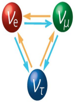

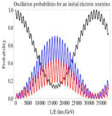

Neutrino Oscillation Induced by Chiral Torsion (NuDM - 2022)

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Introduction

- Neutrino mixing and oscillation.
- Matter (normal matter) effect.
- Effect of Torsion.
- NSI.

Neutrino Oscillation in Vacuum

A flavor eigenstate ν_l can be written in terms of mass eigenstate ν_α as

$$|\nu_l\rangle = \sum_\alpha U_{l\alpha}|\nu_\alpha\rangle.$$

After a time t the flavor eigenstate will be

$$|\nu_l(t)\rangle = \sum_\alpha e^{-\iota E_\alpha t} U_{l\alpha}|\nu_\alpha\rangle,$$

where $\quad E_\alpha = E + \frac{m_\alpha^2}{2E}.$
Then

$$P(\nu_l \to \nu_l') = \sum_{\alpha,\beta} |U^*_{l'\alpha} U_{l\alpha} U_{l'\beta} U^*_{l\beta}| \cos\left(\frac{(m_\alpha^2 - m_\beta^2)t}{2E} - \phi_{ll'\alpha\beta}\right),$$

where $\quad \phi_{ll'\alpha\beta} = arg\left(U^*_{l'\alpha} U_{l\alpha} U_{l'\beta} U^*_{l\beta}\right) \quad$ and $\quad E_\alpha - E_\beta = \frac{m_\alpha^2 - m_\beta^2}{2E}.$

Neutrino Oscillation in Vacuum

For 2 flavors of neutrino

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$\nu_e \to \nu_\mu$ conversion probability

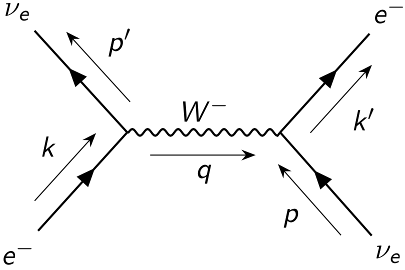
$$P_{\nu_e \to \nu_\mu} = \sin^2 2\theta \;\; \sin^2\left(\frac{\Delta m^2_{21}}{4E}\right) L$$

ν_e survival probability

$$P_{\nu_e \to \nu_e} = 1 - \sin^2 2\theta \;\; \sin^2(\frac{\Delta m^2_{21}}{4E}) L$$

Weak Interaction

The Feynman diagram for charged current interaction is



S matrix element for this process $(e\nu_e \to e\nu_e)$

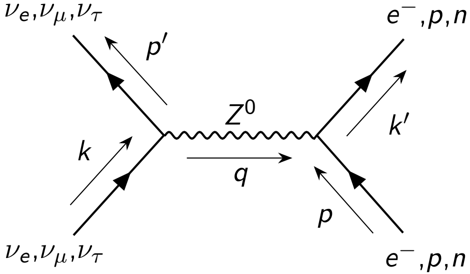
$$S_{CC} = \frac{G_F}{\sqrt{2}}\delta^4\left(p+k-p'-k'\right)\bar{u}^{s'}_{\nu_e}(p')\gamma^\mu L u^s_{\nu_e}(p)\,\bar{u}^{r'}_e\left(k'\right)\gamma_\mu L u^r_e(k)$$

Effective charged-current Hamiltonian becomes

$$H_{eff}\,(CC) = \sqrt{2}G_F n_e \psi^{s'\,\dagger}_{\nu_e} L \psi^s_{\nu_e}$$

Weak Interaction

The Feynman diagram for neutral current process



Background averaged S matrix element

$$\langle S_{NC} \rangle = \sqrt{2} G_F \left[n_e \left(-\frac{1}{2} + 2 \sin^2 \theta_W \right) + n_p \left(\frac{1}{2} - 2 \sin^2 \theta_W \right) + n_n \left(-\frac{1}{2} \right) \right] \bar{\psi}^{s'}_{\nu_e} \gamma^0 L \psi^s_{\nu_e}.$$

Effective neutral current contribution

$$H_{eff}(NC) = \sqrt{2} G_F \left(-n_n \frac{1}{2} \right) \psi^{s' \dagger}_{\nu_e} L \psi^s_{\nu_e}$$

Effect of Torsional 4-Fermion Interaction

Fermions give rise to space time torsion. This torsion is added to Levi-Civita connection.

