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Impact of non-perturbative Effects in Simplified *t*-Channel Dark Matter Models

in collaboration with

Emanuele Copello, Julia Harz, Kirtimaan Mohan and Dipan Sengupta

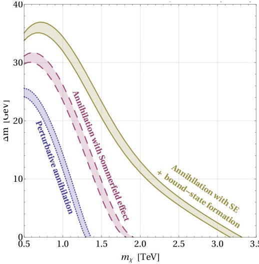
based on 2203.04326, published in JHEP 08 (2022)

supported by DFG Emmy Noether Grant No. HA 8555/1-1.

Mathias Becker

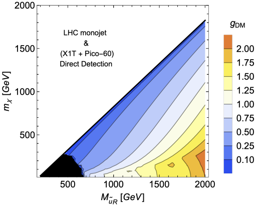
Motivation I

Figure from [Harz,Petraki (2018)]



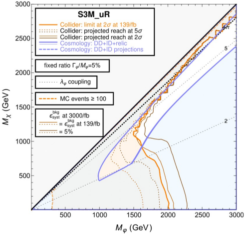
BSF in the early universe

Figure from [Mohan et. al (2019)]



RGE improved Direct Detection (DD)

Figure from [Arina et. al (2020)]



Experimental constraints from the LHC t-channel DM working group

⇒ How impactful are non-perturbative Effects for experimental exclusion limits

What is new?

- Bound State Formation (BSF) effects on the relic density in a *realistic* model of colored coannihilation
- Study of the interplay of a large variety of experimental searches, considering the SE and BSF.
- We point out the possibilities of Bound state searches at the LHC
- Correct implemenation of BSF in micrOMEGAs (including three-gauge boson vertex contribution and an estimate of bound state three-body decays)

What is new?

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- Correct implemenation of BSF in micrOMEGAs (including three-gauge boson vertex contribution and an estimate of bound state three-body decays)

$\rightarrow$ A flat correction factor for non-perturbative effects is unapplicable

$\rightarrow$ Corrections on the exclusion limits can be as large as $\mathcal{O}(100\%)$

$\rightarrow$ Bound State searches close gap between prompt and long-lived searches!

Simplified t-Channel Dark Matter

Universal framework for t-channel DM models [Arina,Fuks,Mantani (2020)]

S3M-uR t-channel Dark Matter

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{kin,BSM}} + g_{\text{DM}}\overline{\chi}(u_R)_i(X^\dagger)_i + h.c.$$

$$\chi = (\mathbf{1},\mathbf{1})_0 \qquad X_i = (\mathbf{3},\mathbf{1})_{2/3}$$

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$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{kin,BSM}} + g_{\text{DM}}\overline{\chi}(u_R)_i(X^\dagger)_i + h.c.$$

$$\chi = (\mathbf{1},\mathbf{1})_0 \qquad X_i = (\mathbf{3},\mathbf{1})_{2/3}$$

- Discrete $\mathcal{Z}_2$: SM fields even, dark sector fields odd
- Majorana fermion DM χ
- 3 generations of mediators X_i

Parameters: $(m_\chi = m_{\text{DM}}, \Delta m = m_X - m_{\text{DM}}, g_{\text{DM}})$

Dark Matter Freeze-Out

Assumptions during DM freeze-out:

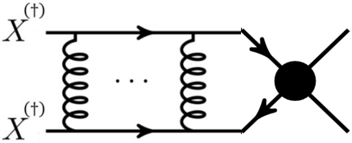
- Dark sector in *kinetic* eq. with the SM.
- Dark sector particles in *chemical* eq. with themselves.

Coannihilation

$$\frac{dn}{dt} + 3Hn = -\langle \sigma_{\text{eff}} v \rangle \left(n^2 - \left(n^{\text{eq}} \right)^2 \right)$$

