Dynamical Mass Generation in RQED

Juan Angel Casimiro Olivares
Cosmology and particles
September 2022

Motivation

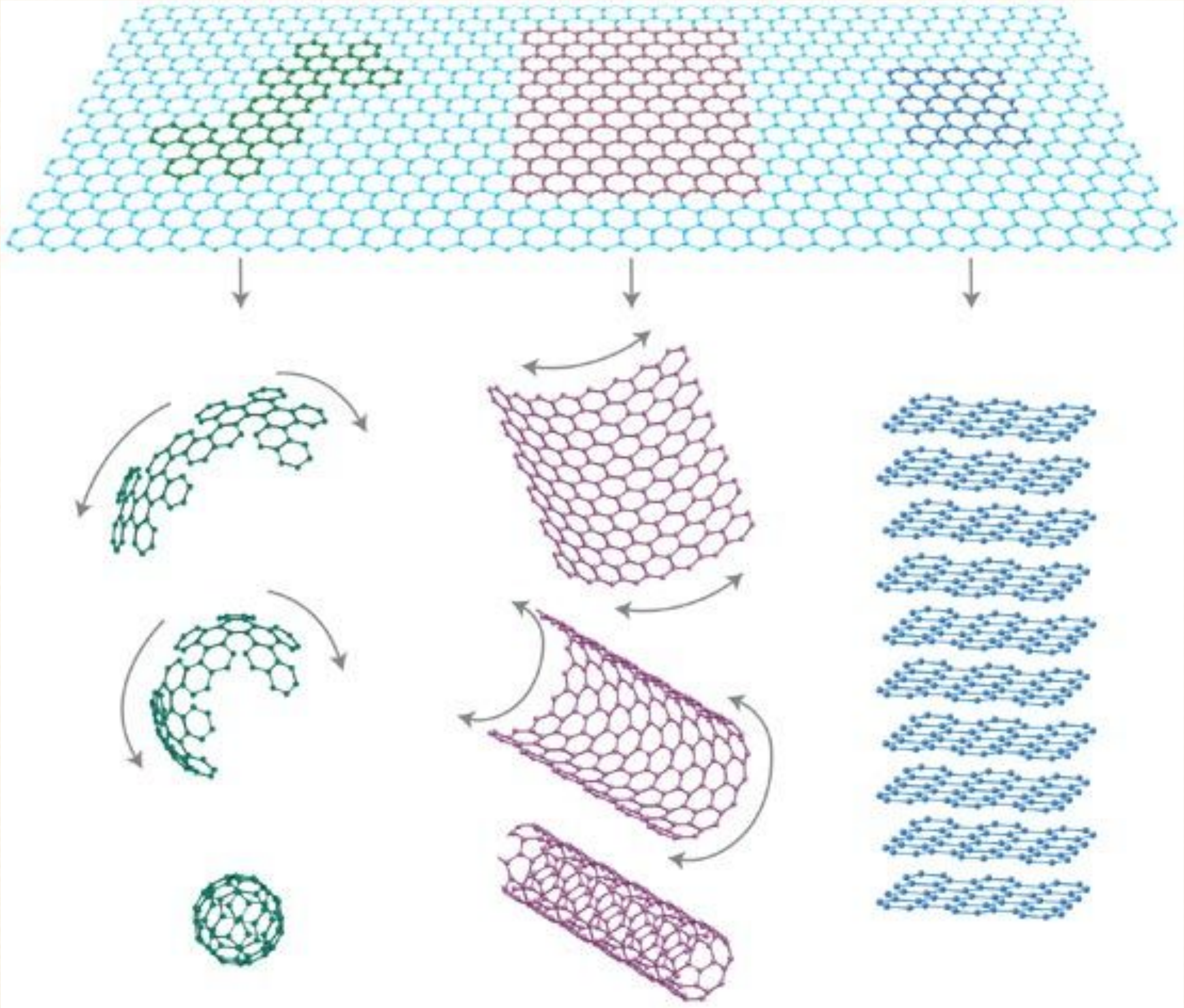
Graphene Physics

Dynamical Mass Generation

Graphene

- The membrane of carbon atoms packed in a hexagonal 2-dimensional lattice is called graphene.
- It is the structural component of the other graphitic elements.
- Graphene is a revolutionary material with a wide variety of potential technological applications while offering an expectation to explore aspects of fundamental physics.

It can be wrapped to form fullerenes, coiled into nanotubes, or stacked on graphite.



Properties

- It is a carbon sheet 1 atom thick.
- Exhibits high thermal and electrical conductivity.
- It is an almost transparent material.
- Their charge carriers behave as "ultra relativistic”.

