Cosmological Scalar Field φCDM and Mass Varying Neutrino Models

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The presentation is supported in part by the Rustaveli National Science Foundation of Georgia under Grant FR-19-8306

Based on papers: Avsajanishvili *et al.*, 2014; Avsajanishvili *et al.*, 2015; Avsajanishvili *et al.*, 2018

International Conference on Neutrinos and Dark Matter, Sharm El Sheikh

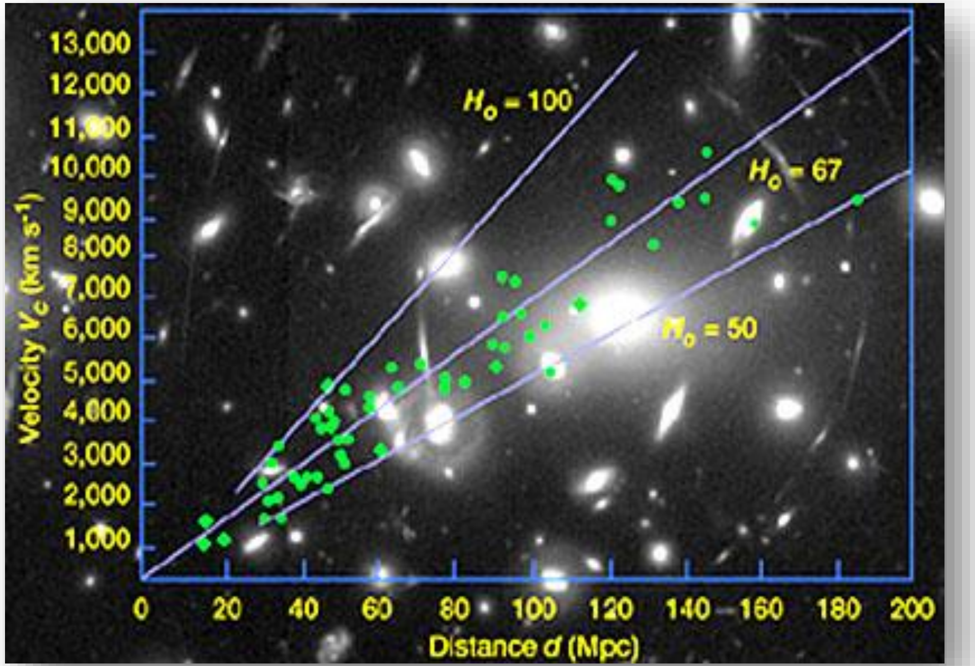
26th September 2022

Hubble law

$$V = H_0 D$$

H_0 - **Hubble constant**

H_0 **= 67.4 km s^{-1}Mpc^{-1}**



Accelerated expansion of the Universe

Nobel Prize in Physics 2011

Photo: Lawrence Berkeley National Lab — Photo: Belinda Pratten, Australian National University — Photo: Scanpix/AFP

Saul Perlmutter | **Brian P. Schmidt** | **Adam G. Riess**

The Nobel Prize in Physics 2011 was awarded *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"* with one half to Saul Perlmutter and the other half jointly to Brian P. Schmidt and Adam G. Riess.

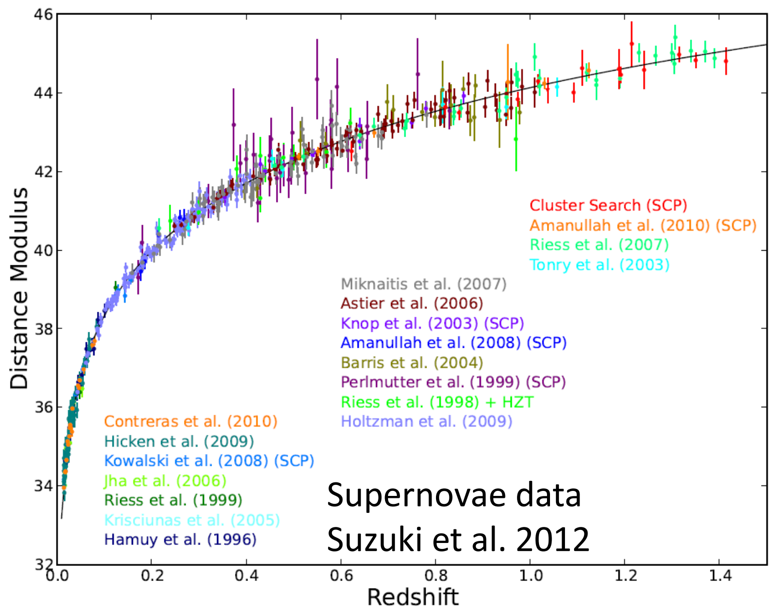
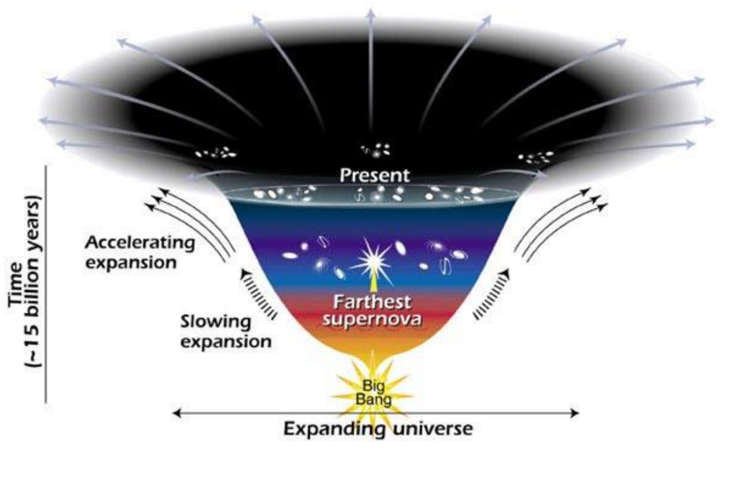


Figure 4. Hubble diagram for the Union2.1 compilation. The solid line represents the best-fit cosmology for a flat ΛCDM Universe for supernovae alone. SN SCP06U4 falls outside the allowed x_1 range and is excluded from the current analysis. When fit with a newer version of SALT2, this supernova passes the cut and would be included, so we plot it on the Hubble diagram, but with a red triangle symbol.



Dark Energy? **Modified Gravity?**

Cosmological constant

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}(R+2\Lambda) + S_M$$

- $\Lambda \rightarrow$ **"concordance" ΛCDM model**

- **ΛCDM model provides the best fit for cosmological observations**

- **ΛCDM model is based on GTR for description of gravity in the Universe on large scales**

- **ΛCDM model has:**
  - **- fine tuning problem**
  - **- coincidence problem**

Dynamical scalar field φCDM models

The equation of state parameter

$$w = p/\rho$$

↙ ↘

$$w_\Lambda = -1 \qquad\qquad w_\phi(t) \neq -1$$

ΛCDM model   **φCDM models**

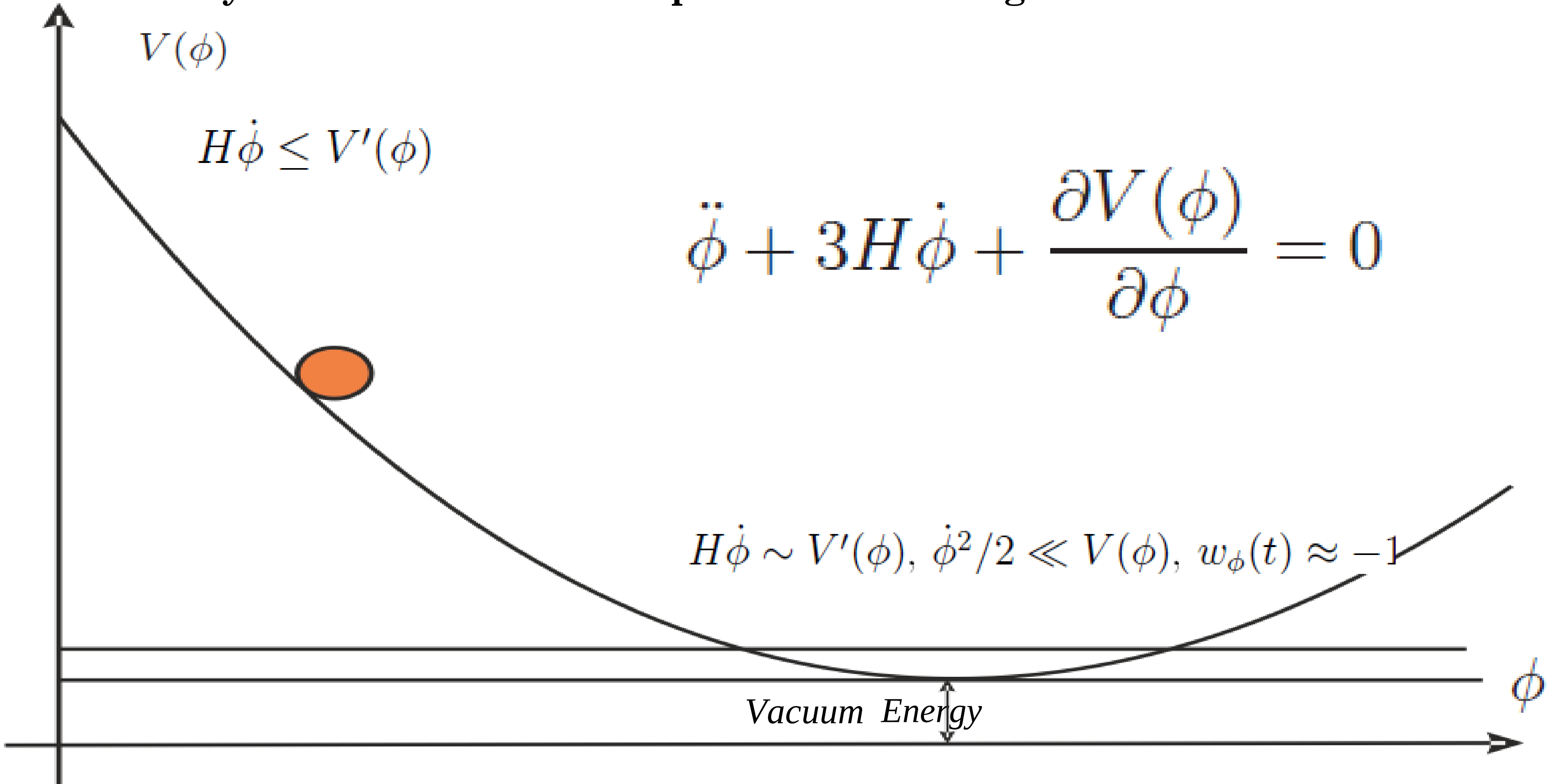
Phantom and quintessence φCDM models

$$S = \int d^4x\sqrt{-g}\Big[-\frac{M_{\rm pl}^2}{16\pi}R + \mathcal{L}_\phi\Big] + S_{\rm M}$$

