International Conference on Neutrinos and Dark Matter (NuDM-2022), Sep 26 online

Is magic texture for Majorana neutrino immanent in Dirac nature for simple Dirac mass matrix?

Yuta Hyodo. with T. Kitabayashi

Graduate School of Science and Technology,

Tokai University

This talk is based on arXiv:2208.11874

Magic texture

Harrison and Scott, PLB594 (2004)

- Majorana flavor neutrino mass matrix
- mixing matrix is Tri-Maximal(TM2) mixing

$$U_{\rm TM} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin\theta \\ -\frac{\cos\theta}{\sqrt{6}} + \frac{e^{-i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\sin\theta}{\sqrt{6}} - \frac{e^{-i\phi}\cos\theta}{\sqrt{2}} \\ -\frac{\cos\theta}{\sqrt{6}} - \frac{e^{-i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\sin\theta}{\sqrt{6}} + \frac{e^{-i\phi}\cos\theta}{\sqrt{2}} \end{pmatrix}$$

→The flavor neutrino mass matrix for Majorana-type neutrinos is magic texture

$$M = \begin{pmatrix} a & b & c \\ b & d & a+c-d \\ c & a+c-d & b-c+d \end{pmatrix} \begin{matrix} \leftarrow a+b+c \\ \leftarrow a+b+c \\ \leftarrow a+b+c \end{matrix}$$

$$\begin{matrix} \uparrow & \uparrow & \uparrow \\ a+b+c & a+b+c & a+b+c \end{matrix}$$

Majorana
⇅
symmetric

equal

equal

equal

The name “magic” is inspired by the nature of the magic square.

Magic textures

Harrison and Scott, PLB594 (2004)

- Majorana flavor neutrino mass matrix
- five independent sums

Y.H. and T.K, PTEP (2021)

- Majorana neutrinos
- Majorana flavor neutrino mass matrix : symmetric

$$\begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$$

$$S_{M1} = a + b + c$$
$$S_{M2} = b + d + e$$
$$S_{M3} = c + e + f$$
$$S_{M4} = a + d + f$$
$$S_{M5} = 2c + d$$

$\mathbf{M1}: S_{\mathrm{M1}} = S_{\mathrm{M2}} = S_{\mathrm{M3}}$, $\mathbf{M2}: S_{\mathrm{M1}} = S_{\mathrm{M2}} = S_{\mathrm{M4}}$,

$\mathbf{M3}: S_{\mathrm{M1}} = S_{\mathrm{M3}} = S_{\mathrm{M4}}$, $\mathbf{M4}: S_{\mathrm{M2}} = S_{\mathrm{M3}} = S_{\mathrm{M4}}$,

⋮

$\mathbf{M9}: S_{\mathrm{M2}} = S_{\mathrm{M4}} = S_{\mathrm{M5}}$, $\mathbf{M10}: S_{\mathrm{M3}} = S_{\mathrm{M4}} = S_{\mathrm{M5}}$

- five independent sums
- **We require that three of the five sums.** ⟶ ${}_5C_3 = 10$ **types**
- **type M1 is a traditional magic texture.**
- **typeⅣ(NO) and typeⅨ(NO)(type I (NO,IO)) are also consistent with neutrino experiments.**

Magic texture for Majorana neutrino immanent in Dirac flavor neutrino mass matrix

- Dirac neutrinos
- The flavor neutrino mass matrix for Dirac-type neutrinos M_D is not symmetric.

$$\begin{aligned} M_{\rm D} &= M_{\rm Mi} + \Delta M_{\rm i} \\ &= M_{\rm Mi} + \begin{pmatrix} 0 & 0 & 0 \\ \epsilon_1 & 0 & \epsilon_1^{\rm Mi} \\ \epsilon_2 & \epsilon_2^{\rm Mi} & \epsilon_3^{\rm Mi} \end{pmatrix} \quad ({\rm i} = 1, 2, 3, 4, 7, 9, 10), \end{aligned}$$

$$\begin{aligned} M_{\rm D} &= M_{\rm M5} + \Delta M_5 \\ &= M_{\rm M5} + \begin{pmatrix} 0 & 0 & 0 \\ \epsilon_1 & \epsilon_1^{\rm M5} & \epsilon_3^{\rm M5} \\ \epsilon_2 & \epsilon_2^{\rm M5} & 0 \end{pmatrix}, \end{aligned}$$