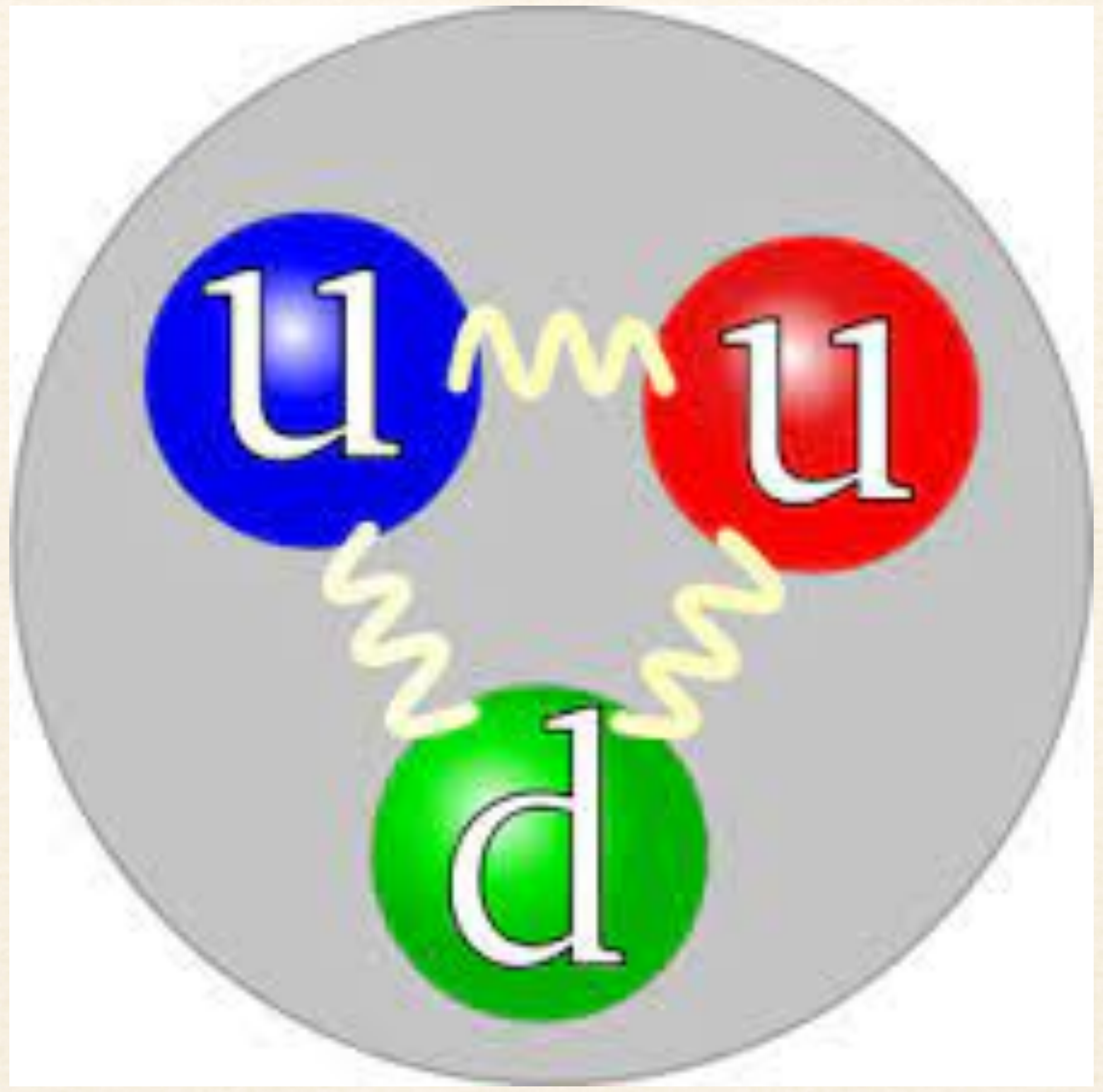
The electromagnetic interactions between the charge carriers of these two-dimensional materials correspond, in the static case, to Coulombian-type interactions and not the typical logarithmic behavior of in-plane quantum electrodynamics. This is easily understood because although it is true that fermions restrict their movement in two dimensions, the electromagnetic field propagates freely throughout space. This scenario has led to the development of a theory called Pseudo or Reduced Quantum Electrodynamics.

Dynamical Mass Generation

- One particular problem to understand is how the mass of a proton ($\sim 1 GeV$) looks like when quarks interact.
- This phenomena is known as Dynamical Mass Generation (DMG) and we want to see if it takes place in graphene.
- With this we would have a more natural way to open and close the energy gap when turning on or off a current.

- The origin of the masses of the electroweak Standard Model particles is explained by the Higgs mechanism.
- In this mechanism the gauge symmetry is spontaneously broken so that the bosons acquire their mass
- Then through the Yukawa couplings, that is, the couplings between the Higgs boson and the fermions, their later acquire their mass.

- On the other hand, we have that at distance scales of the order of the size of the proton, the quarks behave as if their masses were about 300 MeV, so that the Higgs mechanism is not "enough" to explain 99% of the total mass of the proton or neutron.



- As a proposal we are working on the theory called reduced quantum electrodynamics (RQED), since it is a simplified model that shares the DMG phenomenon with QCD.

- In RQED we want to see how photons, which move throughout space, interact with electrons moving in the plane from the point of view of the plane.

Lagrangian *RQED*

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}^2 - ej^\mu A_\mu + \mathscr{L}_M + \mathscr{L}_{GF}$$

$$\mathscr{L}_{RQED} = -\frac{1}{4}F^{\mu\nu}\left[\frac{2}{\sqrt{\Box}}\right]F_{\mu\nu} - ej^\mu A_\mu + \mathscr{L}_M + \mathscr{L}_{GF}\,.$$

