NATIONAL SCIENCE CENTRE
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DARK MATTER FREEZE-OUT AND FREEZE-IN BEYOND KINETIC EQUILIBRIUM

Andrzej Hryczuk

NCBJ

based on:
A.H. & M. Laletin 2204.07078
A.H. & M. Laletin 2104.05684
and **T. Binder, T. Bringmann, M. Gustafsson & A.H.** 1706.07433, 2103.01944

NuDM 2022

andrzej.hryczuk@ncbj.gov.pl

28th September 2022

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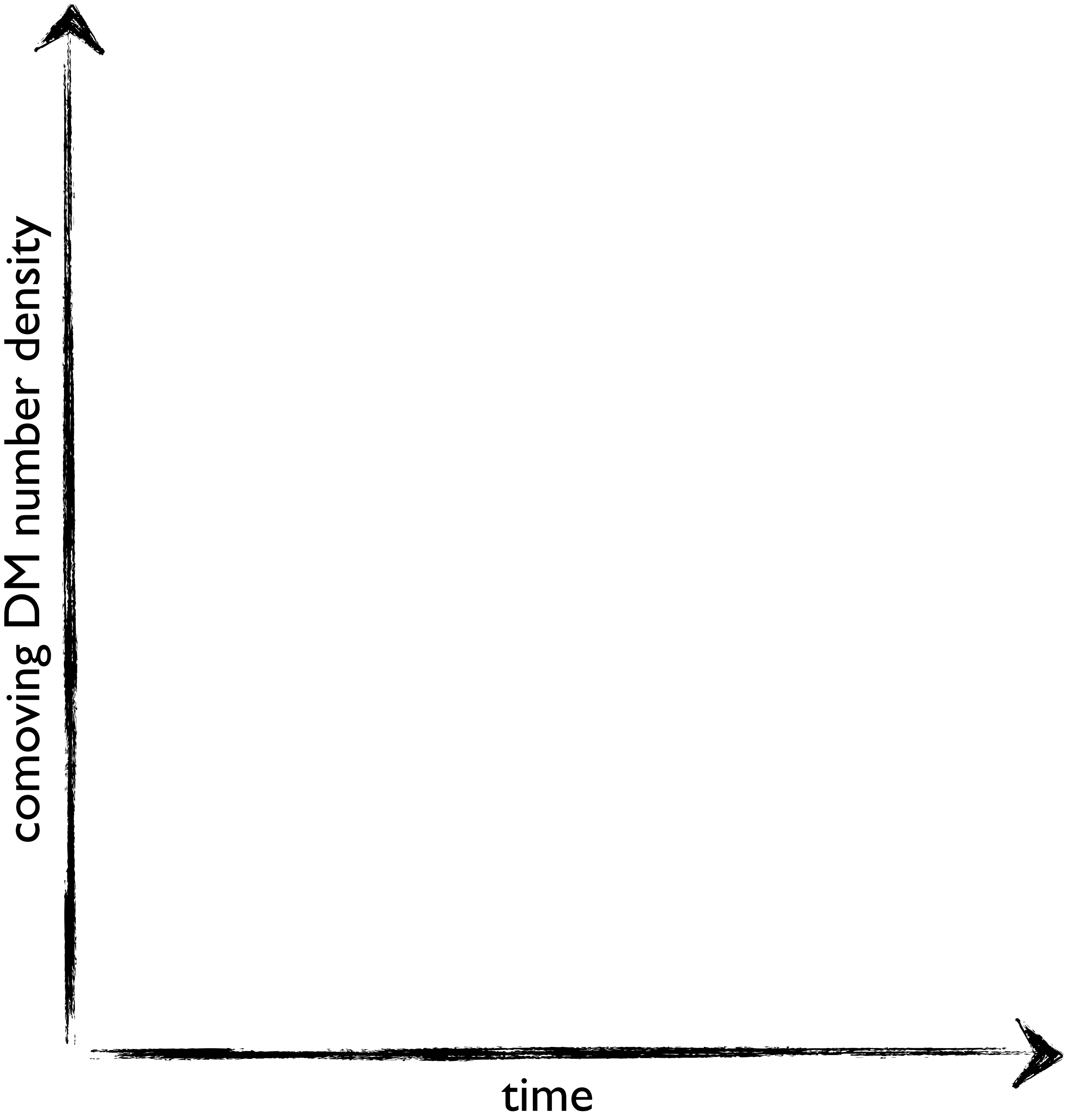

