Right-handed Sneutrino Dark Matter

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based on

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Outline

SUSY $B-L$ Extension of the SM

- BLSSM is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$.
- This type of extension implies the existence of extra 3 superfields, one per generation with one unit of $B-L$ charge, in order to cancel the associate $B-L$ triangle anomaly.
- These superfields are identified with the right-handed neutrinos and will be denoted N_i.
- In addition, in order to break the $B-L$ symmetry at TeV scale, two Higgs superfields $\hat{\chi}_{1,2}$ with ± 2 $B-L$ charges are required.
- The superpotential of the BLSSM is given by

$$W = (Y_u)_{ij}\hat{Q}_i\hat{H}_2\hat{U}_j^c + (Y_d)_{ij}\hat{Q}_i\hat{H}_1\hat{D}_j^c + (Y_e)_{ij}\hat{L}_i\hat{H}_1\hat{E}_j^c + \mu\,\hat{H}_1\hat{H}_2 \\ + (Y_N)_{ij}\hat{N}_i^c\hat{\chi}_1\hat{N}_j^c + (Y_\nu)_{ij}\hat{L}_i\hat{H}_2\hat{N}_j^c + \mu'\,\hat{\chi}_1\hat{\chi}_2.$$

- The first line in equation represents the conventional yukawa interaction as well as μ term in MSSM; while the second line stands for the additional interaction induced by extended gauge group $U(1)_{B-L}$.

Higgs Bosons in BLSSM

- In BLSSM, we have 2 Higgs doublet and 2 Higgs singlet superfields, i.e. 12 degrees of freedom:
  - ⋆ 4 have been eaten by $W^{\pm}$, Z, and Z'.
  - ⋆ 2 neutral pseudoscalar Higgs bosons A, A'.
  - ⋆ 2 charged Higgs bosons $H^{\pm}$.
  - ⋆ 4 neutral scalar Higgs bosons h, h', H, H'.

- The mass matrix of BLSSM CP-even neutral Higgs at tree level is given by

$$M^2 = \begin{pmatrix} M^2_{hH} & M^2_{hh'} \\ {M^2_{hh'}}^T & M^2_{h'H'} \end{pmatrix}$$

- M^2_{hH} is MSSM neutral CP-even Higgs mass matrix $\Rightarrow m_h \sim 125$ GeV $\& m_H \sim m_A \sim \mathcal{O}(1 \text{ TeV})$.

- $$M^2_{h'H'} = \begin{pmatrix} m^2_{A'} c^2_{\beta'} + g^2_{BL} v'^2_1 & -\frac{1}{2} m^2_{A'} s_{2\beta'} - g^2_{BL} v'_1 v'_2 \\ -\frac{1}{2} m^2_{A'} s_{2\beta'} - g^2_{BL} v'_1 v'_2 & m^2_{A'} s^2_{\beta'} + g^2_{BL} v'^2_2 \end{pmatrix}$$

$$\Rightarrow m^2_{h',H'} = \frac{1}{2}\left[(m^2_{A'} + M^2_{Z'}) \mp \sqrt{(m^2_{A'} + M^2_{Z'})^2 - 4 m^2_{A'} M^2_{Z'} \cos^2 2\beta'}\right]$$

$$\Rightarrow m_{h'} \simeq \left(\frac{m^2_{A'} M^2_{Z'} \cos^2 2\beta'}{m^2_{A'} + M^2_{Z'}}\right)^{\frac{1}{2}} \simeq \mathcal{O}(100 \text{ GeV})$$

Right-handed Snetrino

- The sneutrino mass matrix, in the basis $(\tilde{\nu}_L, \tilde{\nu}_L^*, \tilde{\nu}_R, \tilde{\nu}_R^*)$, is approximately given by a 2×2 block diagonal matrix, where the element 11(22) of this matrix is given by the diagonal LH (RH) sneutrino mass matrix, M_{RR}, defined as

$$M_{RR}^2 = \begin{pmatrix} M_N^2 + m_{\tilde{N}}^2 + m_D^2 + \frac{M_{Z'}^2}{2}\cos 2\beta' & M_N(A_N - \mu' \cot\beta') \\ M_N(A_N - \mu' \cot\beta') & M_N^2 + m_{\tilde{N}}^2 + m_D^2 + \frac{M_{Z'}^2}{2}\cos 2\beta' \end{pmatrix}$$