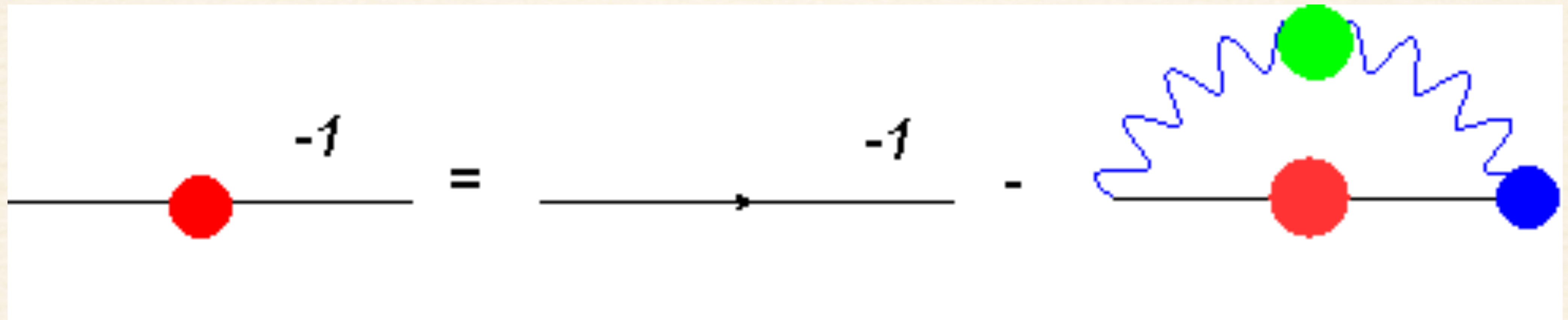
It is necessary to have a potential of the following form in the plane

$$V(r) = -\frac{a}{r}$$

For this to be true, we must choose a propagator with the following form

$$\Delta^{\mu\nu}(q^2) = \frac{-i}{\sqrt{q^2}} \left[g^{\mu\nu} - \left(1 - \xi\right) \frac{q^\mu q^\nu}{p^2} \right].$$

Schwinger–Dyson equations



- $$S_F^{-1}(p) = S_{0F}^{-1}(p) - \pi\alpha \int \frac{d^3k}{(2\pi)^3} \Gamma^\mu(k,p) S_F(k) \gamma^\nu \Delta_{\mu\nu}(p-k)$$

- An alternative to studying the DMG is the use of the Schwinger-Dyson (SDE) equations.
- These equations are an infinite tower of relations between the Green functions of a given quantum field theory, whose nature is non-perturbative.

Gap equation in RQED

$$\frac{1}{F(p)} = 1 + \frac{\alpha}{2\pi p^2}\int_0^\Lambda \frac{dkF(k)k^2}{k^2+M(k)^2}\left\{\frac{\theta(k-p)}{k}\left[-(2+\zeta)p^2+(1-\zeta)\frac{p^4}{k^2}\right]+\frac{\theta(p-k)}{p}\left[-(2+\zeta)k^2+(1-\zeta)\frac{k^4}{p^2}\right]\right\},$$

$$\frac{M(p)}{F(p)} = \frac{\alpha(2+\zeta)}{2\pi}\int_0^\Lambda dk\frac{k^2F(k)M(k)}{k^2+M^2(k)}\left[\frac{\theta(k-p)}{k}+\frac{\theta(p-k)}{p}\right],$$

Nevertheless, as a first approximation, let us take $F(p) = 1$ and explore the solution of the gap equation. Then, the gap equation becomes the non-linear integral equation for the mass function

$$M(p) = \frac{2\alpha}{\pi p}\int_0^p dk\ \frac{k^2 M(k)}{k^2 + M^2(k)} + \frac{2\alpha}{\pi}\int_p^\Lambda dk \frac{k^2 M(k)}{[k^2 + M^2(k)]k}$$

This expression can be straightforwardly converted into the following differential equation

$$p^2 M''(p) + 2pM'(p) + \frac{2\alpha}{\pi}\frac{p^2 M(p)}{p^2 + M^2(p)} = 0$$

Restricted to the boundary conditions

$$\lim_{p \to \Lambda} \left(p \frac{dM(p)}{dp} + M(p) \right) = 0$$

$$\lim_{p \to 0} p^2 \frac{dM(p)}{dp} = 0$$

Upon linearizing the differential equation when $p \gg M(p)$, we can again write the resulting equation in the Euler-Cauchy form

$$\frac{d}{dp} \left(p^2 \frac{dM(p)}{dp} \right) + \frac{\alpha}{\pi} M(p) = 0$$