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{i,j} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{\text{eq}}}{n^{\text{eq}}} \frac{n_j^{\text{eq}}}{n^{\text{eq}}}$$

$$n = \sum_i n_i \quad \text{and} \quad i,j = \{\chi, X_1, X_2, X_3\}$$

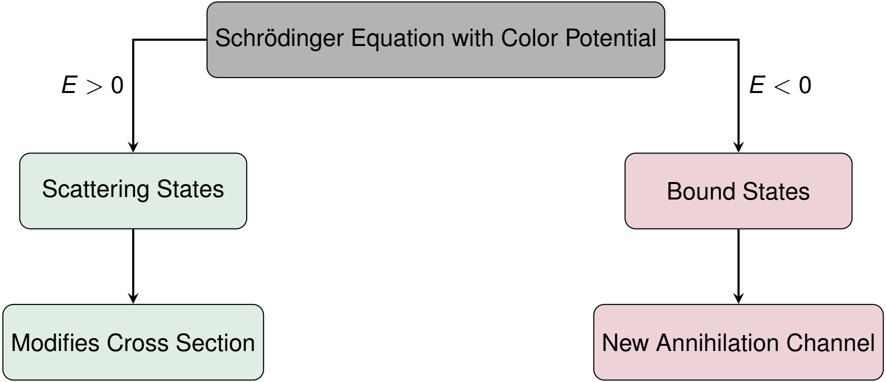


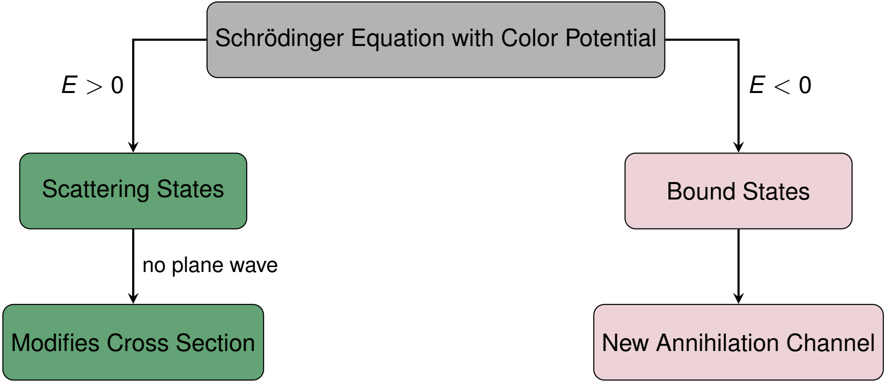
n-gluon exchanges contribute with $\left(\frac{\alpha}{v}\right)^n$ for $\alpha \sim v$

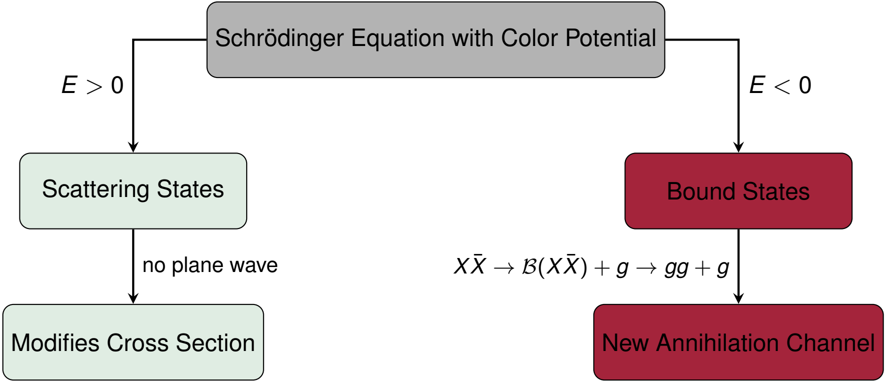
$\rightarrow$ Resummation required since $\alpha_s \sim v \sim 0.1$

$\rightarrow$ Reduces to Schrödinger Equation for $v \ll 1$. For details [Petraki,Postma,Wiechers(2015)]

Figure from Talk by J.Harz @ DM Working Group


Schrödinger Equation with Color Potential
$E > 0$
$E < 0$
Scattering States
Bound States
Modifies Cross Section
New Annihilation Channel


Schrödinger Equation with Color Potential
$E > 0$
$E < 0$
Scattering States
Bound States
no plane wave
Modifies Cross Section
New Annihilation Channel


Schrödinger Equation with Color Potential
$E > 0$
$E < 0$
Scattering States
Bound States
no plane wave
$X\bar{X} \to \mathcal{B}(X\bar{X}) + g \to gg + g$
Modifies Cross Section
New Annihilation Channel