Phantom models	Quintessence models
$w_0 < -1$	$-1 < w_0 < -1/3$
$\mathcal{L}_\phi = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$	$\mathcal{L}_\phi = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$
$\ddot{\phi} + 3H\dot{\phi} - \frac{\partial V(\phi)}{\partial\phi} = 0$	$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial\phi} = 0$

Quintessence

- **Spatially uniform dynamical scalar field**
- **Slowly rolls down to the minimum of its almost flat potential**
- **This model avoids the fine tuning problem**
- **Plays a role of the time-dependent cosmological constant**



Subdivision of the quintessence models

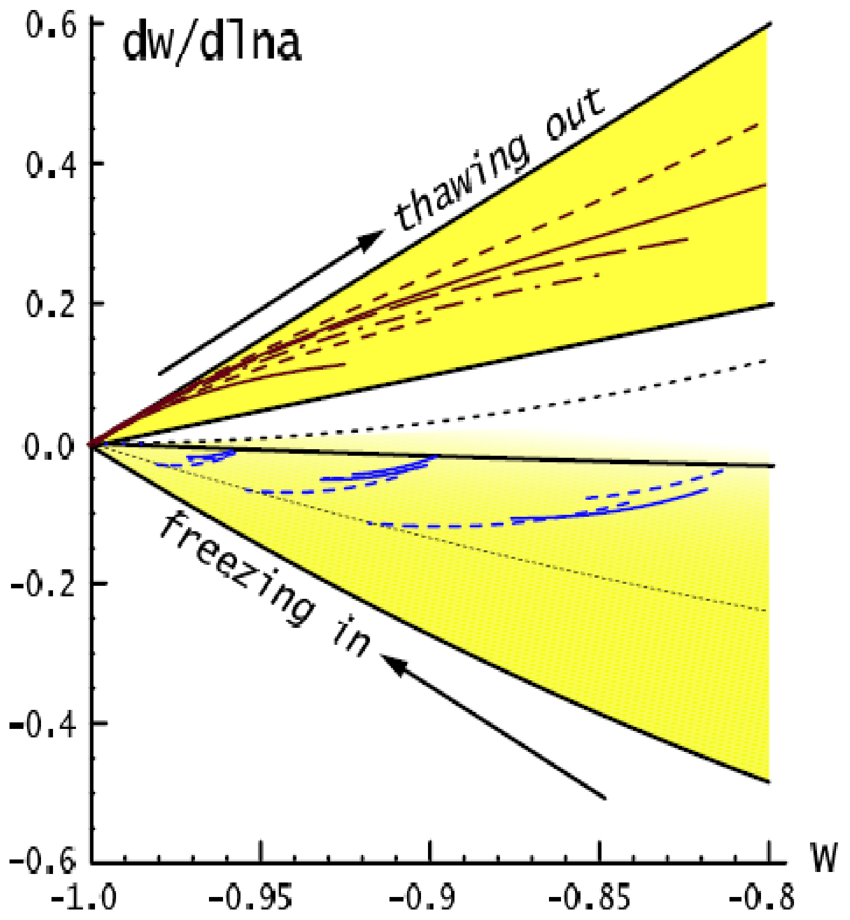
• **thaving models**
for which the evolution of the scalar field is fast compared to the Hubble expansion:

$H < \sqrt{V''(\phi)}$ - **underdamped**

• **freezing (tracking) models**
for which the evolution is slow compared to the Hubble expansion:

$H > \sqrt{V''(\phi)}$ - **overdamped**

$$V(\phi) = \frac{\kappa}{2} M_{\rm pl}^2 \phi^{-\alpha}$$



R. R. Caldwell and E. V. Linder, Phys. Rev. Lett. **95**, 141301 (2005)

Ratra-Peebles potential

The Ratra-Peebles potential: $$V(\phi) = \frac{\kappa}{2} M_{\rm pl}^2 \phi^{-\alpha}$$

B. Ratra and P. J. E. Peebles, Phys. Rev. **D37**, 3406 (1988)

Model parameters: $$\alpha > 0 \qquad k > 0$$

$$0 < \alpha \leq 0.7$$ L. Samushia, arXiv:0908.4597

$$\alpha = 0 \qquad \phi\text{CDM} \longrightarrow \Lambda\text{CDM}$$

Ratra-Peebles φCDM model

- **The first Friedmann equation** $$E(a) = \left(\Omega_{r0}a^{-4} + \Omega_{m0}a^{-3} + \frac{1}{12H_0^2}\left(\dot{\phi}^2 + \kappa M_{pl}^2\phi^{-\alpha}\right)\right)^{1/2}$$
- **The scalar field equation** $$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{2}\kappa\alpha M_{pl}^2\phi^{-(\alpha+1)} = 0$$
- **The scalar field density parameter** $$\Omega_\phi(a) = \frac{1}{12H_0^2}\left(\dot{\phi}^2 + \kappa M_{pl}^2\phi^{-\alpha}\right)$$
- **The energy density of the scalar field** $$\rho_\phi = \frac{M_{pl}^2}{32\pi}\left(\dot{\phi}^2 + \kappa M_{pl}^2\phi^{-\alpha}\right)$$
- **The pressure of the scalar field** $$p_\phi = \frac{M_{pl}^2}{32\pi}\left(\dot{\phi}^2 - \kappa M_{pl}^2\phi^{-\alpha}\right)$$
- **The equation of state parameter** $$w_\phi = \frac{\dot{\phi}^2 - \kappa M_{pl}^2\phi^{-\alpha}}{\dot{\phi}^2 + \kappa M_{pl}^2\phi^{-\alpha}}$$
- **We consider the flat universe**

Equations and initial conditions for background dynamics

- **We solved the system of the equations:**

from $a_{\rm in} = 5 \cdot 10^{-5}$ (radiation domination epoch, $a \sim t^{1/2}$) to the present epoch, $a_0 = 1$

$$E(a) = \left(\Omega_{\rm r0} a^{-4} + \Omega_{\rm m0} a^{-3} + \frac{1}{12H_0^2}\left(\dot{\phi}^2 + \kappa M_{\rm pl}^2 \phi^{-\alpha}\right)\right)^{1/2}, \quad \text{for} \quad M_{\rm pl} = 1$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{2}\kappa\alpha M_{\rm pl}^2 \phi^{-(\alpha+1)} = 0$$

- **The initial conditions**:

$$\kappa = \left(\frac{\alpha+6}{\alpha+2}\right)\left[\frac{1}{2}\alpha(\alpha+2)\right]^{\alpha/2}$$

$$\phi_{\rm in} = \left[\frac{1}{2}\alpha(\alpha+2)\right]^{1/2} t_{\rm in}^{\frac{4}{\alpha+2}}$$

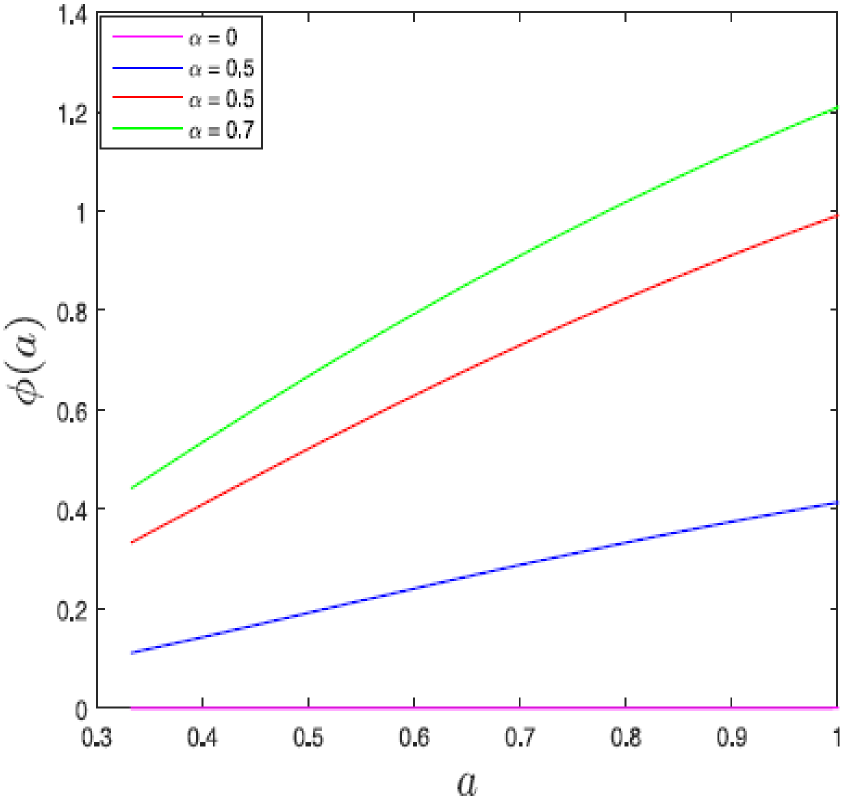
$$\dot{\phi}_{\rm in} = \left(\frac{8\alpha}{\alpha+2}\right)^{1/2} t_{\rm in}^{\frac{2-\alpha}{2+\alpha}}$$