Which, admits a general solution of the form

$$M(p) = C_1 p^{n_+} + C_2 p^{n_-},$$

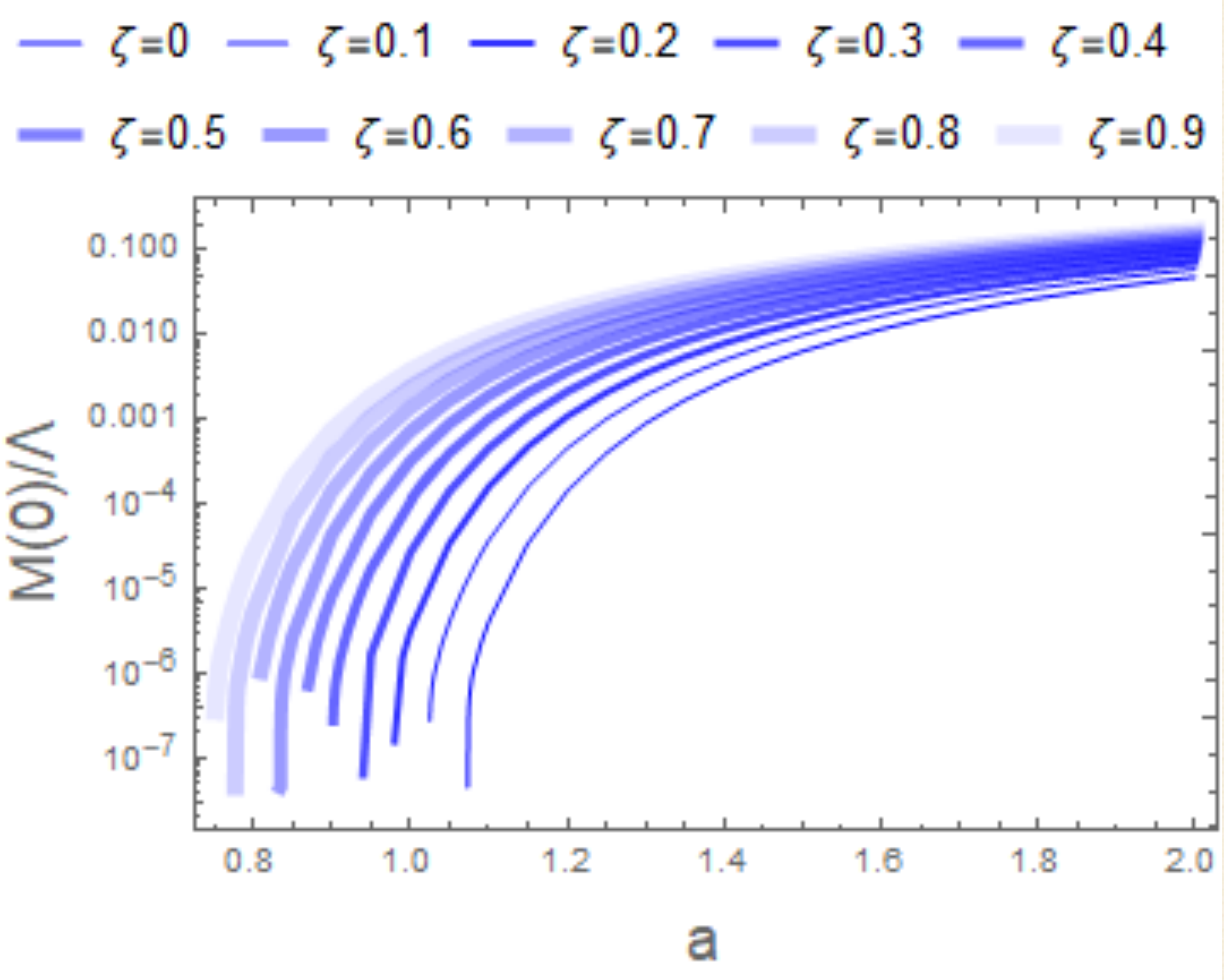
$$n_\pm = -\frac{1}{2} \pm \frac{1}{2}\sqrt{1 - \frac{8\alpha}{\pi}}\,.$$

The dynamical mass obeys the Miransky scaling

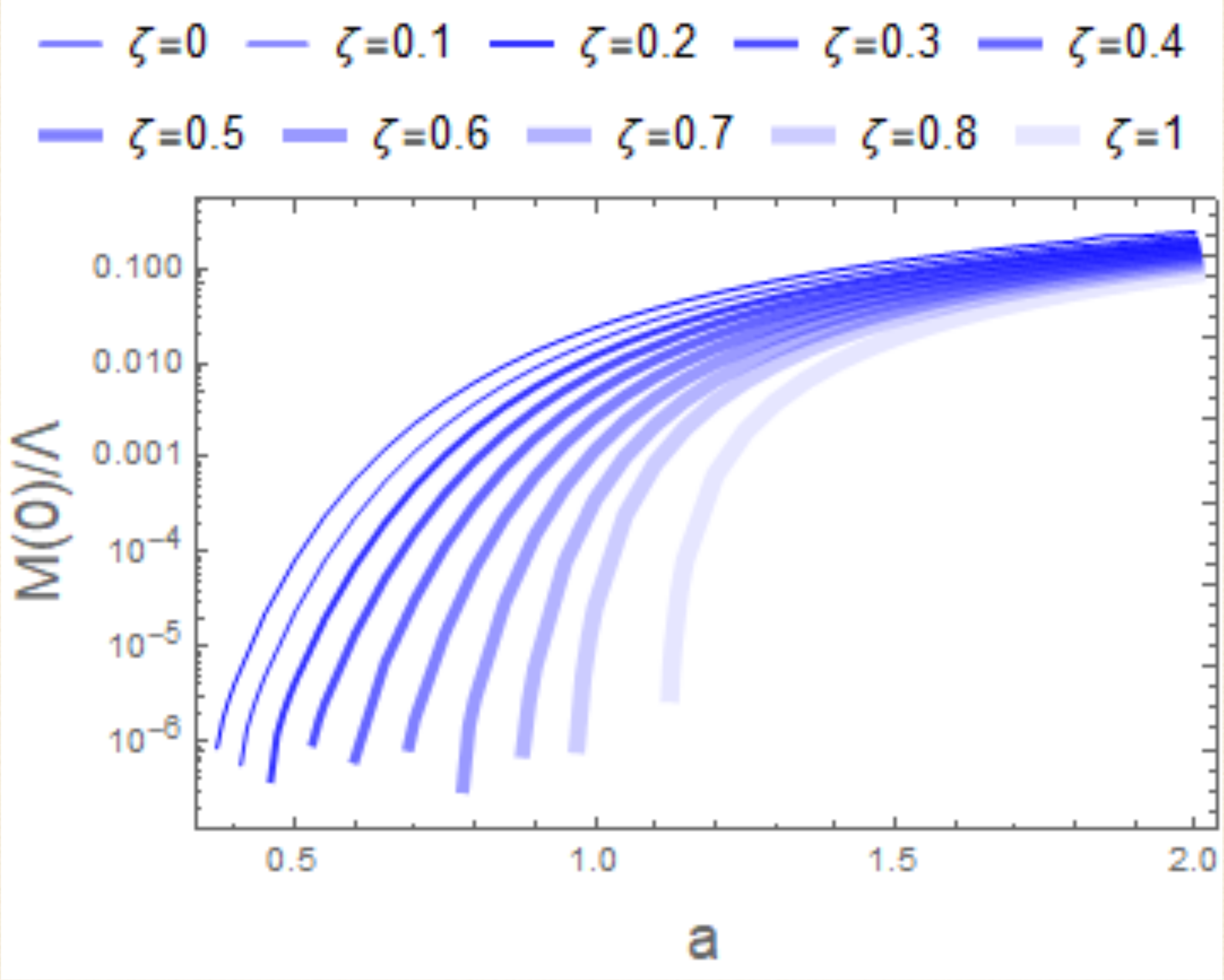
$$\frac{\Lambda}{\kappa} = \exp\left(\frac{A}{\sqrt{\frac{\alpha}{\alpha_c} - 1}} + \delta\right)$$