SE vs BSF

Modified Coannihilation [Ellis,Luo,Olive(2015)]

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{i,j \in \{\chi, X\}} \langle S(\alpha / v_{ij}) \cdot \sigma_{ij} v_{ij} \rangle \frac{n_i^{\text{eq}}}{n^{\text{eq}}} \frac{n_j^{\text{eq}}}{n^{\text{eq}}} + \langle \sigma_{\text{BSF}} v \rangle_{\text{eff}} \left(\frac{n_X^{\text{eq}}}{n^{\text{eq}}} \right)^2$$

$\langle \sigma_{\text{eff}} v \rangle$	Sommerfeld Effect	Bound State Formation
$g_{\text{DM}} \gg g_s$	$-$	0
$g_{\text{DM}} \ll g_s$	$+$	$++$

$\rightarrow$ No flat factor

BSF vs SE

$$\langle \sigma_{\rm eff} v \rangle = \sum_{i,j \in \{\chi, X\}} \langle S(\alpha / v_{ij}) \cdot \sigma_{ij} v_{ij} \rangle \frac{n_i^{\rm eq}}{n^{\rm eq}} \frac{n_j^{\rm eq}}{n^{\rm eq}} + \langle \sigma_{\rm BSF} v \rangle_{\rm eff} \left(\frac{n_X^{\rm eq}}{n^{\rm eq}} \right)^2$$

Sommerfeld Effect:

- Has an effect independently of the hierarchy between $g_{\rm DM}$ and g_s
- Tends to lower $\langle \sigma_{\rm eff} v \rangle$ for $g_{\rm DM} > g_s$
- Tends to increase $\langle \sigma_{\rm eff} v \rangle$ for $g_{\rm DM} < g_s$

Bound State Formation:

- BSF is purely mediated by g_s $\rightarrow$ less important for $g_{\rm DM} \gg g_s$
- Always increases $\langle \sigma_{\rm eff} v \rangle$
- For $g_{\rm DM} < g_s$ more sizable than the Sommerfeld effect.

Calculation of the Relic Density

We adjusted micrOMEGAs 5.2.7 such that

- the Sommerfeld Effect is included for colored scalars up to the adjoint representation
- Bound State effects are included for colored scalars up to the adjoint representation

Determine $g_{DM,0}$ for each data point $(m_{\text{DM}}, \Delta m)$ such that DM is *not* overproduced.

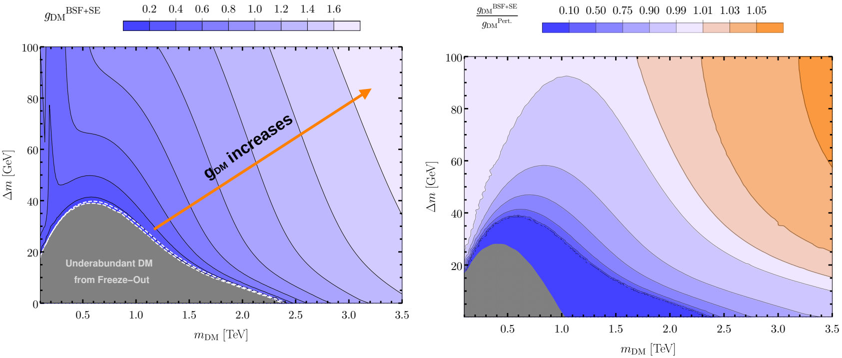
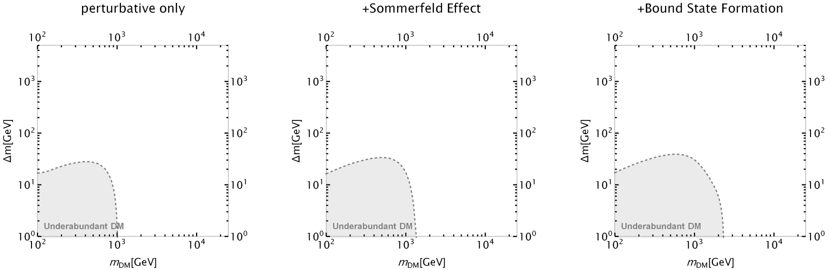


Figure from [MB,Copello,Harz,Mohan,Sengupta(2022)]



→ Bound State Formation increases the area where the strong interaction deplete relic density significantly!