We can split the spin connection as

$$A_\mu{}^{ab} = \omega_\mu{}^{ab} + \Lambda_\mu{}^{ab} \,.$$

The Einstein-Cartan action

$$S = \frac{1}{2\kappa}\int |e| d^4x \left(R(\widehat{\Gamma}) + e^\mu_a e^\nu_b \,\partial_{[\mu}\Lambda_{\nu]}{}^{ab} + e^\mu_a e^\nu_b \left[\omega_{[\mu}, \Lambda_{\nu]}{}^{ab}\right]_- \right) + \int |e| d^4x \left(\frac{1}{2\kappa} e^\mu_a e^\nu_b \left[\Lambda_{[\mu}, \Lambda_{\nu]}{}^{ab}\right]_- + \mathscr{L}_\psi \right)$$

$$\mathscr{L}_\psi = \bar{\psi}\left(i\partial_\mu\psi + \omega_\mu + \Lambda_\mu\right)\psi$$

Λ is taken to couple chirally.

Geometrical contribution to neutrino mass matrix, Shbhasish Chakrabarty and Amitabha Lahiri, Eur. Phys. J.C.(2019) 79:697.

Effect of Torsional 4-Fermion Interaction

The solution is of the form

$$\Lambda_\mu{}^{ab} = \frac{\kappa}{4}\epsilon^{abcd}e_{c\mu}\sum_i\left(\lambda_L^i\bar{\psi}_{iL}\gamma_d\psi_{iL} + \lambda_R^i\bar{\psi}_{iR}\gamma^d\psi_{iR}\right)$$

Insert this solution back into the action, the effective quartic interaction term comes

$$-\frac{3\kappa}{16}\left(\sum_i\left(\lambda_L^i\bar{\psi}_L^i\gamma_a\psi_L^i + \lambda_R^i\bar{\psi}_R^i\gamma_a\psi_R^i\right)\right)^2$$

It is the field in the mass basis which couple to torsion, since torsion appears with the geometric connection. Interaction of neutrinos with background is

$$-\left(\sum_{i=1,2}\left(\lambda_i^L\bar{\nu}_i\gamma_a L\nu_i + \lambda_i^R\bar{\nu}_i\gamma_a R\nu_i\right)\right)\times\left(\sum_{f=e,p,n}\left(\lambda_f^V\bar{f}\gamma_a f + \lambda_f^A\bar{f}\gamma_a\gamma^5 f\right)\right).$$

Effect of Torsional 4-Fermion Interaction

Like for weak interactions, the background factor is replaced by its average value

$$\sum_{f=e,p,n} \left\langle \lambda_f^V \bar{f} \gamma_a f + \lambda_f^A \bar{f} \gamma_a \gamma^5 f \right\rangle$$

$$\tilde{n} = \sum \lambda_f^V n_f$$

We also consider maximal violation of chirality, so $\lambda_i^R = 0$ for neutrinos. The contribution to the effective Hamiltonian is

$$\sum_{i=1,2} \left(\lambda_i \nu_i^\dagger L \nu_i \right) \tilde{n}\,.$$

Two Flavors

Including all the effects, we find in the mass basis,

$$i\frac{d}{dx}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \left[E\mathbb{I} + \frac{1}{2E}\begin{pmatrix}m_1^2 & 0\\ 0 & m_2^2\end{pmatrix} + \begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}\tilde{n} - \frac{G_F}{\sqrt{2}}(n_n - n_e) + U^T\begin{pmatrix}A & 0\\ 0 & -A\end{pmatrix}U^*\right]\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

where $A = \frac{G_F}{\sqrt{2}} n_e$. In the flavor basis this equation becomes

$$i\frac{d}{dx}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \left[E_0\mathbb{I} + \frac{1}{4E}\begin{pmatrix}-\Delta m_s^2\cos 2\theta + D & \Delta m_s^2 \sin 2\theta\\ \Delta m_s^2 \sin 2\theta & \Delta m_s^2 \cos 2\theta - D\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix},$$

where $D = 4AE = 2\sqrt{2}G_F n_e E$,

$$E_0 = E + \frac{m_1^2 + m_2^2}{4E} + \frac{\lambda_1 + \lambda_2}{2}\tilde{n} - \frac{G_F}{\sqrt{2}}(n_n - n_e)\,,$$

$$\Delta m_s^2 = \Delta m^2 + 2\tilde{n}E\Delta\lambda\,,$$

$$\Delta\lambda = \lambda_2 - \lambda_1\,.$$

Two Flavors

θ_M is the mixing angle in matter, modified by the torsional interaction,

$$\tan 2\theta_M = \frac{\tan 2\theta}{1 - \frac{D}{\Delta m_s^2 \cos 2\theta}} .$$

The conversion probability

$$P_{\nu_e \to \nu_\mu} = \sin^2(2\theta_M) \sin^2 \left(\frac{\Delta m_M^2}{4E} L \right) ,$$

where

$$\Delta m_M^2 = \sqrt{(\Delta m_s^2 \cos 2\theta - D)^2 + (\Delta m_s^2 \sin 2\theta)^2} .$$

The survival probability

$$P_{\nu_e \to \nu_e} = 1 - \sin^2(2\theta_M) \sin^2 \left(\frac{\Delta m_M^2}{4E} L \right)$$

Three Flavors

This can be generalized to three neutrinos. The mixing matrix is

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$.(Ignoring Majorana phases.)

