Very Special Linear Gravity: A Gauge-Invariant Graviton Mass

Jorge Alfaro

Profesor Titular,PUC

Cosmology and Particles,
September 12-14, 2022
UBB, Chillán

Fondo Gemini Astro20-0038

Outline

- Very Special Relativity
- Lagrangian for Very Special Linear Gravity
- Gauge fixing
- Solution to the Equations of Motion (E.o.M.)
- Gravitational waves and geodesic deviation
- Order of magnitude of the new effects
- Multipolar nature of gravitational radiation
- Conclusions
- Ref: J.A. and A. Santoni, PLB829(2022)137080

Very Special Relativity

A. Cohen and S. Glashow, Phys.Rev.Lett.97:021601,2006:
All kinematical effects associated to invariance under the Lorentz group(6 parameters) can be obtained from four parameters subgroups of the Lorentz group, opening the road to new predictions which violate Lorentz symmetry, but preserve the symmetry under such subgroups.

- VSR implies special relativity (SR) in the context of local quantum field theory or of CP conservation.
- Most interesting Subgroup of the Lorentz Group:Hom(2), 3 parameters; Sim(2) , 4 parameters.
  There are no invariant tensors for these cases. So SR kinematics is preserved.
  No local Lorentz symmetry-breaking operator preserving either of these groups exists.

$$T_1 = K_x + J_y, T_2 = K_y - J_x \quad (1)$$

$$Hom(2) generators : T_1, T_2, K_z \quad (2)$$

$$Sim(2) generators : T_1, T_2, K_z, J_z \quad (3)$$

$n = (1, 0, 0, 1)$, $n.n = 0$, n is invariant under T_1, T_2, J_z, but under boosts in the z-direction (generated by K_z), $n \to e^{\phi} n$
p_1, p_2 particle momenta: $\frac{p_1.n}{p_2.n}$ is VSR but not SR invariant.

Neutrino mass in VSR:

$$\left(\not{p} - \frac{m_v^2}{2} \frac{\not{n}}{n.p}\right) \nu_L = 0, \left(\not{p} - \frac{m_v^2}{2} \frac{\not{n}}{n.p}\right)^2 \nu_L =$$

$$(p^2 - m_\nu^2) \nu_L = 0$$

Massive graviton

- Dark matter
- Cosmology
- Bounds on the graviton mass
- Until now, massive gravity models were usually constructed as Lorentz invariant.
- Nevertheless, as in the case of Electromagnetism and the Proca Theory, there is no way of trivially preserving both Lorentz and Gauge invariance when giving mass to the graviton.
- Giving up on the Gauge invariance leads to the appearance of three degrees of freedom (D.o.F.) more than General Relativity (GR), which are responsible for different pathologies of these theories, like the vDVZ discontinuity and ghost modes (i.e. the Boulware-Deser ghost).
- Solution: Vainshtein Mechanism and the fine-tuned dRGT action to avoid ghosts.Instabilities in Cosmology.

VSR Lorentz invariance violation

- Giving up on Lorentz invariance, by implementing VSR, is the other viable possibility for massive gravity.
- Experience with VSR Electrodynamics and VSR massive Neutrinos tell us that VSR extensions avoid the introduction of ghosts in the spectrum.
- In fact, gauge invariance of our formulation does not allow for new additional D.o.F. other than the usual two of the massless graviton, getting round most of the problems cited above.
- Nevertheless, these advantages come at the price of considering new non-local terms in the theory and assuming a preferred space-time null direction, represented by the lightlike four-vector n^{μ}.

Lagrangian of the Field $h_{\mu\nu}$

- We want to find the general expression for the lagrangian quadratic in the graviton field $h_{\mu\nu}$

$$\mathcal{L}_g = h^{\mu\nu} O_{\mu\nu\alpha\beta} h^{\alpha\beta} \,. \tag{4}$$

- To built the most generic form of the operator O, we restricted our search to terms containing up to two derivatives and with a dimensionless or a mass-squared dimensioned parameter.
- In momentum space, the objects we can put together to build up terms for $O_{\mu\nu\alpha\beta}$ are: $\eta_{\mu\nu}$, p_μ and $N_\mu = \frac{n_\mu}{n \cdot p}$.
- Symbolically speaking, the operator O assumes the structure

$$O = 3\,\eta\eta + 9\,pp\eta + 12\,pN\eta + 12\,ppNN \;, \tag{5}$$

where the numbers indicate how many different terms we can construct with each set of objects.