$$A = -2\pi, \delta = -4$$

- Case without wavefunction renormalization
  - We take rainbow approximation
  - We work in arbitrary gauge
  - The wavefunction renormalization is set equal to 1


ζ=0
ζ=0.1
ζ=0.2
ζ=0.3
ζ=0.4
ζ=0.5
ζ=0.6
ζ=0.7
ζ=0.8
ζ=0.9
0.100
0.010
0.001
10^-4
10^-5
10^-6
10^-7
M(0)/Λ
0.8
1.0
1.2
1.4
1.6
1.8
2.0
a

- Case with wavefunction renormalization
  - We take rainbow approximation
  - We work in arbitrary gauge
  - We take the corrections of the wave function renormalization



ζ=0
ζ=0.1
ζ=0.2
ζ=0.3
ζ=0.4
ζ=0.5
ζ=0.6
ζ=0.7
ζ=0.8
ζ=1
0.100
0.010
0.001
10⁻⁴
10⁻⁵
10⁻⁶
M(0)/Λ
0.5
1.0
1.5
2.0
a

Considering the wavefunction renormalization, the increase in the gauge dependence of the critical coupling is noticeable. In this way we can see that it is necessary to give a better truncation of our SDE tower

- Ward Identity case
  - We work in arbitrary gauge
  - We contract the vertex with Ward's identity
  - We split the SDE in two parts, in one of them we place the gauge dependence

Gap equation Ward Identity

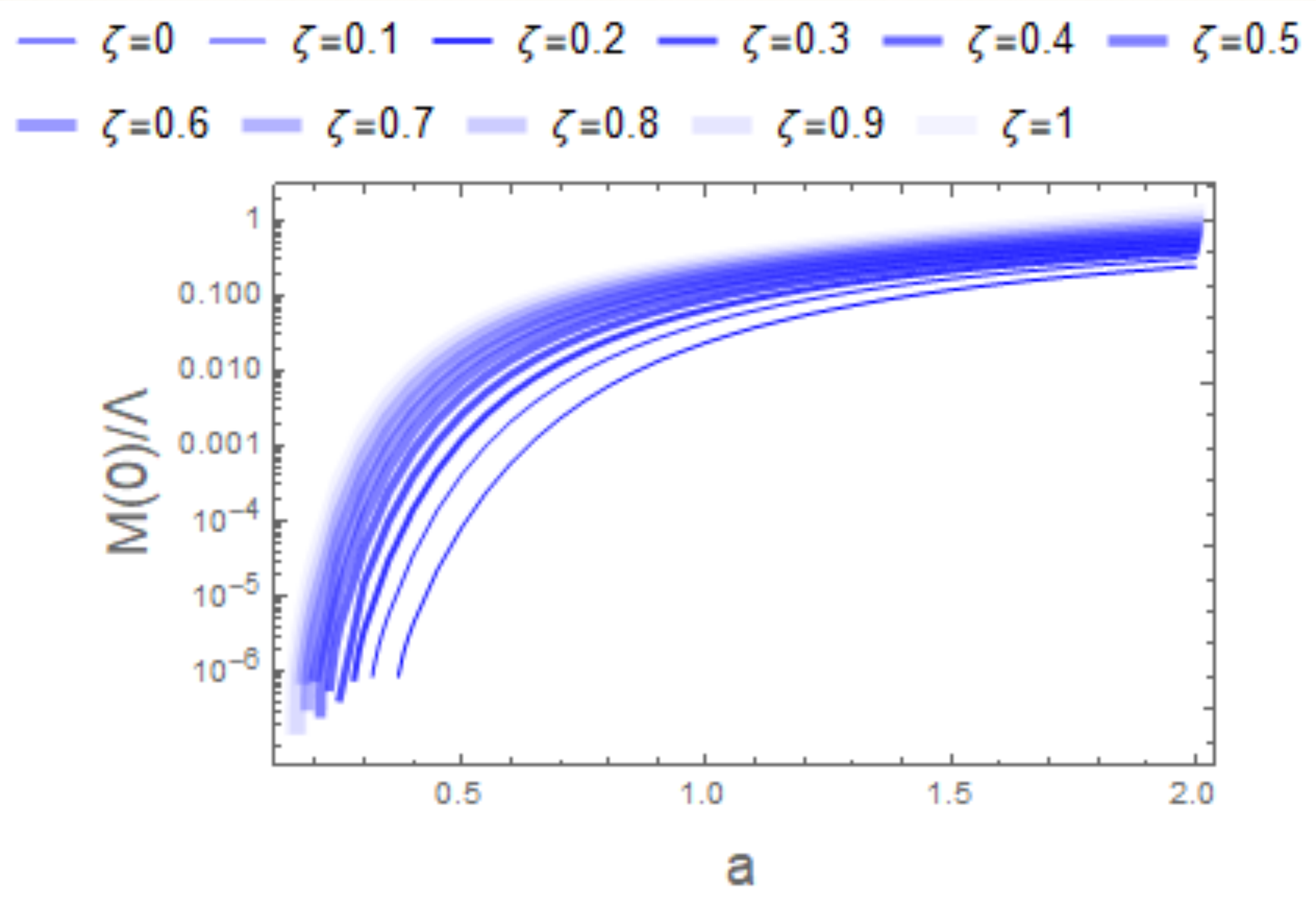
$$\frac{1}{F(p)} = 1 + \frac{\alpha}{2\pi p^2}\int_0^{\Lambda} \frac{dkF(k)k^2}{k^2+M(k)^2}\left\{\frac{\theta(k-p)}{k}\left[-2p^2-\zeta\frac{p^4}{k^2}\right]+\frac{\theta(p-k)}{p}\left[-2k^2-\frac{k^4}{p^2}\right]\right\} + \frac{\alpha\zeta}{4\pi p^2}\int_0^{\lambda}\frac{dkF(k)k^2}{F(p)(k^2+M^2(p)}$$