- Here M_N is the right-handed neutrino mass, which is proportional to the $B-L$ symmetry breaking VEV, *i.e.* $M_N = Y_N v_1' \sim \mathcal{O}(1)$ TeV, and $m_D = Y_\nu \langle H_2 \rangle = Y_\nu v_2$, with $Y_\nu \lesssim \mathcal{O}(10^{-6})$

- The soft SUSY breaking parameters $m_{\tilde{N},\tilde{L}}$ and $A_{\nu,N}$ are the sneutrino, slepton scalar masses and trilinear couplings, respectively & $\tan\beta'$ is defined as $\tan\beta' = v_1'/v_2'$.

- The mixing between the right-handed sneutrinos and right-handed anti-sneutrinos is quite large, since it is given in terms of $Y_N \sim \mathcal{O}(1)$. Thus, the eigenvalues of the right-handed sneutrino are given by

$$m_{\tilde{\nu}_\mp}^2 = M_N^2 + m_{\tilde{N}}^2 + m_D^2 + \frac{1}{2} M_{Z'}^2 \cos 2\beta' \mp \Delta m_{\tilde{\nu}_R}^2,$$

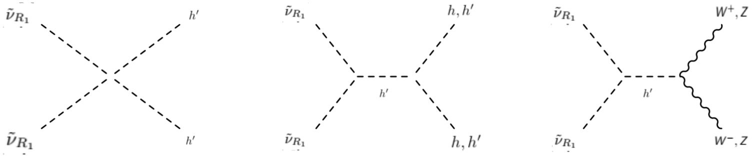
where $\Delta m_{\tilde{\nu}_R}^2 = |M_N(A_N - \mu' \cot\beta')|$. It is clear that the lightest $\tilde{\nu}_- \equiv \tilde{\nu}_{R_1}$ is the lightest sneutrino and can be even the LSP for a wide region of parameter space, hence it can be stable and a viable candidate for DM.

The interactions of sneutrino DM

- In the BLSSM, the relevant interaction terms of lightest right-handed sneutrino are given by the following Lagrangian:

$$\begin{aligned}\mathcal{L} &= Y(\tilde{\upsilon}_{R_1})^2 h' + \lambda_4 (\tilde{\upsilon}_{R_1})^2 h'^2 + \lambda_2 h' h^2 + g_{W\pm} h' W^+ W^- \\ &+ \lambda_3 h' h' h' + g_{ZZ} h' ZZ,\end{aligned} \tag{1}$$

- The annihilations of the $\tilde{\nu}_{R_1}$ are given the following:



- With couplings

$$g_{h'\,W^+\,W^-} \simeq i\; g_2 M_W \left(\Gamma_{32}\sin\beta + \Gamma_{31}\cos\beta\right), \qquad g_{h'\,ZZ} \simeq i\; g_z\, M_Z \left(\Gamma_{32}\sin\beta + \Gamma_{31}\cos\beta\right),$$

$$Y_{\tilde{\upsilon}_1\tilde{\upsilon}_1 h'} \simeq i\; (\Gamma^R_{14})^2 \left[\frac{g_{BL}^2}{2}\Big(v'_1\Gamma_{33} - v'_2\Gamma_{34}\Big) + \sqrt{2}\Big(\Gamma_{34}\mu' Y_x - \Gamma_{33} T_x\Big) - 4v'_1\Gamma_{33}Y_x^2\right],$$

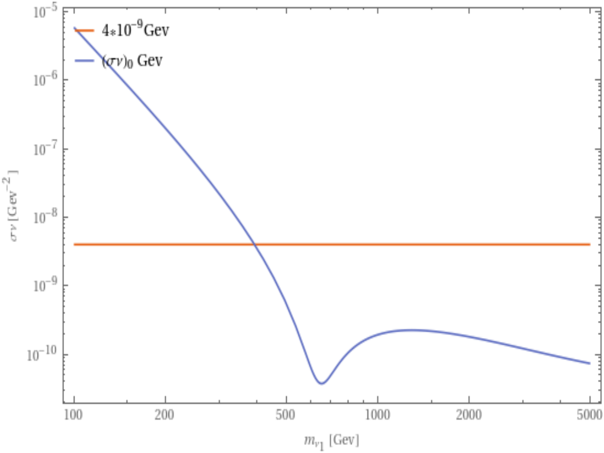
$$\lambda_{\tilde{\upsilon}_1\tilde{\upsilon}_1 h' h'} \simeq i\; (\Gamma^R_{14})^2 \left[\frac{g_{BL}^2}{2}\Big(\Gamma_{33}^2 - \Gamma_{34}^2\Big) + \frac{g_{BL}\tilde{g}}{4}\Big(\Gamma_{31}^2 - \Gamma_{32}^2\Big) - 4\Gamma_{33}^2 Y_x^2\right],$$