Parameters' Restrictions

- Gauge invariance of the Lagrangian:$\delta h_{\mu\nu} = p_\mu \xi_\nu + p_\nu \xi_\mu$. , i.e. $O_{\mu\nu\alpha\beta} p^\alpha = 0$.
- Furthermore, we must impose for O the index symmetries $\mu \Longleftrightarrow \nu$, $\alpha \Longleftrightarrow \beta$, $\mu\nu \Longleftrightarrow \alpha\beta$.
- This way, we reduce the 36 parameters of the expression (5) to only two free parameters: a constant global factor χ, that we can identify with the Einstein-Hilbert constant $\chi = \frac{1}{2\kappa} = \frac{c^4}{16\pi G}$, and m_g^2, that has dimensions of a mass squared and, as we will see, plays the role of a graviton mass.
- The operator O is very long. So we skip it.

Gauge choices

- The E.o.M for the graviton field in momentum space is:

$$O_{\mu\nu\alpha\beta}h^{\alpha\beta} = 0\,. \tag{6}$$

In the following, we will use the definition $h = \eta^{\mu\nu}h_{\mu\nu}$, to simplify the notation.

- Firstly, we use the linearized diffeomorphism gauge freedom to fix a Lorentz gauge $p^{\mu}h_{\mu\nu} = 0$. This way, using the Lorentz condition and contracting the E.o.M with p_{μ}, n_{μ} and $\eta_{\mu\nu}$ we get two useful equations

$$p^2 h = 0\,, \tag{7}$$

$$p_{\nu}h + p^2 N^{\mu}h_{\mu\nu} = 0\,. \tag{8}$$

- The Lorentz gauge does not fix the gauge uniquely. We can still make a new gauge transformation satisfying

$$p^{\mu}h'_{\mu\nu} = 0 = p^{\mu}h_{\mu\nu} + p^2\xi_{\nu} + p\cdot\xi p_{\nu} = p^2\xi_{\nu} + p\cdot\xi p_{\nu}\,, \tag{9}$$

Fixing $h = 0$ and $n^\mu h_{\mu\nu} = 0$

- i.e.

$$\xi_\nu = (c_\nu - \frac{1}{2}\frac{p \cdot c}{p^2} p_\nu)\delta(p^2)\,, \tag{10}$$

with c_ν arbitrary, meaning we still have the freedom to impose at least four conditions on $h_{\mu\nu}$ by fixing c_μ.

- We can use this additional gauge freedom to fix $h = 0$ and $n^\mu h_{\mu\nu} = 0$.
- Our E.o.M with gauge conditions

$$p^\mu h_{\mu\nu} = 0\,, \tag{11}$$

$$n^\mu h_{\mu\nu} = 0\,, \tag{12}$$

$$h = 0\,, \tag{13}$$

simply becomes a Klein-Gordon equation for the field h, that in momentum space reads:

$$(p^2 - m_g^2) h_{\mu\nu} = 0\,, \tag{14}$$

where m_g plays the role of a mass for the graviton.

Solution of the E.o.M.

- From (11) and (12) we observe that

$$h_{0\beta} = -\frac{n^i}{n^0} h_{i\beta} = -\frac{p^i}{p^0} h_{i\beta} \to (\frac{n^i}{n^0} - \frac{p^i}{p^0}) h_{i\beta} = 0\,, \tag{15}$$

- Choose as a three-dimensional spatial basis the orthogonal set $\{\vec{u}\,,\ \vec{M}\,,\ \frac{\vec{n}}{n^0} - \frac{\vec{p}}{p^0}\}$, where we have defined the adimensional and $SIM(2)$-invariant vectors

$$\vec{u} = \frac{\vec{n}}{n^0} \times \frac{\vec{p}}{p^0}\,, \quad \vec{M} = \vec{u} \times (\frac{\vec{n}}{n^0} - \frac{\vec{p}}{p^0})\,. \tag{16}$$

