Observing the topological magnetoelectric effect in classical and quantum electrodynamics

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Work done in collaboration with

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&

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Based on:

with SF - Classical effects: PhD Thesis and manuscript in preparation

with LCE & AMR - Quantum effects: manuscript under revision.

Outline

- Motivation
  - On topological matter and the search for ~~magnetic monopoles~~ new physics!
  - Magnetoelectric effect.
  - Applications of "evading Earnshaw's theorem".
  - Making topological magnetoelectric effect (TME) as comparable to the expected "optical" (quantum) response.
- Topological phases of matter:
  - Quantum Hall (QH) state.
  - Topological insulators (TIs) ~~and Weyl semimetals (WSMs)~~.
  - Topological band theory vs Topological field theory (TFT).
- Electromagnetic (EM) response of magnetoelectric media:
  - EM response of TIs, ~~WSMs~~ and chiral metamaterials.
- On TEM modes in a "complex" TI single conductor waveguide - S. Filipini
  - Cylindrical Boundary-value problem.
  - Results and interpretation.
  - Measurability of TMEP.
- On q-entanglement in a TI Q-Dot hybrid immersed in external B field- L. Castro-Enríquez
  - Quantisation of the TI response and Hamiltonian of the system.
  - Open systems approach for the dynamics of dissipative system. Lindblad's equation.
  - Peres' criterion and prediction of "topological entanglement".
- Summary and conclusions

Motivation - I

- **Topological matter is a new kind of matter!**
  $\longrightarrow$ new toys to play with ... and ~~not~~ surprisingly(*) sheds light on new physics.

> ### *6.11 On the Question of Magnetic Monopoles*
>
> At the present time (1998) there is no experimental evidence for the existence of magnetic charges or monopoles. But chiefly because of an early, brilliant theoretical argument of Dirac,† the search for monopoles is renewed whenever a new energy region is opened up in high-energy physics or a new source of matter, such as rocks from the moon, becomes available. Dirac's argument, outlined below, is that the mere existence of one magnetic monopole in the universe would offer an explanation of the discrete nature of electric charge. Since the quantization of charge is one of the most profound mysteries of the physical world, Dirac's idea has great appeal. The history of the theoretical ideas and experi-

Figura: JD. Jackson "Classical electrodynamics". Magnetic monopole $\rightarrow$ new physics.

(*) It turns out that the EM response of topological insulators TIs (one of the paramount examples of "topological matter" is as if a magnetic-monopole-like $\mathbf{B}$ field were induced.)

A static charge q near a TI's surface, it induces a B field that is **as if there was an image** g monopole!

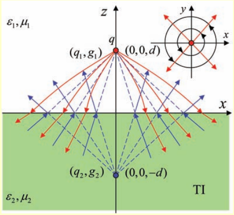


Figura: Qi, Li, Zang, Zhang, Science **323** (2009).

Motivation - II

- **Magnetoelectric effect:**

$$\mathbf{D}(\mathbf{r},\omega) = \epsilon(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega) + \frac{\alpha}{\pi}\theta(\mathbf{r},\omega)\mathbf{B}(\mathbf{r},\omega). \quad (1)$$

$$\mathbf{H}(\mathbf{r},\omega) = \frac{1}{\mu(\mathbf{r},\omega)}\mathbf{B}(\mathbf{r},\omega) - \frac{\alpha}{\pi}\theta(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega). \quad (2)$$

Either new vacuum properties (classical or quantum) or magnetoelectric media can intertwine $\mathbf{E} \leftrightarrow \mathbf{B}$ fields, **even in the static case!**

Such constitutive relations $\longleftarrow$ E-L field equations of modified Wilczek's axion electrodynamics, dubbed θ-ED.

$$S = S_0 + S_\theta = \frac{1}{8\pi}\int dt d^3\mathbf{r}\left[\epsilon\mathbf{E}^2 - (1/\mu)\mathbf{B}^2 + \frac{2\alpha}{\pi}\theta(\mathbf{r},t)\mathbf{E}\cdot\mathbf{B}\right]. \quad (3)$$

This action provides theoretical insight in many contexts:

- TFT for the QHE.
- EM response of topological insulators and Weyl semimetals.
- Dark matter proposals.
- Astrophysical scenarios.

Motivation - III

- **Evading Earnshaw theorem. Opens the door for TEM modes in guided EM waves:**
  - The E-B field **solutions** depend (obviously) on the boundary conditions (BCs)!
  - The θ term in the modified Maxwell theory change the BCs.
  - $\longrightarrow$ Solutions that were not allowed in ordinary Maxwell theory, are now allowed due to modified BCs?

- In Maxwell's theory with ordinary matter/vacua transverse electromagnetic **(TEM) waves cannot exist in confined media with less than 2 conductors**.

- TEM waves have practical importance: (i) No cuttof frequency, (ii) nor bounds on operational bandwidth, (iii) less Ohmic dissipations.

TEM waves basically mean:

- $\mathbf{E(r,t)} = \mathbf{E_t} + \mathbf{E_z}\hat{\mathbf{z}}$, with $\mathbf{E}_z = 0$. Similar for $\mathbf{B}$.
- Free space dispersion relation: $k = |\mathbf{k}| = \pm\frac{\omega}{\mathbf{c}}\sqrt{\mu\epsilon}$.
- Mutually transverse and transverse to direction of propagation $\mathbf{B} = \pm\sqrt{\mu\epsilon}\,\mathbf{z}\times\mathbf{E}$.

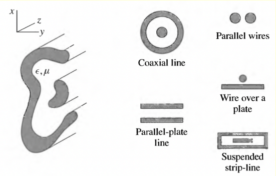


Figura: Waveguide profile. A. Zangwill "Modern electrodynamics".