U can be expressed in terms of rotation matrices O_{ij} and $U_\delta = (1 \quad 1 \quad e^{i\delta})$ as

$$U = \mathcal{O}_{23}\mathcal{U}_\delta\mathcal{O}_{13}\mathcal{U}_\delta^\dagger\mathcal{O}_{12}\,.$$

Three Flavors

In normal matter (uniform or slowly varying density) the evolution equation is

$$i\frac{d}{dx}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \left[E_0'\mathbb{I} + \frac{1}{2E}\left(U^*\begin{pmatrix}0 & 0 & 0\\ 0 & \Delta\tilde{m}_{12}^2 & 0\\ 0 & 0 & \Delta\tilde{m}_{31}^2\end{pmatrix}U^T + \begin{pmatrix}D & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right)\right]\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

We have used the definition

$$E_0' = E + \frac{m_1^2 + 2\lambda_1\tilde{n}E}{2E} - \frac{G_F}{\sqrt{2}}n_n\,,$$

$$\Delta\tilde{m}_{ij}^2 = \Delta m_{ij}^2 + 2\tilde{n}E\Delta\lambda_{ij}\,,$$

$$\Delta m_{ij}^2 = m_i^2 - m_j^2$$

$$\Delta\lambda_{ij} = \lambda_i - \lambda_j\,.$$

Three Flavors

Then the evolution equation can be written as follows

$$
i\frac{d}{dx}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \frac{\Delta\tilde{m}^2_{31}}{2E}\left[U^*\begin{pmatrix}0&0&0\\0&\alpha&0\\0&0&1\end{pmatrix}U^T + \begin{pmatrix}\tilde{A}&0&0\\0&0&0\\0&0&0\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}
$$

$$
= \frac{\Delta\tilde{m}^2_{31}}{2E}\mathcal{O}_{23}\mathcal{U}^*_\delta M\mathcal{U}^T_\delta\mathcal{O}^T_{23}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} .
$$

We have written

$$
M = \mathcal{O}_{13}\mathcal{O}_{12}\begin{pmatrix}0&0&0\\0&\alpha&0\\0&0&1\end{pmatrix}\mathcal{O}^T_{12}\mathcal{O}^T_{13} + \begin{pmatrix}\tilde{A}&0&0\\0&0&0\\0&0&0\end{pmatrix}
$$

$$
\tilde{A} = D/\Delta\tilde{m}^2_{31}
$$

$$
\alpha = \Delta\tilde{m}^2_{21}/\Delta\tilde{m}^2_{31} \equiv (\Delta m^2_{21} + 2\tilde{n}E\Delta\lambda_{21})/(\Delta m^2_{31} + 2\tilde{n}E\Delta\lambda_{31})
$$

Three Flavors

$$
\begin{aligned}
P_{\nu_e \to \nu_\mu} =& \alpha^2 c_{23}^2 \sin^2(2\theta_{12}) \frac{\sin^2 \tilde{A}\Delta}{\tilde{A}^2} + 4 s_{13}^2 s_{23}^2 \frac{\sin^2((\tilde{A}-1)\Delta)}{(\hat{A}-1)^2} \\
&+ 2\alpha s_{13} \sin(2\theta_{23}) \sin(2\theta_{12}) \cos(\Delta - \delta) \frac{\sin(\tilde{A}\Delta)}{\tilde{A}} \frac{\sin((\tilde{A}-1)\Delta)}{(\tilde{A}-1)} \,,
\end{aligned}
$$

where $\quad \Delta = (\Delta \tilde{m}_{31}^2 L)/(4E)\,.$

$$
\begin{aligned}
P_{\nu_e \to \nu_e} =& 1 - P_{\nu_e \to \nu_\mu} - P_{\nu_e \to \nu_\tau} \\
=& 1 - \alpha^2 \sin^2(2\theta_{12}) \frac{\sin^2 \tilde{A}\Delta}{\tilde{A}^2} - 4 s_{13}^2 \frac{\sin^2((\tilde{A}-1)\Delta)}{(\tilde{A}-1)^2}\,.
\end{aligned}
$$

These results need to be fitted to data to estimate the λ.

Thank you.