- We can write

$$h_{i\beta} = A_\beta u_i + B_\beta M_i\,,\ h_{0\beta} = -\frac{n^i}{n^0} h_{i\beta} = \frac{n_i}{n_0} h_{i\beta}\,,$$
$$h_{ij} = A_j u_i + B_j M_i = A_i u_j + B_i M_j = h_{ji}\,. \tag{17}$$

- Since we want $(\frac{n^i}{n^0} - \frac{p^i}{p^0})h_{ij} = 0$, then A_i, B_i have to be linear combinations of u_i and M_i

$$A_i = au_i + bM_i \; , \quad B_i = cu_i + dM_i \, ,$$

where imposing $h_{ij} = h_{ji}$ we find $b = c$.

- The coefficient A_0, B_0 are constrained by $h_{0i} = h_{i0}$, from which

$$A_0 = \frac{\vec{M}\cdot\vec{n}}{n_0} b \; , \quad B_0 = \frac{\vec{M}\cdot\vec{n}}{n_0} b \, .$$

- Traceless condition:

$$a = \frac{(\vec{M}\cdot\vec{n})^2 - n_0^2 \vec{M}^2}{n_0^2 \vec{u}^2} d = \tilde{a}\, d \, . \qquad (18)$$

Finally, we get the expressions for the components of $h_{\mu\nu}$ in function of the two physical degrees of freedom b, d

$$h_{00} = \left(\frac{\vec{M}\cdot\vec{n}}{n_0}\right)^2 d \; , \quad h_{0i} = \left(\frac{\vec{M}\cdot\vec{n}}{n_0}\right)(bu_i + dM_i) \, ,$$

$$h_{ij} = (\tilde{a} u_i u_j + M_i M_j)\, d + (u_i M_j + u_j M_i) b \, . \qquad (19)$$

Gravitational Waves and Geodesic Deviation

As a first application, we want to study the modifications produced by VSR to the known geodesic deviation equations for a gravitational wave, represented by the space-time perturbation $h_{\mu\nu}$.

- The expression of the geodesic deviation equation depends on the linearized Riemann Tensor $R_{\mu\nu\alpha\beta}$ in the following way

$$\partial_0^2 \delta\xi^\mu = R^\mu_{\ 00\gamma}\delta\xi^\gamma = \eta^{\mu\delta}R_{\delta 00\gamma}\delta\xi^\gamma = \eta^{\mu\mu}R_{\mu 00\gamma}\delta\xi^\gamma \,,$$
$$R_{i00j} = \frac{1}{2}(h_{0j,0i} + h_{0i,0j} - h_{00,ij} - h_{ij,00}) \tag{20}$$

- The case $\mu = 0$ is trivial, since $R_{000\gamma} = 0$. We have no temporal displacement
- For the spatial component of the equation, we see that

$$\partial_0^2 \delta\xi^i = \frac{1}{2}(h_{00,ij} + h_{ij,00} - h_{0i,0j} - h_{0j,0i})\delta\xi^j \,. \tag{21}$$

- Reference Choice and Plane Wave Ansatz:

$$h_{\mu\nu} = \mathcal{RE}(A_{\mu\nu}e^{i(Et-pz)}) \,, p^\mu A_{\mu\nu} = 0 = n^\mu A_{\mu\nu} = A^\mu_\mu \tag{22}$$

i=1,2,3

- The equation (21) for $i = 1, 2$ becomes

$$\partial_0^2 \delta\xi^i = \frac{1}{2}\partial_0^2 h_{i1}\delta\xi^1 + \frac{1}{2}\partial_0^2 h_{i2}\delta\xi^2 + \frac{1}{2}\frac{m_g^2}{E^2}\partial_0^2 h_{i3}\delta\xi^3 \,. \qquad (23)$$