General Idea

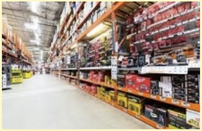
- Search TME effects. If enhancement is possible, better!
- Describe/characterise the interaction between radiation and matter with topological phases.
- Open the door for a new kind of photonics.
- Find concrete applications of a very much cherished topological field theory in hep, namely, axion electrodynamics.
- Looking for TEM solutions, that were otherwise precluded in Maxwell's theory, since 2015 when we (A. Martín-Ruiz, <u>MC</u>, L.F. Urrutia: PRD**92**125015) reported a possibility to evade Earnshaw's thm. i.e., with conductors or trivial insulators this was a "no-go" thm, now we are opening a door into a variety of new possibilities. L. Cancino (undergrad thesis unpublished) also made some numerical progress hinting in this direction.
- ... many other motivations: Concrete scenario for realizing Weyl fermions, for studying relativistic like excitations on "table-top" experiments [CV, AR, AJM, ML]...

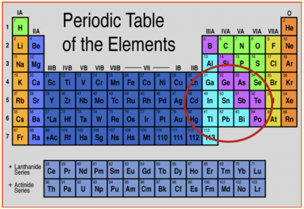
Topological insulators - I

Qualitative description

Topological insulators - What are they?

In the real world:



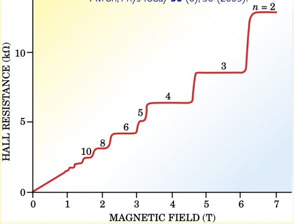


Material that is insulating in bulk, in its core,

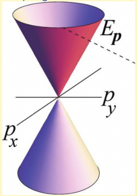
that, however, exhibits conducting/metallic states on the boundary.

Ok, but not like a brick of glass with a thin copper coating on the outside!

Avron, Phys Today **56** (8), 38 (2003).



Tkachov, "Topological Insulators", '16.



Qi, Science **323**, 1184 (2009).

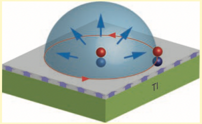
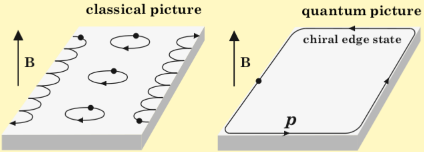


Fig. 3. Illustration of the fractional statistics induced by image monopole effect. Each electron forms a dyon with its image monopole. When two electrons are exchanged, an AB phase factor is obtained (which is determined by half of the image monopole flux) and leads to statistical transmutation.

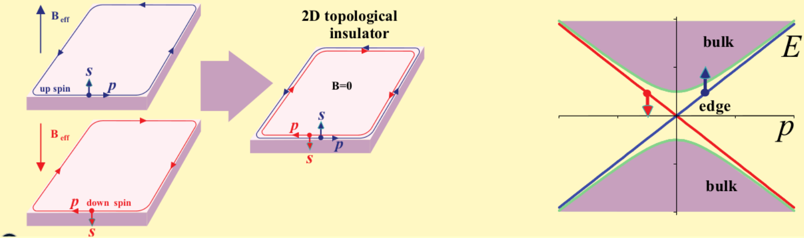
Figura: TIs are insulating in the bulk, and have robust metallic states on the edge.

Topological insulators - II

Direction of **p** is tied to **B**.
(smoking gun of broken TRS)*



Due to SOC, effective **B** for each spin state. But two copies have no total **B** (thus TRS)! And 2 helical (spin-momentum orientation locked)



figures from G. Tkachov (2016)

Figura: TIs are insulating in the bulk, and have robust metallic states on the edge.

Weyl semimetals (WSMs - I)

- Can be thought of as 3D analogues of graphene.
- Topological phases with broken time-reversal (or inversion) symmetry.
- Band structure containing two Weyl nodes in the Brillouin zone when the Fermi level is close to the Weyl nodes. Weyl node:= band crossing point (conduction-valence).
- WSM possess protected gapless surface states on disconnected Fermi arcs with end points at the projection of the bulk nodes onto the surface of the BZ.

WSMs - II

Low-energy physics of two-node WSM is described by

$$H = v_F \hbar \tau^z \boldsymbol{\sigma} \cdot (\mathbf{k} + \tau^z \mathbf{b}) + \hbar \tau^z b_0, \qquad \mathbf{k} = -i\nabla$$

$\boldsymbol{\tau}$ Node d.o.f.

(Comment role of chiral anomaly)

v_F Fermi velocity

$\Delta E \propto 2\hbar b_0$

$\boldsymbol{\sigma}$ conduction-valence band d.o.f.

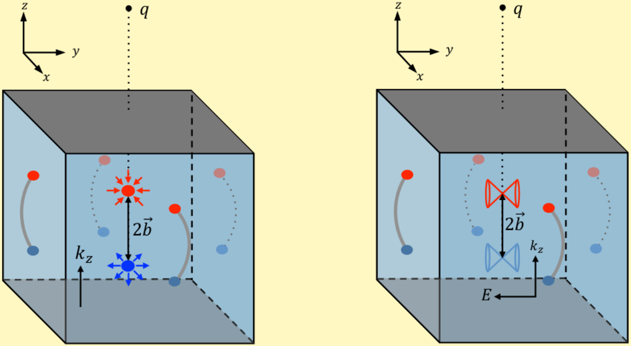


Figura: Mixture of momentum space and coordinate space depicted. Sources and sinks of Berry curvature are shown on the left. On the right the Weyl nodes are shown. In both, the Fermi arc states on the boundary are also shown.

Topological band theory (TBT) vs Topological field theory (TFT)

Currently there are two different and accepted general theories for TIs: **TBT** and **TFT**.

Topological Band Theory: **e.g., TKNN**

- Valid for noninteracting systems without disorder. (For some cases, the stability of topological phases has been studied with mild disorder and interaction.)
- However, it provides a simple and important criteria to evaluate which band insulators are topological nontrivial.

Topological Field Theory

- Generally valid for interacting systems including disorder.
- It therefore identifies the physical **response** associated with topological order.
- TFT reduces to TBT in the noninteracting limit.

To describe the interaction of "topo-matter" with EM radiation, let us review TFT.