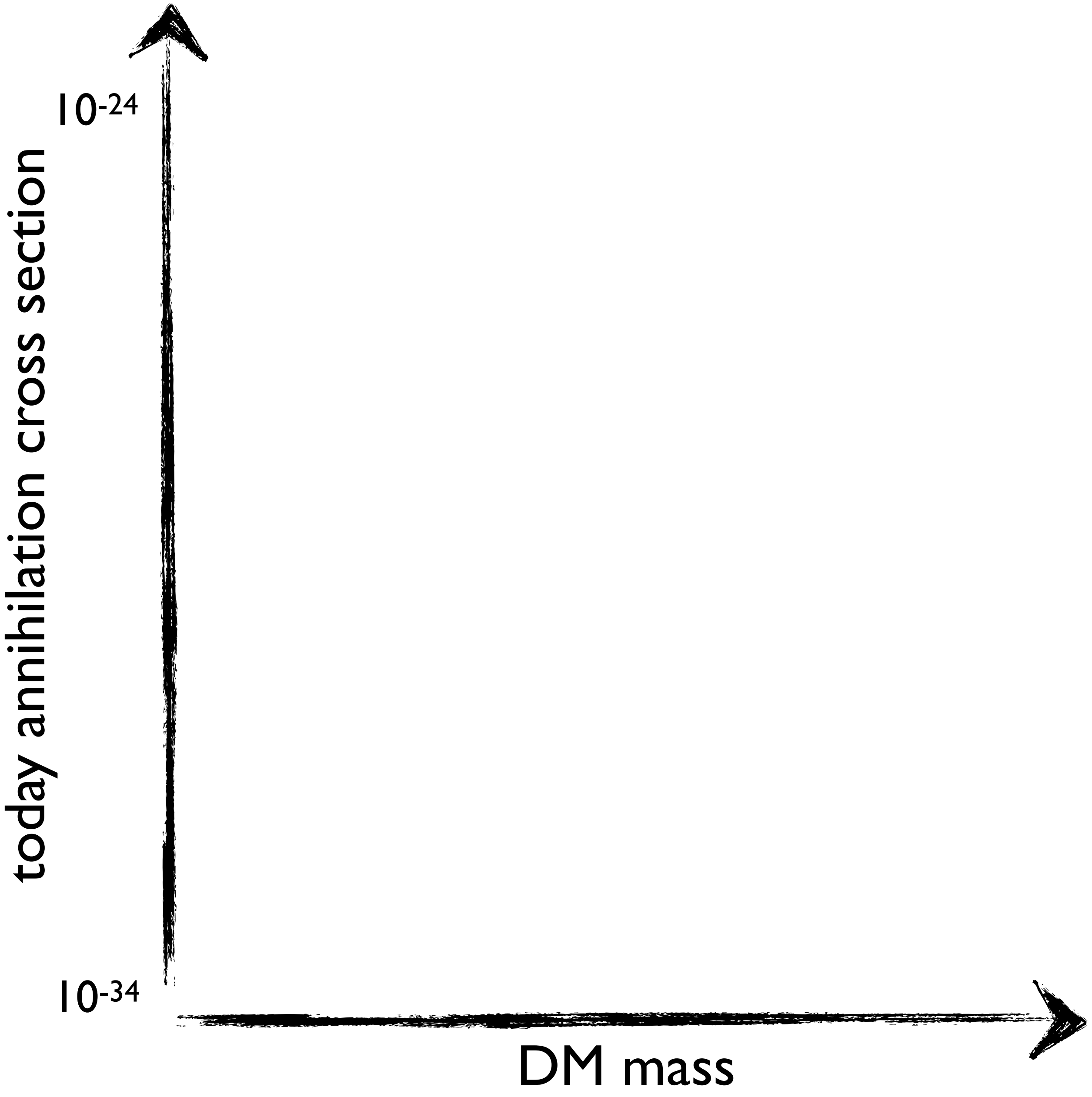

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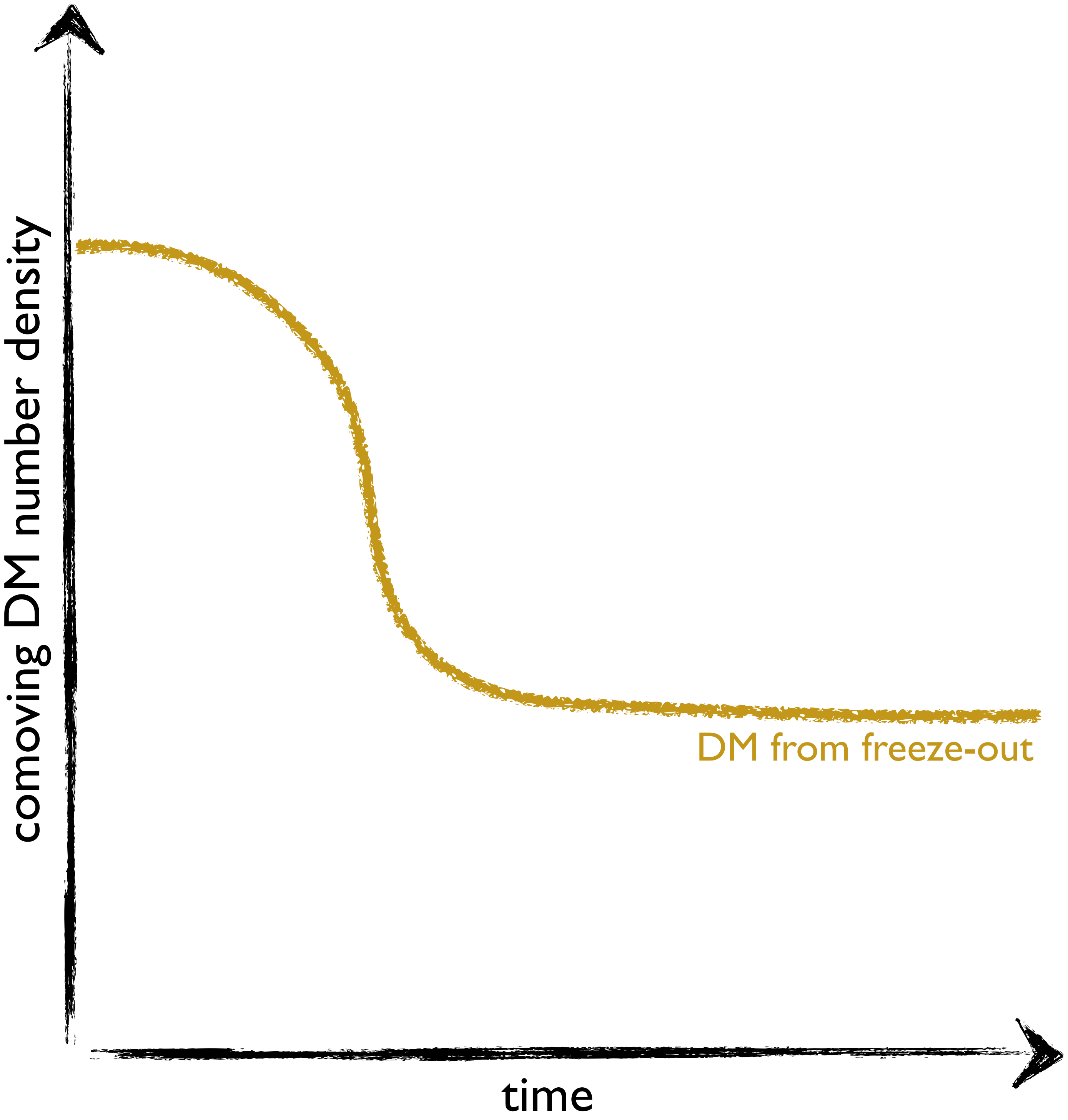

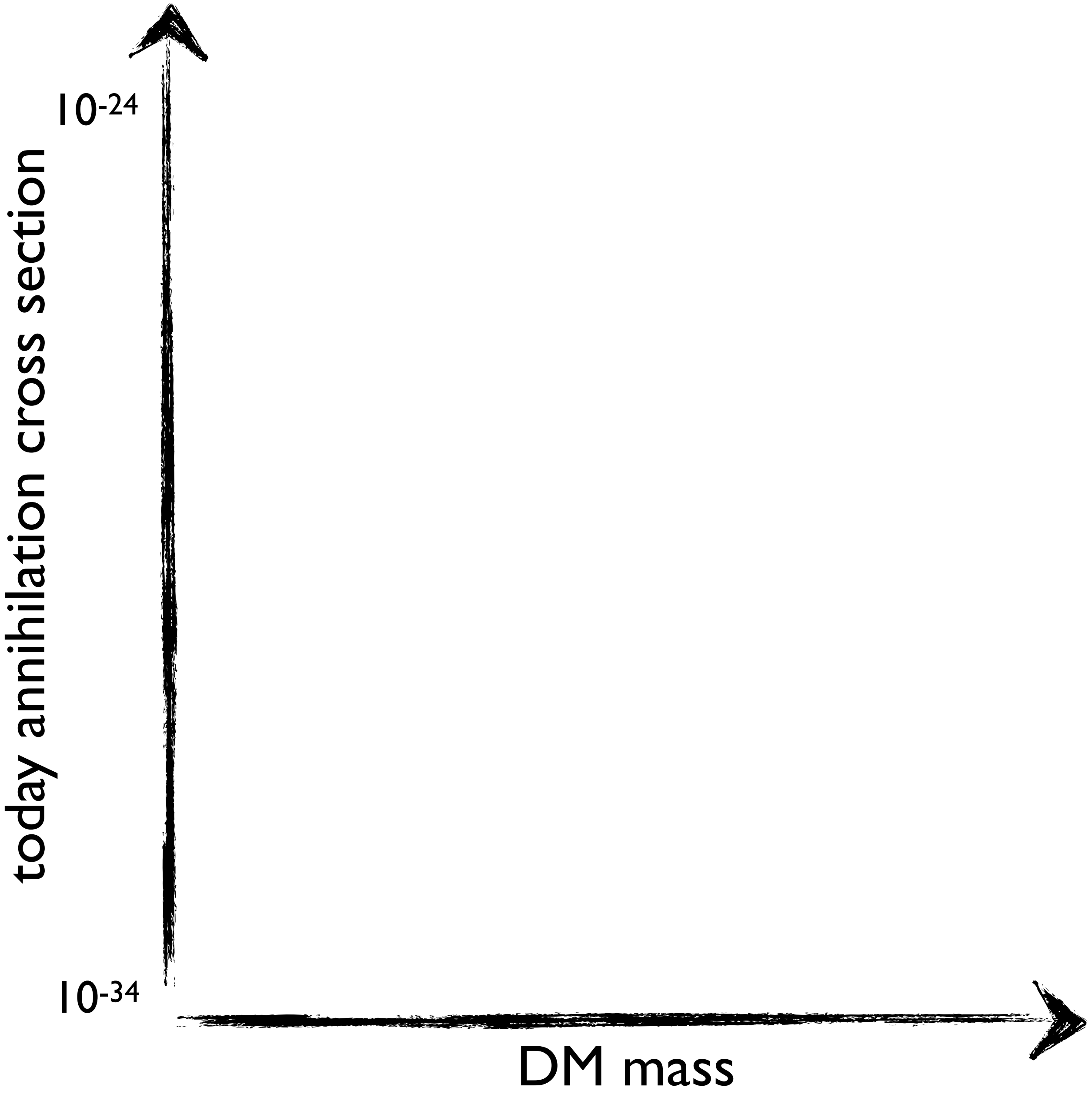

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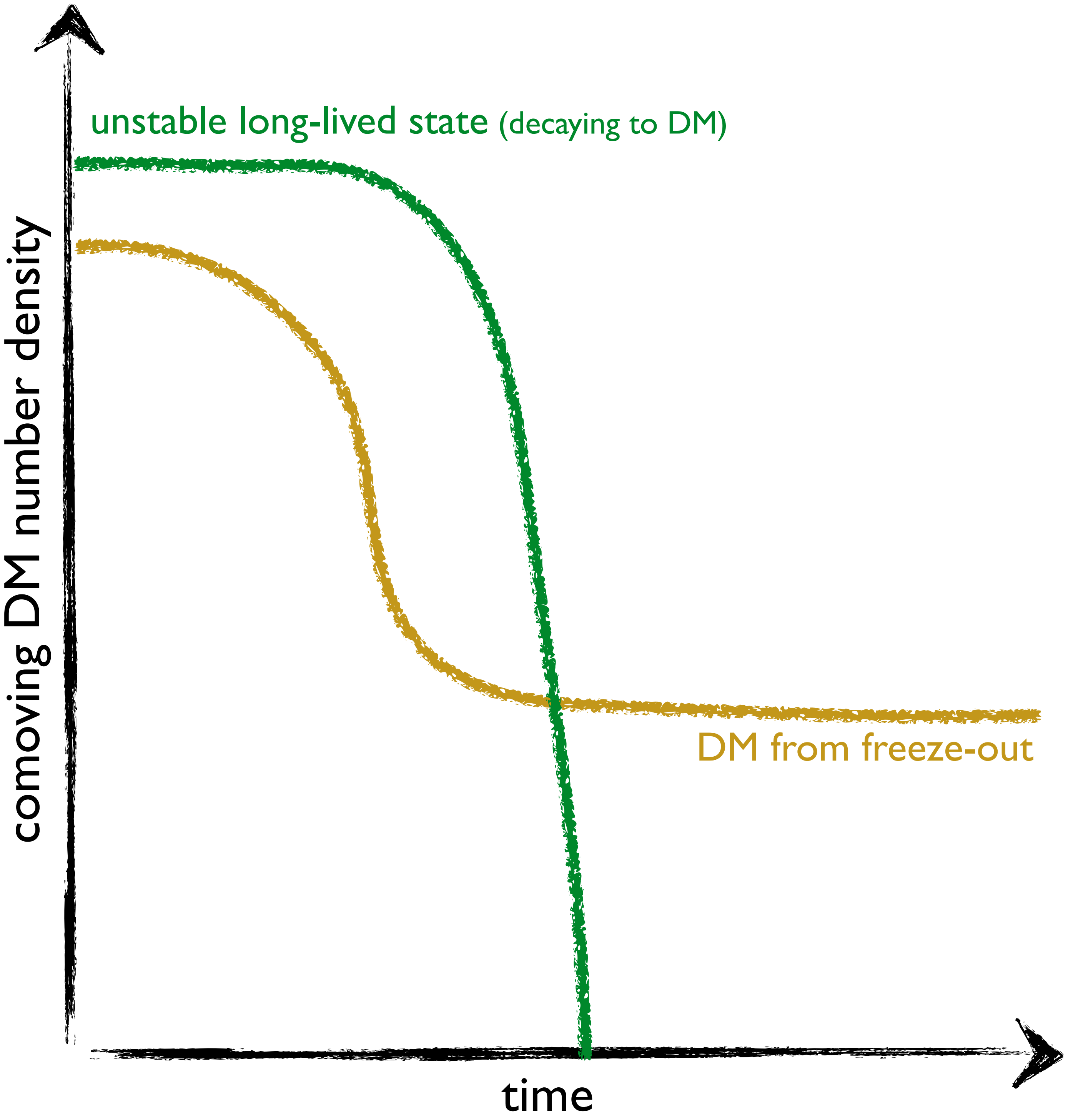

