Effect of spacetime torsion on fermion dynamics (NuDM 2022)

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Geometric action in terms of spin connection

We can express the curvature tensor in terms of spin connection using the tetrad postulate as

$$R^{\lambda}{}_{\sigma\mu\nu} = e^{j}_{\sigma} e^{\lambda}_{i} (\partial_\mu A_\nu{}^{i}{}_{j} - \partial_\nu A_\mu{}^{i}{}_{j} + A_\mu{}^{i}{}_{k} A_\nu{}^{k}{}_{j} - A_\nu{}^{i}{}_{k} A_\mu{}^{k}{}_{j}) = e^{j}_{\sigma} e^{\lambda}_{i} F_{\mu\nu}{}^{i}{}_{j}. \tag{1}$$

$$S_{EH} = \frac{1}{2\kappa} \int d^4 x e R \tag{2}$$

$$R = e^{\mu}_{a} e^{\nu}_{b} F_{\mu\nu}{}^{ab}$$

$$F_{\mu\nu}{}^{ab} = \partial_\mu A_\nu{}^{ab} - \partial_\nu A_\mu{}^{ab} + A_\mu{}^{a}{}_{c} A_\nu{}^{cb} - A_\nu{}^{a}{}_{c} A_\mu{}^{cb}.$$

Matter action in terms of spin connection

Covariant derivative of spinor fields in minimal substitution,

$$D_\mu \psi = \partial_\mu \psi - \frac{i}{4} A_\mu{}^{ab} \sigma_{ab} \psi \,, \qquad \sigma_{ab} = \frac{i}{2} \left[\gamma_a \,, \gamma_b \right] . \tag{3}$$

The Dirac Lagrangian after minimal substitution is

$$\mathcal{L}_\psi = \frac{i}{2} \left(\bar{\psi} \gamma^\mu \partial_\mu \psi - \partial_\mu \bar{\psi} \gamma^\mu \psi - \frac{i}{4} A_\mu^{ab} \, \bar{\psi} \{ \sigma_{ab}, \gamma_c \} \psi \, e^{\mu c} \right) + i m \bar{\psi} \psi \,. \tag{4}$$

Then the action of gravity plus a fermion field can be written as

$$S = \int |e| d^4 x \left(\frac{1}{2\kappa} F_{\mu\nu}{}^{ab} e_a^\mu e_b^\nu + \mathcal{L}_\psi \right) , \tag{5}$$

Fermions under gravity

In presence of fermions the action looks like

$$S = \frac{1}{2\kappa}\int d^4x\ e e_a^\mu e_b^\nu (\partial_\mu A_\nu{}^{ab} - \partial_\nu A_\mu{}^{ab} + A_\mu{}^a{}_c A_\nu{}^{cb} - A_\nu{}^a{}_c A_\mu{}^{cb}) + \int d^4x\ e[\frac{i}{2}(\bar{\psi}\gamma^\mu\partial_\mu\psi - \partial_\mu\bar{\psi}\gamma^\mu\psi) - \frac{i}{4}A_\mu{}^{ab}\bar{\psi}\{\sigma_{ab},\gamma_c\}\psi e^{\mu c} + im\bar{\psi}\psi]. \tag{6}$$

Extremising the action leads to the equation of motion

$$A_\mu{}^{ac}e^{\mu b} + A_\mu{}^{cb}e^{\mu a} = e_\nu^c(e^{\mu a}\partial_\mu e^{\nu b} - e^{\mu b}\partial_\mu e^{\nu a}) - \frac{\kappa}{4}\bar{\psi}\{\sigma^{ab},\gamma^c\}\psi. \tag{7}$$

Fermions under gravity contd.

We finally get

$$A_\mu{}^{ab} = \omega_\mu{}^{ab} + \frac{\kappa}{8}\bar{\psi}\{\sigma^{ab}, \gamma_c\}\psi e^c_\mu \qquad (8)$$
$$= \omega_\mu{}^{ab} + \Lambda_\mu{}^{ab}.$$

This is spin connection in presence of the fermions.
$\Lambda_\mu{}^{ab}$ is antisymmetric in all of its indices.
$\omega_\mu{}^{ab}$ is the spin connection of the Levi-Civita connection.