$$\begin{aligned} M_{\rm D} &= M_{\rm Mi} + \Delta M_{\rm i} \\ &= M_{\rm Mi} + \begin{pmatrix} 0 & 0 & 0 \\ \epsilon_1 & \epsilon_1^{\rm Mi} & 0 \\ \epsilon_2 & \epsilon_2^{\rm Mi} & \epsilon_3^{\rm Mi} \end{pmatrix} \quad ({\rm i} = 6, 8) \end{aligned}$$

$$|\varepsilon| = |\varepsilon_1| + |\varepsilon_2| = 0 \text{ and } |\varepsilon^{M_i}| = |\varepsilon_1^{M_{\rm i}}| + |\varepsilon_2^{M_{\rm i}}| + |\varepsilon_3^{M_{\rm i}}| = 0$$

↓

We obtain type Mi magic textures in the neutrino sector.

$$|\varepsilon| = |\varepsilon_1| + |\varepsilon_2| \ll 1 \text{ and } |\varepsilon^{M_i}| = |\varepsilon_1^{M_{\rm i}}| + |\varepsilon_2^{M_{\rm i}}| + |\varepsilon_3^{M_{\rm i}}| \ll 1 \quad (\Delta M_i \text{ is small})$$

↓

M_D is approximately the same as type Mi magic texture.

numerical calculation

The flavor neutrino mass matrix for Dirac-type neutrinos

$$\begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{\mu e} & M_{\mu\mu} & M_{\mu\tau} \\ M_{\tau e} & M_{\tau\mu} & M_{\tau\tau} \end{pmatrix} = \begin{pmatrix} U_{e1}m_1 & U_{e2}m_2 & U_{e3}m_3 \\ U_{\mu 1}m_1 & U_{\mu 2}m_2 & U_{\mu 3}m_3 \\ U_{\tau 1}m_1 & U_{\tau 2}m_2 & U_{\tau 3}m_3 \end{pmatrix}$$

$$U_{\alpha\beta} = f(\theta_{ij}, \delta)$$

$$\Delta m_{ij}^2 = m_i^2 - m_j^2$$

$m_1 = 0 - 0.12\text{eV}$(NO:$m_1 < m_2 < m_3$)

$m_3 = 0 - 0.12\text{eV}$(IO :$m_3 < m_1 < m_2$)

Esteban et al., NuFIT 5.1 (2021)

NuFIT 5.1 (2021)

with SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 7.0$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.45^{+0.77}_{-0.75}$	$31.27 \to 35.87$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.450^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.570^{+0.016}_{-0.022}$	$0.410 \to 0.613$
$\theta_{23}/^\circ$	$42.1^{+1.1}_{-0.9}$	$39.7 \to 50.9$	$49.0^{+0.9}_{-1.3}$	$39.8 \to 51.6$
$\sin^2\theta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02060 \to 0.02435$	$0.02241^{+0.00074}_{-0.00062}$	$0.02055 \to 0.02457$
$\theta_{13}/^\circ$	$8.62^{+0.12}_{-0.12}$	$8.25 \to 8.98$	$8.61^{+0.14}_{-0.12}$	$8.24 \to 9.02$
$\delta_{CP}/^\circ$	230^{+36}_{-25}	$144 \to 350$	278^{+22}_{-30}	$194 \to 345$
$\frac{\Delta m_{21}^2}{10^{-5}\ \text{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m_{3\ell}^2}{10^{-3}\ \text{eV}^2}$	$+2.510^{+0.027}_{-0.027}$	$+2.430 \to +2.593$	$-2.490^{+0.026}_{-0.028}$	$-2.574 \to -2.410$

$$|\varepsilon| = |\varepsilon_1| + |\varepsilon_2|$$

$$\downarrow$$

$$|\varepsilon^{M_i}| = |\varepsilon_1^{M_\text{i}}| + |\varepsilon_2^{M_\text{i}}| + |\varepsilon_3^{M_\text{i}}|$$

①We calculate the $|\varepsilon|= \varepsilon_1 + \varepsilon_2$.

②We calculate the $|\varepsilon^{M_i}|$ when the minimum value of $|\varepsilon|_{min}$.