- Perturbative solution:$\delta\xi^\mu(t) = \delta\xi_0^\mu + \delta\xi_1^\mu(t)$,with initial conditions $\delta\xi_1^i(t=0) = \partial_0\delta\xi_1^i(t=0) = 0$,

$$\delta\xi^i = \delta\xi_0^i + \frac{1}{2}h_{i1}\delta\xi_0^1 + \frac{1}{2}h_{i2}\delta\xi_0^2 + \frac{1}{2}\frac{m_g^2}{E^2}h_{i3}\delta\xi_0^3 \,. \qquad (24)$$

- To work out the case $\mu = 3$, we follow the same procedure as $\mu = 1, 2$. We get the complete displacement in the z-direction

$$\delta\xi^3 = \delta\xi_0^3 + \frac{1}{2}\frac{m_g^2}{E^2}h_{13}\delta\xi_0^1 + \frac{1}{2}\frac{m_g^2}{E^2}h_{23}\delta\xi_0^2 + \frac{1}{2}\frac{m_g^4}{E^4}h_{33}\delta\xi_0^3 \,. \qquad (25)$$

VSR Effects' Magnitude are proportional to the factor $\frac{m_g^2}{E^2}$.

- From the time lag measured between the gravitational and electromagnetic signal of GW170817, we get $m_g \lesssim 10^{-19}\ eV$
- From Binary Pulsar, we get an upper bound of $m_g \sim 10^{-28}\ eV$
- From Solar system's tests we get $m_g \sim 10^{-24}\ eV$
- Therefore, using $10^{-24}\ eV$ as an average upper bound of the graviton mass, in the range of frequencies spanned by the interferometers LIGO and VIRGO, $10 Hz$ to $10 kHz$, the upper bound for our perturbative parameter will be approximately of $\frac{m_g^2}{E^2} \sim 10^{-20}$, making VSR effects probably too small to be detected for our current generation of gravitational wave detectors.
- Future interferometers like LISA, will explore a lower frequency range $[0.1 mHz, 1 Hz]$.Then the parameter upper bound increases to $\frac{m_g^2}{E^2} \sim 10^{-10}$.Combined with larger dimensions of future interferometers and the anisotropic nature of VSR could lead to observable effects.

Multipolar Nature of Gravitational Radiation

- The E.o.M. (6) in presence of sources in momentum space reads as

$$O_{\mu\nu\alpha\beta}h^{\alpha\beta} = \kappa\, T_{\mu\nu}\,, \tag{26}$$

  where κ is a constant.

- The gauge invariance, therefore, implies

$$p^{\mu} O_{\mu\nu\alpha\beta}h^{\alpha\beta} = 0 \to p^{\mu} T_{\mu\nu} = 0\,, \tag{27}$$

  which represents for $T_{\mu\nu}$ the usual equation leading to conservation of total energy E and momentum $\vec{P}$ of the sources.

- GR and VSR: being $\rho(\vec{x})$ the energy density of the system, the gravitational monopole M and dipole $\vec{D}$ of a system are proportional respectively to $\int d^3x\rho(\vec{x})$ and $\int d^3x\rho(\vec{x})\vec{x}$, which are the total energy E and momentum $\vec{P}$ of the system, and since both quantities are conserved the second derivative of the gravitational monopole and dipole will be zero $d_t^2 M = d_t^2 \vec{D} = 0$, implying a null monopolar and dipolar radiation.

Conclusions

- In this work, we studied the theory of linearized gravity in the framework of VSR, finding it allows for a graviton mass while preserving gauge invariance.
- We then explored the solution to the new E.o.M. in a specific gauge, and applied it to compute the geodesic deviation due to a gravitational wave. Remarkably, the presence of graviton mass produces a motion also on the propagation direction, in contrast to what happens in the massless case;motions along the two transverse directions are modified too. Hidden in h_{ij} there are anisotropic effects depending on the direction of $\vec{n}$.
- We have coupled the linear VSR gravity to matter and verified that dipole radiation of gravitational waves is absent.
- Further work must be done to give precise numerical predictions on distinct gravitational phenomena. Nevertheless, the presence of a graviton mass would affect almost every gravitational area of study, making the VSR massive gravity formulation worth exploring.

THANK YOU