Trimaximal neutrino mixing with a vanishing minor

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- The mass eigen state of neutrino field can be related to flavor state by unitary transformation.
- This unitary matrix is known as Pontecorvo-Maki-Nakagawa (PMNS) matrix U, parameterized by three mixing angles: solar mixing angle θ_{12}, atmospheric mixing angle θ_{23} and reactor mixing angle θ_{13}, one Dirac CP violating phase δ.

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \quad (1)$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$; θ_{ij} is mixing angle, δ is Dirac CP violating phase.

Neutrino Oscillation

- The experimental value of various neutrino oscillation parameters are listed in table below [*I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz and A. Zhou*, JHEP 09, 178 (2020). NuFIT 5.0(2020)]

parameter	Normal ordering(best fit)		inverted ordering ($\Delta\chi^2 = 7.1$)	
	bfp$\pm 1\sigma$	3σ ranges	bfp$\pm 1\sigma$	3σ ranges
θ_{12}°	$33.44^{+0.77}_{-0.74}$	31.27$\to$ 35.86	$33.45^{+0.78}_{-0.75}$	31.27$\to$ 35.87
θ_{23}°	$49.2^{+0.9}_{-1.2}$	40.1$\to$ 51.7	$49.3^{+0.9}_{-1.1}$	40.3$\to$ 51.8
θ_{13}°	$8.57^{+0.12}_{-0.12}$	8.20$\to$ 8.93	$8.60^{+0.12}_{-0.12}$	8.24$\to$ 8.96
δ°	197^{+27}_{-24}	120$\to$ 369	282^{+26}_{-30}	193$\to$ 352
$\frac{\Delta m^2_{21}}{10^{-5} eV^2}$	$7.42^{+0.21}_{-0.20}$	6.82$\to$ 8.04	$7.42^{+0.21}_{-0.20}$	6.82$\to$ 8.04
$\frac{\Delta m^2_{3l}}{10^{-3} eV^2}$	$+2.517^{+0.026}_{-0.028}$	+2.435$\to$ +2.598	$-2.498^{+0.028}_{-0.028}$	-2.581$\to$ -2.414

Tribimaximal mixing

- Harrison, Perkins and Scott introduced the tribimaximal (TB) mixing pattern.
- TB mixing matrix have two types of symmetries: $\mu - \tau$ symmetry and magic symmetry.
- The tribimaximal mixing pattern is given by

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}. \quad (2)$$

- According to this mixing pattern, the value of maximal atmospheric mixing angle $\theta_{23} = 45^\circ$, solar mixing angle $\theta_{12} = 35.26^\circ$ and reactor mixing angle $\theta_{13} = 0^\circ$.

Trimaximal Mixing

- Tribimaximal mixing pattern does not allow CP violation in neutrino oscillation as $\theta_{13} = 0^\circ$.
- Experiments like T2K, MINOS confirm non-zero value of θ_{13}.
- The TB mixing pattern is modified by multiplying rotation matrix U_{13}, called trimaximal mixing (TM) matrix.

$$U_{TM} = U_{TB} U_{13}. \tag{3}$$

Trimaximal Mixing

- The trimaximal mixing matrix is given by

$$U_{TM} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin\theta \\ -\frac{\cos\theta}{\sqrt{6}} + \frac{e^{-i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\sin\theta}{\sqrt{6}} - \frac{e^{-i\phi}\cos\theta}{\sqrt{2}} \\ -\frac{\cos\theta}{\sqrt{6}} - \frac{e^{-i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\sin\theta}{\sqrt{6}} + \frac{e^{-i\phi}\cos\theta}{\sqrt{2}} \end{pmatrix}. \quad (4)$$

- Here θ and ϕ are free parameters.
- The neutrino mass matrix corresponding to trimaximal mixing matrix is

$$M = V^* M_{diag} V^\dagger = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{\mu e} & M_{\mu\mu} & M_{\mu\tau} \\ M_{\tau e} & M_{\tau\mu} & M_{\tau\tau} \end{pmatrix}, V = U_{TM} P \quad (5)$$

- P contain two CP violating Majorana phases α and β.

Neutrino oscillation parameters

- The neutrino mixing angles corresponding to trimaximal mixing are

$$s_{12}^2 = \frac{1}{3 - 2\sin^2\theta}, \qquad s_{23}^2 = \frac{1}{2}\left(1 + \frac{\sqrt{3}\sin 2\theta\cos\phi}{3 - 2\sin^2\theta}\right), \qquad s_{13}^2 = \frac{2}{3}\sin^2\theta. \quad (6)$$

  here $s_{ij} = \sin\theta_{ij}$

- The allowed value of neutrino oscillation parameters θ_{12}, θ_{23} and θ are $(35.68^\circ - 35.76^\circ)$, $(39.80^\circ - 50.19^\circ)$ and $(10.06^\circ - 10.95^\circ)$ respectively.
- The best fit value of θ is 10.50° corresponding to the minimum value of χ^2

$$\chi^2(\theta, \phi) = \sum_{i=1}^{3}\left[\frac{\theta_i^{cal} - \theta_i^{exp.}}{\sigma_i^{exp.}}\right]^2 \quad (7)$$

  Here $\theta_i = (\theta_{12}, \theta_{13}, \theta_{23})$, θ_i^{cal} are the calculated values of θ_i and $\theta_i^{exp.}$ are the experimental values of θ_i.