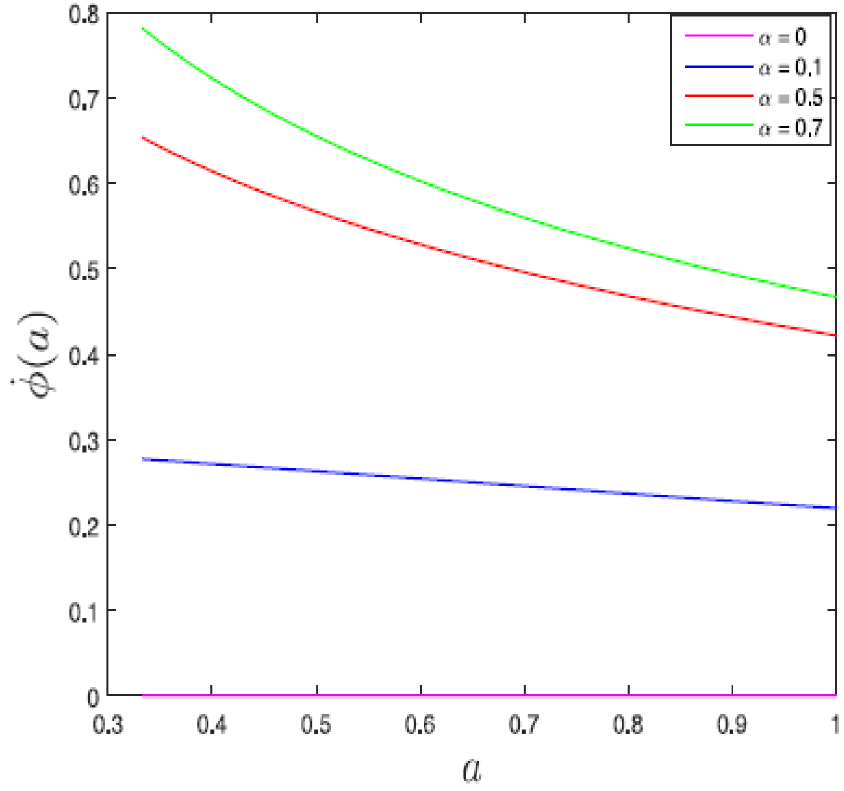
M. O. Farooq, arXiv:1309.3710

O. Avsajanishvili, N. A. Arkhipova, L. Samushia and T. Kahniashvili, Eur. Phys. J. C **74**, 11, 3127 (2014)

$\Omega_{\rm m0} = 0.315$, $\Omega_{\phi 0} = 0.685$, $h = 0.673$, $H_0 = 100h$ km c^{-1} Mpc^{-1} (Planck 2013 results. XVI)

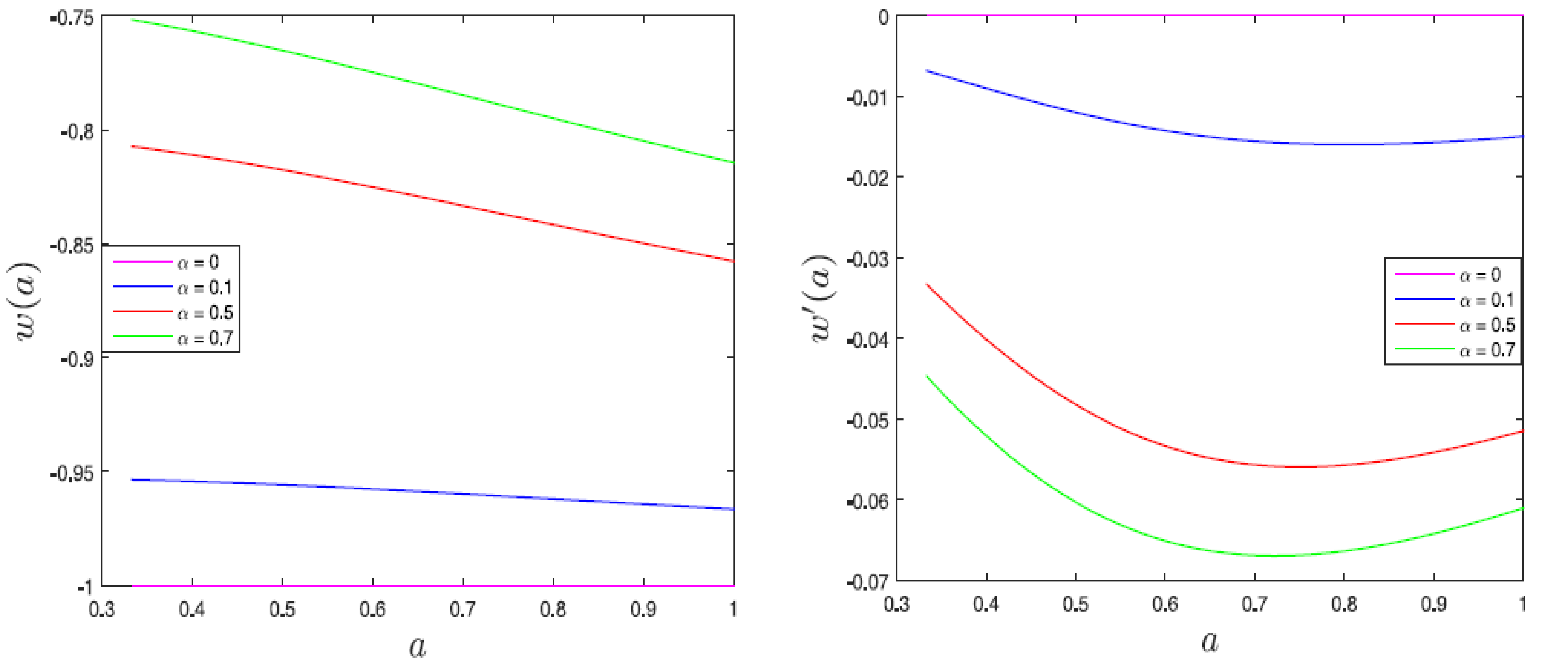
Background dynamics





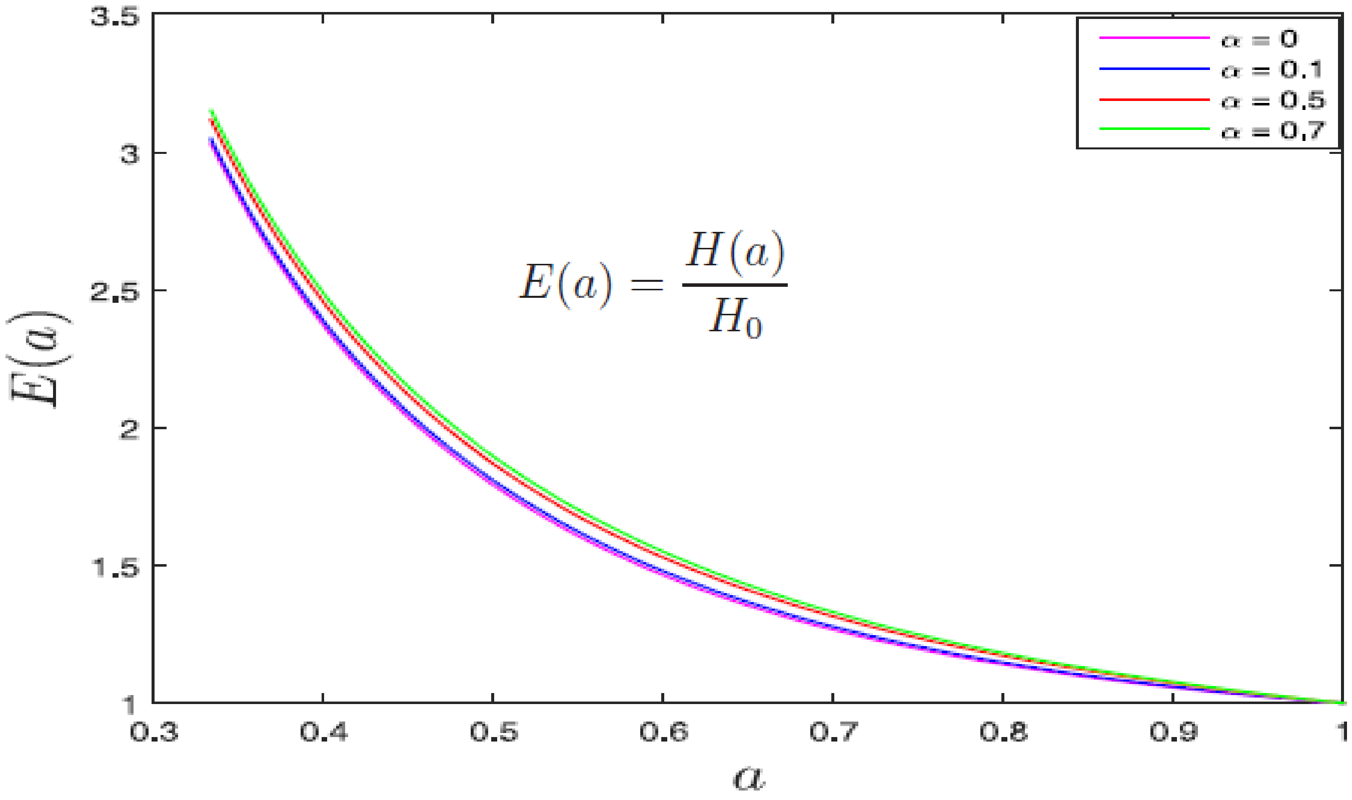
- A larger value of α induces a stronger time dependence of the scalar field.

Background dynamics



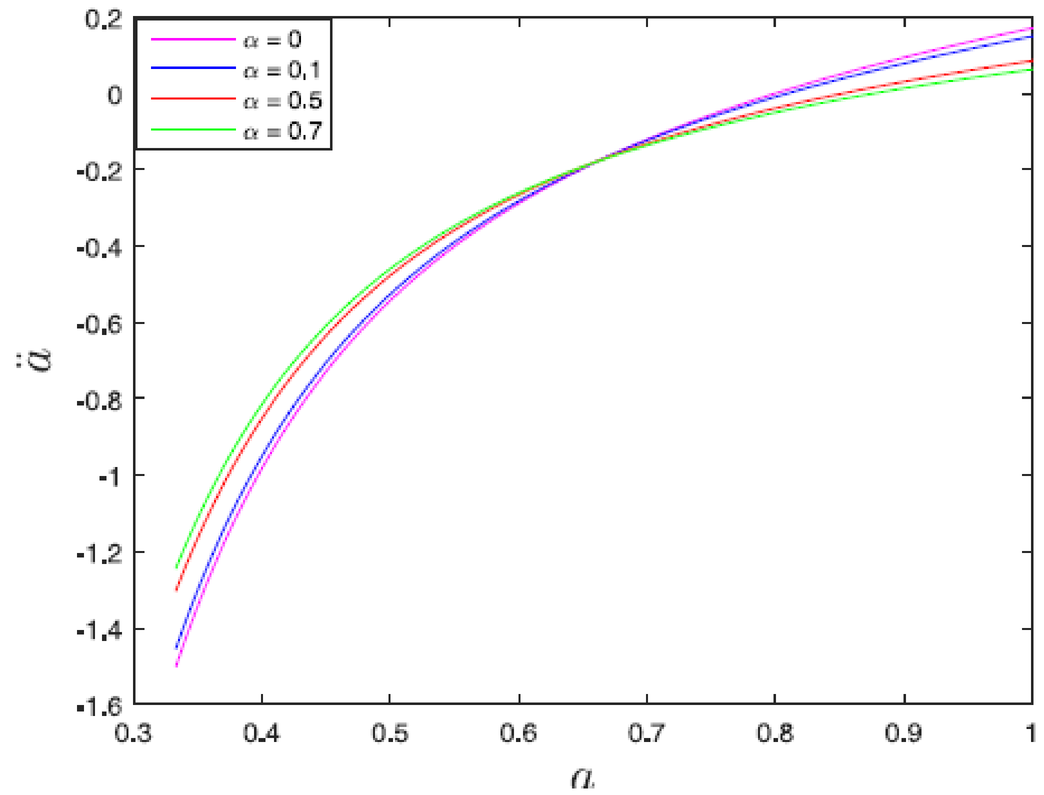
- A larger value of α induces a stronger time dependence of the EoS parameter $w(a)$ and its scale factor derivative $w'(a)$.
- As expected, for the ΛCDM model, $w(a) = -1$ and $w'(a) = 0$.