$$\times\left\{\frac{\theta(k-p)}{k}\left[\frac{2p^2k^2}{k^2}-\frac{p^4}{k^2}-\frac{M(k)M(p)p^2}{k^2}+\frac{2p^2M(k)M(p)}{k^2}\right]+\frac{\theta(p-k)}{p}\left[\frac{2p^2k^2}{p^2}-\frac{p^2k^2}{p^2}-\frac{M(k)M(p)k^2}{p^2}\right.\right.$$

$$\left.\left.+\frac{2p^2M(k)M(p}{p^2}\right]\right\}$$

$$\frac{M(p)}{F(p)} = \frac{\alpha}{\pi}\int_0^{\Lambda} dk\frac{k^2F(k)M(k)}{k^2+M^2(k)}\left[\frac{\theta(k-p)}{k}+\frac{\theta(p-k)}{p}\right]+\frac{\alpha\zeta}{2\pi}\int_0^{\lambda}\frac{dkF(k)k^2}{F(p)(k^2+M^2(p)}$$

$$\times\left\{\frac{\theta(k-p)}{k}\left[M(p)\left(1-\frac{p^2}{k^2}\right)-M(k)\frac{p^2}{2k^2}\right]+\frac{\theta(p-k)}{p}\left[-M(k)\left(1-\frac{k^2}{p^2}\right)-M(p)\frac{k^2}{2p^2}\right]\right\}.$$



ζ=0
ζ=0.1
ζ=0.2
ζ=0.3
ζ=0.4
ζ=0.5
ζ=0.6
ζ=0.7
ζ=0.8
ζ=0.9
ζ=1
1
0.100
0.010
0.001
10^-4
10^-5
10^-6
M(0)/Λ
0.5
1.0
1.5
2.0
a

