Aspects of Chiral Symmetry Breaking in RQED

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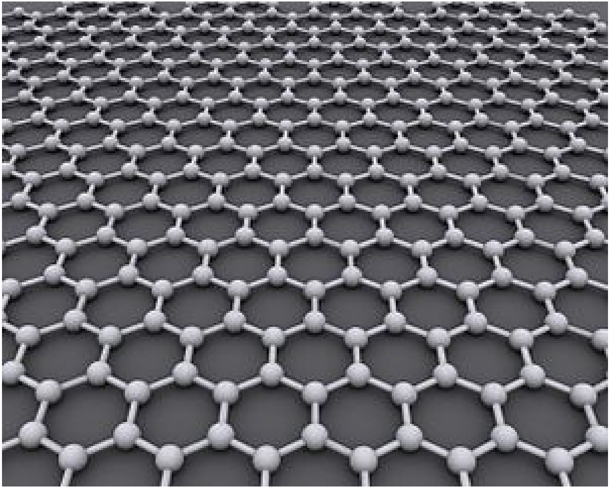
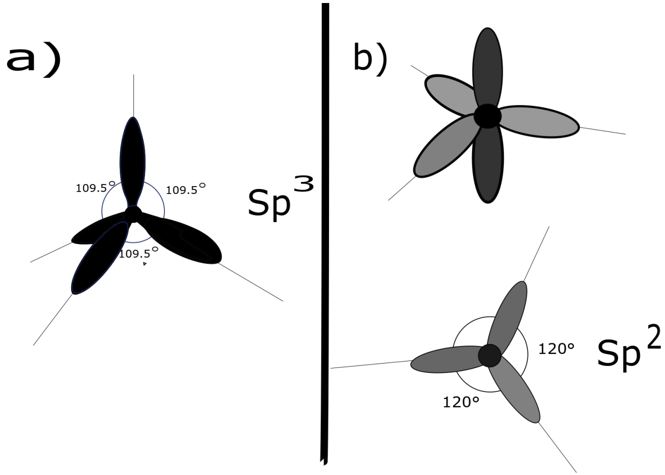


Mother of all graphitic structures

Mother of all graphitic structures *a la mexicana*



a)
109.5°
109.5°
109.5°
Sp3
b)
120°
120°
Sp2

Crystalline structure

Graphene

$$a_1 = \frac{a}{2}\left(3; \sqrt{3}\right);$$

$$a_2 = \frac{a}{2}\left(3 \quad \sqrt{3}\right);$$

$$_1 = \frac{a}{2}\quad 1; \sqrt{3}\quad ; \qquad\qquad _2 = \frac{a}{2}\quad 1; \quad \sqrt{3}\quad ; \qquad\qquad _3 = (\quad a; 0):$$

Reciprocal lattice

Graphene

$$a_i \quad b_j = 2 \quad {}_{ij}.$$

$$b_1 = \quad \frac{4}{a\sqrt{3}}, 0 \quad ; \qquad b_2 = \quad \frac{2}{a\sqrt{3}}, \frac{2}{a} \quad .$$

Dirac points

Reciprocal lattice

$$K' = \frac{2}{3a}; \frac{2}{3\sqrt{3}a} \quad ; \qquad K = \frac{2}{3a}; \frac{2}{3\sqrt{3}a} \quad ;$$

Lineal combination

$$E \;\; {}_A(r_A) = t \;\; {}_B(r_B) + t \;\; {}_B(r_B \quad a_2) + t \;\; {}_B(r_B \quad a_1);$$
$$E \;\; {}_B(r_B) = t \;\; {}_A(r_A) + t \;\; {}_A(r_A + a_1) + t \;\; {}_A(r_A + a_2);$$

Lineal combination

$$E\ \ {}_A(r_A) = t\ \ {}_B(r_B) + t\ \ {}_B(r_B\quad a_2) + t\ \ {}_B(r_B\quad a_1);$$
$$E\ \ {}_B(r_B) = t\ \ {}_A(r_A) + t\ \ {}_A(r_A + a_1) + t\ \ {}_A(r_A + a_2);$$

Bloch's Theorem

$${}_k(r + T) = e^{ik\ \ T}\ \ {}_k(r);$$

T is a crystal translation

Lineal combination

$$E \quad _A(r_A) = t \quad _B(r_B) + t \quad _B(r_B \quad a_2) + t \quad _B(r_B \quad a_1);$$
$$E \quad _B(r_B) = t \quad _A(r_A) + t \quad _A(r_A + a_1) + t \quad _A(r_A + a_2);$$