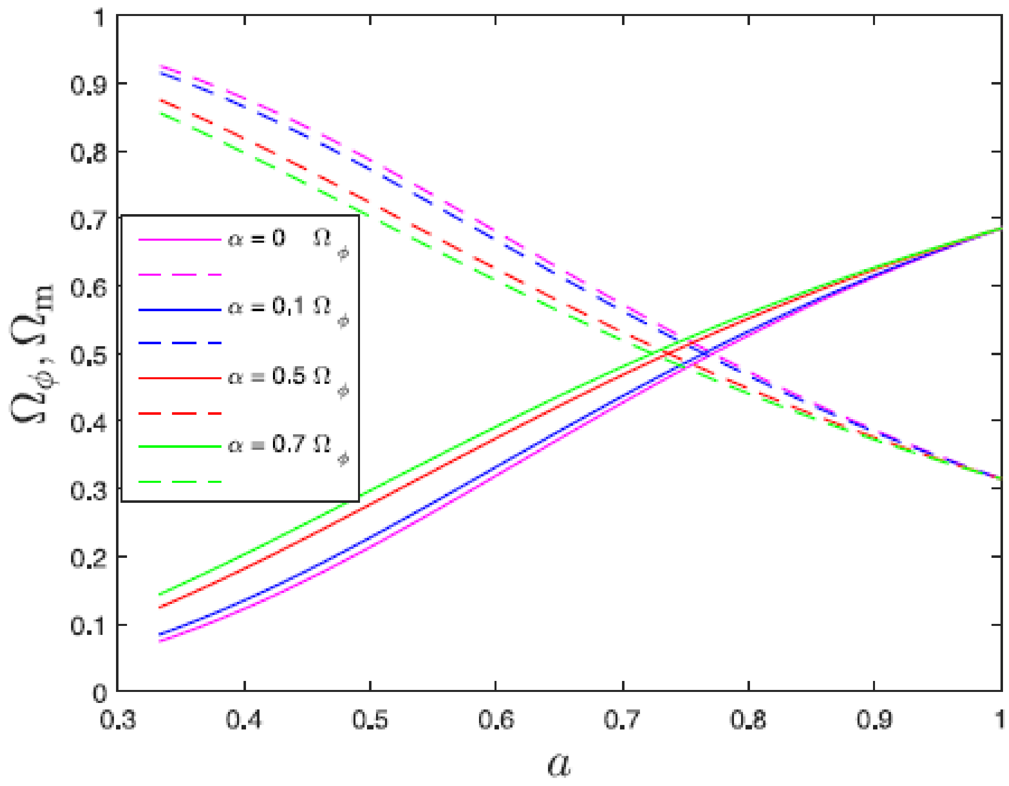
Hubble expansion of the Universe



- The expansion occurs more rapidly with increase in the value of the α parameter.
- ΛCDM limit corresponds to the slowest rate of the Hubble expansion.

Dynamics and energy components of the Universe





- The dynamics domination of DE began earlier than the energetic domination for the fixed value of the parameter α.

- With larger value of α, the quintessence energetic domination began earlier, and vice versa.

How does the quintessence model affect the large-scale structure in the Universe?

$$\delta'' + \left(\frac{3}{a} + \frac{E'}{E}\right)\delta' - \frac{3\Omega_{\rm m0}}{2a^5E^2}\delta = 0$$

$$\delta = \frac{\rho(\vec{r},t) - \langle\rho\rangle}{\langle\rho\rangle}$$

$$\delta(a_{\rm in}) = \delta'(a_{\rm in}) = 5\cdot 10^{-5}$$

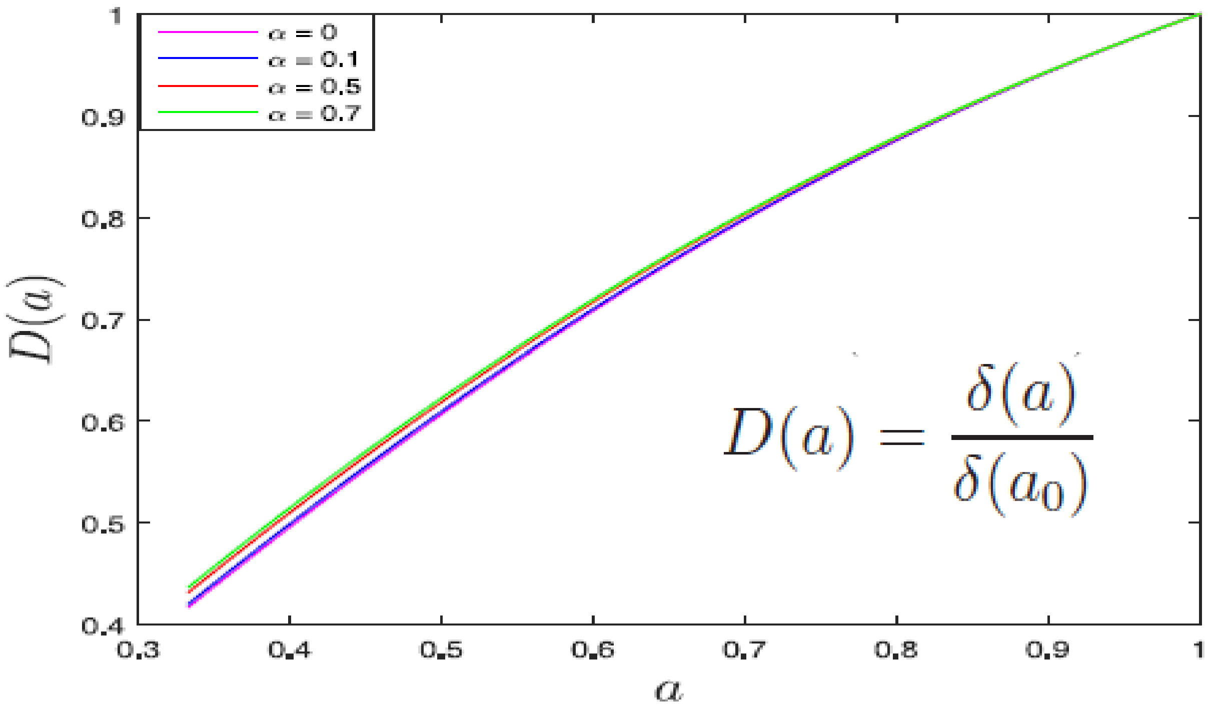
F. Pace, J.-C. Waizmann and M. Bartelmann, Mon. Not. Roy. Astron. Soc. **406**, 1865 (2010)

$$\frac{\partial\rho}{\partial t} + \nabla(\rho\vec{v}) = 0$$

$$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} + \nabla\Phi + \frac{\nabla p}{\rho} = 0$$