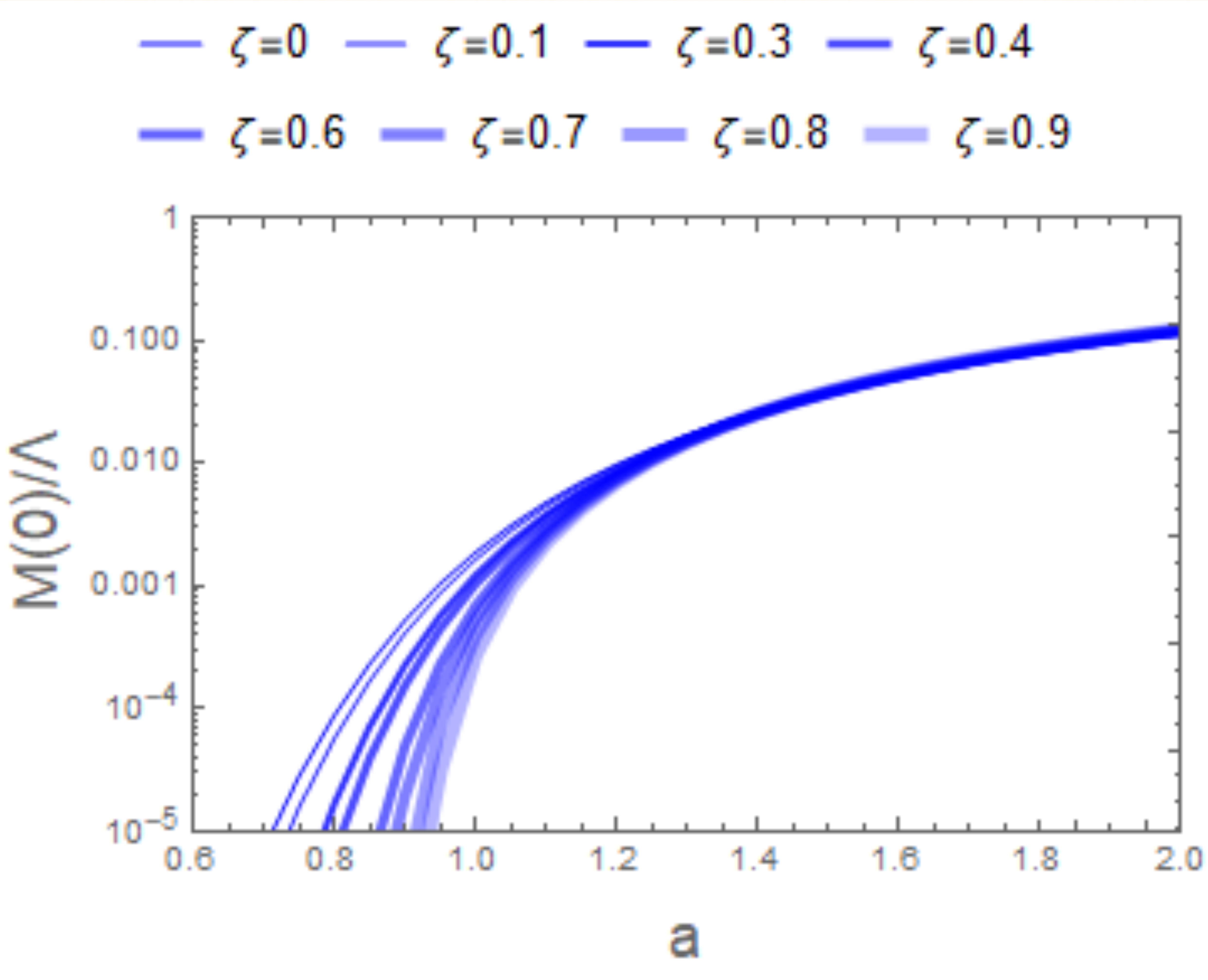
- Ball-Chiu vertex case
  - We take only the central part of the vertex of Ball-Chiu
  - We work in arbitrary gauge
  - All SDE has dependency on the gauge

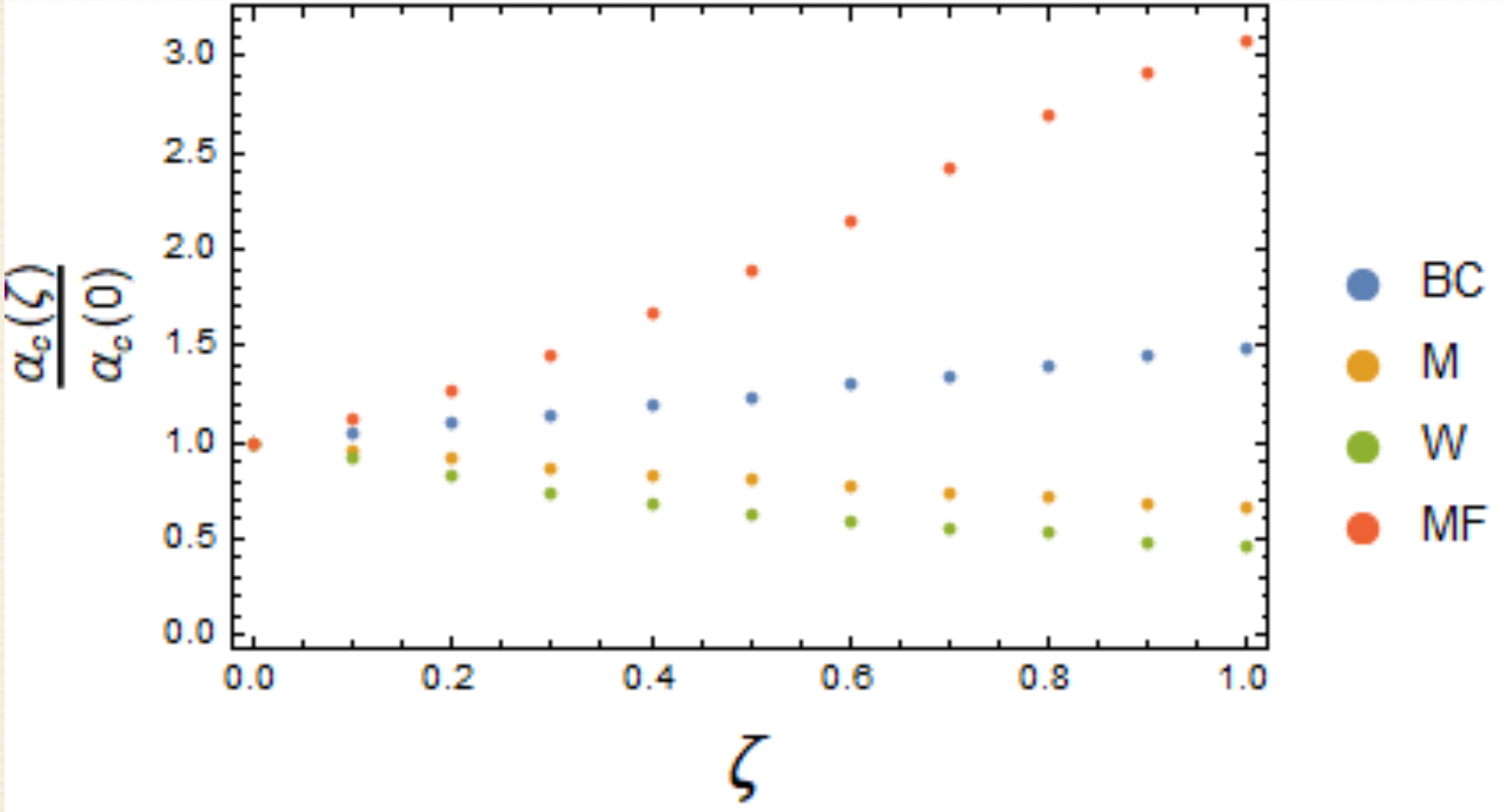
Gap equation Ball-Chin Vertex (Central part)

