} .$$

This entropy was also understood by using CFT description of the defects with central charge

$$c_q = \frac{3\ell^2}{G_4}\left(1-\frac{1}{q}\right)$$

and using Cardy entropy [Arias, D, Olea, Sundell 20].

Asymptotic Symmetry Algebra

Other CFT that we have not analyze is the one living at $\mathcal{H}$ introduced by Carlip [Carlip 99] showing that the algebra of asymptotic charges of GR, under suitable classes of boundary condition, correspond to a copy of the Virasoro algebra

In order to see this we can use the Barnich–Brandt formalis where the charges [Barnich, Brandt 01]

$$\mathcal{Q}_\zeta[h;g] = \frac{1}{8\pi G_4}\oint k_\zeta[h,g]$$

where k_ζ is a covariant two-form and h is a metric g fluctuation, satisfy the Dirac bracket algebra

$$\{\mathcal{Q}_\zeta, \mathcal{Q}_{\zeta'}\} = \mathcal{Q}_{[\zeta,\zeta']} + \frac{1}{8\pi G_4}\oint k_\zeta[\mathcal{L}_{\zeta'}g, g]$$

Asymptotic Symmetry Algebra

A one-parameter family of $\mathrm{Diff}(S^1)$ vector fields which preserve the horizon structure [Carlip 99, Silva 02]

$$\zeta_m^\mu = T_m \chi^\mu + R_m \rho^\mu \ , \qquad \rho_\mu = -\frac{1}{2\kappa} \nabla_\mu \chi^2$$

satisfying the boundary conditions

$$\delta\chi^2 = 0 \ , \qquad \chi^\mu t^\nu \delta g_{\mu\nu} = 0 \ , \qquad \delta\rho_\mu = 0 \ ,$$

ρ_μ is orthogonal to the orbits of χ^μ , and t^μ is unit spacelike vector tangent to the horizon.

This condition ensure that $\vec{\chi}$ remains null and normal at the horizon.

This are horizon analogs to the fall-off conditions one usually imposes at infinity

Asymptotic Symmetry Algebra

This asymptotic KV's satisfy the de Witt algebra

$$[\zeta_m, \zeta_n]^\mu = (m-n)\zeta^\mu_{m+n}$$

and

$$T_m = -\frac{\ell}{\alpha}\exp(im(\phi - \alpha t/\ell))\ , \qquad R_m = \frac{\alpha}{\ell\kappa} im T_m\ , \qquad \alpha \in \mathbb{R}$$

to satisfy the aforementioned conditions

Asymptotic Symmetry Algebra

The resulting symmetry group is a single copy of Virasoro algebra with central extension

$$\frac{1}{8\pi G_4}\oint_{\mathcal{H}} k_{\zeta_m}[\mathcal{L}_{\zeta_n} g; g] = -i\left(\frac{\ell^2}{4q\alpha G_4}\right)\left(\alpha^2 m^3 + 2m\right)\delta_{m+n,0} \ ,$$

such that promoting Dirack brackets to quantum commutators $\{\cdot,\cdot\} \to \frac{1}{i\hbar}[\cdot,\cdot]$ and defining quantum commutators

$$\hbar L_m = \mathcal{Q}_m + \frac{3\ell^2}{8G_4}\frac{\alpha}{q}\delta_{m,0}$$

yields to

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}$$

where the central charge

$$c = 12i \lim_{r\to\ell} \mathcal{Q}_{\zeta_m}[\mathcal{L}_{\zeta_n} g; g]|_{m^3} = \frac{3\ell^2}{\hbar G_4}\frac{\alpha}{q}$$

and the α parameter has been choose to unity by hand in many different solutions in order to recover the Killing horizon entropy via Cardy formula

$$S_{\mathrm{C}} = 2\pi\sqrt{\frac{c}{6}\Delta} = \frac{\pi^2}{3}cT\,,$$

where T is the temperature of chiral modes near the horizon, and Δ is the eigenvalue of the zero mode L_0 .

In dS we can use the near horizon temperature [Bunch, Davies 78]

$$T_{\mathrm{BD}} = \frac{1}{2\pi} \Rightarrow S_{\mathrm{C}} = \frac{\alpha}{q}S_{\mathrm{GH}}$$

Instead of choosing this value, we recover the previous result of interpreting GH entropy as the Renyi entropy of a maximally mixed entangled state by re-scaling

$$\alpha = 2\gamma(q-1) \Rightarrow S_{\mathrm{C}} = \left(1 - \frac{1}{q}\right)\gamma S_{\mathrm{GH}} \Rightarrow S_q = \gamma S_{\mathrm{GH}}$$

As was obtained previously by replica methods

Log Corrections

Cardy's result consist on noticing that the quantity

$$\mathrm{Tr}\exp\left(2\pi i(L_0-\frac{c}{24})\tau\right)=\exp\left(\frac{\pi c}{6}\tau_2\right)Z(\tau)$$

is modular invariant, and invariant under $\tau\to -1/\tau$ exchanging the torus radius. Using same ingredients, Carlip has shown that the first Gaussian fluctuation to the CFT partition function leads to

$$S_{\mathrm{C}}\sim S_{\mathrm{C}}^{(0)}-\frac{3}{2}\log S_{\mathrm{C}}^{(0)}+\log c$$