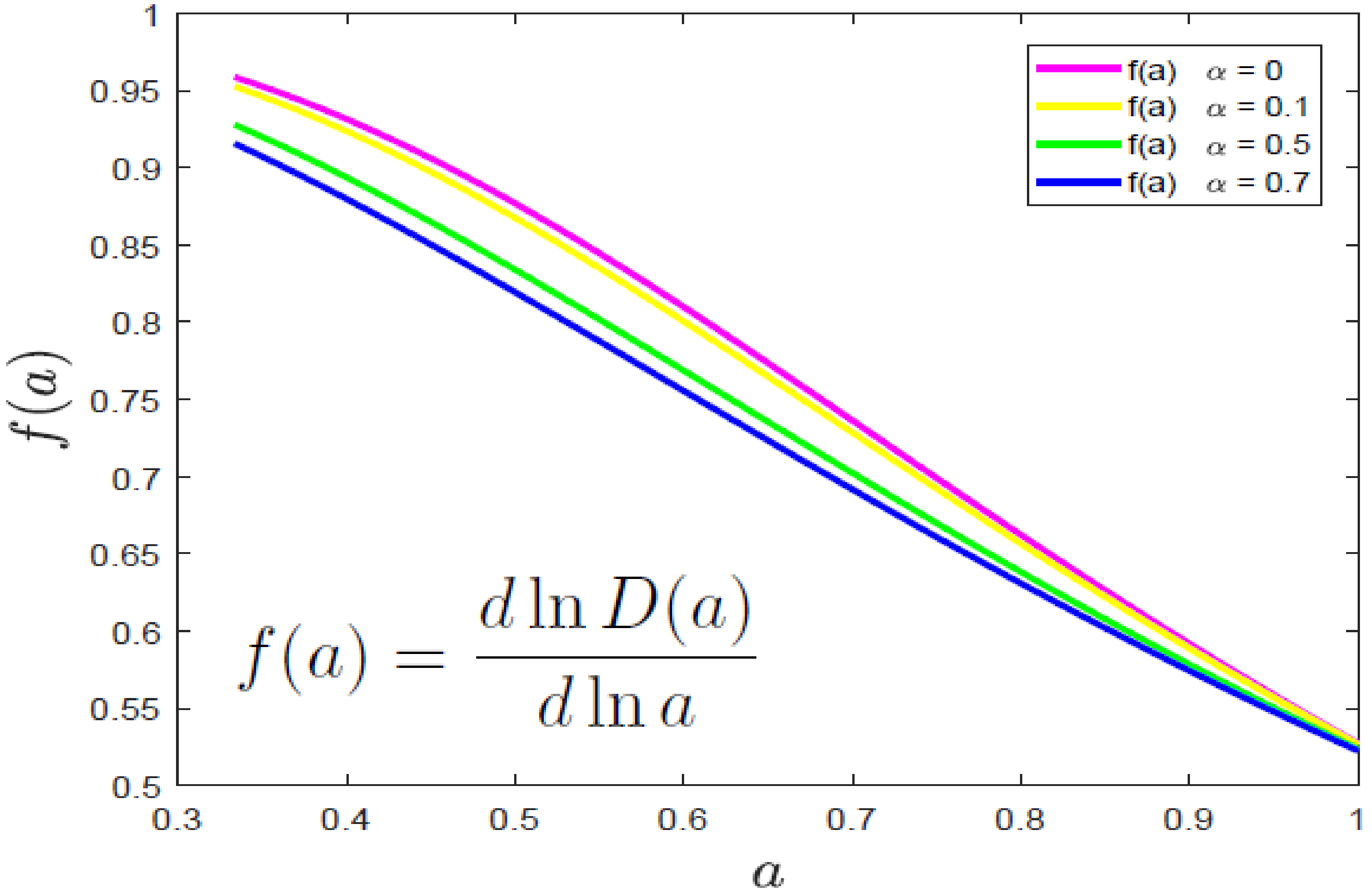
$$\nabla^2\Phi = 4\pi G\,(\rho + 3p)$$

Growth of the matter density fluctuations



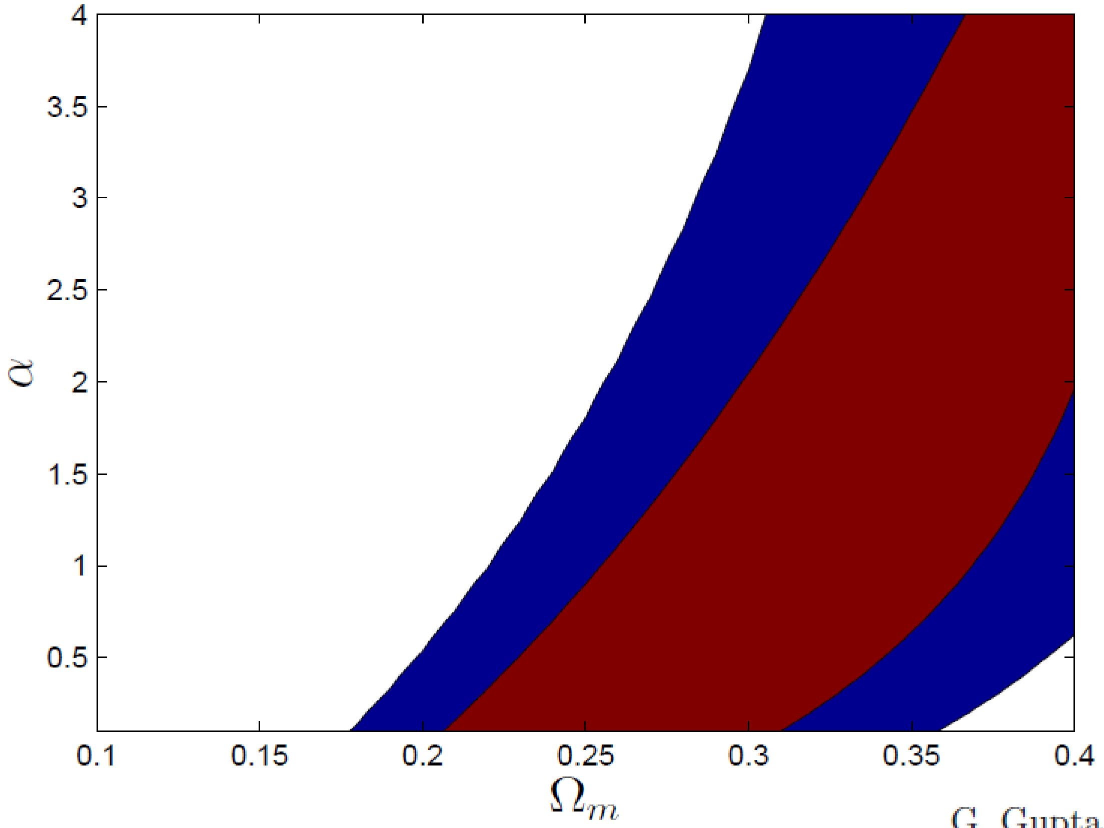
- The scalar field with larger value of α parameter induces the larger values of the matter density fluctuations at the origin moment and at all scale factors until nowadays.

The evolution of the large-scale structure growth rate



Increasing the value of the α parameter slows down the large-scale structure growth rate. This is due to the fact that with an increase in the value of the α parameter, the Hubble expansion occurs faster, which leads to the suppression of the large-scale structure growth rate in the Universe.

Constraints from the growth rate data



$$\chi^2(\alpha, \Omega_{\rm obs}) = \frac{[f_{\rm obs} - f_{\rm th}(\alpha, \Omega_{\rm m})]^2}{\sigma^2}$$

$$\mathcal{L}^{\rm f}(\alpha, \Omega_{\rm m}) \propto \exp[-\chi^2(\alpha, \Omega_{\rm m})/2]$$

G. Gupta, S. Sen and A. A. Sen, JCAP **1204**, 028 (2012)

- Using the growth rate data alone, we have got the highly degenerated likelihood contours in the $\alpha - \Omega_m$ plane.

- If we fix $\alpha = 0$, we get the best fit value of $\Omega_{\rm m} = 0.278 \pm 0.03$, which is within 1σ confedence level of the Planck 2013 data.

Constraints from the BAO data

$$\mathcal{L}^{\mathrm{B}}(\alpha, \Omega_{\mathrm{m}}, H_0) \propto \exp(-\chi^2_{\mathrm{B}}/2)$$

$$\chi^2_{\mathrm{B}} = \boldsymbol{X}^{\mathrm{T}} \boldsymbol{C}^{-1} \boldsymbol{X} \quad \boldsymbol{X} = \eta_{\mathrm{th}} - \eta_{\mathrm{obs}}$$

$$\eta(z) \equiv d_{\mathrm{A}}(z_{\mathrm{dec}})/D_{\mathrm{V}}(z_{\mathrm{BAO}})$$

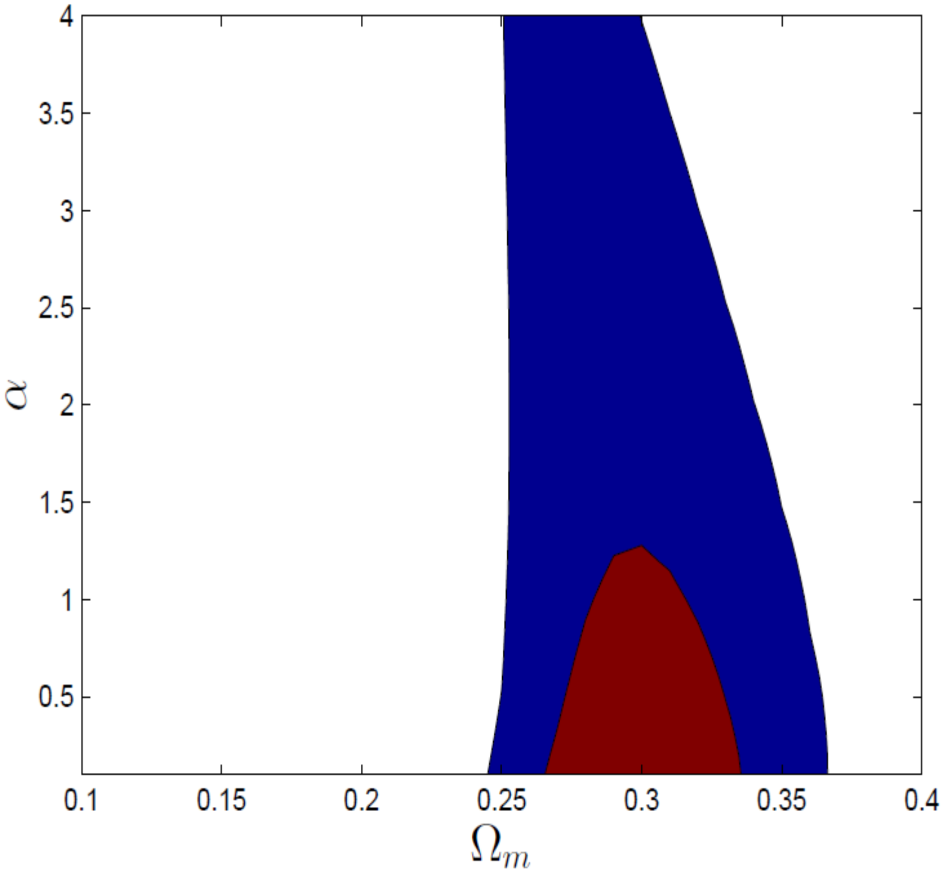
Angular diameter distance:

$$d_{\mathrm{A}}(z, \alpha, \Omega_{\mathrm{m}}, H_0) = \int_0^z \frac{dz'}{H(z', \alpha, \Omega_{\mathrm{m}}, H_0)}$$

Distance scale: $D_{\mathrm{V}}(z, \alpha, \Omega_{\mathrm{m}}, H_0) =$

$$[d^2_{\mathrm{A}}(z, \alpha, \Omega_{\mathrm{m}}, H_0) z / H(z, \alpha, \Omega_{\mathrm{m}}, H_0)]^{1/3}$$