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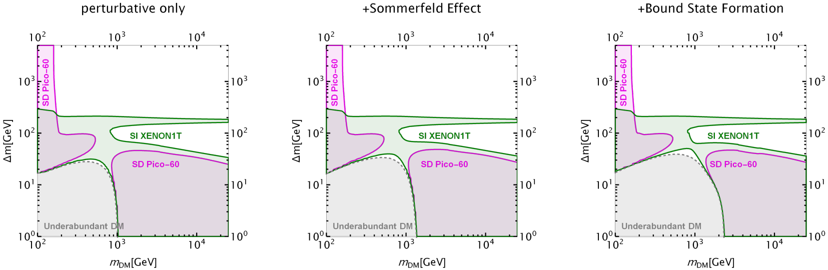
Direct Detection Constraints



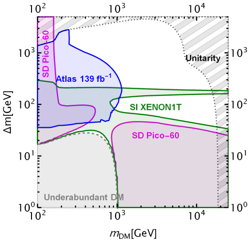
- SI stronger than SD for $\Delta m < m_{\rm DM}$
- Strong constraints on the coannihilating area
- Inclusion of BSF opens up parameter space in this region

Prompt Collider Searches

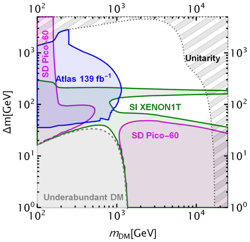
mono-jet + ETmiss search by ATLAS [arXiv:1711.03301]

multi-jets + ETmiss search by CMS [arXiv:1704.07781]

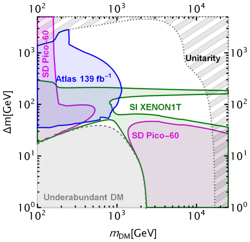
perturbative only



+Sommerfeld Effect



+Bound State Formation



- mainly constrains larger Δm
- Thus non-perturbative effects are mild/absent.

How to constrain the "gray" area ?

Freeze-out leads to underabundant DM $\rightarrow$ correct abundance requires alternative production

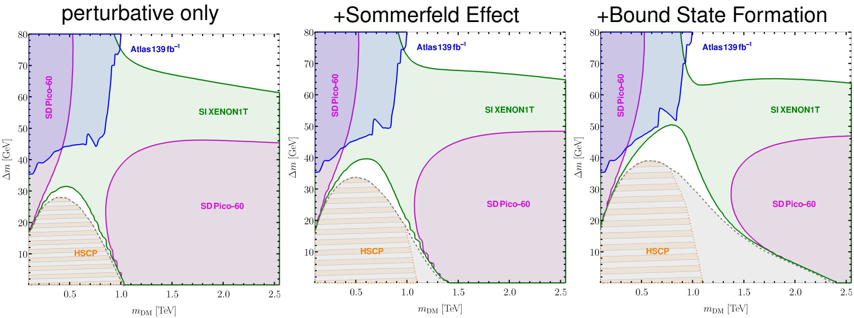
Out-of-chemical equilibrium estimate

$$\frac{\Gamma_{X\leftrightarrow\chi}\left(\tilde{g}_{\mathrm{DM}}\right)}{H} \lesssim 1\,, \text{ at freeze-out}$$

$$\rightarrow \tilde{g}_{\mathrm{DM}} \lesssim \sqrt{\frac{m_{\mathrm{DM}}}{GeV}}\left(10^{-9} + 6.8\cdot 10^{-11}\frac{\Delta m}{m_{\mathrm{DM}}}\right)$$

For $g_{\mathrm{DM}} < \tilde{g}_{\mathrm{DM}}$ DM production is non-thermal

Long-Lived-Particle (LLP) searches constrain large lifetimes $\rightarrow g_{\mathrm{DM}} \geq g_{\mathrm{DM}}^{\mathrm{LLP}}$



→ Region can not be fully tested when including non-perturbative effects

→ A precise treatment, assuming *conversion driven freeze-out*, has been analyzed in [Garny,Heisig (2021)]

