Perturbative higher curvature corrections of black holes in string-inspired scenarios

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Based on:
[*arXiv* : 2207.13214] with M. Chernicoff, G. Giribet and J. Oliva

Introduction

- Higher-curvature corrections are a natural generalization of the Einstein-Hilbert action. It gives a improvement in UV behaviour of the theory. [Stelle '77]

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- [Metsaev and Tseytlin '86,'87] higher curvature corrections appears in the heterotic string theory.

Two facts about string theory

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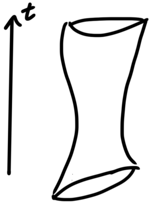
Classically, we can propagate a string in background

$$g_{\mu\nu}(x) \quad , \quad \phi(x) \quad , \quad B_{\mu\nu}(x)$$



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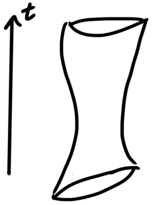

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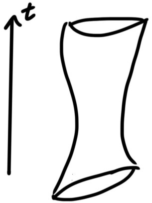

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- For example: In the bosonic string $\Longrightarrow$ $D = 26$.
- The restrictions of the background fields are equations coming from an action principle

$$\delta I\left[g_{\mu\nu}, \phi, B_{\mu\nu}\right] = 0$$

$$I[g_{\mu\nu}, \phi, B_{\mu\nu}] = I^{(0)} + \alpha' I^{(1)} + \alpha'^2 I^{(2)} + \ldots$$

Perturbative scenario

In the perturbative regime, where string fluctuations are neglected

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- Second order field equations are given by

$$\frac{1}{4}\delta^{\mu\nu\rho\sigma}_{\lambda\sigma\delta\eta}R^{\lambda\sigma}{}_{\mu\nu}R^{\delta\eta}{}_{\rho\sigma} = R^{\mu\nu}{}_{\rho\sigma}R^{\rho\sigma}{}_{\mu\nu} - 4R^{\mu}{}_{\nu}R^{\nu}{}_{\mu} + R^2$$

A tool: Field redefinition

Simple example without scalar field

$$I = \int d^D x \sqrt{-g} \left[R + \alpha \left(R^{\mu\nu}_{\ \ \rho\sigma} R^{\rho\sigma}_{\ \ \mu\nu} - 4 R^{\mu}_{\ \nu} R^{\nu}_{\ \mu} + R^2 \right) \right] + \mathcal{O}\left(\alpha^2\right)$$

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$$\delta I \left[g_{\mu\nu} \right] = \int d^D x \sqrt{-g} \left(G_{\mu\nu} + \alpha H_{\mu\nu} \right) \delta g^{\mu\nu} + \mathcal{O}\left(\alpha^2\right)$$

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then

$$I \mapsto \int \sqrt{-g} d^D x \; R + \alpha \left[R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\mu\nu} - (C_1 + 4) R^{\mu}{}_{\nu} R^{\nu}{}_{\mu} + \left(\frac{C_1}{2} - C_2 + \frac{C_2 D}{2} + 1 \right) R^2 \right] + \mathcal{O}\left(\alpha^2\right)$$

Back to the low energy limit of strings

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The answer is yes, and it is given by

$$\begin{aligned} \phi &\mapsto \phi-\frac{\alpha}{2}\left(R+4\left(2D-5\right)\partial_\mu\phi\partial^\mu\phi\right)\\ g_{\mu\nu} &\mapsto g_{\mu\nu}-4\alpha\left(R_{\mu\nu}-4\partial_\mu\phi\partial_\nu\phi+4g_{\mu\nu}\partial_\rho\phi\partial^\rho\phi\right) \end{aligned}$$

After the field redefinition and up to $\mathcal{O}\left(\alpha^2\right)$, the low energy limit of string theory becomes

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The equations of motion are schematically given by

$$\begin{aligned} G_{\mu\nu} + (\cdots) + \alpha\mathcal{H}_{\mu\nu} &= 0 \ , \\ \Box\phi + (\cdots) + \alpha(\cdots) &= 0 \ . \end{aligned}$$

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- One way to parametrize the solutions is

$$\begin{aligned} g_{\mu\nu} &= g^{(0)}_{\mu\nu} + \alpha g^{(1)}_{\mu\nu} + \mathcal{O}\left(\alpha^2\right) \ , \\ \phi &= \phi^{(0)} + \alpha\phi^{(1)} + \mathcal{O}\left(\alpha^2\right) \ . \end{aligned}$$

What about the background solution?