$$\lambda_{h'h'h'} \simeq i\; g_{BL}^2 \left[v'_1\Big(-3\Gamma_{33}^3 + 3\Gamma_{33}\Gamma_{34}^2\Big) + v'_2\Big(3\Gamma_{33}^2\Gamma_{34} - 3\Gamma_{34}^3\Big)\right],$$

$$\lambda_{h'hh} \simeq i\; g_{BL}^2 \left[v'_1\Big(-3\Gamma_{33}^2\Gamma_{13} + \Gamma_{13}\Gamma_{34}^2 + 2\Gamma_{34}\Gamma_{33}\Gamma_{13}\Big) + v'_2\Big(\Gamma_{33}^2\Gamma_{14} + 2\Gamma_{33}\Gamma_{34}\Gamma_{13} - 3\Gamma_{34}^2\Gamma_{14}\Big)\right],$$

Annihilation Cross Section

- By choose the benchmark point for parameter space for computing annihilation cross section:

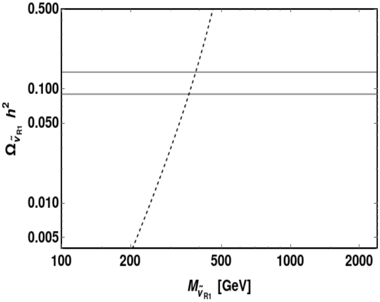


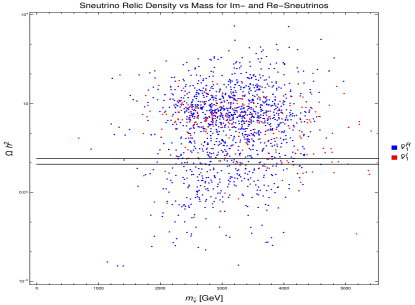
- The suppressed of $\langle \sigma^{\mathrm{ann}}_{\tilde{\nu}_{R_1}} v \rangle$

Sneurino Relic Density

- The value of these annihilation cross sections determine the relic abundance, which is given by

$$\Omega_{\tilde{\nu}_{R_1}} h^2 = \frac{2.1 \times 10^{-27}\ \mathrm{cm}^3\mathrm{s}^{-1}}{\langle \sigma^{\mathrm{ann}}_{\tilde{\nu}_{R_1}} v \rangle} \left(\frac{x_F}{20}\right) \left(\frac{100}{g_*(T_F)}\right)^{\frac{1}{2}}$$





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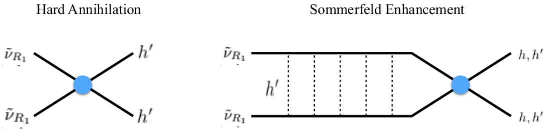
- $x_F = m_{\tilde{\nu}_{R_1}} / T_F \simeq \mathcal{O}(20)$, $g_*(T_F) \simeq \mathcal{O}(100)$ is the number of degrees of freedom at freeze-out.

- The suppressed of $\langle \sigma^{\mathrm{ann}}_{\tilde{\nu}_{R_1}} v \rangle \Rightarrow$ large relic abundance $\Omega_{\tilde{\nu}_{R_1}} h^2$. For few points in the parameter space it can be within the 2σ allowed region by the Planck collaboration: $0.09 < \Omega h^2 < 0.14$

Bound state and Sommerfeld Enhancement

- If the annihilation of the DM (with no self-interactions) into SM final states is a localized interaction at the origin. Then the probability of finding the DM particles at the origin is just $|\psi_0(0)|^2$, where ψ_0 is the incoming wave function and a solution to the non-relativistic Schrodinger equation.



- If self-interactions are allowed by theory, then one possibility is that DM particle can interact with itself via long range force before annihilating.The exchange of multiple mediators change the wave function of incoming DM particle so that the probability of finding them is now $|\psi(0)|^2$, where ψ is the modified wave function in the presence in the interaction potential. This is known as the Sommerfeld enhancement.