R. Giostri, M. V. dos Santos, I. Waga, R. R. R. Reis, M. O. Calvo and B. L. Lago, JCAP **1203**, 027 (2012)

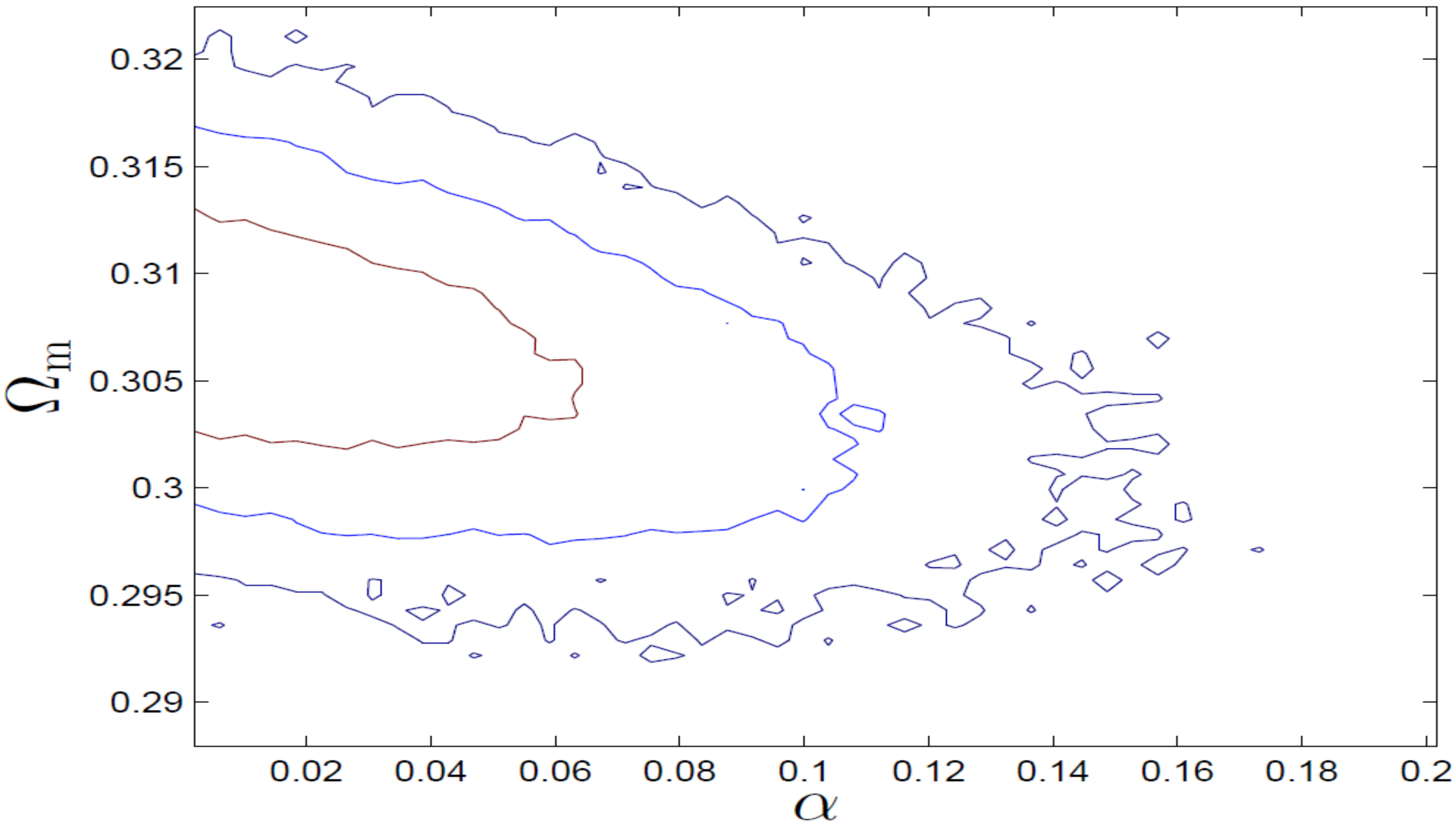


The addition of the BAO data and distance prior from CMB broke the degeneracy:

$0 \leq \alpha \leq 1.3$ at 1σ confidence level

$0.26 \leq \Omega_{\mathrm{m}} \leq 1.34$ at 1σ confidence level

MCMC analysis with upcoming DESI data



Carrying out the MCMC analysis with upcoming DESI data, we obtained ranges of α and Ω_m parameters, $0 < \alpha \leq 0.16$ and $0.296 < \Omega_m < 0.32$ at 3σ confidence level, where the Ratra-Peebles ϕCDM model is compliance with the ΛCDM model.

Mass Varying Neutrino Model

MaVaN model

$$S_B^E = \int_0^\beta d\tau \int a(t)^3 d^3x \left[\frac{1}{2}\left(\frac{\partial\tau}{\partial\phi}\right)^2 + \frac{1}{2a^2}(\nabla\phi)^2 + V(\phi)\right]$$

$$S_D^E = \int_0^\beta d\tau \int a(t)^3 d^3x \; \bar{\psi}(\mathbf{x},\tau)\left(\gamma^o \frac{\partial}{\partial\tau} - \frac{\imath}{a}\gamma\cdot\boldsymbol{\nabla} + m_\nu - \mu\gamma^o\right)\psi(\mathbf{x},\tau)$$

$$\mathcal{S} = S_B^E + S_D^E\big|_{m_\nu=0} + g\int_0^\beta d\tau \int a^3 d^3x \; \phi\bar{\psi}\psi$$

R. Fardon, A. E. Nelson and N. Weiner, JCAP **0410**, 005 (2004)

Saddle point approximation

- **Path integral**

$$\mathcal{Z} = \int \mathcal{D}\phi \, \mathrm{e}^{-\mathcal{S}(\phi)} = \int \mathcal{D}\phi \, \exp\left[- S_B^E + \log \mathrm{Det}\hat{D}(\phi) \right] ,$$

$$\hat{D}(\phi) = \gamma^o \frac{\partial}{\partial \tau} - \frac{\imath}{a}\gamma \cdot \boldsymbol{\nabla} + g\phi(\mathbf{x},\tau) - \mu\gamma^o$$

- **The total thermodynamic potential**

$$V_{\phi\nu} = V(\phi) + V_\nu(\varphi) = V(\phi) - \frac{2N_F}{3\pi^2}\int_{\beta\varphi}^{\infty} \frac{(E^2-\varphi^2)^{3/2}}{e^{\beta E}+1} dE.$$

- **Conditions for the minimum of the total thermodynamic potential at equilibrium**

$$\left.\frac{\partial V_{\phi\nu}(\phi)}{\partial \phi}\right|_{\mu,\beta;\phi=\phi_{\mathrm{cr}}} = 0, \qquad \left.\frac{\partial^2 V_{\phi\nu}(\phi)}{\partial \phi^2}\right|_{\mu,\beta;\phi=\phi_{\mathrm{cr}}} > 0$$

$$\Downarrow$$

$$\left.\frac{\partial V_{\phi\nu}(\phi)}{\partial \phi}\right|_{\phi=\phi_{\mathrm{cr}}} = \left.\frac{\partial V(\phi)}{\partial \phi}\right|_{\phi=\phi_{\mathrm{cr}}} + \left.\frac{\partial V_\nu(\phi)}{\partial \phi}\right|_{\phi=\phi_{\mathrm{cr}}} = 0, \;\; \phi_{\mathrm{cr}} = \langle\varphi\rangle, \;\; m_\nu = g\phi_{\mathrm{cr}}, \;\; g = 1$$

G. Y. Chitov, T. August, A. Natarajan and T. Kahniashvili, Phys. Rev. D **83**, 045033 (2011)

Saddle point approximation for the Ratra-Peebles potential

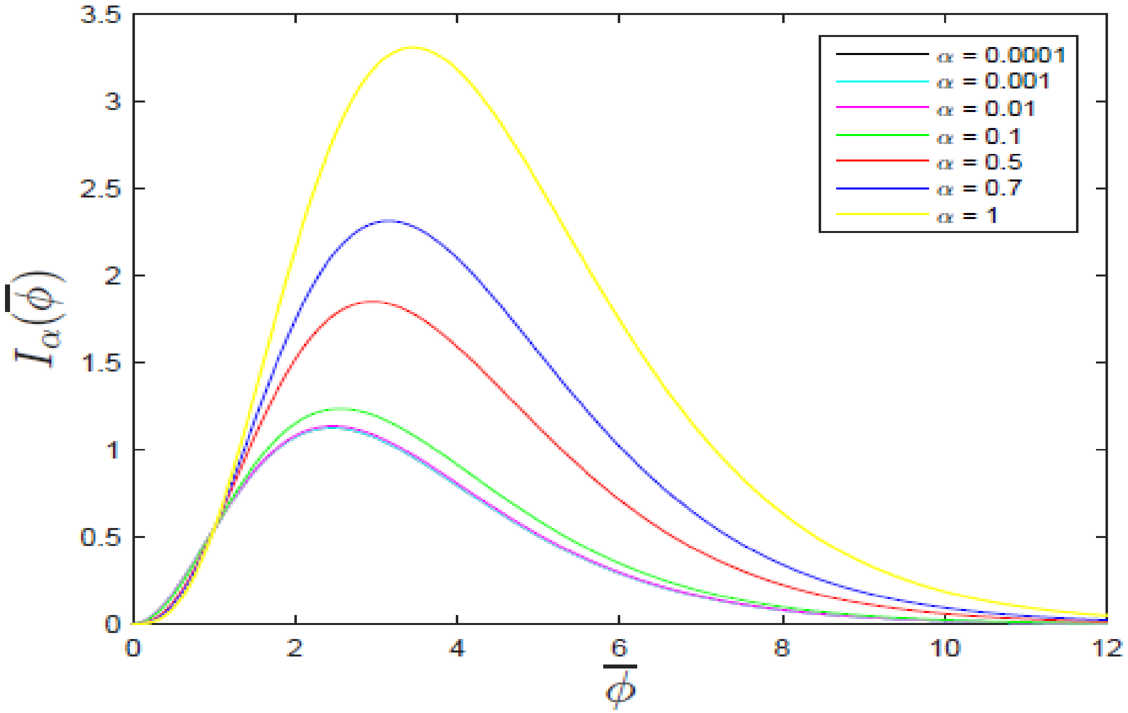
- **The total potential**

$$V_{\phi\nu}(\phi) = \frac{M_\phi^{\alpha+4}}{\phi^\alpha} - \frac{2gN_F}{3\pi^2}\int_{\beta\phi}^{\infty}\frac{(E^2-\phi^2)^{3/2}}{e^{\beta E}+1}dE$$

- **Conditions of minimum**

$$\left.\frac{\partial V_{\phi\nu}(\phi)}{\partial\phi}\right|_{\phi=\phi_{\rm cr}} = \frac{\alpha M_\phi^{\alpha+4}}{\phi^{\alpha+1}} - \frac{2gN_F}{\pi^2}\int_{\beta\phi}^{\infty}\frac{(E^2-\phi^2)^{1/2}}{e^{\beta E}+1}dE = 0$$