Chern-Simons-like action - I

In 1992 Zhang found a correct description for the QH effect in these terms. In particular, his description captures the desired features:

1. Quantisation of Hall conductance.
2. Fractional charge and statistics of quasiparticles.
3. Ground-state degeneracy.

Zhang's TFT for the QH effect is given by the effective action.

$$S_{eff} = \frac{C_1}{4\pi} \int d^2x \int dt A_\mu \epsilon^{\mu\nu\tau} \partial_\nu A_\tau .$$

Where C_1 can be cast as (the TKNN invariant) the integral of the Berry curvature in $2D$ momentum space:

$$C_1 = \frac{1}{2\pi} \int d^2k f_{xy}(\mathbf{k}) \ \in \ \mathbb{Z}, \quad \text{with} \quad f_{xy} = \partial_{k_x} a_y(\mathbf{k}) - \partial_{k_y} a_x(\mathbf{k})$$

and $a_i(\mathbf{k})$ the Berry connection. From the effective Lagrangian one finds:

$$j_\mu = \frac{C_1}{2\pi} \epsilon^{\mu\nu\tau} \partial_\nu A_\tau , \quad j_i = \frac{C_1}{2\pi} \epsilon^{ij} E_j \quad \text{and} \quad j_0 = \frac{C_1}{2\pi} B.$$

The QH response with conductance $\sigma_H = C_1/2\pi$!

E-filed induces a transverse current and a B-field induces charge accumulation.

Chern-Simons-like action - II

- Zhang's TFT for the 2+1 dimensional QH is... only 2+1. Ok that was handy for the QH effect which is a planar phenomenon. But how about other material with topological order for which we want a TFT if they are higher dimensional, say a 3 spatial dimensional material?

- As all CS theories Zhang's is in odd dimensions. However it was soon generalised to 4+1, and so on.

- One can obtain a 3+1 version by means of dimensional reduction:

$$A_\mu(x) = \begin{cases} A_\mu(x_0, x_1, x_2, x_3) & \text{for} \quad \mu \neq 4 \\ A_4(x_0, x_1, x_2, x_3, x_4) & \text{for} \quad \mu = 4 \end{cases}$$

- Compactifying the extra dimension on a circle results in :

$$S_\theta = \frac{\alpha}{32\pi^2} \int d^3x dt \theta(x,t) \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu}(x,t) F_{\lambda\rho}(x,t), \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

Where $\theta(x,t)$ is proportional to the flux due to A_4 through the compactified extra dimension.

TFT for electromagnetic (EM) response of topologically ordered media

The action for the EM response of TIs and WSMs reads (non-covariantly):

$$S = S_0 + S_\theta = \frac{1}{8\pi}\int dt d^3\mathbf{r}\left[\epsilon \mathbf{E}^2 - (1/\mu)\mathbf{B}^2 + \frac{2\alpha}{\pi}\theta(\mathbf{r},t)\mathbf{E}\cdot\mathbf{B}\right]. \qquad (4)$$

The parameters ϵ, μ and θ are effective macroscopic parameters that actually encode information about the microscopic (band) structure of the material. In the case of ϵ, μ it is "optical" properties, while θ is purely topological property.

Modified EM field equations are:

$$\begin{aligned} \nabla\cdot(\varepsilon\mathbf{E}) &= 4\pi\rho - \frac{\alpha}{\pi}\nabla\theta\cdot\mathbf{B}, \quad \nabla\times\mathbf{E} = -c^{-1}\partial\mathbf{B}/\partial t, & (5)\\ \nabla\cdot\mathbf{B} &= 0, \quad \nabla\times(\mathbf{B}/\mu) - \frac{1}{c}\frac{\partial(\varepsilon\mathbf{E})}{\partial t} = \frac{4\pi}{c}\mathbf{J} + \frac{\alpha}{\pi}\nabla\theta\times\mathbf{E} + \frac{\alpha}{c\pi}\frac{\partial\theta}{\partial t}\mathbf{B}. & (6)\end{aligned}$$

* For 3D time reversal (TR) invariant TIs, $\theta(\mathbf{r},t)$ is piecewise constant with a finite discontinuity at the interface of between topologically trivial band insulator $\theta = 0$ and a non-trivial one $\theta = (2n+1)\pi$.

* For 3D WSMs with broken TR and inversion symmetries, $\theta(\mathbf{r},t) = 2\mathbf{b}\cdot\mathbf{r} - 2b_0 t$. The quantity $\mathbf{b}$ reflects the splitting of the Dirac cones into pairs of Weyl nodes in momentum space, b_0 is related to the energy offset of the Weyl nodes.

EM response of Topological Insulators (TIs).

Key issue is that piecewise constant θ implies $\nabla\theta = \tilde{\theta}\delta(\Sigma)\hat{\mathbf{n}}$ and $\partial_t\theta = 0$, where $\tilde{\theta}$ is the discontinuity of θ across the boundary and Σ is the interface, with unit normal $\hat{\mathbf{n}}$. With this e.g., the BCs that lead to the TME effect are:

$$\Delta\mathbf{E}\cdot\hat{\mathbf{n}}|_\Sigma = 4\pi\sigma + \tilde{\theta}\mathbf{B}\cdot\hat{\mathbf{n}}|_\Sigma, \qquad \Delta\mathbf{B}\cdot\hat{\mathbf{n}}|_\Sigma = -\tilde{\theta}\mathbf{E}\cdot\hat{\mathbf{n}}. \tag{7}$$

Previously reported results of considerable interest: (i) Adapted Green's Theorem for new BCs (Dirichlet or Neumann); (ii) General Green's function for the corresponding boundary value problems for planar, spherical and cylindrical TI interfaces; (iii) Earnshaw's theorem circumvented; (iv) Conservation laws; (v) Contribution to the Casimir effect, ...

One key observation for the EM response is to focus in situations highly sensitive to boundary conditions.

What about propagation of θ-EM waves in confined spaces?

Could it be that the modified BCs allow new/different modes?

e.g., are TEM waves allowed or not in hollow pipes with θ boundaries?

EM response of WSMs.