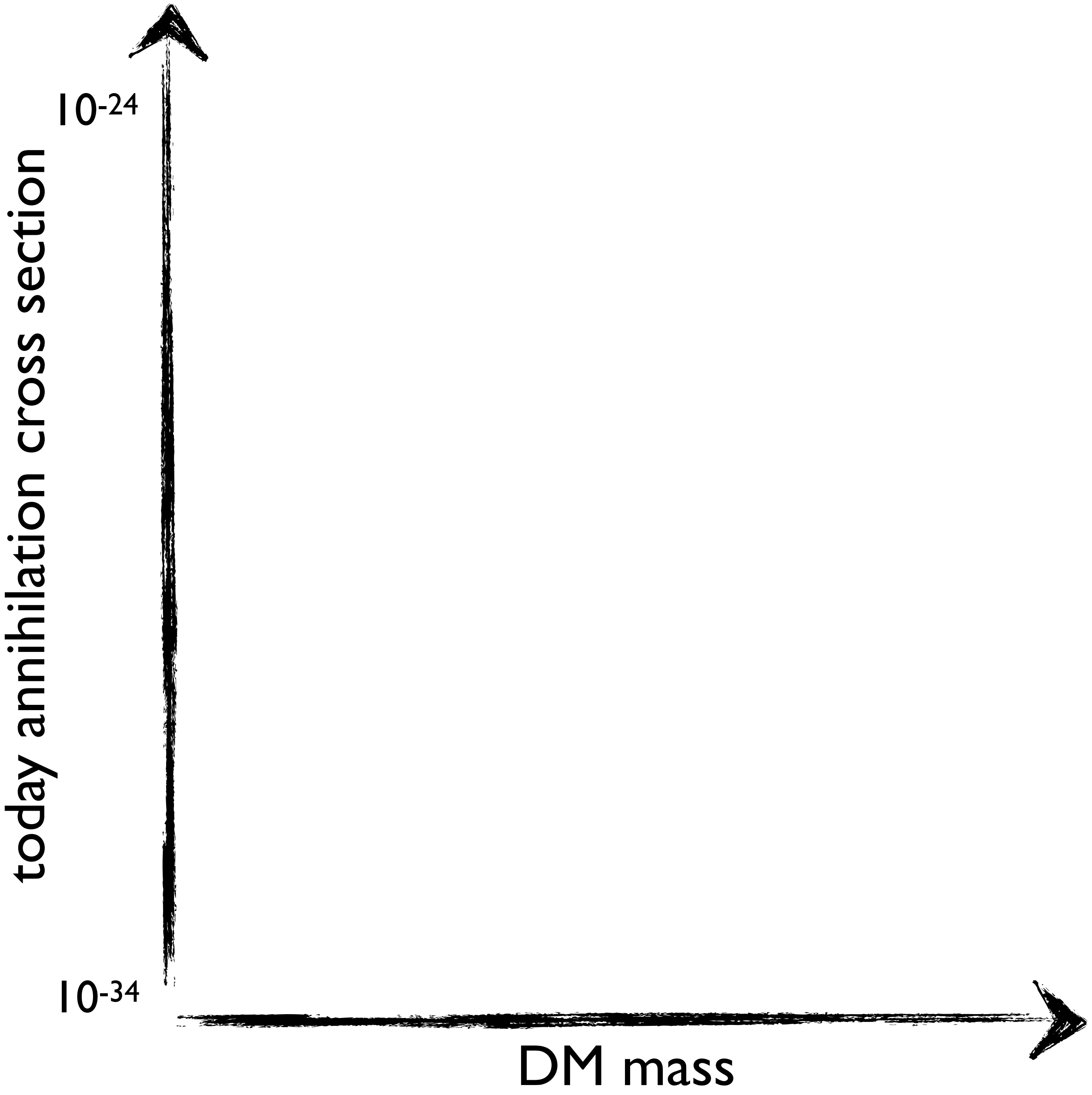

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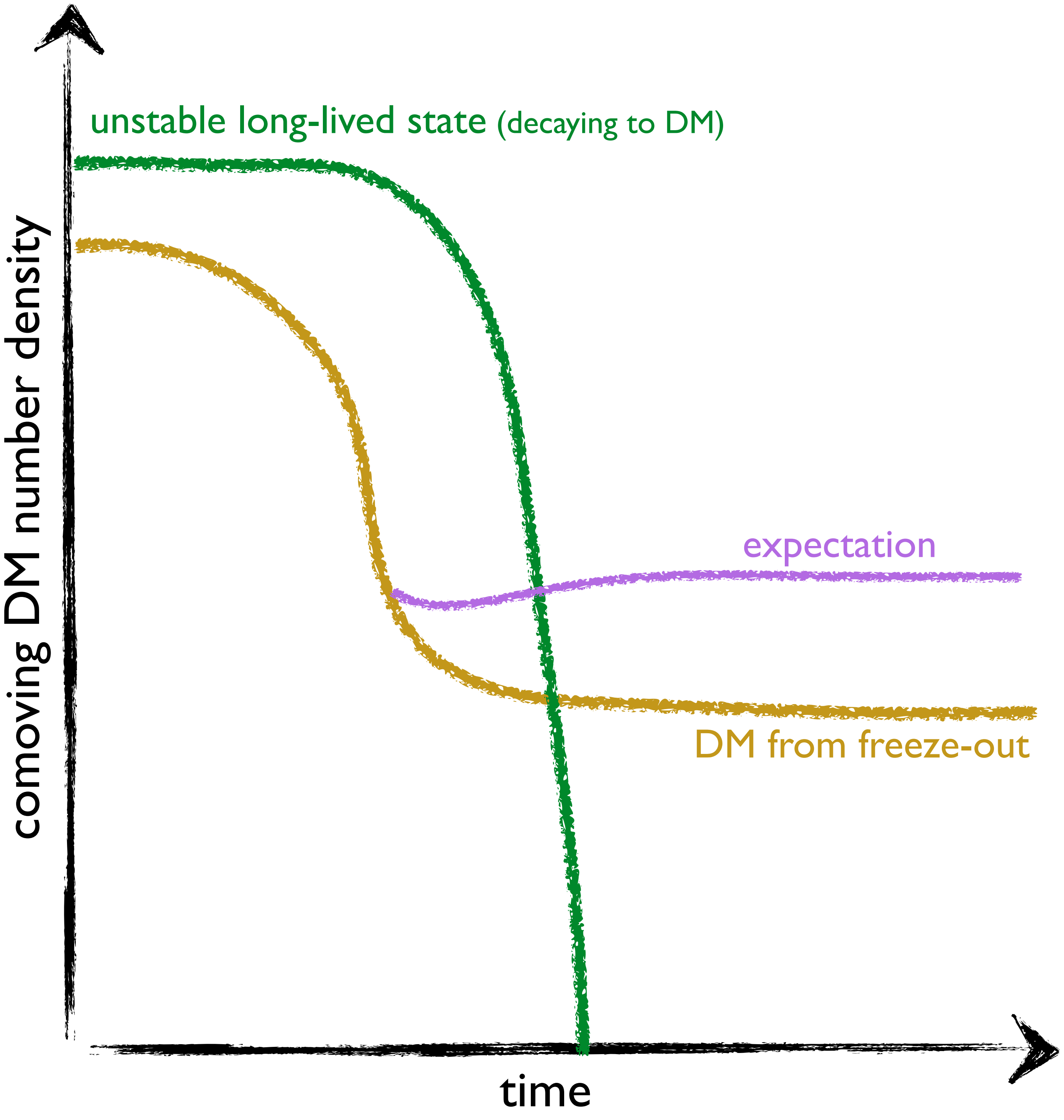