$$\frac{1}{F(p)} = 1 + \frac{\alpha}{2\pi p^2} \int_0^{\Lambda} \frac{dkF(k)k^2}{k^2 + M(k)^2} \left[\frac{1}{F(k)} + \frac{1}{F(p)}\right] \left\{ \frac{\theta(k-p)}{k} \left[-(2+\zeta)p^2 + (1-\zeta)\frac{p^4}{k^2}\right] + \frac{\theta(p-k)}{p} \right.$$

$$\left. \times \left[-(2+\zeta)k^2 + (1-\zeta)\frac{k^4}{p^2}\right] \right\},$$

$$\frac{M(p)}{F(p)} = \frac{\alpha(2+\zeta)}{2\pi} \int_0^{\Lambda} dk \, \frac{k^2 F(k) M(k)}{k^2 + M^2(k)} \left[\frac{1}{F(k)} + \frac{1}{F(p)}\right] \left[\frac{\theta(k-p)}{k} + \frac{\theta(p-k)}{p}\right],$$


ζ=0
ζ=0.1
ζ=0.3
ζ=0.4
ζ=0.6
ζ=0.7
ζ=0.8
ζ=0.9
M(0)/Λ
1
0.100
0.010
0.001
10^-4
10^-5
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
a



$\frac{\alpha_c(\zeta)}{\alpha_c(0)}$
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
ζ
BC
M
W
MF

At first glance, it might seem surprising that the simplest scenario of neglecting wavefunction renormalization effects and solving the mass function alone yields the least gauge parameter dependent results.
This is better understood from a renormalization group analysis, that exploits the similarities of the $1/N$ approximation in QED_3 and RQED itself.

Research OR Education of Physics Revista Mexicana de Física **??** (*?*) ???–??? MES? AÑO?

Non-perturbative field theoretical aspects of graphene and related systems

Juan Angel Casimiro Olivares
Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo, Edificio C-3, Ciudad Universitaria, Morelia, Michoacán 58040, México.

Ana Julia Mizher
Instituto de Física Teórica, Estadual Paulista, Rua Dr. Bento Teobaldo Ferraz, 271 - Bloco II, 01140-070 São Paulo, SP, Brazil.

Centro de Ciencias Exactas, Universidad del Bío-Bío,
Avda. Andrés Bello 720, Casilla 447, 3800708, Chillán, Chile.

Alfredo Raya
Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo, Edificio C-3, Ciudad Universitaria, Morelia, Michoacán 58040, México.

Centro de Ciencias Exactas, Universidad del Bío-Bío,
Avda. Andrés Bello 720, Casilla 447, 3800708, Chillán, Chile.

Received 10 May 2021; accepted 23 May 2021

In this article, we review the dynamics of charge carriers in graphene and related 2D systems from a quantum field theoretical point of view. By allowing the electromagnetic fields to propagate throughout space and constraining fermions to move on a 2D manifold, the effective theory of such systems becomes a non-local version of quantum electrodynamics (QED) dubbed in literature pseudo or reduced QED. We review some aspects of the theory assuming the coupling arbitrary in strength. In particular, we focus on the chiral symmetry breaking scenarios and the analytical structure of the fermion propagator in vacuum and under the influence of external agents like a heat bath, in the presence of a Chern-Simons term, anisotropy and in curved space. We briefly discuss the major advances and some new results on this field.

En este artículo revisamos la dinámica de los portadores de carga en grafeno y otros sistemas 2D relacionados desde un punto de vista de la teoría de campos cuánticos. Permitiendo que los campos electromagnéticos se propaguen en el espacio, pero restringiendo a los fermiones a moverse en una variedad bidimensional, la teoría efectiva para estos sistemas se vuelve una versión no local de la electrodinámica cuántica (QED por sus siglas en inglés) llamada en la literatura pseudo QED o QED reducida. Hacemos un recuento de algunos aspectos de la teoría asumiento un acomplamiento de intensidad arbitraria. En particular, nos enfocamos en los escenarios de rompimiento dinámico de la simetría quiral y la estructura analítica del propagador del fermión en el vacío y bajo la influencia de agentes externos como un baño térmico, en la presencia de un término de Chern-Simons, anisotropía y en espacio curvo. Discutimos brevemente los mayores avances y algunos nuevos resultados en este campo.

Final Remarks

- The principle of gauge invariance is very important in modern physics. It is the key to understanding the origin and nature of fundamental interactions.
- If we solve SDE we can notice that depends on the approximation the dependence on the gauge of the critical coupling increases or decreases.
- We need to give a better truncation vertex for our SDE to try to recover the gauge invariance.

Gracias