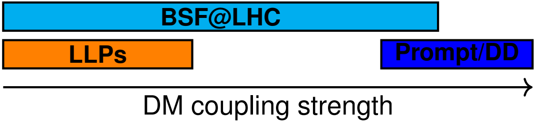
Bound State Formation at the LHC

Production Cross Section[Martin(2008)]

$$\sigma\left(pp \to \mathcal{B}(XX^\dagger)\right) = \frac{\pi^2}{8m_\mathcal{B}^3}\Gamma\left(\mathcal{B}(XX^\dagger) \to gg\right)\mathcal{P}_{gg}\left(\frac{m_\mathcal{B}}{13\text{ TeV}}\right)$$

$\to$ try to observe the bound state resonance in $\gamma\gamma$ final state. ATLAS (2017)

Efficient for all g_DM small enough such that $\Gamma_X < E_B$, roughly speaking $g_\text{DM} \lesssim g_s$.



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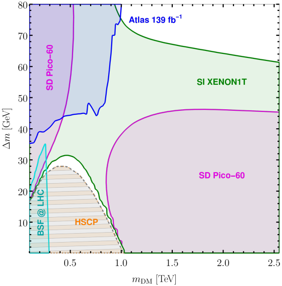
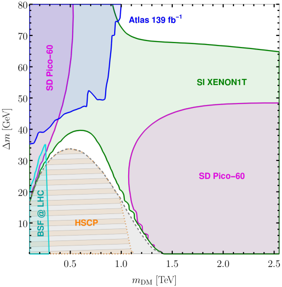
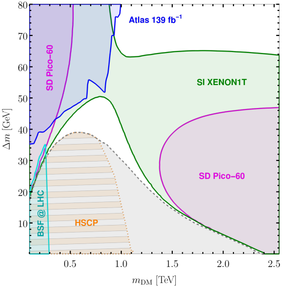
perturbative only



+Sommerfeld Effect



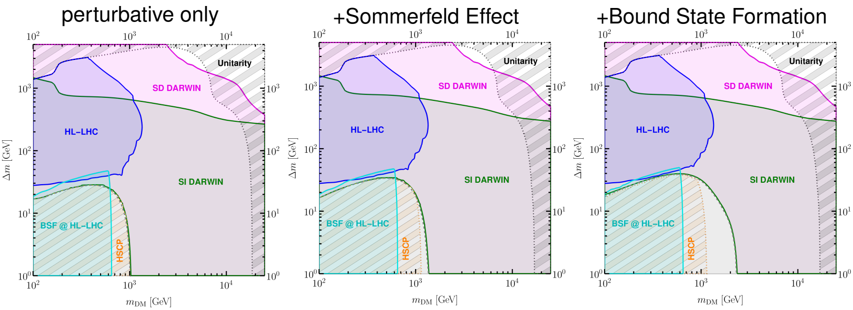
+Bound State Formation



Limits at 37 fb^{-1} relatively weak in mass ($\sim$ 300 GeV)

But huge potential: Closes the gap between prompt and LLP searches

Expected Future Limits



- Highly testable: Parameter space almost completely probed
- Remember: HSCP not a strict exclusion here (BSF@LHC is!)
- Bound State effects enlarge the area still necessary to test

When is BSF relevant?

No coannihilation required!
$\rightarrow$ Expect potentially large non-perturbative effects for $\alpha \sim v \sim 0.1$

Case I: Massless/light mediator (for instance colored annihilation)

$$\sigma_{\text{ann}} \sim \frac{\alpha^2}{m^2} \xrightarrow{\Omega_{\text{DM}} \sim 1/\sigma_{\text{ann}}} \alpha \sim 0.1 \frac{m}{\text{TeV}}$$

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$$\sigma_{\text{ann}} \sim \frac{\alpha^2}{m^2} \xrightarrow{\Omega_{\text{DM}} \sim 1/\sigma_{\text{ann}}} \alpha \sim 0.1 \, \frac{m}{\text{TeV}}$$

Case II: Massive mediator with mass M (Yukawa potential)

$$\sigma_{\text{ann}} \sim \alpha^2 \frac{m^2}{(m^2 + M^2)^2} \xrightarrow{\Omega_{\text{DM}} \sim 1/\sigma_{\text{ann}}} \alpha \sim 0.1 \left(\frac{m}{\text{TeV}} \right) \left(1 + \frac{M^2}{m^2} \right)$$

But Yukawa suppression sizable if $\alpha m \lesssim M$

Conclusion

- Non-perturbative Effects can increase or decrease the annihilation cross section of DM
  $\rightarrow$ Cannot be handled by a flat correction factor!