Bloch's Theorem

$$_k(r + T) = e^{ik \quad T} \quad _k(r);$$

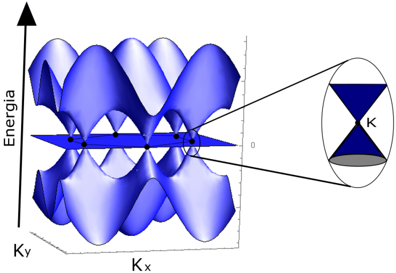
T is a crystal translation

$$\hat{H}(k) \quad \begin{matrix} A \\ B \end{matrix} \quad = \quad \begin{matrix} 0 & t \, \mathrm{P} \; e^{\quad ik} \\ t \, \mathrm{P} \; e^{+ik} & 0 \end{matrix} \quad \begin{matrix} A \\ B \end{matrix} \; :$$

Graphene

$$E = t \sum e^{ik}$$

$$= t \sqrt{3 + \cos(\sqrt{3}k_y a) + 4\cos(\sqrt{3}k_y a/2)\cos(3k_x a/2)}.$$



Effective Halmitonian
Graphene

Taylor expanding the Hamiltonian around the Dirac points $K = k + q$ with $|q| \ll |k|$,

$$\hat{H}_{K,K'} = \hbar v_F \begin{pmatrix} 0 & q_x \mp i q_y \\ q_x \pm i q_y & 0 \end{pmatrix},$$

where

$$v_F = \frac{3at}{2\hbar} \simeq \frac{1}{300} c.$$

Taylor expanding the Hamiltonian around the Dirac points $K = k + q$ with $|q| \quad |k|$,

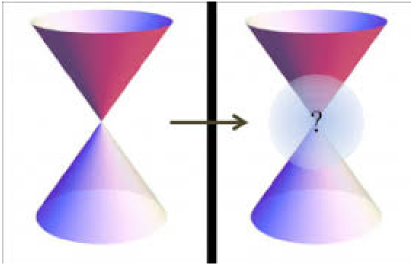
$$\hat{H}_{K;K'} = \hbar v_F \begin{pmatrix} 0 & q_x \quad iq_y \\ q_x \quad iq_y & 0 \end{pmatrix} ;$$

where

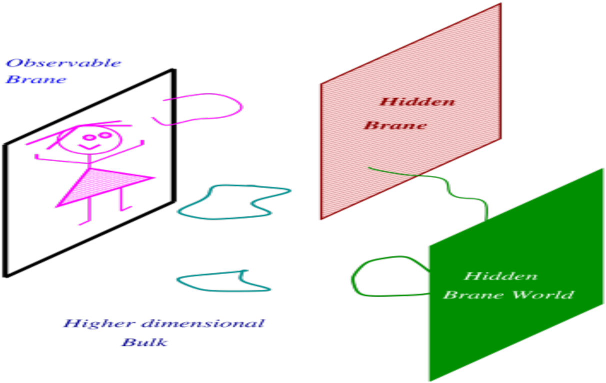
$$v_F = \frac{3at}{2\hbar} \simeq \frac{1}{300} c .$$

$$\hat{H}_{K'} = {}_1 H_K \, {}_1$$

Big question
Mind the gap

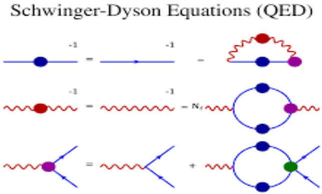


From QED to RQED
Braneworld inspired



SDEs
Infinite tower

SDE form an infinite, tower of nonlinear relations among the Green functions of a given QFT



We express the electron propagator $S(k)$ as

$$S(k) = \frac{F(k^2;\ \ ^2)}{i \quad k + M(k^2;\ \ ^2)}:$$

(Tree level $!$ $F(k^2;\ \ ^2) = 1$ and $M(k^2;\ \ ^2) = m_0$).

The photon propagator takes the form

$$\begin{aligned} (q) &= \quad ^T (q) + \ ^L (q) \\ ^T (q) &= \frac{\%}{q} \qquad \frac{q \ q}{q^2} \quad ; \\ ^L (q) &= \% \ \frac{q \ q}{q^3} ; \end{aligned}$$

where $1 \quad _e,$ $\quad , \% = \quad [\ \ ] = (4 \quad) \ ^e$. It is important to notice that RQED ($_e = 1=2$) yields $\ = \% = 1=2$ and $\ = \ =2$.