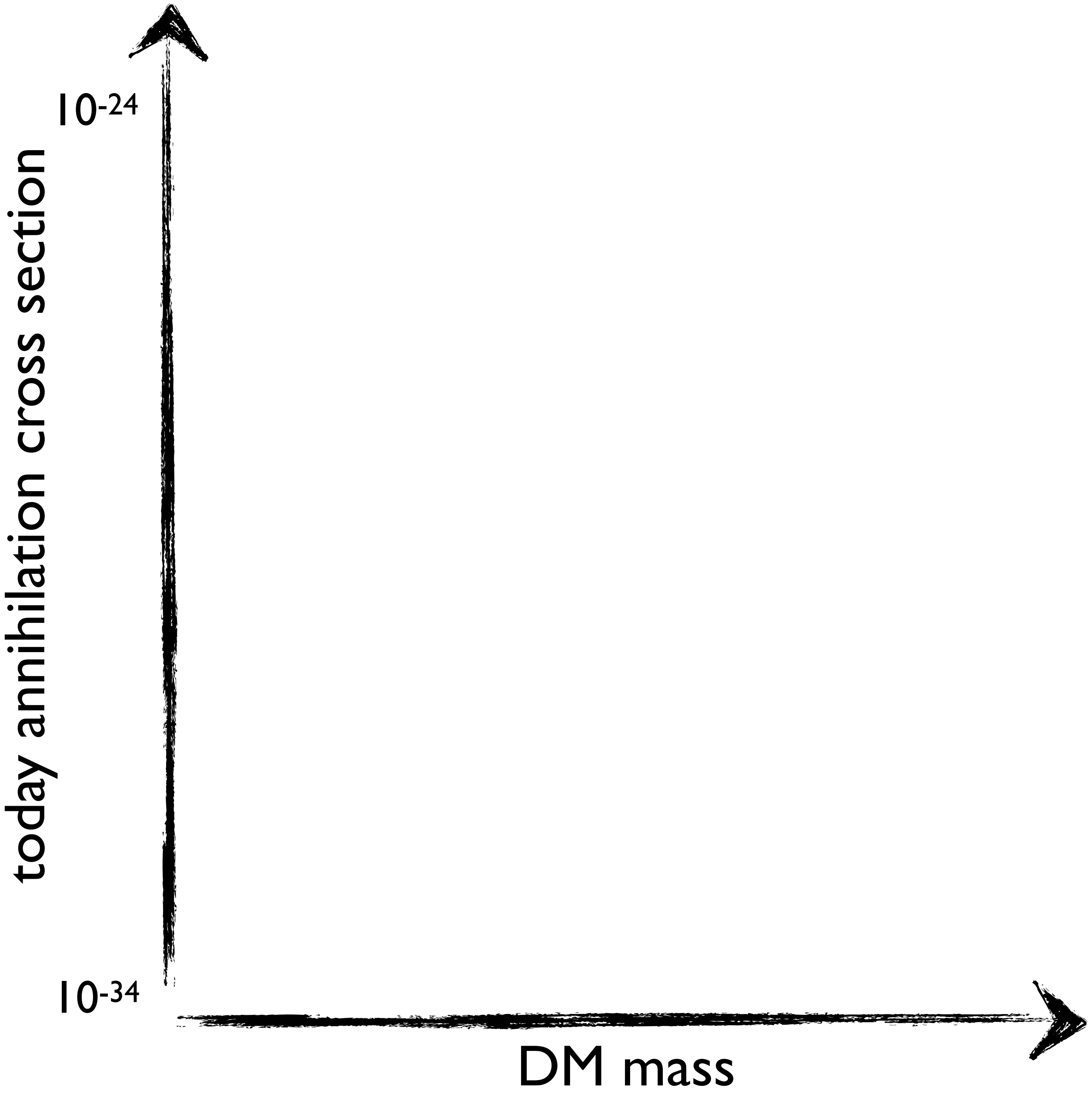

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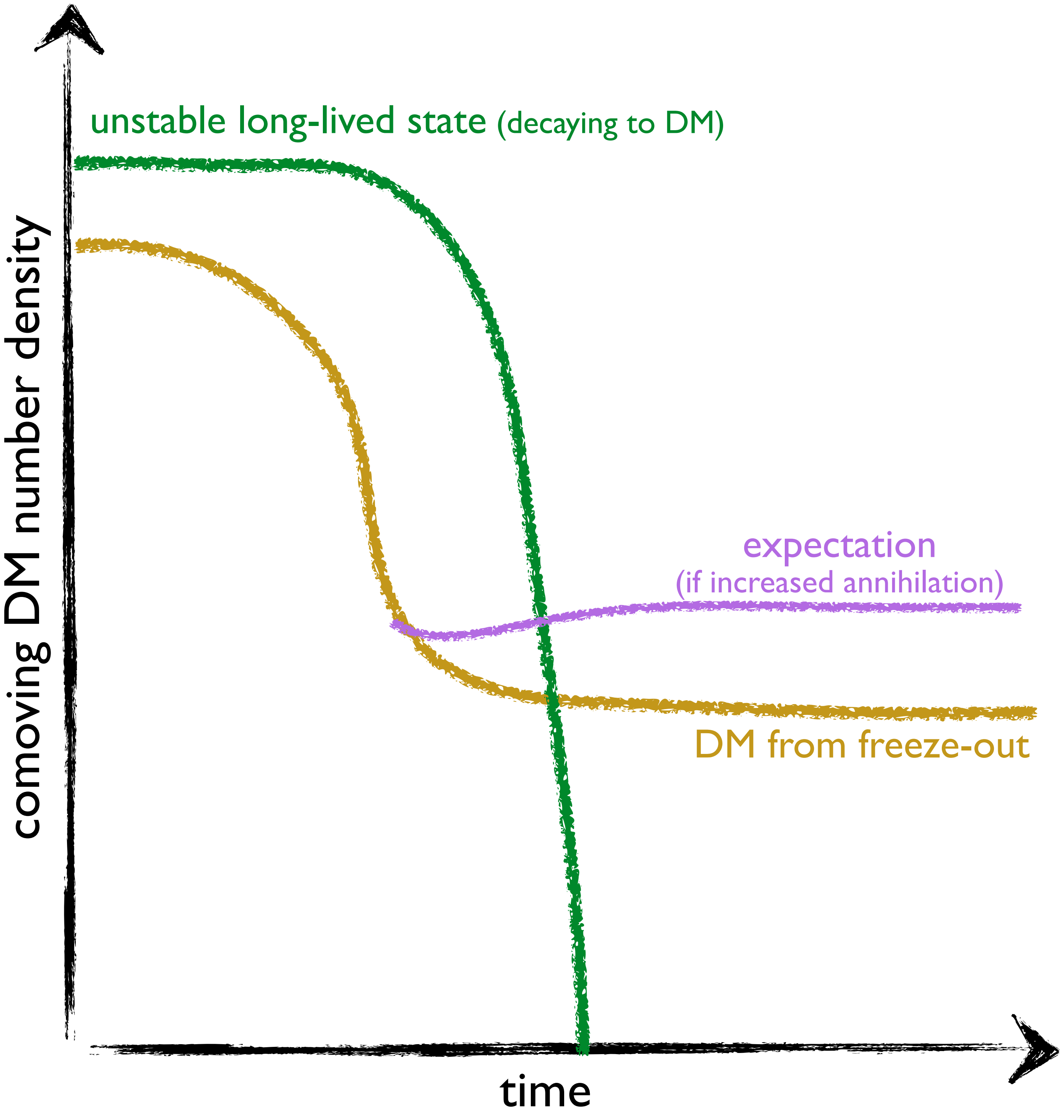