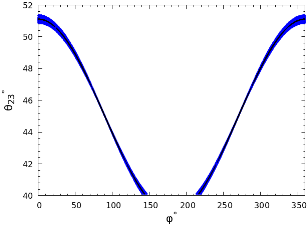
Neutrino oscillation parameters

- The Jarlskog rephasing invariant measure of CP violation can be expressed as

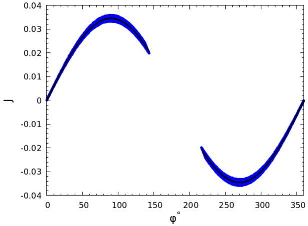
$$J = Im\left[U_{11} U_{12}^* U_{21}^* U_{22}\right] \tag{8}$$

- For trimaximal mixing, the Jarlskog rephasing invariant is

$$J = \frac{1}{6\sqrt{3}} \sin 2\theta \sin\phi. \tag{9}$$



(a) variation of θ_{23} with respect to ϕ



(b) variation of J with respect to ϕ

Neutrino oscillation parameters

- The Dirac CP violating parameter δ can be obtained from

$$\csc^2\delta = \csc^2\phi - \frac{3\sin^2 2\theta \cot^2\phi}{(3-2\sin^2\theta)^2}. \tag{10}$$

- The Dirac CP violating phase δ is restricted to two regions around 90° and 270°.
- The neutrino oscillation experiments like T2K, MINOS perfer the δ value around 270° and the bestfit value corresponding to minimum value of χ^2 is 313.03°.
- For trimaximal mixing matrix effective Majorana mass term is

$$|M_{ee}| = |\frac{1}{3}(2m_1\cos^2\theta + m_2 e^{2i\alpha} + 2m_3\sin^2\theta e^{2i\beta})|. \tag{11}$$

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vanishing minor and neutrino masses

- The vanishing minor condition is

$$M_{(ab)}M_{(cd)} - M_{(uv)}M_{(wx)} = 0; \quad a, b, c, u, v, w \text{ and } x \to e, \mu \text{ and } \tau. \tag{12}$$

which gives

$$m_1 m_2 X_3 e^{2i\alpha} + m_2 m_3 X_1 e^{2i(\alpha+\beta)} + m_3 m_1 X_2 e^{2i\beta} = 0 \tag{13}$$

where

$$X_k = (U_{ai}U_{bi}U_{cj}U_{dj} - U_{ui}U_{vi}U_{wj}U_{xj}) + (i \leftrightarrow j).$$

(k,i,j) as the cyclic permutation of (1,2,3).

- Two mass ratios are

$$\frac{m_1}{m_2} = \frac{Re(X_3 e^{2i\alpha})Im(X_1 e^{2i\alpha+\beta}) - Re(X_1 e^{2i(\alpha+\beta)})Im(X_3 e^{2i\alpha})}{Re(X_2 e^{2i\beta})Im(X_3 e^{2i\alpha}) - Re(X_3 e^{2i\alpha})Im(X_2 e^{2i\beta})} \tag{14}$$

and

$$\frac{m_2}{m_3} = \frac{Re(X_1 e^{2i(\alpha+\beta)})Im(X_2 e^{2i\beta}) - Re(X_2 e^{2i\beta})Im(X_1 e^{2i(\alpha+\beta)})}{Re(X_3 e^{2i\alpha})Im(X_1 e^{2i(\alpha+\beta)}) - Re(X_1 e^{2i(\alpha+\beta)})Im(X_3 e^{2i\alpha})}. \tag{15}$$

vanishing minor and neutrino masses

- The value of m_1, m_2 and m_3 are

$$m_1 = \sqrt{\Delta m^2_{21}}\sqrt{\frac{(\frac{m_1}{m_2})^2}{|1-(\frac{m_1}{m_2})^2|}}, m_2 = \sqrt{\Delta m^2_{21}}\sqrt{\frac{1}{|1-(\frac{m_1}{m_2})^2|}} \textit{and} \tag{16}$$

$$m_3 = \sqrt{|\Delta m^2_{32}|}\sqrt{\frac{1}{|1-(\frac{m_2}{m_3})^2|}}. \tag{17}$$

where Δm^2_{21} and Δm^2_{32} are squared mass differences.