- The development of a DM bound state through a scalar field, ϕ, requires the screening length, $1/m_\phi$, to be much larger than the Bohr radius of bound state $\sim 2/\alpha\, m_{\rm DM}$. Thus, one gets the following condition.

$$\alpha\, m_{\rm DM}/m_\phi \gtrsim 1.64,$$

- The Sommerfeld enhancement is defined as the ratio of probabilities of finding the DM at the origin in the presence of the potential, relative to no potential:

$$S = \frac{|\psi(0)|^2}{|\psi^0(0)|^2}. = |\psi(0)|^2 \quad \text{where } |\psi^0(0)|^2 = 1.$$

- Since $v_{rel} \ll 1$ we do not need to use relativistic quantum field theory for this calculation.
- We can compute the same v-dependent correction to particle scattering using non-relativistic quantum mechanics.
- For $V(r)$ potential, the wave function $\psi(\vec{r})$ can be expanded in terms spherical harmonics, combined with an energy-dependent radial function $R(r; E)$.

$$\psi_k(\vec{r}) = \sum_{l=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi) R_\ell(r; E)$$

- As in the hydrogen atom, the radial, time-independent Schrodinger equation, is given as

$$\left[-\frac{1}{2mr^2} \frac{d}{dr}(r^2 \frac{d}{dr}) + \frac{\ell(\ell+1)}{2mr^2} + V(r) - E \right] R_\ell(r; E) = 0$$

- For Coulomb potential, one finds that $R_{k\ell} = j_\ell(\rho)$, where $j_\ell(\rho)$ is spherical Bessel functions.
- At the origin, $\vec{r} = \vec{0}$, we get

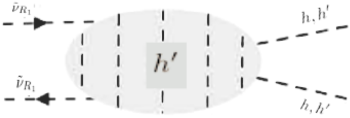
$$\left|\psi_k(\vec{0})\right|^2 = \frac{2\pi e^2}{v_{rel}} \frac{1}{1 - e^{-2\pi e^2 / v_{rel}}} \approx \begin{cases} \frac{2\pi e^2}{v_{rel}} & \text{for } v_{rel} \to 0 \\ 1 & \text{for } v_{rel} \to \infty \end{cases}$$

- This increased probability measure is called the Sommerfeld enhancement.
- The annihilation decay of bound state into XY can be calculated in terms of the amplitude of annihilation $\phi\phi^* \longrightarrow XY$ and the bound state wave function at the origin.
- Thus, one finds

$$\sigma = S\sigma_0$$

RH-Sneutrino Sommerfeld effect

- As h' and $\tilde{\nu}_{R_1}$ are belonging to the B-L sector, the strength of their interaction is fairly large.
- Thus, a bound state for $\tilde{\nu}_{R_1}$ may be formed by mediating the scalar boson h'.



- A scalar particle exchange mediates Yukawa potential. The sneutrino bound state mediated by h' Higgs boson, yields a Yukawa potential with range $1/m_{h'}$, is only possible if

$$\frac{\alpha_\nu m_{\tilde{\nu}_{R_1}}}{m_{h'}} > 1.6, \qquad \text{where} \;\; \alpha_\nu = (Y_{\tilde{\nu}_{R_1}\tilde{\nu}_{R_1}h'})^2/16\pi m^2_{\tilde{\nu}_{R_1}}.$$

- Due large m'_h, sneutrino bound state can be achieved only for heavy sneutrino or/and a quite strong coupling.
- The internal motion of the bound state of $\tilde{\nu}_{R_1}$ is non-relativistic, described by Schrodinger equation

$$\Big(-\frac{\nabla^2}{2\mu_r} + V(r)\Big)\psi(r) = E\psi(r),$$

where $\mu_r = m_{\tilde{\nu}_{R_1}}/2$ is the reduced mass and $V(r)$ is the Yukawa potential, obtained from h'-exchange:

$$V(r) = -\alpha_\nu \frac{e^{-m_{h'}r}}{r},$$

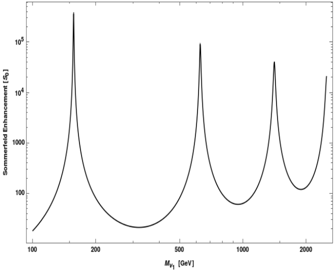
- The exact analytical solution of Schrodinger equation with Yukawa potential is not available. However, an approximate one can be found based on the scaled Hulthen potential

$$V_H(r) = -\alpha m_* \frac{e^{-m_* \; r}}{1 - e^{-m_* \; r}}, \quad \text{where} \quad m_* = \frac{\pi^2}{6} m_{h'} .$$

- This type of potential is called Hultzen potential. Now, it is possible to obtain analytical solutions for the $\ell = 0$ modes of the wavefunctions.