When $\phi^{(0)} = \phi_0 = cte$, the "background" metric $g^{(0)}_{\mu\nu}$ fulfills the Einsteins' equations

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- α' correction to Schwarzschild black hole in $D = 4$ [22' Chernicoff, Giribet, Oliva, MO]
- α' correction to Kerr in $D = 4$.
- Gregory-Laflame instability of black string in $D = 5$ Work in progress!

Dilatonic black hole

Let us consider the $D = 4$ case and the correction of the Schwarzschild black hole

$$\begin{aligned}\phi(r) &= \phi_0 + \alpha\phi_1(r)\\ ds^2 &= -\left(f_0(r) + \alpha f_1(r)\right)dt^2 + \frac{dr^2}{g_0(r) + \alpha g_1(r)} + r^2 d\Omega^2\end{aligned}$$

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The equations can be solved analytically

$$\phi(r) = \phi_0 + \alpha\left[c_1 \ln\left(1 - \frac{\mu}{r}\right) - \frac{2}{\mu r} - \frac{1}{r^2} - \frac{2\mu}{3r^3}\right] + O\left(\alpha^2\right)$$

$$g^{rr} = 1 - \frac{\mu}{r} + \alpha\left(-\frac{\mu c_1}{r} \ln\left(1 - \frac{\mu}{r}\right) + \frac{2}{r^2} + \frac{\mu}{r^3} - \frac{10\mu^2}{3r^4}\right) + O\left(\alpha^2\right)$$

$$g_{tt} = -1 + \frac{\mu}{r}$$
$$-\alpha\left[\frac{c_1(2r - 3\mu)}{r} \ln\left(1 - \frac{\mu}{r}\right) + \frac{4}{r^2} + \frac{5\mu}{3r^3} + \frac{2\mu^2}{r^4} - \frac{2\mu^2 c_1 + 8}{\mu r}\right] + O\left(\alpha^2\right)$$

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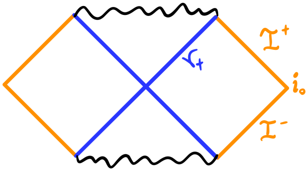
In order
to have a black hole. In this coordinates,
it means that there exists r_+ such that

$$g^{rr}(r_+) = 0 = g_{tt}(r_+) \quad .$$



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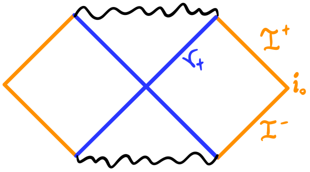

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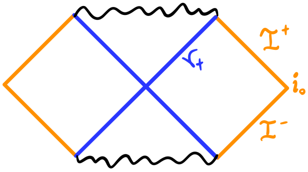

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The Hawking temperature is a geometric quantity that requires the explicit form of the spacetime at least

$$T = \frac{1}{4\pi r_+}\left(1 - \frac{11}{3}\frac{\alpha}{r_+^2}\right) + \mathcal{O}\left(\alpha^2\right)$$

Thermodynamic of dilatonic black hole

Using the Noether-wald formalism [93' Wald] [94' Iyer and Wald] to compute the entropy

$$S = \frac{\beta}{4} \int_{\mathcal{H}} \sqrt{-g}\epsilon_{\mu\nu\rho\sigma} q^{\mu\nu} dx^{\rho} \wedge dx^{\sigma} \ .$$

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The correction to the Entropy is

$$S = \frac{\pi r_+^2}{G} + \frac{46\pi\alpha}{3G} + \mathcal{O}\left(\alpha^2\right) \ .$$

The Noether-Wald formalism also provides a tool to compute the energy of the spacetime but we have to supplement the bulk action with the appropriated boundary terms [18' Deruelle, Merino and Olea]

$$
\begin{aligned}
I_{BT} &\equiv \int_{\partial M} d^3x\sqrt{-h}e^{-2\phi}\left[2K + 4\alpha\delta^{\mu_1\mu_2\mu_3}_{\nu_1\nu_2\nu_3}K^{\nu_1}_{\mu_1}\left(\frac{1}{2}\mathcal{R}^{\nu_2\nu_3}_{\quad\mu_2\mu_3} - \frac{1}{3}K^{\nu_2}_{\mu_2}K^{\nu_3}_{\mu_3}\right)\right] \\
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The mass is given by