Effective four fermi theory

Which finally gives us

$$S = \frac{1}{2\kappa}\int d^4x \; eR(\hat{\Gamma}) + \int d^4x e\Big(\frac{i}{2}(\bar{\psi}\gamma^\mu \widehat{D}_\mu\psi - \widehat{D}_\mu\bar{\psi}\gamma^\mu\psi) + im\bar{\psi}\psi - \frac{3\kappa}{16}(\bar{\psi}\gamma^i\gamma^5\psi)^2\Big). \tag{9}$$

We get an effective theory with a quartic interaction lagrangian. It has been recently proposed that the most generic form of contortion is

$$\Lambda_\mu{}^{ab} = \frac{\kappa}{4}\epsilon^{abcd}e_{c\mu}\sum_i\left(\lambda^i_L\bar{\psi}_{iL}\gamma_d\gamma^5\psi_{iL} + \lambda^i_R\bar{\psi}_{iR}\gamma^d\gamma^5\psi_{iR}\right).[1] \tag{10}$$

[1] Eur. Phys. J. C **79** (2019) 697 [arxiv:1904.06036 [hep-ph]].

Chiral torsion

As before, we can insert this solution back into the action and get an effective quartic interaction term

$$S = \frac{1}{2\kappa}\int d^4xeR(\hat{\Gamma}) + \int d^4xe\left[\frac{i}{2}(\bar{\psi}\gamma^\mu\widehat{D}_\mu\psi - \widehat{D}_\mu\bar{\psi}\gamma^\mu\psi) + im\bar{\psi}\psi\right.$$
$$\left. - \frac{3\kappa}{16}\left(\sum_i\left(\lambda_L^i\bar{\psi}_L^i\gamma_a\gamma^5\psi_L^i + \lambda_R^i\bar{\psi}_R^i\gamma_a\gamma^5\psi_R^i\right)\right)^2\right]. \tag{11}$$

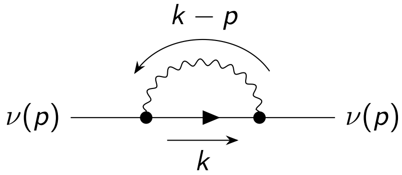
where the sum runs over all species of fermions.

Finite temperature field theory

1. Real-time formalism of finite temperature field theory, the fermion propagator gets an effective thermal contribution as

$$D(p) = \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon} - 2\pi n_f(p^0)\delta(p^2 - m^2)(\not{p} + m). \quad (12)$$

2. For a neutrino, medium it shifts the pole of the propagator due to the thermal contribution. This will change the effective mass of the neutrino (concerned fermion here) due to weak interaction and torsion interaction.



This diagram will change the neutrino propagator as

$$\frac{i}{\not{p}}(-i\Sigma)\frac{i}{\not{p}}. \tag{13}$$

A straightforward calculation yields

$$\begin{aligned}-i\Sigma = -\frac{g^2}{2}\int \frac{d^4k}{(2\pi)^4}\gamma^\lambda \mathbb{L}[\frac{i(\not{k}+m)}{k^2-m^2+i\epsilon} \\ -2\pi n_f(k^0)\delta(k^2-m^2)(\not{k}+m)]\gamma^\sigma \mathbb{L}\frac{-ig_{\lambda\sigma}}{(k-p)^2-M_W^2}.\end{aligned} \tag{14}$$

In the contact approximation

$$
\begin{aligned}
-i\Sigma = & \frac{ig^2}{2M_W^2} \int \frac{d^4k}{(2\pi)^4}(2\pi)\gamma^\lambda \mathbb{L}(\not{k}+m)\gamma_\lambda \mathbb{L}[\delta(k^0-\omega_k)+\delta(k^0+\omega_k)] \\
& - \frac{ig^2}{2M_W^2} \int \frac{d^4k}{(2\pi)^3} \frac{\gamma^0 k^0}{\omega_k} \mathbb{L} n_F(k^0)[\delta(k^0-\omega_k)+\delta(k^0+\omega_k)] \\
= & -i\sqrt{2}G_F n_e \gamma^0 \mathbb{L}.
\end{aligned}
\tag{15}
$$

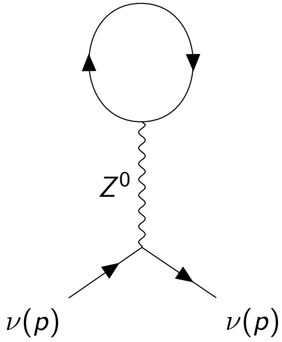
The spatial components drop out on momentum integration. We have used the definition

$$
n_e = 2\int \frac{d^3k}{(2\pi)^3} n_f^e(\omega_k). \tag{16}
$$

Now, the total propagator is

$$\begin{aligned}&\frac{i}{\not{p}}+\frac{i}{\not{p}}(-i\Sigma)\frac{i}{\not{p}}\\ =&\frac{i}{\not{p}-\Sigma}=\frac{i}{\gamma^0(p^0-\sqrt{2}G_F n_e\mathbb{L})-\vec{\gamma}\cdot\vec{p}}.\end{aligned}\tag{17}$$