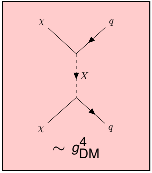
- Non-perturbative Effects are non-neglible in scenarios of colored coannihilation and open up small mass parameter space:

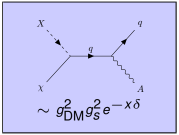
  Viable Parameter space shifts from $(m_{\text{DM}}, \Delta m) < (1\,TeV, 30\,GeV)$ to $(1.4\,TeV, 40\,GeV)$ (Sommerfeld Effect) and $(2.4\,TeV, 50\,GeV)$ (Bound State Formation)
  $\rightarrow$ Sommerfeld Effect alone not a good approximation!

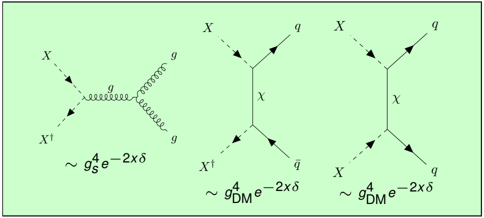
- Bound State searches at colliders close the gap in "coupling space" between prompt and long-lived-particle searches

Annihilation Channels

NPE = Non-Perturbative Effects







$\Rightarrow$ No NPE $\Rightarrow$ No NPE $\Rightarrow$ Subject to NPE

Color Decomposition

Process: $(X_1)_{\mathbf{R_1}} + (X_2)_{\mathbf{R_2}} \rightarrow SM + SM$

Color Potential

$$V(r) = -\frac{\alpha_s}{2r}\left[C_2(\mathbf{R_1}) + C_2(\mathbf{R_2}) - C_2(\mathbf{R})\right] = -\frac{\alpha_{\text{eff},[\mathbf{R}]}}{r}$$

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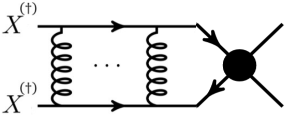
Color Configurations

$$\mathbf{3} \times \overline{\mathbf{3}} = \mathbf{1} + \mathbf{8} \rightarrow \alpha_{\text{eff},[\mathbf{1}]} = \frac{4}{3},\ \alpha_{\text{eff},[\mathbf{8}]} = -\frac{1}{6}$$

$$\mathbf{3} \times \mathbf{3} = \overline{\mathbf{3}} + \mathbf{6} \rightarrow \alpha_{\text{eff},[\overline{\mathbf{3}}]} = \frac{2}{3},\ \alpha_{\text{eff},[\mathbf{6}]} = -\frac{1}{3}$$



n-gluon exchanges contribute with $\left(\frac{\alpha}{v}\right)^n$ for $\alpha \sim v$

Sommerfeld Effect

$$\sigma(X_1 X_2 \to SM\, SM) = S\left(\frac{\alpha_{\text{eff}}}{v}\right)\sigma_{\text{pert.}}$$

Sommerfeld Factor

$$S\left(\frac{\alpha_{\text{eff}}}{v}\right) = \begin{cases} 1 & \text{, if } \left|\frac{\alpha_{\text{eff}}}{v}\right| \ll 1, \\ \frac{\alpha_{\text{eff}}}{v} & \text{, if } \left|\frac{\alpha_{\text{eff}}}{v}\right| \gg 1 \wedge \alpha_{\text{eff}} > 0, \\ \exp\left(2\pi \frac{\alpha_{\text{eff}}}{v}\right) & \text{, if } \left|\frac{\alpha_{\text{eff}}}{v}\right| \gg 1 \wedge \alpha_{\text{eff}} < 0 \end{cases}$$

Figure from Talk by J.Harz @ DM Working Group

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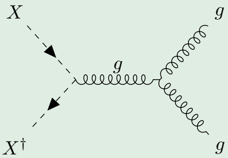
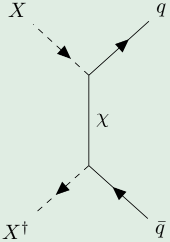
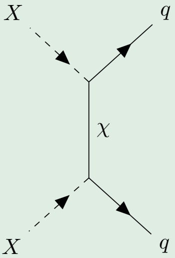
Annihilation Channels II