In general the electric current depends both on ***E*** and ***B*** field. Plus chiral fermions in ***B*** field with chemical potentials $\mu_{\rm L}$ and $\mu_{\rm R}$ Two additional ***B*** dependent contribs. to current:

$$\mathbf{J}^{(\rm E)} = \sigma_{ij}(\omega) E_j \hat{\mathbf{e}}_i \qquad \mathbf{J}^{(\rm B)} = \frac{\alpha}{2\pi^2}\mu_5 \mathbf{B} \quad , \quad \mathbf{J}_5^{(\rm B)} = \frac{\alpha}{2\pi^2}\mu \mathbf{B},$$

$\mu_5 = \dfrac{\mu_L - \mu_r}{2}$ chiral and electric chemical potentials respectively. $\mu = \dfrac{\mu_L + \mu_r}{2}$

It turns out that the effective action leading to low-energy physics of WSM is...?

$$S_\theta = \frac{\alpha}{4\pi^2}\int \theta(\mathbf{r},t)\,\mathbf{E}\cdot\mathbf{B}\,dt d^3x, \qquad \theta(\mathbf{r}, t = i\tau) = 2\mathbf{b}\cdot\mathbf{r} - 2ib_0\tau$$

Field eqs: (sourceless eqs. unaltered). And corresponding constitutive relations.

$$\nabla\cdot\mathbf{D} = 4\pi\left(\rho - \frac{\alpha}{2\pi^2}\mathbf{b}\cdot\mathbf{B}\right) \qquad \mathbf{B} = (1+\chi_m)\,\mathbf{H}$$

$$\nabla\times\mathbf{H} - \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t} = \frac{4\pi}{c}\left(\mathbf{J} + \frac{\alpha}{2\pi^2}c\mathbf{b}\times\mathbf{E} - \frac{\alpha}{2\pi^2}b_0\mathbf{B}\right) \qquad \mathbf{D} = \tilde{\epsilon}\,\mathbf{E} \qquad \tilde{\epsilon} = \epsilon + i\,\epsilon(b)$$

AHE (as what happens for WSM with brokenTR)

Contributes to the CME additional to $\boldsymbol{J}^{(\mathbf{B})}$ above.

Are you hungry?

Following Fabrizio's pasta and Alfredo's Tacos, may I add other cylindrically layered dishes.

On TEM modes in a "complex" TI single conductor waveguide - S. Filipini

- Consider a cylindrical conducting boundary at $\rho = R_2$.
- The space $0 \leq \rho \leq R_1$ filled with medium $\epsilon_1, \mu_1 = 1, \theta_1$.
- The space $R_1 \leq \rho \leq R_2$ filled with $\epsilon_2, \mu_2 = 1, \theta = 2$.
- A first condition is $\epsilon_1 = \epsilon_2$. In fact, for the usual coaxial cable this is a condition too.

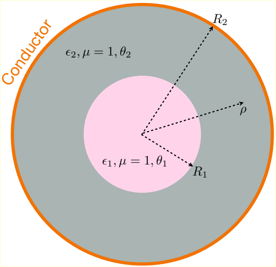


Figura: There is no conductor at R_1. In ordinary Maxwell, this config does not allow TEM waves.

- TEM waves only possible iff:

$$1 + \frac{\eta_\pm^2 \tilde{\theta}^2}{4}\left(1 - \left(\frac{R_1}{R_2}\right)^{2\nu}\right)^2 = 0 \quad \Longrightarrow R_2 = R_2(\eta_\pm, \tilde{\theta}, \nu, R_1). \qquad (8)$$

$\eta_\pm \equiv \pm\sqrt{\mu/\epsilon}$, the $\pm$: propagation in the $\pm\,\hat{\mathbf{z}}$ direction, $\tilde{\theta} \equiv \theta_2 - \theta_1$, and ν is a "mode" index coming from the separation constant.

- Either:
  - Negative index material (NIM): $\epsilon \leq 0$.
  - θ purely imaginary.

- This model can describe TIs when frequency range is slightly away the Fermi energy and within the energy of the gap. where θ may acquire imaginary part.
- NIM à la Veselago (1968), are plausible but in that case it would not necessarily be related to a topological magnetoelectric effect.
- The TEM wave solutions for arbitrary ν are cumbersome. To get an idea, for $\nu = 1$:

$$\mathbf{E}_1^I(\rho,\phi) = E_0 \frac{\rho}{|\Delta_i| R_1^2} e^{i\,\mathrm{sgn}(\Delta_i)\phi} (i\,\mathrm{sgn}(\Delta_i)\hat{\boldsymbol{\rho}} - \hat{\boldsymbol{\phi}}) \tag{9}$$

$$\mathbf{E}_1^{II}(\rho,\phi) = E_0 \frac{1}{\rho^2 R_2^2} e^{i\,\mathrm{sgn}(\Delta_i)\phi} \left(-i\,\mathrm{sgn}(\Delta_i)\left(\rho^2 + R_2^2\right)\hat{\boldsymbol{\rho}} + (\rho^2 - R_2^2)\hat{\boldsymbol{\phi}}\right) \tag{10}$$

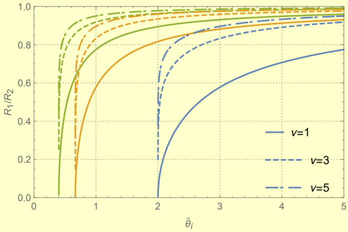
where R_2 is a (fixed by existence of TEM BCs) function of $R_1, \eta_\pm$ and $\tilde{\theta}$ and $\mathbf{B}^i = \pm\sqrt{\mu\epsilon}\,\mathbf{z} \times \mathbf{E}^i$.

- EM responses are “classical”, hence these TME responses, being of $\mathcal{O}(\alpha)$, are suppressed compared to usual optical or other transport properties. Yet,

I will show that Filipini’s proposal, does yield potentially observable effects.

On TEM modes in a "complex" TI single conductor waveguide - S. Filipini

- TEM boundary value problem $\rightarrow$ a non trivial condition between the geometry & "optical" properties.
- For given $\eta_{\pm}, \tilde{\theta}$, for each value of $\nu = 0, 1, 2, \ldots$ only some $R_2 = R_2(R_1)$ are admissible.