- It turns out that the Sommerfeld enhancement is given by

$$S = \left(\frac{2\pi\alpha}{\upsilon_{rel}}\right) \frac{sinh\left(\frac{6 m_{\tilde{\nu}_{R_1}} \upsilon_{rel}}{\pi m_{h'}}\right)}{cosh\left(\frac{6 m_{\tilde{\nu}_{R_1}} \upsilon_{rel}}{\pi m_{h'}}\right) - cos\left[\sqrt{\frac{24 m_{\tilde{\nu}_{R_1}} \alpha_{\nu}}{m_{h'}} - \frac{36 m^2_{\tilde{\nu}_{R_1}} \upsilon^2_{rel}}{\pi^2 m^2_{h'}}}\right]},$$



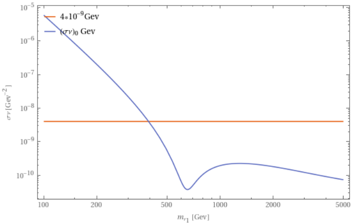
where υ_{rel} is the relative velocity between the $\tilde{\nu}_{R_1}$ DM particles.

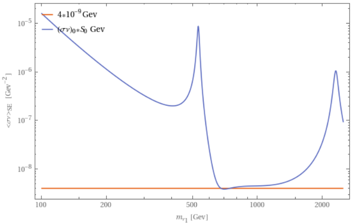
Sneurino Annihilation Cross Section with SE

- The generic Sneutrino DM annihilation cross section times the relative velocity before including the Sommerfeld enhancement has the form $(\sigma \upsilon_{rel})_0 = a + b\nu_{rel}^2 + O(\nu_{rel}^4)$.

- With Sommerfeld effect, the thermally averaged cross section of S-wave at a temperature T or $x \equiv m_\chi / T$ can be written as

$$\langle \sigma \upsilon_{rel} \rangle = a \langle S_0(\upsilon_{rel}) \rangle (x) + b \langle S_1(\upsilon_{rel}) \upsilon_{rel}^2 \rangle (x)$$





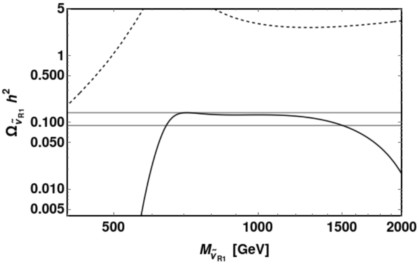
- The thermally averaged annihilation cross section depends on the parameters α and $m_{h'}$.

Sneurino Relic Density with SE

- This enhancement of annihilation cross section will have a significant effect on computing the relic abundance, Ωh^2 of $\tilde{\nu}_{R_1}$ DM.

- With Sommerfeld effect, the thermally averaged cross section of S-wave at a temperature T or $x \equiv m_\chi / T$ can be written as

$$\langle \sigma \upsilon_{rel} \rangle = a \langle S(\upsilon_{rel}) \rangle (x) = \frac{(\sigma \upsilon_{rel})_0 \; x^{3/2}}{2\sqrt{\pi}} \int_0^\infty \; S(\upsilon_{rel}) \; e^{-\frac{x \upsilon_{rel}^2}{4}} \upsilon_{rel}^2 \; d\upsilon_{rel}$$



- Relic density of the dark matter mass $\tilde{\nu}_{R_1}$ with Sommerfeld enhancement with $m_{h'} = 28$GeV, where horizontal lines correspond to the Planck limits for the relic abundance.

Conclusions

- The corresponding Sommerfeld Effects have important impacts on Right-handed Sneutrino DM.
- With SE, the $\tilde{\nu}_{R_1}$ annihilation cross section is enhanced significantly and hence relic abundance can be within the observed limits.
- It relaxes the sever constraints imposed on the parameter space of this SUSY model and $\tilde{\nu}_{R_1}$ is now viable DM candidate.