We get

$$S_{\mathrm{C}}\sim\left(1-\frac{1}{q}\right)\gamma S_{\mathrm{GH}}-\frac{3}{2}\log\left[\left(1-\frac{1}{q}\right)^{\frac{1}{3}}\gamma S_{\mathrm{GH}}\right],$$

Log corrections and Reny entropy

Now the tensionless limit has a divergent log term in the Free energy. Inverting Baez relation as before, we get the Renyi entropy

$$S_q = \gamma S_{\mathrm{GH}} - \frac{3}{2}\left(\frac{q}{q-1}\right)\log\left[\left(1-\frac{1}{q}\right)^{\frac{1}{3}}\gamma S_{\mathrm{GH}}\right],$$

with limits

$$S_{\mathrm{E}} = \lim_{q\to 1} S_q = \gamma S_{\mathrm{GH}} - \frac{3}{2}\lim_{q\to 1}\frac{q}{q-1}\log\left[\left(1-\frac{1}{q}\right)^{\frac{1}{3}}\gamma S_{\mathrm{GH}}\right],$$

$$S_{\infty} = \lim_{q\to\infty} S_q = \gamma S_{\mathrm{GH}} - \frac{3}{2}\log\gamma S_{\mathrm{GH}},$$

$$S_0 = \lim_{q\to 0} S_q = \gamma S_{\mathrm{GH}},$$

Log Corrections and Renyi Entropy

The limits $q \to \infty$ and $q \to 0$ of the Renyi entropy, referred to, respectively, as the min-entropy and the max-, or Hartley, entropy, yield information of the dimensionality of the density matrix ρ, namely

$$S_\infty = -\log \lambda_1 \ ,$$
$$S_0 = \log \mathcal{D} \ ,$$

where λ_1 is the largest eigenvalue that is active in ρ, and $\mathcal{D}$ corresponds to the number of non-vanishing elements of ρ. In terms of and we get

$$\lambda_1 = (\gamma S_{\mathrm{GH}})^{\frac{3}{2}} \exp\{-\gamma S_{\mathrm{GH}}\} \ ,$$
$$\mathcal{D} = \exp\{\gamma S_{\mathrm{GH}}\} \ ,$$

We have obtained the Banks proposal

$$\gamma S_{\mathrm{GH}} = \log \mathcal{D} \ ,$$

when we consider this first quantum correction.

The $q \to 1$ limit correspond to the Entanglement entropy which now acquires a log divergence that can be rewritten as

$$S_{\mathrm{E}} = \frac{A_{\mathcal{H}}}{\delta_q^2} + \gamma S_{\mathrm{GH}} \,,$$

where δ_q can be identified with the UV cutoff appearing in EE of QFT's [Srednicki 93] that reads

$$\delta_q^2 = \frac{2\gamma^2 A_{\mathcal{H}}}{3}\left(\frac{1-q}{q}\right)\log\left[\left(1-\frac{1}{q}\right)^{\frac{1}{3}}\frac{\gamma A_{\mathcal{H}}}{4\hbar G_4}\right]^{-1}$$

or

$$\left(1-\frac{1}{q}\right) = \frac{\delta_q^2}{2\gamma A_{\mathcal{H}}} W_0\left(\frac{128\hbar^3 G_4^3}{\gamma^2 A_{\mathcal{H}}^2 \delta_q^2}\right)$$

UV Cutoff

$W_0(x)$ is the Lambert W function, such that the tensionless limit $q \to 1$ corresponds to vanishing cutoff, viz.

$$\lim_{q \to 1} \delta_q = 0 \ ,$$

and the IR limit

$$\lim_{q \to 0} \delta_q \to \infty \ ,$$

leads to the Banks formula previosuly mentioned.

Cod2 dS Holography

In the large q limit, the geometry reduces to a three-dimensional global dS spacetime where the defects $\Sigma_{N,S}$ are mapped to the spacelike asymptopia $\mathcal{I}^{\pm}$

$$ds^2 = -d\tau^2 + \cosh^2(\tau/\ell)d\Omega_2^2$$

and the central charge of the defects recovers the result of the dS/CFT correspondence [Strominger 01]

$$c(\Sigma_N \cup \Sigma_S) = \frac{3\ell}{2\hbar G_3}\,, \qquad G_4 = 4\ell G_3$$

such that the Cardt entropy recovers the entropy of dS_3 and the dual CFT has zero temperature $T = q^{-1} \to 0$ in agreement with previous dS/CFT results [Klemm 01, Klemm, Vanzo 02; Dyson, Lindesay, Susskind 02]

Now considering the log corrections one gets

$$S_{\mathrm{C}} \xrightarrow{q\to\infty} \frac{\pi\ell}{2\hbar G_3} - 3\log\frac{\pi\ell}{2\hbar G_3} + \mathrm{const}\,,$$

in agreement with recent results [Anninos, Denef, Law, Sun 20; Silverstein et al. 21, Witten et al. 22]

Conclusions

- Considering the manifold dS/$\mathbb{Z}_q$ allows to use Replica tricks to compute Renyi entropy
- Considering Log corrections to horizon's CFT and computing the Renyi entropy one gets a UV divergent EE
- The IR limit recovers the Banks proposal for the dimensionality of the Hilbert space
- The proposed cod2 holography also recovers the right log corrections of dS_3

Gracias

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