Results1:The |ε|correlates with the neutrino masses(NO)

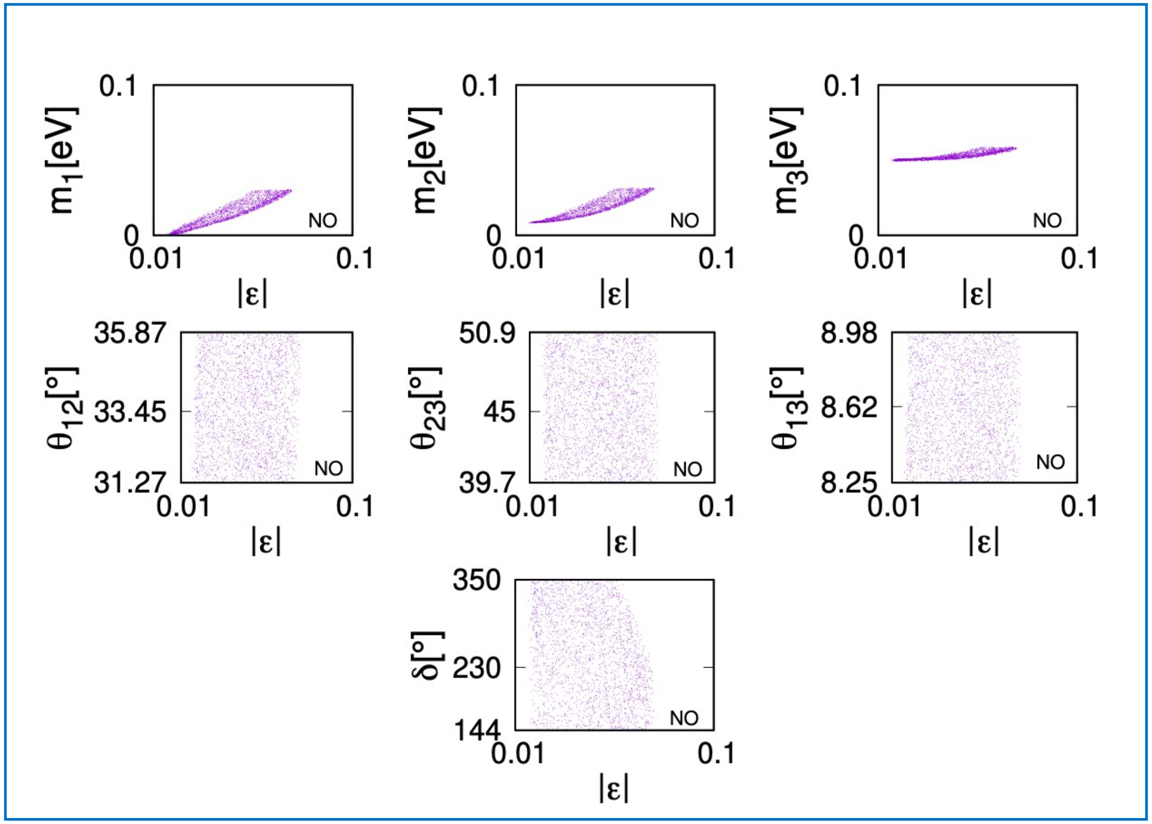

Figure 1:Dependence of the neutrino parameters m_i in eV,θ_{ij}, andδ in degree on $|\varepsilon|$ in the case of NO.

- The |ε| **positively** correlates with the neutrino masses

The minimum of |ε| are obtained for small neutrino masses .

- The |ε| does not correlate with the mixing angles θ_{ij} and Dirac CP-violating phase δ.

- Therefore, |ε| may become the minimum value for best-fit values of mixing angles θ_{ij} and Dirac CP-violating phase δ.

Results2:weak correlation between $|\varepsilon|$ and the mixing angle θ_{12} (IO)