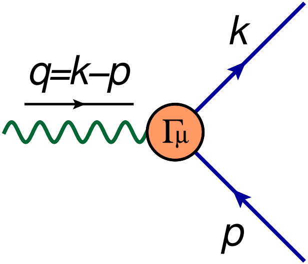


Figure: Diagrammatic representation of the full electron-photon vertex $_{\mu}(k; p)$, with momentum flow indicated.

The electron-photon vertex can be written in terms of twelve independent spin structures.

The WFGTI associated with this vertex takes the form

$$iq \quad (k;p) = S \ {}^{1}(k) \quad S \ {}^{1}(p)\,; \qquad q = k \quad p:$$

This identity allows us to split

$$(k;p) = \ {}^{L}(k;p) + \ {}^{T}(k;p):$$

The longitudinal part ${}^{L}(k;p)$

$${}^{L}(k;p) = \ {}_{1}(k;p) \quad + \ {}_{2}(k;p)t \quad t \quad i \ {}_{3}(k;p)t \ ;$$

with $t = k + p$.

Thus,

$$
\begin{aligned}
{}_1(k;p) &= \frac{1}{2}\ \frac{1}{F(k^2;\ \ ^2)} + \frac{1}{F(p^2;\ \ ^2)}\ ; \\
{}_2(k;p) &= \frac{1}{2}\ \frac{1}{F(k^2;\ \ ^2)} \quad \frac{1}{F(p^2;\ \ ^2)} \quad \frac{1}{k^2 \quad p^2}\ ; \\
{}_3(k;p) &= \quad \frac{M\ (k^2;\ \ ^2)}{F(k^2;\ \ ^2)} \quad \frac{M\ (p^2;\ \ ^2)}{F(p^2;\ \ ^2)} \quad \frac{1}{k^2 \quad p^2}\ ;
\end{aligned}
$$

The transverse part ${}^{T}(k;p)$ of the vertex satis es

$$q \quad {}^{T}(k;p) = 0 :$$

It can be expanded out as

$$ {}^{T}(k;p) = \sum_{i=1}^{8} {}_{i}(k;p) T^{i}(k;p) :$$

Using

$$ = \frac{i}{2} [\quad ; \quad] ;$$

we use

$$T^1(k;p) = i\,[p\ (k\ \ q)\ \ \ k\ (p\ \ q)]\ ;$$

$$T^2(k;p) = [\,p\ (k\ \ q)\ \ \ k\ (p\ \ q)]\,(\ \ \ t)\ ;$$

$$T^3(k;p) = q^2\ \ \ \ q\ (\ \ \ q)\ ;$$

$$T^4(k;p) = iq^2\,[\ \ (\ \ t)\ \ t\ ] + 2q\ p\ k\ \ \ ;$$

$$T^5(k;p) = \ \ q\ \ ;$$

$$T^6(k;p) = \ \ k^2\ \ p^2\ + t\ (\ \ \ q)\ ;$$

$$T^7(k;p) = \frac{i}{2}(k^2\ \ p^2)\,[\ \ (\ \ t)\ \ t\ ] + t\ p\ k\ \ \ ;$$

$$T^8(k;p) = \ i\ \ p\ k\ \ \ \ p\ (\ \ k) + k\ (\ \ p)\ ;$$

The full vertex must have the same transformation properties as the bare vertex under C.

This requires all the $_i$ should be symmetric under $k \,\$\, p$, except $_4$ and $_6$, which are

$$_i(k;p) = \; _i(p;k)\,; \qquad i = 1;2;3;5;7;8;$$
$$_i(k;p) = \quad _i(p;k)\,; \qquad i = 4;6:$$

For the longitudinal components, $_1(k;p)$, $_2(k;p)$ and $_3(k;p)$ are symmetric.