$\sim g_s^4 e^{-2x\delta}$
$\frac{2}{7}[\mathbf{1}] + \frac{5}{7}[\mathbf{8}]$



$\sim g_{\mathrm{DM}}^4 e^{-2x\delta}$
$\frac{1}{3}[\overline{\mathbf{3}}] + \frac{2}{3}[\mathbf{6}]$



$\sim g_{\mathrm{DM}}^4 e^{-2x\delta}$
$f_1(g_{\mathrm{DM}}, g_s)[\mathbf{1}] + f_8(g_{\mathrm{DM}}, g_s)[\mathbf{8}]$

As a rule of thumb, we find:

$g_{\mathrm{DM}} > g_s \rightarrow$ Sommerfeld effect reduces annihilation cross section

$g_{\mathrm{DM}} < g_s \rightarrow$ Sommerfeld effect increases annihilation cross section

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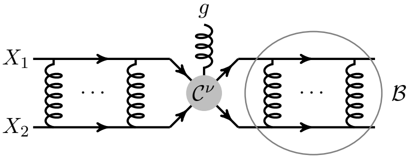
SE vs BSF

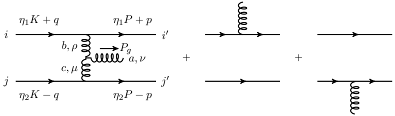
Modified Coannihilation [Ellis,Luo,Olive(2015)]

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{i,j \in \{\chi, X\}} \langle S(\alpha / v_{ij}) \cdot \sigma_{ij} v_{ij} \rangle \frac{n_i^{\text{eq}}}{n^{\text{eq}}} \frac{n_j^{\text{eq}}}{n^{\text{eq}}}$$

$\langle \sigma_{\text{eff}} v \rangle$	Sommerfeld Effect	Bound State Formation
$g_{\text{DM}} \gg g_s$	$-$	
$g_{\text{DM}} \ll g_s$	$+$	

Bound State Formation





Bound State Formation (BSF)

$$\sigma(X_1 X_2 \to \mathcal{B}\,(X_1 X_2)\ g) = \sigma_{\text{BSF}} \sim \frac{\alpha_s^2}{m_X^2} S_{\text{BSF}} \left(\frac{\alpha}{v}\right)$$

Bound state as an additional particle in the thermal bath.
$\Rightarrow$ Boltzmann Equation needs to be modified

Figures from [Harz,Petraki (2018)]

Modified Coannihilation [Ellis,Luo,Olive(2015)]

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{i,j \in \{\chi, X\}} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{\text{eq}}}{n^{\text{eq}}} \frac{n_j^{\text{eq}}}{n^{\text{eq}}} + \langle \sigma_{\text{BSF}} v \rangle_{\text{eff}} \frac{n_X^{\text{eq}}}{n^{\text{eq}}} \frac{n_X^{\text{eq}}}{n^{\text{eq}}}$$

Bound states effectively provide an additional annihilation channel.

Modified Coannihilation [Ellis,Luo,Olive(2015)]

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{i,j \in \{\chi, X\}} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{\text{eq}}}{n^{\text{eq}}} \frac{n_j^{\text{eq}}}{n^{\text{eq}}} + \langle \sigma_{\text{BSF}} v \rangle_{\text{eff}} \frac{n_X^{\text{eq}}}{n^{\text{eq}}} \frac{n_X^{\text{eq}}}{n^{\text{eq}}}$$

Bound states effectively provide an additional annihilation channel.

Bound State contribution to $\langle \sigma_{\text{eff}} v \rangle$

$$\langle \sigma_{\text{BSF}} v \rangle_{\text{eff}} = \langle \sigma_{\text{BSF}} v \rangle \frac{\Gamma_{\mathcal{B} \to SM}}{\Gamma_{\mathcal{B},\text{ion}} + \Gamma_{\mathcal{B} \to SM}}$$

$\rightarrow$ BSF only contributes to the annihilation cross section of DM if the bound states decay into SM particles!