- "Topological" charges and current densities are induced at the θ interface and these do satisfy a continuity equation.

$$\sigma_\theta(t, z, \phi) = \pm \frac{\epsilon E_0}{2\pi} \mathrm{sgn}(\tilde{\theta}_i) \frac{\nu}{R_1^{1+\nu}} \sin(kz - \omega t + \mathrm{sgn}(\Delta_i)\nu\phi) \tag{11}$$

$$\mathbf{K}_\theta(t, z, \phi) = \frac{cE_0}{2\pi} \mathrm{sgn}(\tilde{\theta}_i) \sqrt{\frac{\epsilon}{\mu}} \frac{\nu}{R_1^{\nu+1}} \sin(kz - \omega t + \mathrm{sgn}(\Delta_i)\nu\phi)\hat{\mathbf{z}}$$

EB fields exhibit interesting "mode" patterns.

- Shower nozzle selector,
- Flux/counterflux
- Field configurations that can be exploited (à la Penning trap).

On TEM modes in a "complex" TI single conductor waveguide - S. Filipini

- Shower nozzle selector. $\mathbf{E}_\nu$ fields for $R_2 = 1, \tilde{\theta}_i = 4, \eta_\pm = +1, z = 0$ and $t = 0$ for different values of ν.

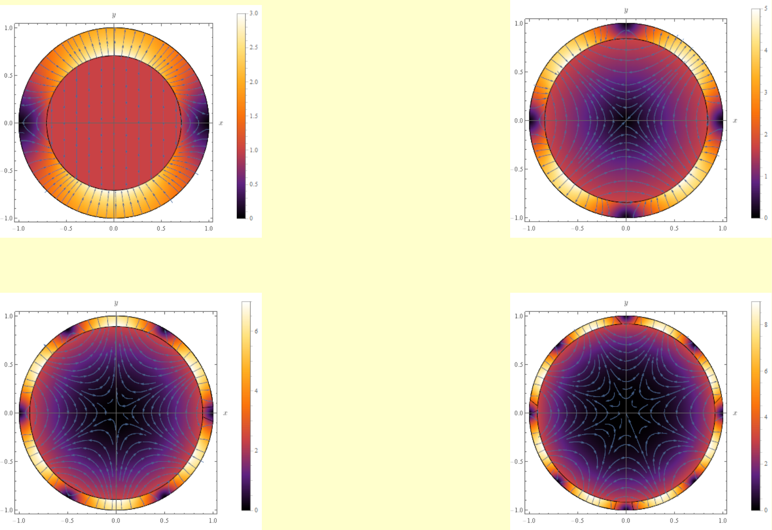


Figura: Density Plots of E for (a) $\nu = 1$, (b) $\nu = 2$, (c) $\nu = 3$ y (d) $\nu = 4$

- One of the figures of merit is the Poynting vector.

$$\left\langle \mathbf{S}_\nu^{(I)} \right\rangle = \frac{c}{4\pi} |\mathsf{E}_0|^2 \nu^2 \frac{1}{\Delta_i^2} (\eta_\pm^{-1} + \mathrm{sgn}(\Delta_i)\theta_{1i}) \left(\frac{\rho^{\nu-1}}{R_1^{2\nu}} \right)^2 \hat{\mathbf{z}} \tag{12}$$

$$\left\langle \mathbf{S}_\nu^{(II)} \right\rangle = \frac{c}{4\pi} |\mathsf{E}_0|^2 \nu^2 \left((\eta_\pm^{-1} - \mathrm{sgn}(\Delta_i)\theta_{2i}) \frac{1}{\rho^{2\nu+2}} + (\eta_\pm^{-1} + \mathrm{sgn}(\Delta_i)\theta_{2i}) \frac{\rho^{2\nu-2}}{R_2^{4\nu}} \right) \hat{\mathbf{z}} \tag{13}$$

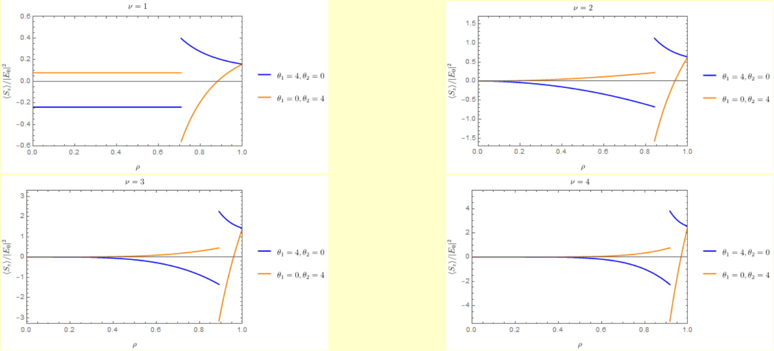


Figura: Poynting in inner and outer regions. Flux/counterflux effect interesting for "light trapping".

Summary and interpretation of c-effects

- **On feasible realization**

  1. Imaginary θ could be associated with chiral metamaterials, but no so satisfactorily. But it can be seen to describe NIM, à la Veselago.
  2. More promising is the case of a TI in the frequency regime where θ is to acquire an imaginary part. This is not untenable, but care must be taken not to go away of the range for proper description in terms of axion electrodynamics.

- **On novelty:**

  1. To the best of our knowledge no TEM waves in hollow waveguides have ever been predicted before. This opens a possibility in the, so far, unknown.
  2. In the literature, at most "quasi-TEM" propagation is a very intense area of research.

- **On possible applications:**

  1. The "shower nozzle selector" could be used to trap and harness light in specific "modal" manner.
  2. Note that the E field profiles have rich structure. One could have resonating cavities with new properties. Also fields are similar to those in Penning traps (relevant for particle experimentalist) or plasma confinement.

- This is a quantum topological magnetoelectric effect.
- Characterising plasmonic nanostructures and quantum dots (QD) has potential applications in nanophotonics and spintronics. Also, advances in the fabrication of TI devices have opened a new avenue towards q-computation and information.
- **Idea:** explore the entanglement of a TI nanostructure and a semiconductor quantum dot. $\rightarrow$ Quantum optical approach.