An educated proposal is

$$
\begin{aligned}
{}_1(k;p) &= \frac{a_1}{(k^2+p^2)}\;{}_3(k;p)\,;\\
{}_2(k;p) &= \frac{2a_2}{(k^2+p^2)}\;{}_2(k;p)\,;\\
{}_3(k;p) &= 2a_3\;\;{}_2(k;p)\,;\\
{}_4(k;p) &= \frac{a_4(k^2\quad p^2)}{4k^2p^2}\;{}_3(k;p)\,;\\
{}_5(k;p) &= \quad a_5\;\;{}_3(k;p)\,;\\
{}_6(k;p) &= \quad\frac{2a_6(k^2+p^2)}{(k^2\quad p^2)}\;{}_2(k;p)\,;\\
{}_7(k;p) &= \quad\frac{a_4q^2}{2k^2p^2}+\frac{a_7}{k^2+p^2}\quad{}_3(k;p)\,;\\
{}_8(k;p) &= 2a_8\;\;{}_2(k;p)\,:
\end{aligned}
$$

Effective modi cation:

$$_6(k;p) \quad = \quad a_6 \frac{(k^2 \quad p^2)}{(k^2 + p^2)} \quad _2(k;p) \; :$$

The one-loop corrections to the electron-photon vertex in the asymptotic limitp^2 k^2 m$_0^2$ is

$$^T(k;p) \quad \overset{p^2}{=} \, ^{k^2} \quad \frac{}{8 \; p^2} \quad \frac{1}{3} \; \log \; \frac{p^2}{k^2} \quad T^{asy} \; ;$$

with

$$T^{asy} = p^2 \qquad p \qquad p \, :$$

Hence, in this limit,

$$^T(k;p) \quad \overset{p^2}{=} \, ^{k^2} \quad \frac{1}{2} \; \frac{1}{F(k^2)} \quad \frac{1}{F(p^2)} \; \frac{T^{asy}}{k^2 \quad p^2} \, :$$

On the other hand, the leading structure of the transverse vertex is

$$^{T}(k;p) \quad \overset{p^2 = k^2}{} \quad (\ _3 + \ _6)\, T^{asy} :$$

Moreover,

$$T^{3\,asy} = T^{6\,asy} \quad T^{asy} :$$

As the simplest construction of the transverse vertex, we can single out $_3(k;p)$ and $_6(k;p)$

$$\begin{aligned} _3(k;p) &= a_3 \quad \frac{1}{F(k^2)} \quad \frac{1}{F(p^2)} \quad \frac{1}{k^2 \quad p^2} ; \\ _6(k;p) &= a_6 \quad \frac{1}{F(k^2)} \quad \frac{1}{F(p^2)} \quad \frac{1}{k^2 + p^2} : \end{aligned}$$

The one-loop behavior of the vertex in the asymptotic limit requires

$$a_3 \quad a_6 = 1{=}2 :$$

The SDE for the electron propagator is

Figure: The gap equation for the electron propagator.

Mathematically,

$$S\ \ ^{1}(k) \quad = \quad S_0\ ^{1}(k) + \frac{\quad}{2\ \ ^{2}} \int_E d^3p \quad S(p) \quad (k;p) \qquad (q)\,;$$

Gap equation

SDE for the fermion propagator

It is equivalent to the coupled integral equations

$$\frac{M(k^2)}{F(k^2)} = m_0 + \frac{\%}{2\ ^2} \mathrm{Z}_E \frac{d^3p}{q^3} \frac{F(p^2)}{p^2 + M^2(p^2)} \frac{1}{F(k^2)} \, \mathrm{n} \; M(p^2)\, q \;\; k \;\; M \;\; (k^2)\, q \;\; p \; \mathrm{o}$$

$$+ \frac{\%}{2\ ^2} \mathrm{Z}_E d^3p \frac{F(p^2)}{p^2 + M^2(p^2)} G_M(k;p);$$

$$\frac{1}{F(k^2)} = 1 \quad \frac{\%}{2\ ^2} \mathrm{Z}_E \frac{d^3p}{q^3} \frac{F(p^2)}{p^2 + M^2(p^2)} \frac{1}{F(k^2)} \, \mathrm{n} \; q \;\; p + M(k^2) M(p^2) \frac{q \;\; k}{k^2} \, \mathrm{o}$$

$$+ \frac{\%}{2\ ^2} \mathrm{Z}_E \frac{d^3p}{k^2} \frac{F(p^2)}{p^2 + M^2(p^2)} G_F(k;p);$$