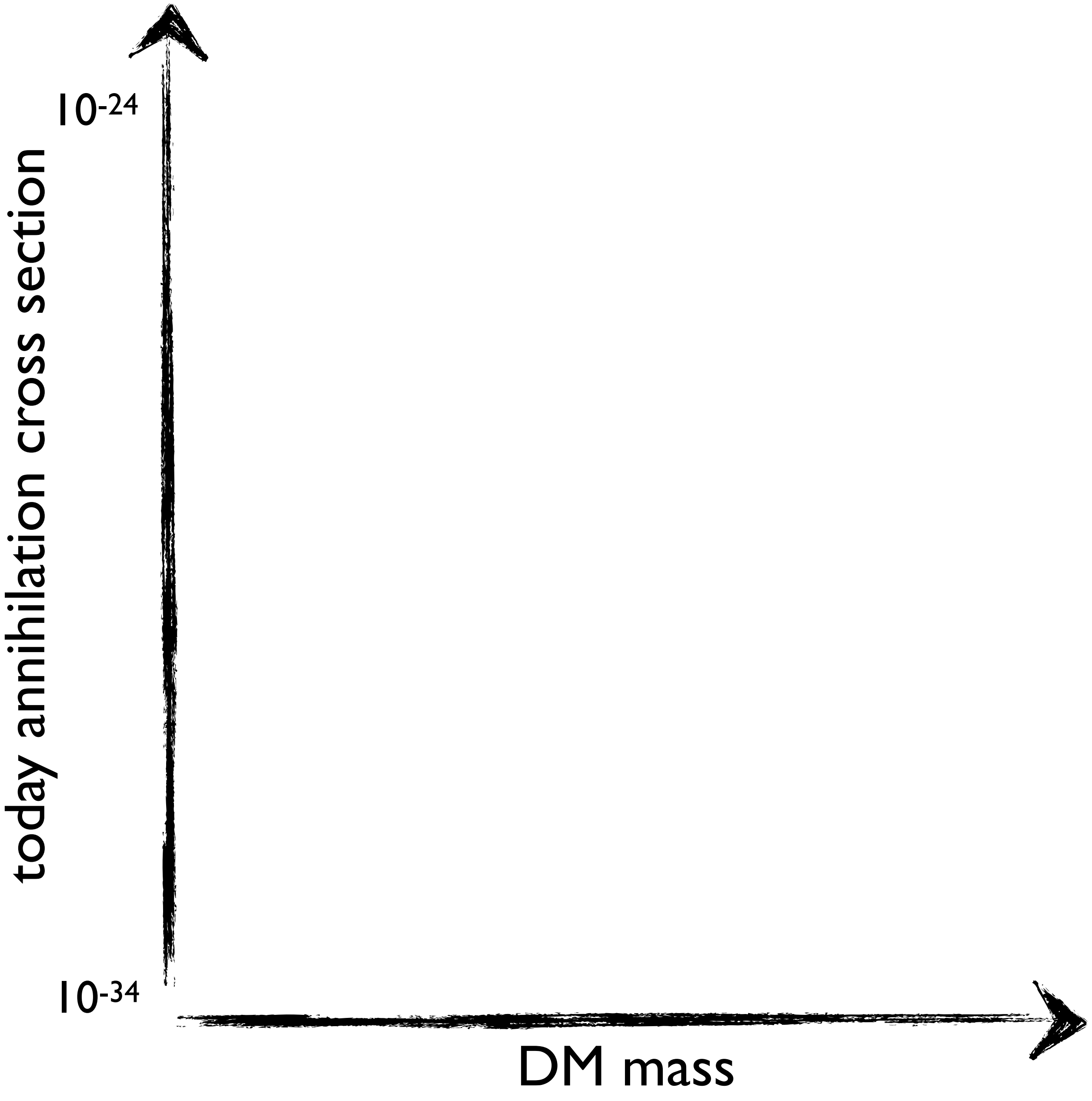

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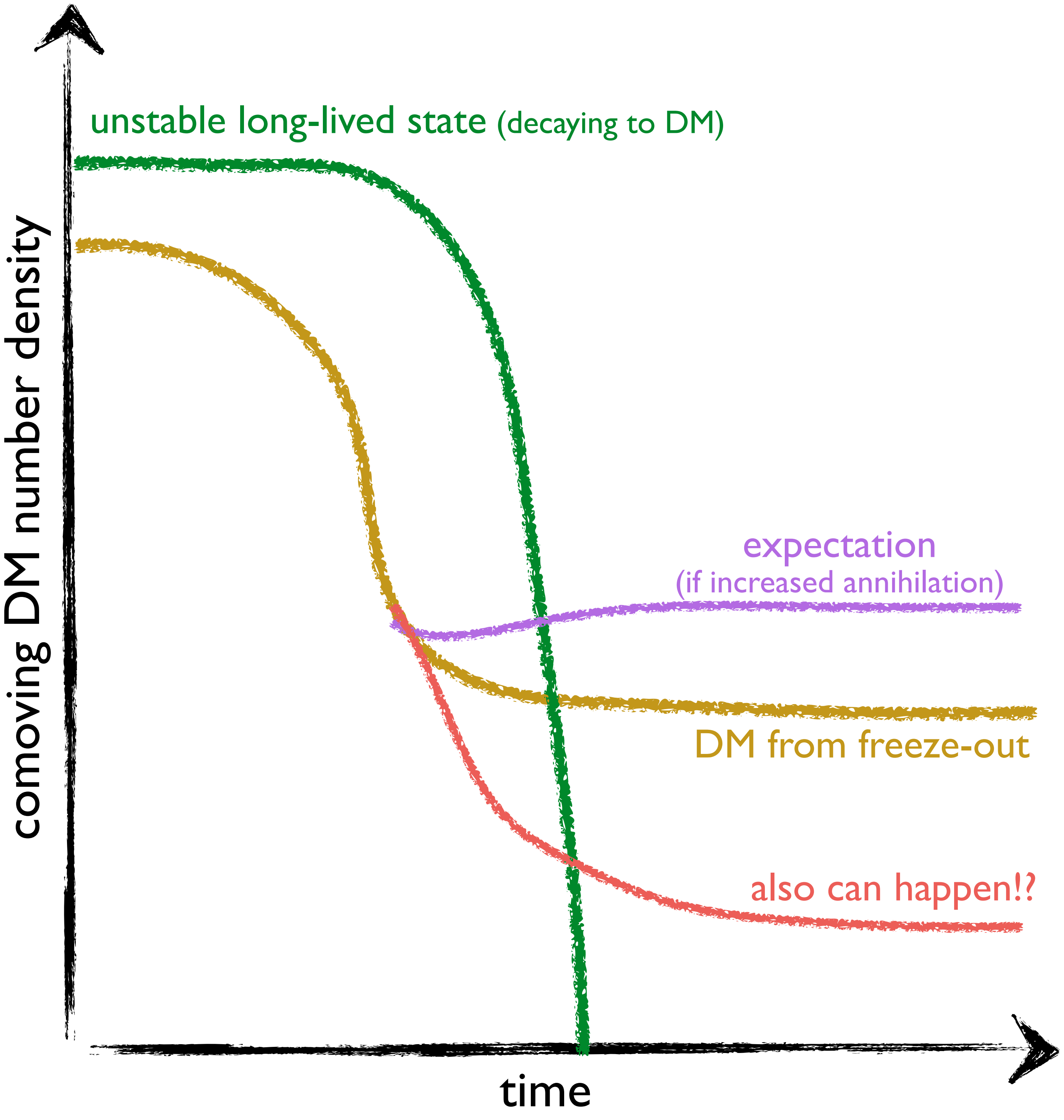