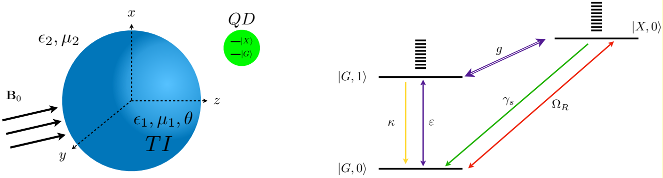


Figura: (L) Hybrid comprised of TI nanoparticle and a semiconductor QD immersed in an external $\mathbf{B}$ field. (R) TI-QD tower of states truncated for $n = 0, 1$, the states are connected by the amplitude ε of the TI $\mathbf{E}$ field, the total decay rate κ of the TI, the coupling strength parameter g, the spontaneous emission rate γ_s of the QD and the Rabi frequency Ω_R.

- Due to the TME effect, the $\mathbf{B}_0(\mathbf{r},t) = \mathrm{Re}\left\{\mathbf{B}_0 e^{-i\omega t}\right\}$ field polarises the TI, which can be described by the electric field

$$\mathbf{E}(\mathbf{r},\omega) = -\frac{\tilde{\alpha} c}{2\epsilon_2 + \epsilon_1(\omega) + (2/3)\tilde{\alpha}^2} \sum_i B_{0i}\,\mathbf{G}_i(\mathbf{r}), \quad \mathbf{G}_i(\mathbf{r}) = \begin{cases} \hat{\mathbf{e}}_i & r < R \\ -\frac{R^3}{r^3}\left[3\left(\hat{\mathbf{e}}_r \cdot \hat{\mathbf{e}}_i\right)\hat{\mathbf{e}}_r - \hat{\mathbf{e}}_i\right] & r > R \end{cases}, \tag{14}$$

where $\tilde{\alpha} = \alpha \cdot (\theta/\pi) = (2l+1)/137$ with $l = 0, 1, 2, \ldots$, c is the speed of light and B_{0i} is the i-th component of the external magnetic field. The $\mathbf{G}_i(\mathbf{r})$ GFs can be obtained from the techniques developed by AMR, MC, LFU.

- Low concentration of free carriers in TI: $\rightarrow \epsilon_1(\omega) - 1 = \omega_e^2[\omega_R^2 - \omega(\omega + i\gamma_0)]^{-1}$, where ω_R and ω_e are the resonant and natural frequencies, respectively. $\gamma_0 \ll \omega_R \sim$ Ohmic losses in the TI. When $\gamma_0 \ll \omega$ the electric field reads

$$\mathbf{E}(\mathbf{r},\omega) = -\eta \frac{\omega_0^2/2\Omega}{\omega - \Omega + i\gamma_0/2} \sum_i c B_{0i}\,\mathbf{G}_i(\mathbf{r}), \qquad \eta = \frac{\tilde{\alpha}}{2\epsilon_2 + 1 + (2/3)\tilde{\alpha}^2}, \tag{15}$$

where $\omega_0 = \omega_e\sqrt{\eta/\tilde{\alpha}}$ and $\Omega^2 = \omega_R^2 + \omega_0^2$. We are interested in the approximation when TI interacts with a dipole whose resonant frequency is close to the plasmon resonance.

Quantum optical model for TI-QD interaction:

- The full Hamiltonian $\hat{H}_{\mathsf{S}} = \hat{H}_{\mathsf{TI}} + \hat{H}_{\mathsf{dip}} + \hat{H}_{\mathsf{int}} + \hat{H}_{\mathsf{exc}}$ is explained as[1] :

$$\begin{aligned} \hat{H}_{\mathsf{TI}} &= \hbar\Delta_{\mathrm{TI}}\hat{a}^{\dagger}\hat{a}, & (16)\\ \hat{H}_{\mathsf{dip}} &= \hbar\Delta_{\mathrm{dip}}\hat{\sigma}^{+}\hat{\sigma}^{-}, & (17)\\ \hat{H}_{\mathsf{int}} &= \hbar g(r)(\hat{\sigma}^{+}\hat{a} + \hat{a}^{\dagger}\hat{\sigma}^{-}), \quad \leftarrow -\hat{\mathbf{p}}\cdot\hat{\mathbf{E}}_{\mathrm{TI}}, \quad \hat{\mathbf{p}} = d(\hat{\sigma}^{+} + \hat{\sigma}^{-})\hat{\mathbf{e}}_i & (18)\\ \hat{H}_{\mathsf{exc}} &= i\hbar(\sqrt{k}\varepsilon\hat{a}^{\dagger} - \sqrt{k}\varepsilon^{*}\hat{a} + \Omega_R\hat{\sigma}^{+} - \Omega_R^{*}\hat{\sigma}^{-}), & (19) \end{aligned}$$

where:

- $\Delta_{\mathrm{TI}} = \Omega - \omega$ and $\Delta_{\mathrm{dip}} = \omega_a - \omega$: detunings between the TI and dipole with the external field.
- $\hat{a}(\hat{a}^{\dagger})$: bosonic creation(annihilation) of plasmon modes induced on the TIs surface.
- ω_a the resonant freq. of the dipole and $\Omega \sim$ freq. of induced $\mathbf{E}$ field.
- d is the dipole moment of the transition and $\hat{\sigma}^{+}$ ($\hat{\sigma}^{-}$) is the Pauli raising (lowering) operator for the TI in the two-level approx.
- $g(r)$ is the TI-QD coupling strength. It depends on the relative orientation of the TIs E field and the QDs dipole. We call this **L**ongitudinal or **T**ransverse **C**ouplings, LC or TC.
- κ the total decay rate of the TI due to Ohmic losses to the environment and to its scattering into free space, $\varepsilon = \varepsilon_0\, e^{i\phi}$ the amplitude of the electric field induced on the TIs surface driven by the external field, and $\Omega_R = iE_0\mu/2\hbar$ the Rabi frequency.

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[1] Some approximations have been made: RWA, Unitary trans, (BCH) to eliminate fastly oscillating terms in the interaction with externall medium.