- If $m_1 < m_2 << m_3$, then neutrino mass spectrum follow normal mass ordering; if $m_3 << m_1 < m_2$, then mass spectrum follow inverted mass ordering and if $m_1 \approx m_2 \approx m_3$, then the mass spectrum is called quasidegenerate mass ordering.
- The six possible patterns of one minor zero in neutrino mass matrix are

Pattern	Constraining equations
I	$C_{33} = 0$
II	$C_{22} = 0$
III	$C_{31} = 0$
IV	$C_{21} = 0$
V	$C_{32} = 0$
VI	$C_{11} = 0$

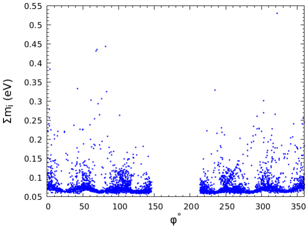
Table: one minor zero patterns

- For $C_{33} = 0$ case, the vanishing minor condition is

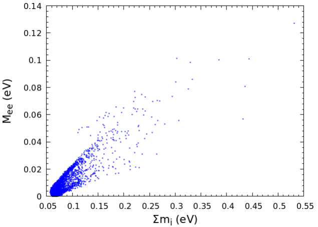
$$M_{ee}M_{\mu\mu} - M_{e\mu}M_{e\mu} = 0. \tag{18}$$

- The total neutrino mass for this pattern is $(0.056 - 0.530)$ eV.
- The effective Majorana mass (M_{ee}) term is $5.697 \times 10^{-6} - 0.127$ eV.
- This pattern follows the normal mass ordering.
- The phenomenology of pattern $C_{22} = 0$ is same under the transformation relation

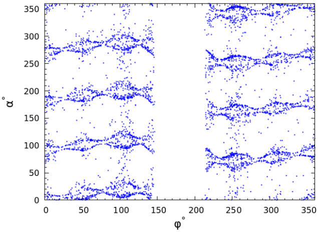
$$\theta_{23} = \frac{\pi}{2} - \theta_{23}, \delta = \pi - \delta. \tag{19}$$



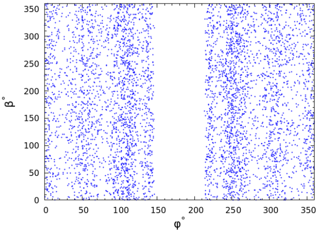
(a) variation of Σm_i with respect to ϕ for C_{33}



(b) variation of m_{ee} with respect to Σm_i for C_{33}

- 

(a) variation of α with respect to ϕ for C_{33}



(b) variation of β with respect to ϕ for C_{33}

- For $C_{31} = 0$ case, the vanishing minor condition is

$$M_{e\mu} M_{\mu\tau} - M_{e\tau} M_{\mu\mu} = 0. \quad (20)$$

- The total neutrino mass for this pattern is $(0.056 - 0.504)$ eV.

vanishing minor and neutrino masses

- For $C_{31} = 0$ case, the effective Majorana mass (M_{ee}) term is $1.163 \times 10^{-4} - 0.165$ eV.
- This pattern is shows with the normal mass ordering.
- The phenomenology of pattern $C_{21} = 0$ is same under the transformation relations (19).
- The vanishing minor condition for $C_{32} = 0$ case is

$$M_{ee} M_{\mu\tau} - M_{\mu e} M_{e\tau} = 0. \tag{21}$$

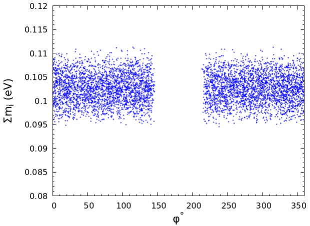
- The total neutrino mass for $C_{32} = 0$ pattern is $(0.056 - 0.624)$ eV.
- The effective Majorana mass (M_{ee}) term is $6.837 \times 10^{-5} - 0.144$ eV for $C_{32} = 0$.
- $C_{32} = 0$ pattern shows the normal mass ordering.

vanishing minor and neutrino masses

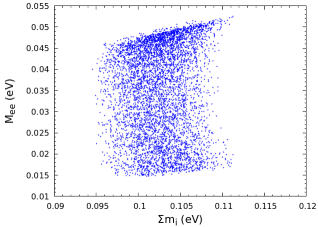
- The vanishing minor condition for $C_{11} = 0$ case is

$$M_{\mu\mu} M_{\tau\tau} - M_{\tau\mu} M_{\mu\tau} = 0. \qquad (22)$$

- The total neutrino mass for this pattern is $0.09 < \sum_{i=1}^{3} m_i < 0.12$.
- This pattern shows inverted mass ordering.



(a) variation of Σm_i with respect to ϕ for C_{11}



(b) variation of m_{ee} with respect to Σm_i for C_{11}

Conclusion

- We obtain unknown parameter θ, mixing angles (θ_{12} and θ_{23}) for the trimaximal mixing .
- We have studied neutrino masses using one vanishing minor condition in neutrino mass matrix using trimaximal mixing matrix. We find all the six patterns under this condition are experimentally allowed.

Thank you