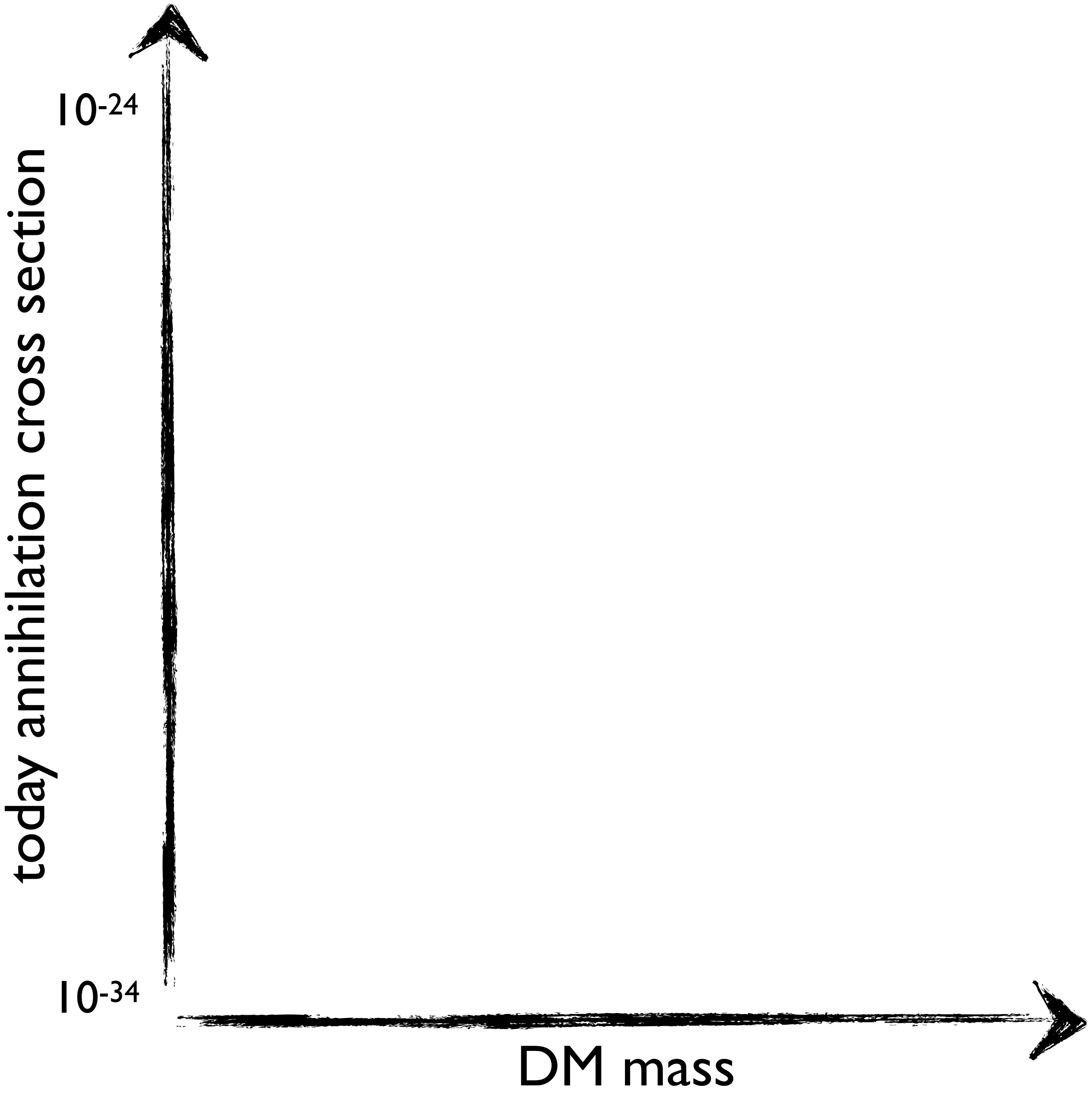

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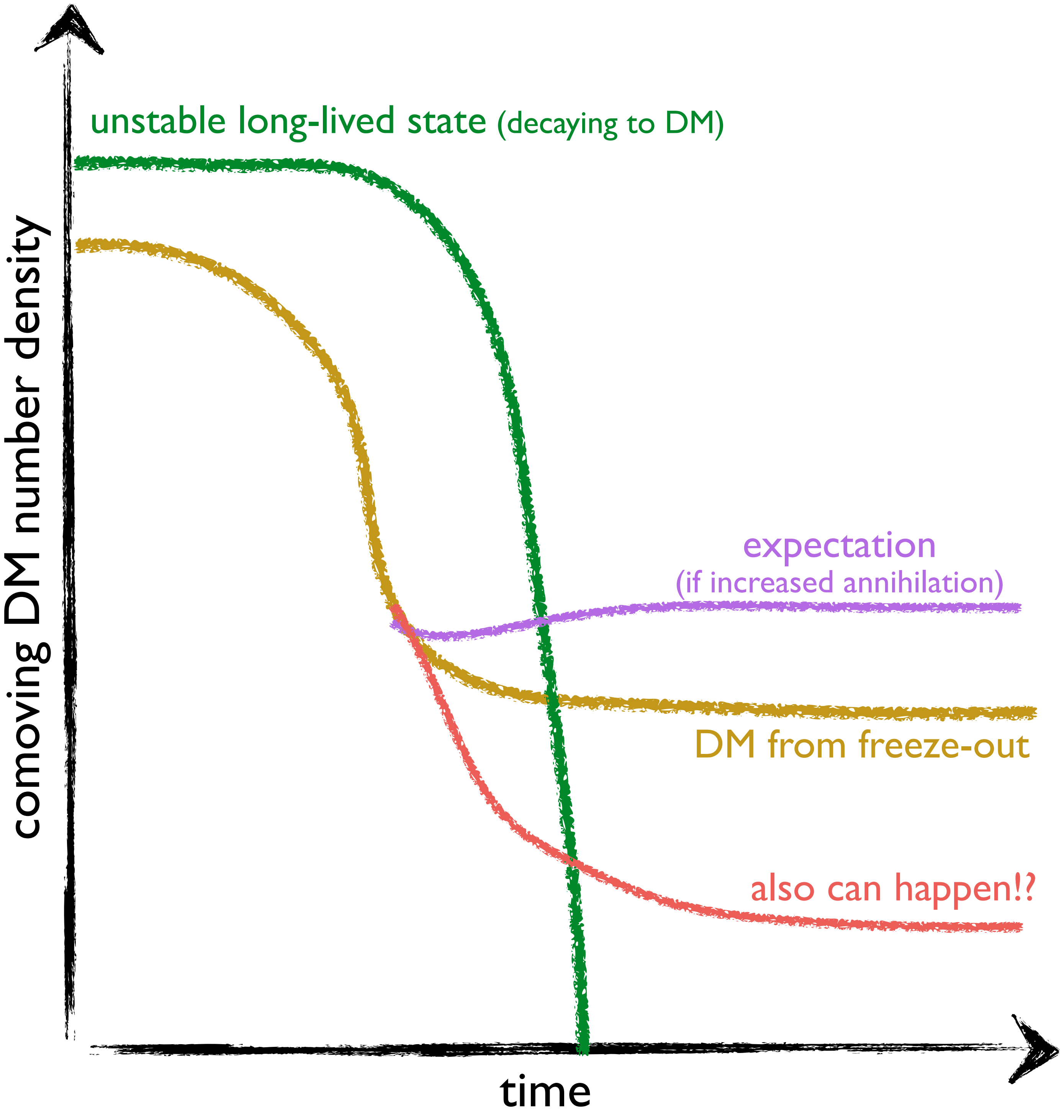

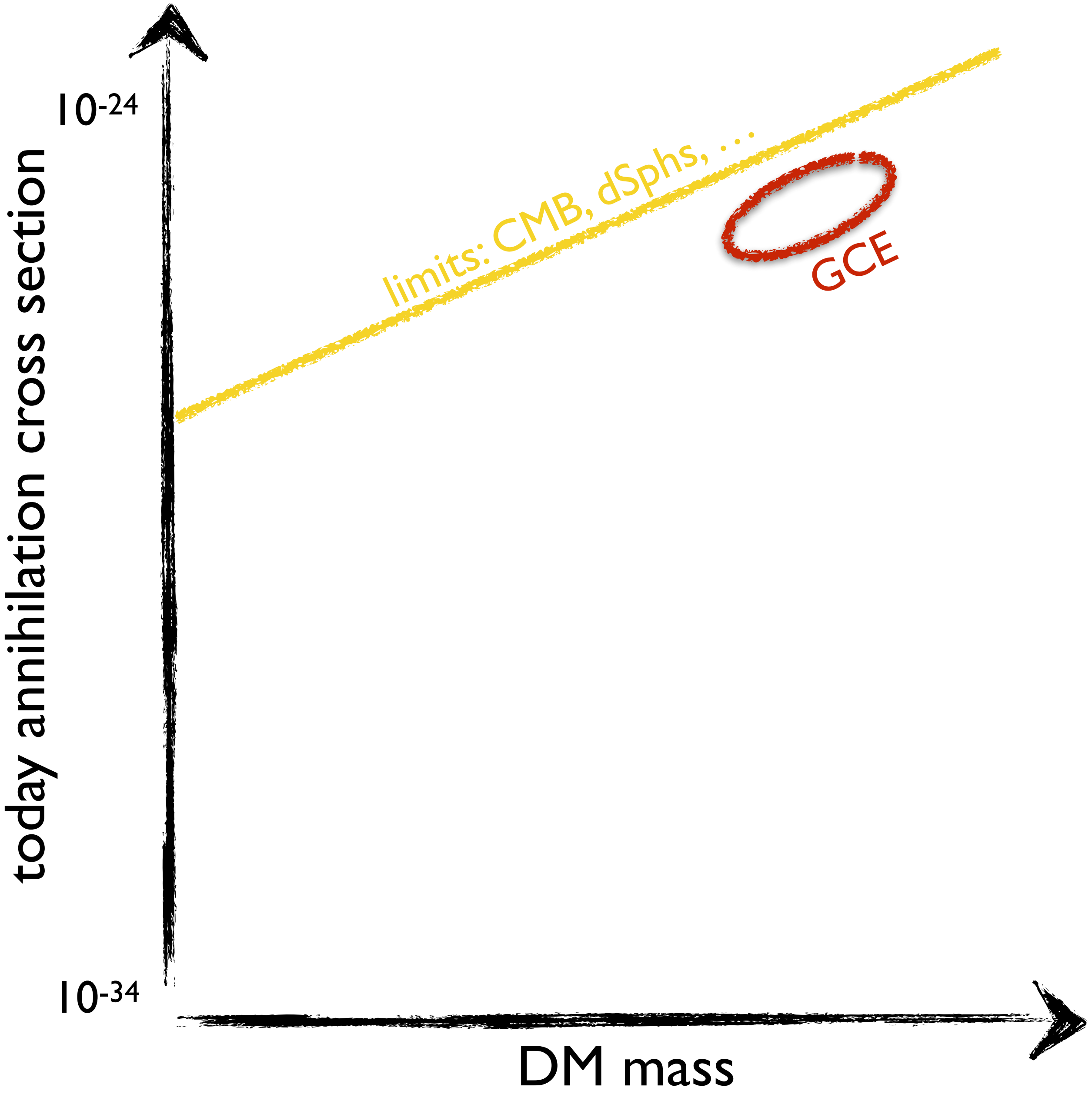

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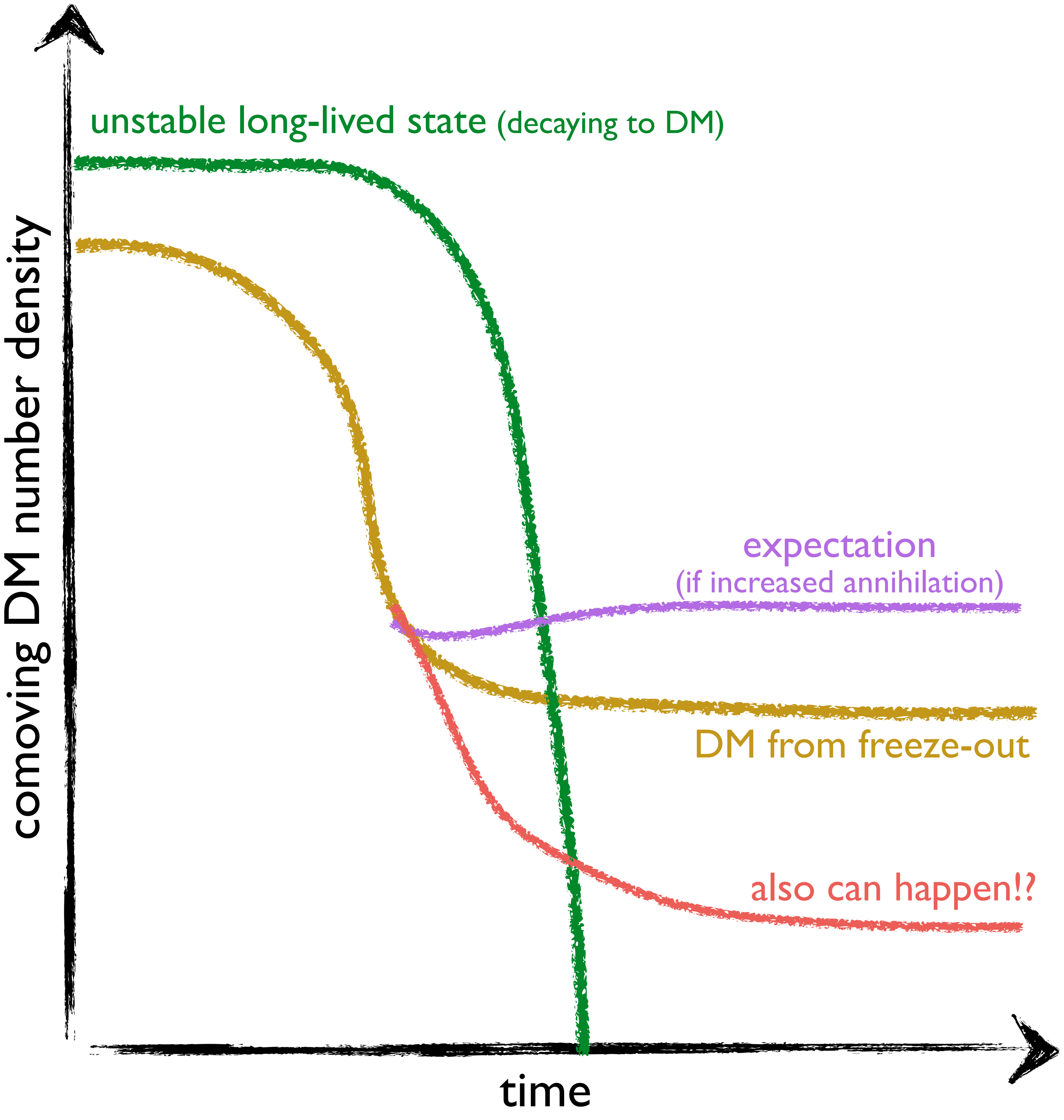