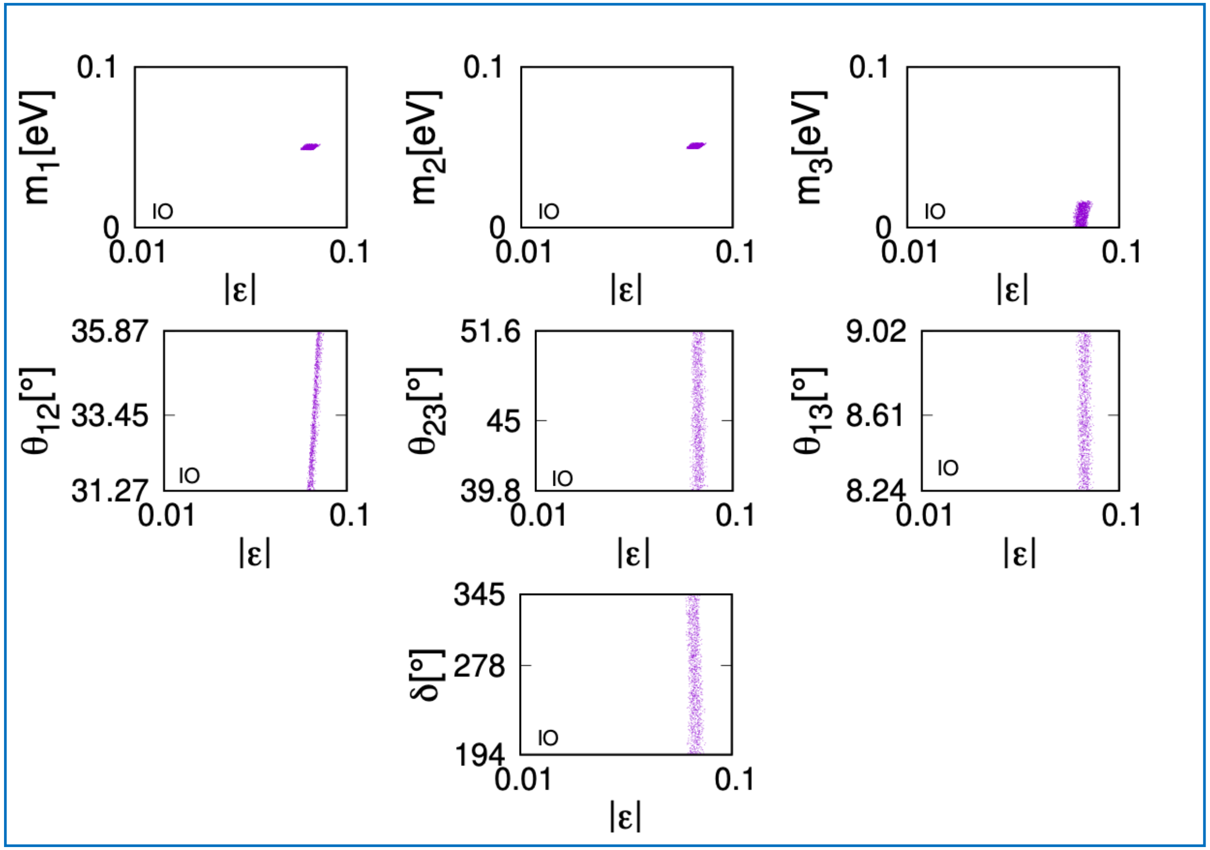

Figure 2:Dependence of the neutrino parameters m_i in eV, θ_{ij}, and δ in degree on $|\varepsilon|$ in the case of IO.

- There is **weak** correlation between $|\varepsilon|$ and the mixing angle θ_{12}.

- $|\varepsilon|$ has no dependence on other parameters $(\theta_{23}, \theta_{13}, \delta, m_i)$.

Results3:NO case is predominant

Table 1:$|\varepsilon|_{\min}$, $|\varepsilon_1|$, $|\varepsilon_2|$, the neutrino parameters m_i in eV, θ_{ij}, and δ in degree when the minimum value of $|\varepsilon|_{\min}$.

	$\lvert\epsilon\rvert_{\min}$	$\lvert\epsilon_1\rvert$	$\lvert\epsilon_2\rvert$	m_1	m_2	m_3	θ_{12}	θ_{23}	θ_{13}	δ
NO	0.01134	0.004264	0.007079	1.116×10^{-5}	0.008284	0.04930	31.30	44.81	8.258	293.7
IO	0.05948	0.04887	0.01062	0.04883	0.04959	0.006853	31.30	40.00	8.815	344.7

- $|\varepsilon_1|$, $|\varepsilon_2|$ in the IO case is one order of magnitude greater than $|\varepsilon_1|$, $|\varepsilon_2|$ in the NO case.