For neutral current mediated process the corresponding diagram is



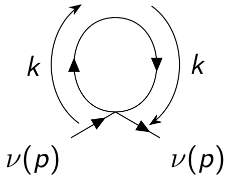
Similar calcuation shows due to neutral current mediated process the propagator modifies as

$$\begin{aligned}
&\frac{i}{\not{p}} + \frac{i}{\not{p}}(-i\Sigma)\frac{i}{\not{p}} \\
=&\frac{i}{\not{p}-\Sigma} \\
=&\frac{i}{\gamma^0(p^0+\sqrt{2}G_F\frac{n_n}{2}\mathbb{L})-\vec{\gamma}\cdot\vec{p}}.
\end{aligned} \tag{18}$$

Hence the total change in propagator pole in the flavor basis is

$$\frac{i}{\gamma^0(p^0-(\sqrt{2}G_F n_e\mathbb{L}-\sqrt{2}G_F\frac{n_n}{2}\mathbb{L}))-\vec{\gamma}\cdot\vec{p}}. \tag{19}$$

Matter due to spin-torsion interaction



Following the same method as in case of NC mediated process we can evaluate the shift in pole of propagator of type i neutrino as

$$\frac{i}{\gamma^0(p^0 - \frac{3\kappa}{8}\sum_f \lambda_V^f n_f \lambda_i \mathbb{L}) - \vec{\gamma}\cdot\vec{p}} = \frac{i}{\gamma^0(p^0 - \lambda_i \tilde{n}\mathbb{L}) - \vec{\gamma}\cdot\vec{p}}. \quad (20)$$

$\tilde{n} = \frac{3\kappa}{8}\sum_f \lambda_V^f n_f$

Neutrino oscillation

The mass eigenstates $|\nu_i\rangle$ and the flavor eigenstates $|\nu_\alpha\rangle$ are related to each other by

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle \ , \tag{21}$$

where U is the mixing matrix, $U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$. We can now write the Schrödinger equation for the neutrinos,

$$\begin{aligned} i\frac{d}{dt}\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \Bigg[E + \frac{1}{2E}\begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} + \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}\tilde{n} \\ - \frac{G_F}{\sqrt{2}}(n_n - n_e) + U^T \begin{pmatrix} A & 0 \\ 0 & -A \end{pmatrix} U^* \Bigg] \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \end{aligned} \tag{22}$$

we have written $A = \frac{G_F}{\sqrt{2}} n_e$. In terms of neutrinos in the flavor basis this equation becomes

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \left[E_0 \mathbb{I} + \frac{1}{4E} \begin{pmatrix} -\Delta m_s^2 \cos 2\theta + D & \Delta m_s^2 \sin 2\theta \\ \Delta m_s^2 \sin 2\theta & \Delta m_s^2 \cos 2\theta - D \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}, \tag{23}$$

where we have written $D = 4AE = 2\sqrt{2} G_F n_e E$,

$$E_0 = E + \frac{m_1^2 + m_2^2}{4E} + \frac{\lambda_1 + \lambda_2}{2} \tilde{n} - \frac{G_F}{\sqrt{2}} (n_n - n_e) \, , \tag{24}$$

and also defined a modified Δm^2 as

$$\Delta m_s^2 = \Delta m^2 + 2\tilde{n} E \Delta\lambda \, , \tag{25}$$

where $\Delta m^2 = m_2^2 - m_1^2$ and $\Delta\lambda = \lambda_2 - \lambda_1$.

Let us write θ_M for the mixing angle in matter, modified by the torsional interaction,

$$\tan 2\theta_M = \frac{\tan 2\theta}{1 - \frac{D}{\Delta m_s^2 \cos 2\theta}} . \qquad (26)$$

Then we can diagonalize Hamiltonian by defining
$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_M & -\sin\theta_M \\ \sin\theta_M & \cos\theta_M \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$
to write the equation for ultrarelativistic neutrinos

$$i\frac{d}{dx}\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \left[E_0 + \frac{\Delta m_M^2}{4E}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\right]\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} , \qquad (27)$$

where we have written
$\Delta m_M^2 = \sqrt{(\Delta m_s^2 \cos 2\theta - D)^2 + (\Delta m_s^2 \sin 2\theta)^2}$. The eigenvalues are $E_0 \mp \frac{\Delta m_M^2}{4E}$,.

Resulting in the survival probability

$$P_{\nu_e \to \nu_e} = 1 - \sin^2(2\theta_M)\sin^2\left(\frac{\Delta m_M^2(E)}{4E}L\right) \tag{28}$$

and the conversion probability

$$P_{\nu_e \to \nu_\mu} = \sin^2(2\theta_M)\sin^2\left(\frac{\Delta m_M^2(E)}{4E}L\right) . \tag{29}$$

So far the effective mass squared difference is only a function of matter density, now is also a function of incident neutrino energy.