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E = \int_{S^2_\infty}\left(\mathbf{Q}\left[\xi\right] - \xi\cdot\mathbf{B}\right) = \frac{r_+}{2G}\left(1 + \frac{11}{3}\frac{\alpha}{r_+^2}\right) .
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The first law is satisfied for this configuration

$$
\delta E = T\delta S .
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Black String in $D = 5$

In GR vacuum black string solution

$$ds^2 = -(1-\mu/r)dt^2 + \frac{dr^2}{1-\mu/r} + r^2(d\theta^2 + cos^2\theta d\varphi^2) + dz^2$$

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The linearized Einstein equations in the transeverse and traceless gauge

$$\mathring{g}^{\mu\nu}\mathring{\nabla}_\mu h_{\nu\rho} = 0 \qquad , \qquad \mathring{g}^{\mu\nu} h_{\mu\nu} = 0$$

leads to the Lichnerowicz operator

$$\Delta_L h_{\mu\nu} = \left(\delta^\rho_\mu \delta^\sigma_\nu \mathring{\Box} + \mathring{R}_\mu{}^\rho{}_\nu{}^\sigma\right) h_{\rho\sigma}$$

Gregory-Laflamme instability in GR

$$h_{\mu\nu} = \varepsilon e^{\Omega t} e^{ikz} \begin{pmatrix} H_{tt}(r) & H_{tr}(r) & 0 & 0 & H_{tz}(r) \\ H_{tr}(r) & H_{rr}(r) & 0 & 0 & H_{tr}(r) \\ 0 & 0 & H(r) & 0 & 0 \\ 0 & 0 & 0 & H(r)\sin^2\theta & 0 \\ H_{tz}(r) & H_{tr}(r) & 0 & 0 & H_{zz}(r) \end{pmatrix}$$

Gregory-Laflamme instability in GR

$$h_{\mu\nu} = \varepsilon e^{\Omega t} e^{ikz} \begin{pmatrix} H_{tt}(r) & H_{tr}(r) & 0 & 0 & H_{tz}(r) \\ H_{tr}(r) & H_{rr}(r) & 0 & 0 & H_{tr}(r) \\ 0 & 0 & H(r) & 0 & 0 \\ 0 & 0 & 0 & H(r)\sin^2\theta & 0 \\ H_{tz}(r) & H_{tr}(r) & 0 & 0 & H_{zz}(r) \end{pmatrix}$$

- Master equation for $H_{tr}(r)$.
- The boundary conditions set $\Omega = \Omega(k)$

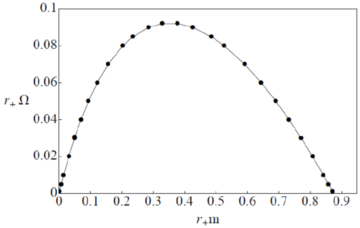


Figure 1.3 A plot of the eigenvalues (m, Ω), scaled by r_+, for which an instability is present.

(m=k) From 1107.5821 [gr-qc]

Black String $D = 5$ corrected with α'

Work in progress in collaboration with J. Oliva C. Herniquez and M. Yañez

Setting

$$\phi = \phi_0 + O\left(\alpha^2\right) \implies I\left[g_\mu\right] = \text{Einstein-Gauss-Bonnet} \qquad (1)$$

The solution is

$$ds^2 = -f\left(r\right)dt^2 + \frac{dr^2}{g\left(r\right)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) + b\left(r\right)dz^2$$

$$f\left(r\right) = 1 - \frac{r_+}{r} - \frac{\left(r - r_+\right)\left(6r_+^2 + 11rr_+ + 23r^2\right)}{9r_+r^4}\alpha + O\left(\alpha^2\right) \ ,$$

$$g\left(r\right) = 1 - \frac{r_+}{r} + \frac{\left(r - r_+\right)\left(r + 5r_+\right)\left(r + 2r_+\right)}{9r_+r^4}\alpha + O\left(\alpha^2\right) \ ,$$

$$b\left(r\right) = 1 + \frac{4\left(6r^2 + 3rr_+ + 2r_+^2\right)}{9r_+r^3}\alpha + O\left(\alpha^2\right) \ .$$