$$
\begin{aligned}
M_{\mathrm{D}} &= M_{\mathrm{Mi}} + \Delta M_{\mathrm{i}} \\
&= M_{\mathrm{Mi}} + \begin{pmatrix} 0 & 0 & 0 \\ \epsilon_1 & 0 & \epsilon_1^{\mathrm{Mi}} \\ \epsilon_2 & \epsilon_2^{\mathrm{Mi}} & \epsilon_3^{\mathrm{Mi}} \end{pmatrix} \quad (\mathrm{i} = 1, 2, 3, 4, 7, 9, 10),
\end{aligned}
$$

Results4:approximately type M1 magic texture

	$\lvert\epsilon_{M1}\rvert$	$\lvert\epsilon_1^{M1}\rvert$	$\lvert\epsilon_2^{M1}\rvert$	$\lvert\epsilon_3^{M1}\rvert$	$\lvert\epsilon_{M2}\rvert$	$\lvert\epsilon_1^{M2}\rvert$	$\lvert\epsilon_2^{M2}\rvert$	$\lvert\epsilon_3^{M2}\rvert$
NO	**0.04990**	0.03688	0.006509	0.006509	0.07632	0.03688	0.006509	0.03292
IO	0.09268	0.007884	0.04240	0.04240	**0.05902**	0.007884	0.04240	0.008740
	$\lvert\epsilon_{M3}\rvert$	$\lvert\epsilon_1^{M3}\rvert$	$\lvert\epsilon_2^{M3}\rvert$	$\lvert\epsilon_3^{M3}\rvert$	$\lvert\epsilon_{M4}\rvert$	$\lvert\epsilon_1^{M4}\rvert$	$\lvert\epsilon_2^{M4}\rvert$	$\lvert\epsilon_3^{M4}\rvert$
NO	0.07554	0.03294	0.009675	0.03292	0.07161	0.03294	0.009675	0.02900
IO	0.1750	0.06588	0.1004	0.008740	0.2155	0.06588	0.1004	0.04928
	$\lvert\epsilon_{M5}\rvert$	$\lvert\epsilon_1^{M5}\rvert$	$\lvert\epsilon_2^{M5}\rvert$	$\lvert\epsilon_3^{M5}\rvert$	$\lvert\epsilon_{M6}\rvert$	$\lvert\epsilon_1^{M6}\rvert$	$\lvert\epsilon_2^{M6}\rvert$	$\lvert\epsilon_3^{M6}\rvert$
NO	0.05730	0.007687	0.01419	0.03542	0.1120	0.007687	0.03955	0.06473
IO	0.07019	0.03567	0.006724	0.02779	0.1273	0.03567	0.03452	0.05715
	$\lvert\epsilon_{M7}\rvert$	$\lvert\epsilon_1^{M7}\rvert$	$\lvert\epsilon_2^{M7}\rvert$	$\lvert\epsilon_3^{M7}\rvert$	$\lvert\epsilon_{M8}\rvert$	$\lvert\epsilon_1^{M8}\rvert$	$\lvert\epsilon_2^{M8}\rvert$	$\lvert\epsilon_3^{M8}\rvert$
NO	0.07861	0.03542	0.01419	0.02900	0.07895	0.007687	0.03955	0.03171
IO	0.08379	0.02779	0.006724	0.04928	0.1146	0.03567	0.03452	0.04440
	$\lvert\epsilon_{M9}\rvert$	$\lvert\epsilon_1^{M9}\rvert$	$\lvert\epsilon_2^{M9}\rvert$	$\lvert\epsilon_3^{M9}\rvert$	$\lvert\epsilon_{M10}\rvert$	$\lvert\epsilon_1^{M10}\rvert$	$\lvert\epsilon_2^{M10}\rvert$	$\lvert\epsilon_3^{D10}\rvert$
NO	0.08132	0.03542	0.01419	0.03171	0.07432	0.03294	0.009675	0.03171
IO	0.07891	0.02779	0.006724	0.04440	0.2107	0.06588	0.1004	0.04440

The flavor neutrino mass matrix for Dirac neutrinos obeys approximately type M1 magic texture for the NO case.

Summary

Summary

- We estimated the relationship between the Dirac flavor neutrino mass matrix and magic texture for Majorana-type neutrinos.

- We obtained that, for the normal mass ordering case, the texture of Dirac flavor neutrino mass matrix obeys approximately type M1 traditional magic texture for Majorana neutrinos.

Future study

- One equality in magic neutrino mass matrix for Majorana neutrinos.

- We would like to study the relation between magic textures and sterile neutrinos.

Thank you for your attention