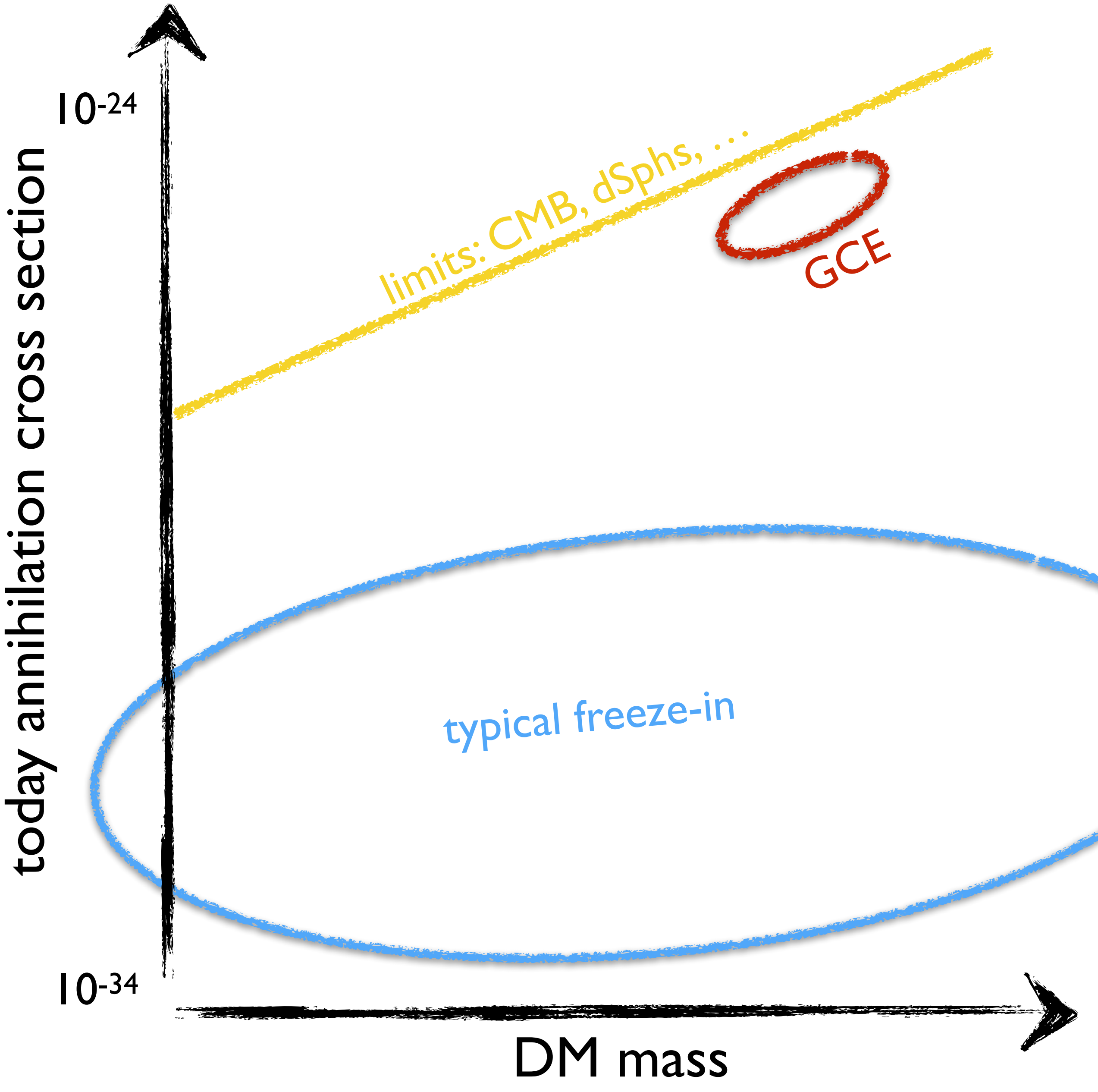

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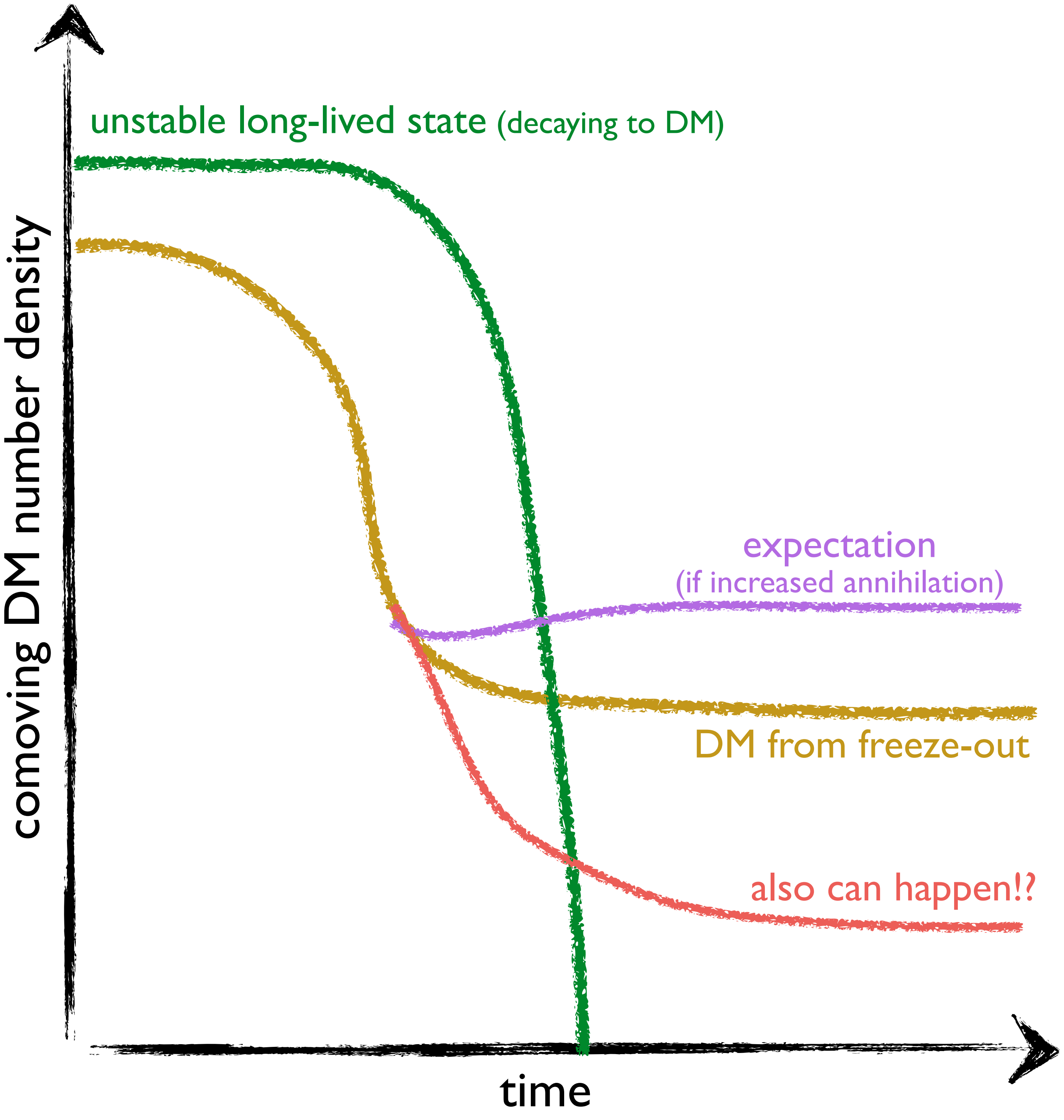