Mass equation: $$\frac{\alpha\pi^2 g^\alpha \overline{M_\phi}^{\alpha+4}}{2N_F} = I_\alpha(\overline{\phi})$$



$$I_\alpha(\overline{\phi}) = \overline{\phi}^{(\alpha+2)}\int_{\overline{\phi}}^{\infty}\frac{(\overline{E}^2-\overline{\phi}^2)^{1/2}}{e^{\overline{E}}+1}d\overline{E}$$

$$g = 1,\ \overline{M}_\phi = \beta M_\phi,\ \overline{\phi} = \beta\phi$$

$N_F = 3,\ \beta = 1/T,\ T = T_{\nu 0}/a,\ T_{\nu 0} = 1.9454\ \mathrm{eV}$

Massless neutrinos

- **Dirac fermions,** $\mu = 0$
- **Fermi distribution function** $$n_F(E) = \frac{1}{e^{\beta E} + 1}$$
- **Fermionic density** $$\rho_\nu = \frac{2N_F}{\pi^2} \int_{\beta\varphi}^{\infty} \frac{(E^2 - \varphi^2)^{1/2}}{e^{\beta E} + 1} dE$$
- **Fermionic pressure** $$p_\nu = \frac{2N_F}{3\pi^2} \int_{\beta\varphi}^{\infty} \frac{(E^2 - \varphi^2)^{3/2}}{e^{\beta E} + 1} dE$$

$N_F = 3,\ \beta = 1/T,\ T = T_{\nu 0}/a,\ T_{\nu 0} = 1.9454\ \mathrm{eV}$

Equations for the neutrinos - DE fluid after the critical point

- **The total potential** $$V_{\rm couple} = \frac{M_\phi^{\alpha+4}}{\phi^\alpha} + \phi\rho_{\rm cr}\left(\frac{a_{\rm cr}}{a}\right)^3$$

- **The first Friedmann equation** $$E(a) = \left(\Omega_{\rm m0}a^{-3} + \frac{1}{\rho_{\rm cr0}}\left(\frac{M_\phi^{\alpha+4}}{\phi^\alpha} + \frac{\dot\phi^2}{2} + \phi\rho_{\rm cr}\left(\frac{a_{\rm cr}}{a}\right)^3\right)\right)^{1/2}$$

- **The scalar field equation** $$\ddot\phi + 3H\dot\phi - \frac{\alpha M_\phi^{\alpha+4}}{\phi^{\alpha+1}} + \rho_{\rm cr}\left(\frac{a_{\rm cr}}{a}\right)^3 = 0$$

- **The neutrino-scalar field fluid density parameter** $$\Omega_\phi(a) = \frac{\frac{\dot\phi^2}{2} + \frac{M_\phi^{\alpha+4}}{\phi^\alpha} + \phi\rho_{\rm cr}\left(\frac{a_{\rm cr}}{a}\right)^3}{E^2(a)\rho_{\rm cr0}}$$

- **The energy density of the neutrino-scalar field fluid** $$\rho_{\rm couple} = \frac{\dot\phi^2}{2} + \frac{M_\phi^{\alpha+4}}{\phi^\alpha} + \phi\rho_{\rm cr}\left(\frac{a_{\rm cr}}{a}\right)^3$$

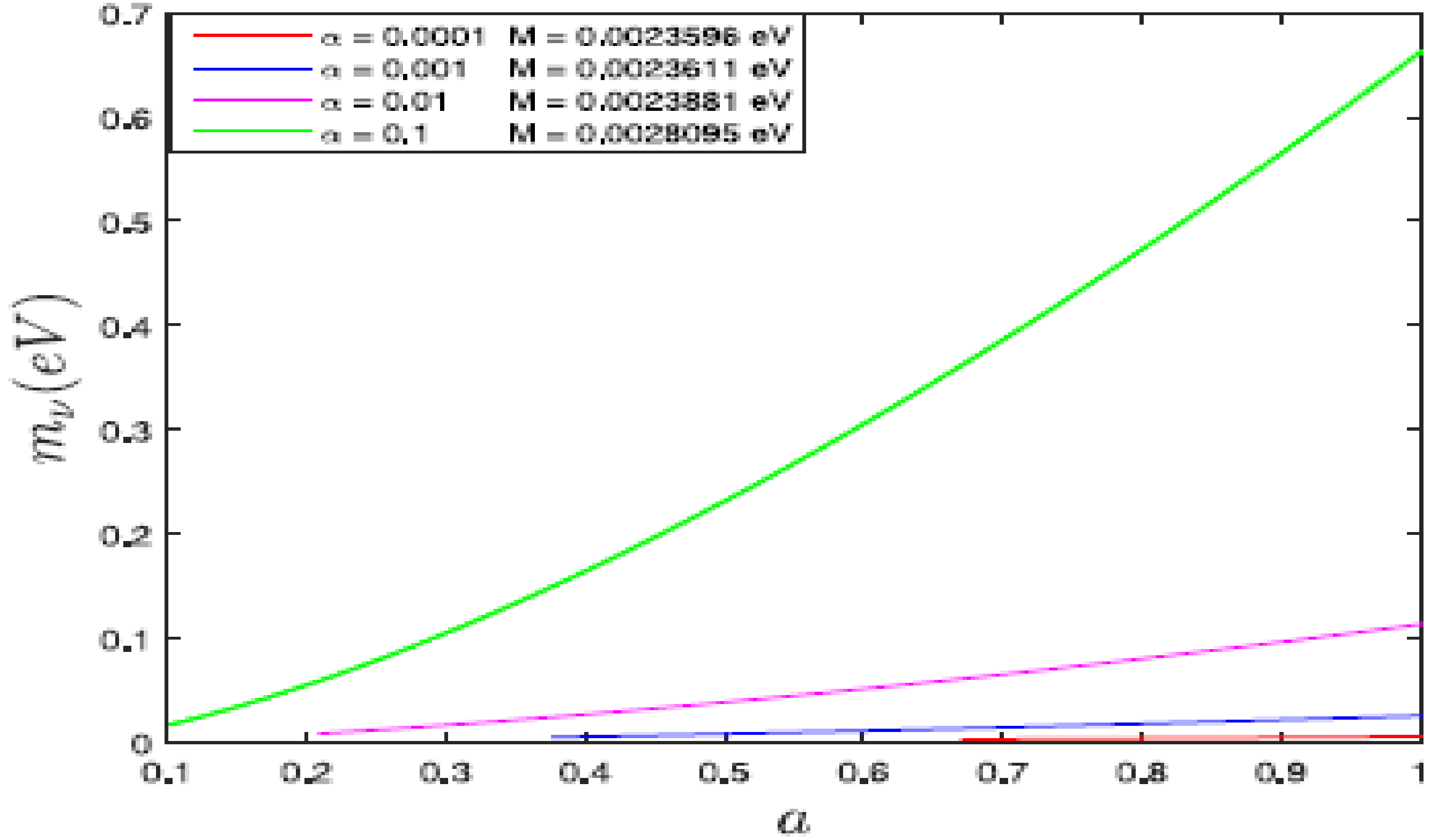
- **The pressure of the of the neutrino-scalar field fluid** $$p_{\rm couple} = \frac{\dot\phi^2}{2} - \frac{M_\phi^{\alpha+4}}{\phi^\alpha} - \phi\rho_{\rm cr}\left(\frac{a_{\rm cr}}{a}\right)^3$$

Values of the scale factor at the critical point and the neutrinos mass depending on the parameter α

α	M	a_{cr}	$m_\nu(a_{cr})$ eV	$m_\nu(a_0)$ eV
10^{-4}	$2.36 \cdot 10^{-3}$	0.67	0.0009	0.006
10^{-3}	$2.36 \cdot 10^{-3}$	0.38	0.0016	0.025
10^{-2}	$2.39 \cdot 10^{-3}$	0.21	0.0028	0.113
10^{-1}	$2.81 \cdot 10^{-3}$	0.10	0.0060	0.663
$1/2$	$6.38 \cdot 10^{-3}$	0.03	0.0208	8.913

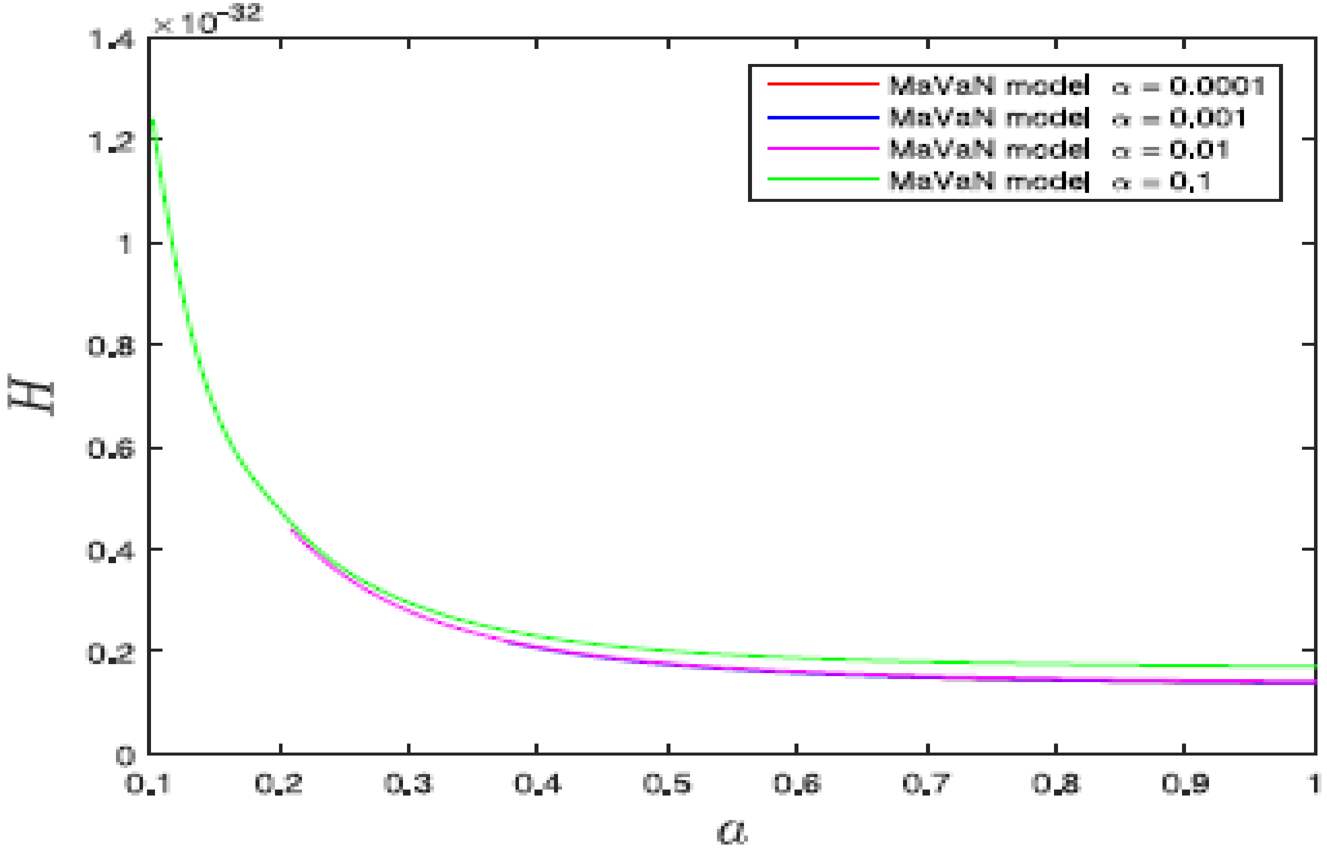
With increase in the value of the model parameter α, the value of the scale factor at the critical point decreases, that is, the moment of the neutrino non-relativization occurs earlier.