Gap equation

SDE for the fermion propagator

with

$$
\begin{aligned}
\frac{G_M\ (k;p)}{{}_0(q^2)} &= \ (3 \qquad)M\ (p^2)\ {}_1 + \quad t^2 \ \frac{(k^2 \quad p^2)^2}{q^2} \quad M\ (p^2)\ {}_2 \\
& \qquad t \quad p \ \frac{(q \quad p)(k^2 \quad p^2)}{q^2} \qquad {}_3 + r\ (k;p)\ {}_1 + 2r\ (k;p)M\ (p^2)\ {}_2 \\
& \quad + 2q^2 M\ (p^2)\ {}_3 \quad 2\ (k^2 \quad p^2)(q \quad p) + r\ (k;p) \quad {}_4 \\
& \qquad 2(q \quad p)\ {}_5 \quad 2(k^2 \quad p^2)M\ (p^2)\ {}_6 \\
& \qquad (k^2 \quad p^2)(t \quad p) \quad r \quad (k;p) \quad {}_7\,;
\end{aligned}
$$

where we have de ned

$$
\begin{aligned}
r\ (k;p) &= k^2 p^2 \quad (k \quad p)^2\,; \\
u(k;p) &= 2\,k \quad p \quad \frac{r\ (k;p)}{q^2}\,:
\end{aligned}
$$

Gap equation

SDE for the fermion propagator

$$
\begin{aligned}
\frac{G_F(k;p)}{{}_0(q^2)} = \; & [(1 \quad 3\;)k \;\; p + 2 \;\; u(k;p)] \;\; {}_1 \\
& (k \;\; p)t^2 + 2r\;(k;p) \quad \frac{(k \;\; p)(k^2 \quad p^2)^2}{q^2} \qquad {}_2 \\
& t \;\; k \quad \frac{(q \;\; k)(k^2 \quad p^2)}{q^2} \quad M\;(p^2) \;\; {}_3 \\
& + r\;(k;p)M\;(p^2)\;{}_1 \quad (k^2 + p^2)r\;(k;p)\;{}_2 \\
& + 2(q \;\; k)(q \;\; p)\;{}_3 + 2\;\; q^2(t \;\; k) \;\; r \;\; (k;p) \;\; M\;(p^2)\;{}_4 \\
& + 2(q \;\; k)M\;(p^2)\;{}_5 \quad 2(k^2 \quad p^2)(k \;\; p)\;{}_6 \\
& + \;\; (k^2 \quad p^2)(t \;\; k) + r\;(k;p) \;\; M\;(p^2)\;{}_7 + r\;(k;p)\;{}_8\,;
\end{aligned}
$$

Solution to the gap equation

Bare vertex

Bare vertex , with $_1 = 1$ and $_{2;3} = {}_{1;2;:::;8} = 0$.

Figure: Dynamically generated mass $M_E =$ as a function of the electromagnetic coupling for the bare vertex ansatz

Our vertex

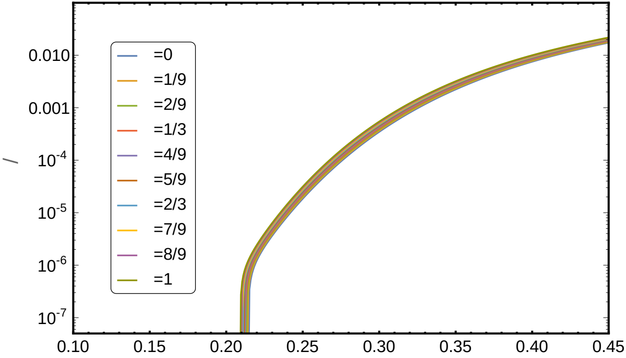


Figure: Dynamically generated mass $M_E =$ as a function of the electromagnetic coupling for our proposed vertex.

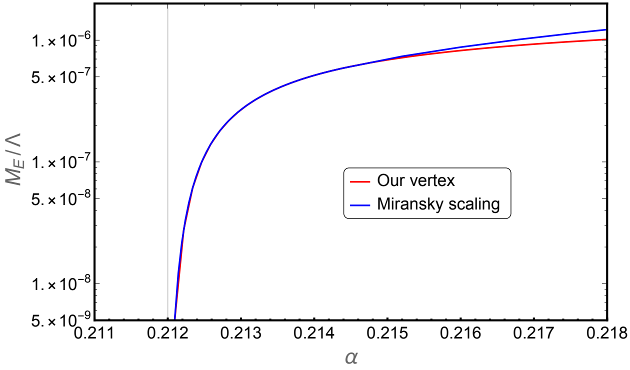


Figure: Miransky Scaling using our vertex for = 1=3.

Final remarks
Concluding

- Symmetry preserving truncations of SDE are important to achieve physically reliable results
- Constraints arising from gauge symmetry guarantee correct physical interpretation
- Chiral symmetry breaking ocurrs in RQED provided the coupling exceeds a critical value
- Educated interactions bring closer to experimentally achievable results

¡GRACIAS!