This solution is not new [Brihaye,Delsate,Radu 1004.2164]

Linearized equations

The linearized Einstein equations in the transeverse and traceless gauge

$$\mathring{g}^{\mu\nu}\mathring{\nabla}_\mu h_{\nu\rho} = 0 \qquad , \qquad \mathring{g}^{\mu\nu} h_{\mu\nu} = 0$$

leads to the α' corrected Lichnerowicz operator

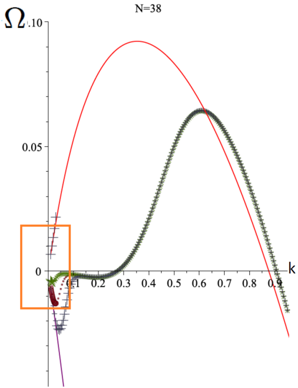
$$0 = \frac{1}{2}\left(-\delta^\rho_\mu\delta^\lambda_\nu\mathring{\Box} + 2\mathring{R}^\rho{}_{\mu\nu}{}^\lambda - 2\mathring{R}\delta^\rho_\mu\delta^\lambda_\nu + \mathring{g}_{\mu\nu}\mathring{R}^{\rho\lambda} + 4\mathring{R}^\lambda{}_{(\mu}\delta^\rho_{\nu)}\right)h_{\rho\lambda}$$
$$-2\alpha\mathring{R}_{\mu\xi\lambda\nu}\mathring{\Box}h^{\lambda\xi}$$
$$+\alpha\left(-\mathring{R}^{\rho\eta\sigma}{}_\lambda\mathring{R}_{\xi\sigma\rho\eta}\mathring{g}_{\mu\nu} - 2\mathring{R}^{\sigma\rho}{}_{\lambda\nu}\mathring{R}_{\sigma\rho\xi\mu} + 4\mathring{R}_\mu{}^{\rho\sigma}{}_\nu\mathring{R}_{\lambda\rho\sigma\xi} + 4\mathring{R}_{\mu\rho\lambda\sigma}\mathring{R}_\nu{}^{\sigma\rho}{}_\xi\right)$$
$$+\alpha\left(4\mathring{g}_{\lambda(\mu}\mathring{R}_{\nu)\rho\sigma\xi}\mathring{\nabla}^\sigma\mathring{\nabla}^\rho h^{\lambda\xi} + 4\mathring{\nabla}^\sigma\mathring{\nabla}_{(\mu}h^{\xi\lambda}\mathring{R}_{\nu)\xi\lambda\sigma} - 2\mathring{g}_{\mu\nu}\mathring{R}^\rho{}_{\lambda\xi\eta}\mathring{\nabla}_\rho\mathring{\nabla}^\eta h^{\lambda\xi}\right)$$

Corrections to the instability: Preliminary results

Only for the scalar perturbation

The parameter α is perturbative when

$$\sqrt{\alpha}k \ll 1$$



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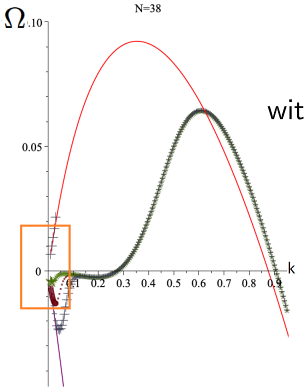

Expanding around $k = 0$

$$\Omega_{\pm}(k) = (c_1(\mu) \pm c_2(\mu)k)\alpha \mp c_3(\mu)k$$

with $\mu = 1$

$$c_1 \sim 10^{-5}$$
$$c_2 \sim 10^{-1}$$
$$c_3 \sim 10^{-1}$$

Conclusions

- In contrast to the 4-dimensional case, the field equations in arbitrary dimension D are more involved, we showed that they can still be solved explicitly in terms of hypergeometric functions.
- The black holes are asymptotically flat black holes with regular event horizons, which behave as thermodynamic objects, just like expected.

Gregory-Laflamme instability:

- It is sensible to α' corrections
- We are trying to decouple tensor and vector modes. Stable as in GR?
- $D = 6, \; D = 7$?

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Thank you!