Evolution of the neutrino mass in the MAVAN model



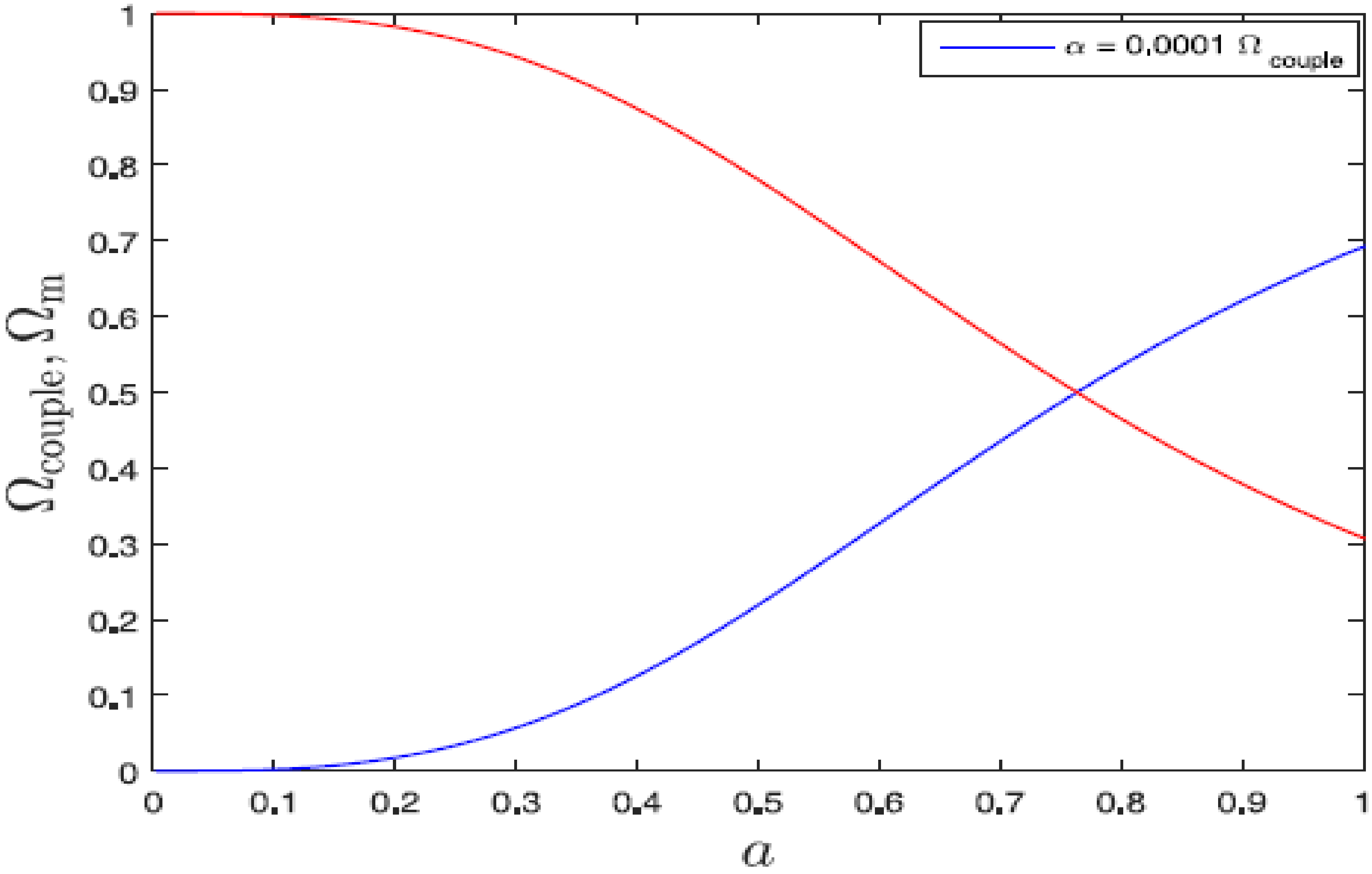
With an increase in the value of the model parameter α, the value of the initial neutrino mass and, accordingly, the value of the neutrino mass increases for all scale factors up to the present epoch.

The evolution of the Hubble parameter in the MAVAN model

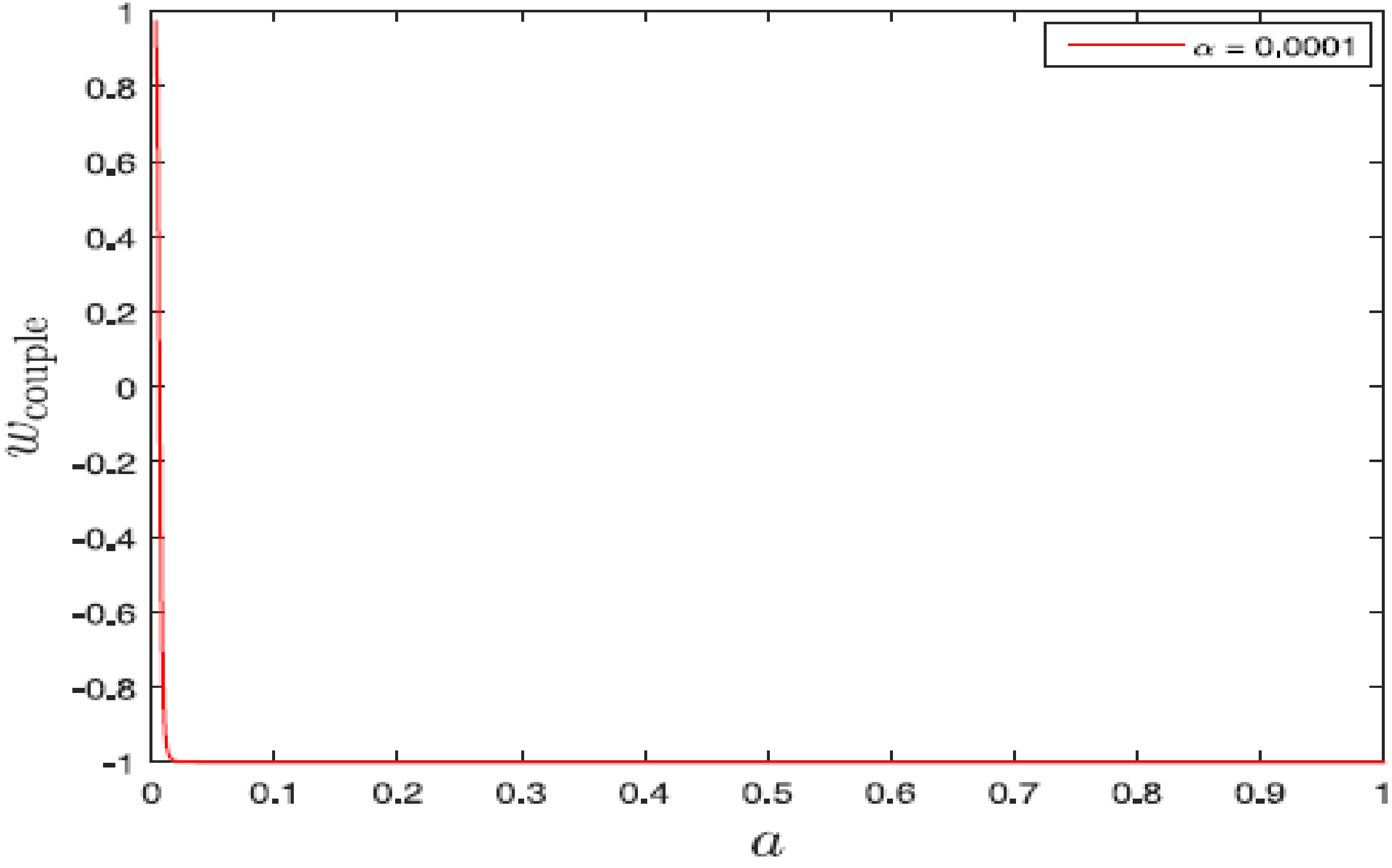


With increase in the value of the α parameter, the expansion of the Universe occurs faster.

Evolution of the density parameters for the matter and DE-neutrinos fluid



Evolution of the EoS parameter for the DE-neutrinos fluid



Conclusion

Scalar field φCDM models

The scalar field φCDM model differs from the ΛCDM model in a number of characteristics, which are generic for this model.

- In the scalar field model,

• the expansion rate of the Universe is always faster than in the ΛCDM model, and

• the DE dominated epoch sets in earlier than in the ΛCDM model

- The scalar field φCDM model predicts a slower large-scale structure growth rate in the Universe than in the ΛCDM model.

Mass Varying Neutrino model

Studying the interaction of the fermionic field and the scalar field with the Ratra-Peebles potential, we found that with increase in the value of the parameter α, that is with strengthening of the scalar field:

• The moment of the neutrinos non-relativization occurs earlier

• The value of the initial neutrino mass and, accordingly, the value of the neutrino mass increases for all scale factors up to the present epoch

• The expansion of the Universe occurs faster.

Thank You for Attention!