Open-systems formalism for q-evol (Lindblad) & entanglement

- Lindblad's master equations takes the form:

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H}_S, \hat{\rho}] + \hat{\mathcal{L}}_{\text{TI}} + \hat{\mathcal{L}}_{\text{QD}}\,, \qquad \hat{\mathcal{L}}_{\text{TI}} = -\frac{\kappa}{2}(\hat{a}^\dagger\hat{a}\hat{\rho} + \hat{\rho}\hat{a}^\dagger\hat{a} - 2\hat{a}\hat{\rho}\hat{a}^\dagger)\,,$$

$$\hat{\mathcal{L}}_{\text{QD}} = -\frac{\gamma_s}{2}(\hat{\sigma}^+\hat{\sigma}^-\hat{\rho} + \hat{\rho}\hat{\sigma}^+\hat{\sigma}^- - 2\hat{\sigma}^-\hat{\rho}\hat{\sigma}^+) + \frac{\gamma_d}{2}(\hat{\sigma}^+\hat{\sigma}^-\hat{\rho} - \hat{\rho}\hat{\sigma}^+\hat{\sigma}^- + \hat{\sigma}^+\hat{\sigma}^-\hat{\rho}\hat{\sigma}^+\hat{\sigma}^- - \hat{\rho}$$

  $\hat{\mathcal{L}}_{\text{TI}}$ and $\hat{\mathcal{L}}_{\text{QD}}$ are the Lindblad operators of the TI and QD. γ_s and γ_d the spontaneous emission rate and dipole dephasing rate of the QD, respectively. κ is the total decay rate of the TI (includes the damping parameter due to Ohmic losses to the environment and its scattering rate into free space).
- Peres' criterion for entanglement: $E(\rho)$ can be quantified by the formula:

$$E(\rho)=\sum_{n=0,1,2\ldots} \max\left\{0,|\rho_{Xn,Gn+1}|-\sqrt{\rho_{Gn,Gn}\rho_{Xn+1,Xn+1}}\right\}. \qquad (21)$$

  Since $|n\rangle = 0, 1$ and $|G,\, X\rangle = 0, 1 \rightarrow E(\rho) = \max\{0, |\rho_{1001}|\}$, the TI-QD entanglement is given by the max value between zero and the absolute value of the density matrix element ρ_{1001} (obtained numerically from the OBEs).
- For a $TlBiSe_2$ TI nanoparticle:
  1. $R = 4$ nm, $\mu = 1$, $\epsilon_1(0) \sim 4$, an energy of $\hbar\Omega = 2{,}2$ eV, damping parameter $\gamma_0 = 0{,}2$ eV, a scattering rate into free-space modes $\gamma_r = 2{,}3 \times 10^{-4}$ eV, hence the total decay rate of the TI is $\kappa = \gamma_0 + \gamma_r = 2{,}5 \times 10^{-4}$ eV.
  2. The TI embedded in a polymer layer of poly(methyl methacrylate) with $\epsilon_2 = 1{,}5$.
- For a spherical CdSe QD of 4 nm:
  1. A resonance energy $\hbar\omega_a = 2{,}0$ eV, spontaneous emission rate $\gamma_s = 6{,}5 \times 10^{-8}$ eV,
  2. dipole dephasing rate $\gamma_d = 4{,}9 \times 10^{-6}$ eV, and transition dipole moment $d = 6{,}4 \times 10^{-28}$ Cm. The external field wavelength $\lambda = 2\pi c/\omega = 550$ nm, $\rightarrow$ TI- QD detunings of $\Delta_{\text{TI}} = 1{,}87$ eV and $\Delta_{\text{dip}} = 1{,}84$ eV, respectively.
  3. Thus the TI driving field amplitude ε approaches the rate of one photon per modified lifetime of the QD, i.e. $|\varepsilon|^2 = \Gamma$, where $\Gamma = \gamma_s + [4g^2\kappa/\kappa^2 + 4(\Omega - \omega_a)^2]$ is defined as the modified spontaneous emission rate. (Refs upon request)

On q-entanglement in a TI Q-Dot hybrid L. Castro-Enríquez

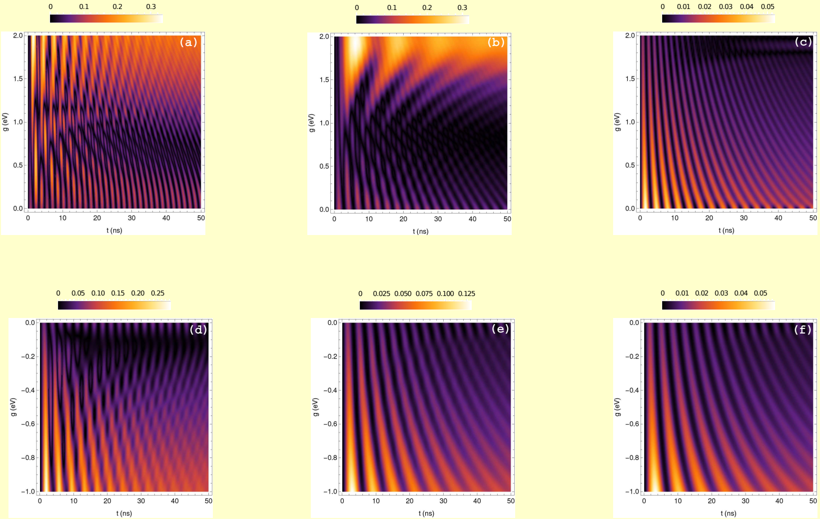

Figura: Density Plots of Entanglement vs coupling strength (g) vs time (t) in LC (up) and TC (bottom) for different values of the parameter $\tilde{\alpha}$: (a, d) $\tilde{\alpha} = 1/137$, (b, e) $\tilde{\alpha} = 3/137$, (c, f) $\tilde{\alpha} = 7/137$.

On q-entanglement in a TI Q-Dot hybrid L. Castro-Enríquez

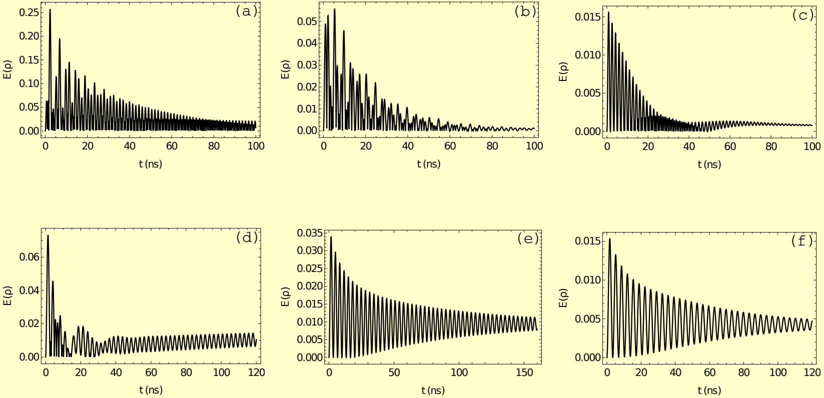

Figura: Entanglement $E(\rho)$ vs time in LC (up) and TC (bottom) with (a, d) $\tilde{\alpha} = 1/137$, (b, e) $\tilde{\alpha} = 3/137$, and (c, f) $\tilde{\alpha} = 7/137$. We fix the coupling strength to (a) $g = 0{,}5$ eV, (b) $g = 0{,}8$ eV, (c) $g = 1{,}8$ eV, (d) $g = -0{,}1$ eV, (e) $g = -0{,}05$ eV, and (f) $g = -0{,}05$ eV.

Summary and interpretation of q-effects

- **Dependence of entanglement on the spatial configurations variables:**

1. $E(\rho)$ decreases with the increasing TI-QD distance (expected).

2. $E(t)$ is determined by the time evolution of the ρ-matrix elements and an oscillatory behaviour is not unexpected. For both $\mathfrak{p}$ (longitudinal L, or transverse T "polarizations" $\mathfrak{p}$) and most ranges of $\tilde{\alpha}$ and g, $E(t)$ attenuates with a generic time profile $\sim \mathcal{E}_{\mathfrak{p}}^{\tilde{\alpha}}(g,t)\, e^{\pm\, i\, \omega_{\mathfrak{p}}^{\tilde{\alpha}}(g) t}$, with $\mathcal{E}_{\mathfrak{p}}^{\tilde{\alpha}}(g,t)$ an attenuating function. For a given g and the same $\tilde{\alpha}$, the angular frequency for LC is $\omega_{LC} \;\dot{\approx}\; 2\,\omega_{TC}$. Now $g(r)$ are interaction energies! Thus, the oscillatory behaviours are governed by $e^{\pm i\omega_{\mathfrak{p}}\, t} = e^{\pm i\,\mathfrak{E}_{\mathfrak{p}}\, t/\hbar}$, where $\mathfrak{E}_{\mathfrak{p}} = \hbar\omega_{\mathfrak{p}}$. Namely, the energy scale of the interaction g sets the time scale for the oscillation of entanglement, thus explaining the factor of $\sim$2 in the frequencies.

3. Regarding the attenuating functions, for $\mathfrak{p}$ =TC attenuates to a constant non-zero value, while for $\mathfrak{p}$ =LC it does to zero. This can be exploited in the following ways. If in a given experimental setup the TI-QD coupling were "unpolarized" but for some reason one can observe whether the states are entangled or not, then this observation can serve to discriminate between different $\mathfrak{p}$. In quantum-information or -computing devices, **decoherence** poses serious problems, mostly due to entanglement. If a setup like ours were appropriate in these contexts, then the a priori choice of $\mathfrak{p}$=TC and the TMEP could foster fault-tolerant quantum computation by masking the effects due to decoherence.

Summary and interpretation of q-effects

- **On temporal evolution of entanglement:**

  As to the different time-behaviour of the attenuating functions, we hypothesize that it might due to either:

  1. The damping effects due to Ohmic losses being larger the stronger the coupling g as the induced excitations on the TI's surface dissipate faster for LC. This is untenable though, as it implies, contrary to what we have found, that we merely have to wait longer for TC entanglement to decay to zero;
  2. A phenomenon related to entanglement sudden death which has been observed occasionally when there is evolution in a dissipative environment or;
  3. Given that for TC the states of the system always keeps probing the TMEP effects of the TI, while for LC it does not, hinging on the fact that the observed entanglement is entirely due to the TMEP, a more speculative hypothesis would be that the quantum mechanical evolution of the interaction between the TI-QD immersed in the external field could trigger a quantum phase transition not necessarily related to one in the band structure of the TI, but rather to the rearrangements of the patterns of entanglement in the full interacting system. The perspective that topological order should be understood from an entanglement point of view (Wen(2019)) endorses this hypothesis.

Summary & Conclusions

- The use of effective topological field theory for the electromagnetic response of topologically ordered media and chiral matter is very powerful.
- Topo and chiral matter have promising applications and also shed lights on fundamental physics, we'd better learn how to characterise, functionalise and control it.
- Photonics with "new" topo toys can lead to applications yet to be discovered.
- In both scenarios c_θ-responses- S. Filipini and q_θ-responses - L.E. Castro we have predicted effects due to the TMEP that are observable with present day resolution.
- Finally we have found sensible and observable applications of evading Earnshaw's theorem. Rotations of the plane of polarisation à la Filipini and "shower nozzle" selector mode could have important applications in harnessing light, cavity resonators and other scenarios.
- TI-QD Entanglement immersed in an external B field, results as a direct consequence of the Z_2 invariant of 3D TIs. To the best of our knowledge, this is the first time that entanglement is given as a direct manifestation of the topological magnetoelectric polarizability. Here, furthermore, "optical" response is already of quantum origin, therefore topo effects are not suppressed with respect to the latter!

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Let us all thank the organisers for the kind invitation, the wonderful Workshop, friendly environment and great trip to *Parque de Agua Nevados de Chillán*and . . . for keeping up the standards!