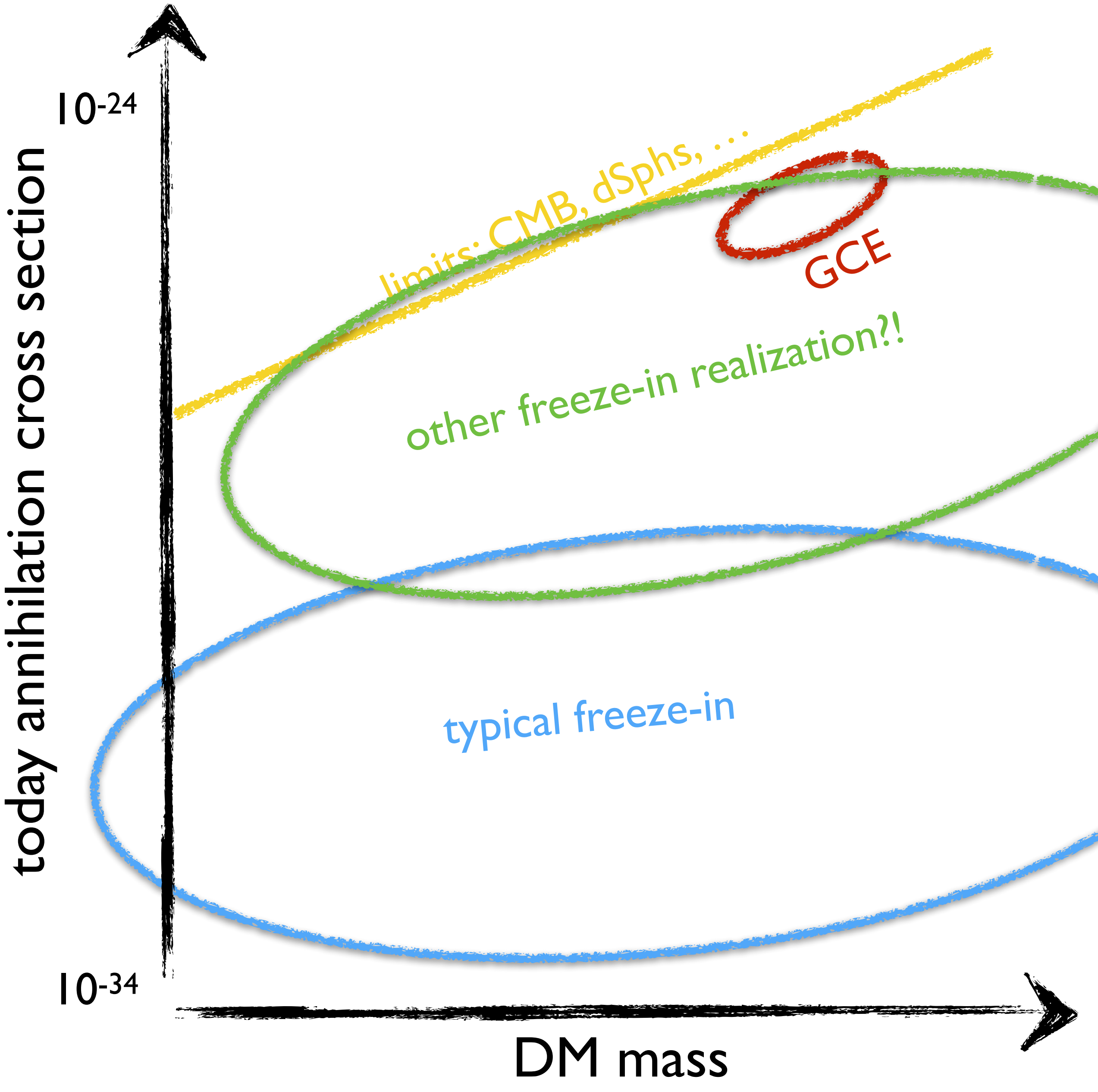

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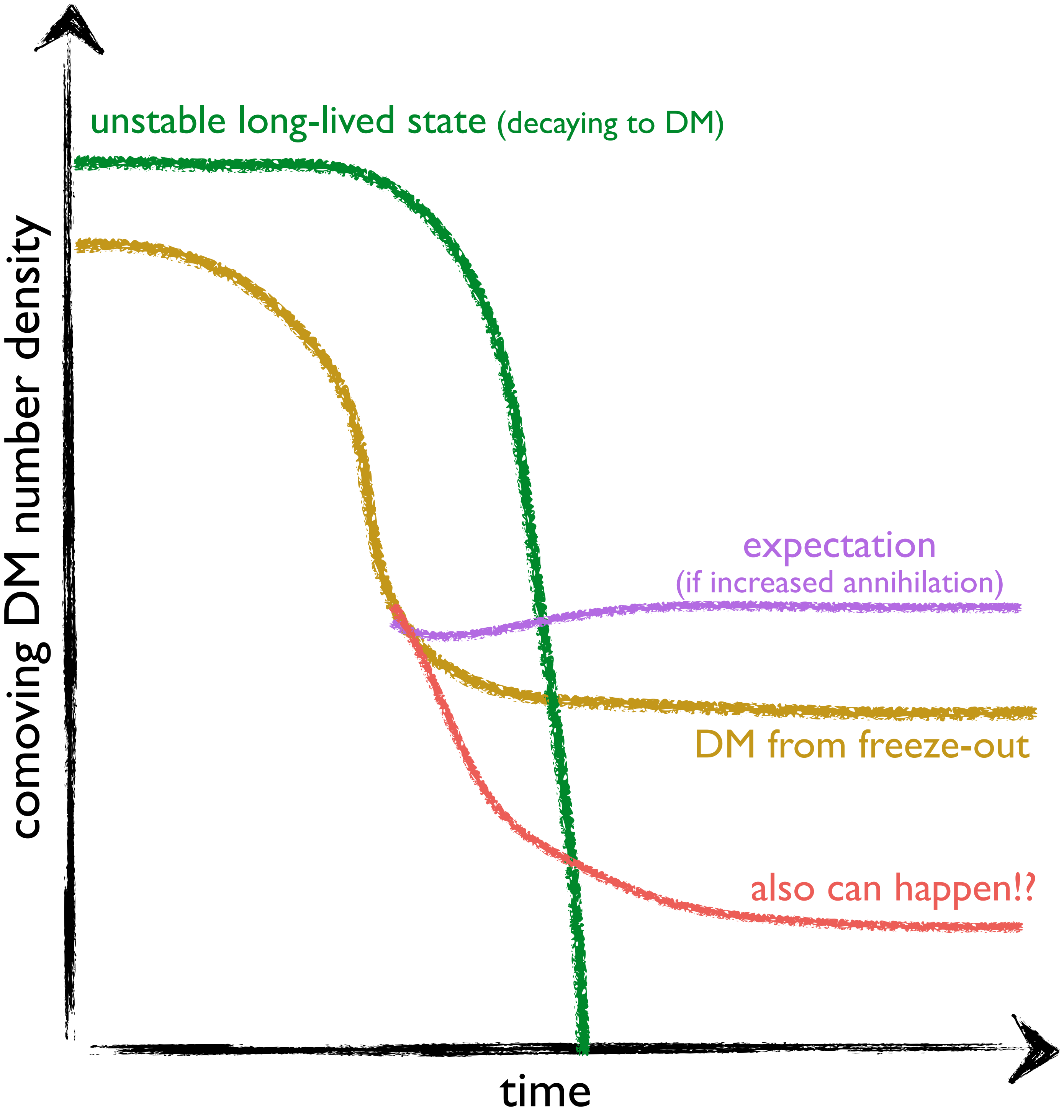
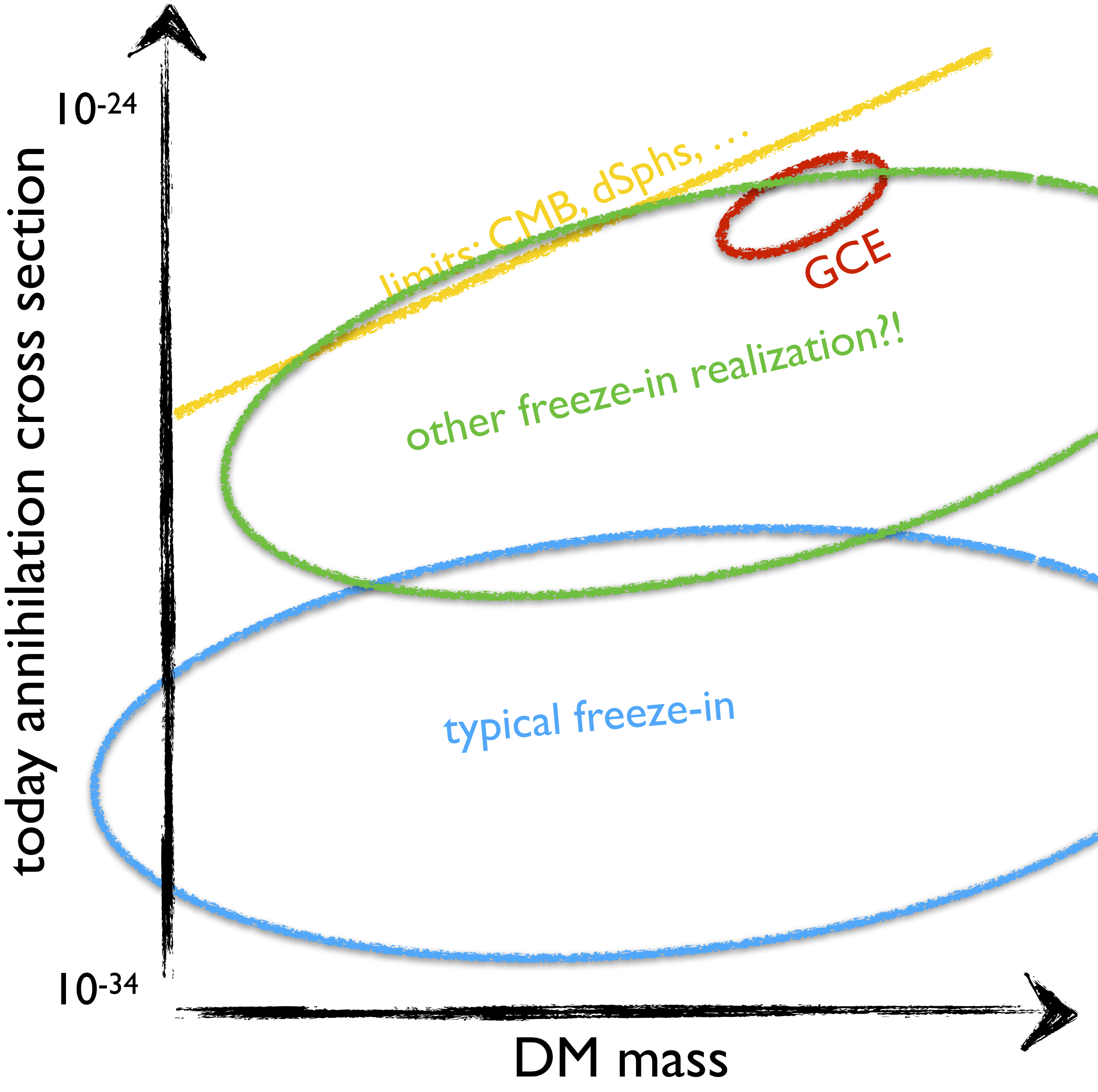




TO SEE WHY AND LEARN MORE STAY TUNED :)

THERMAL RELIC DENSITY

STANDARD SCENARIO

(chemical and kinetic)

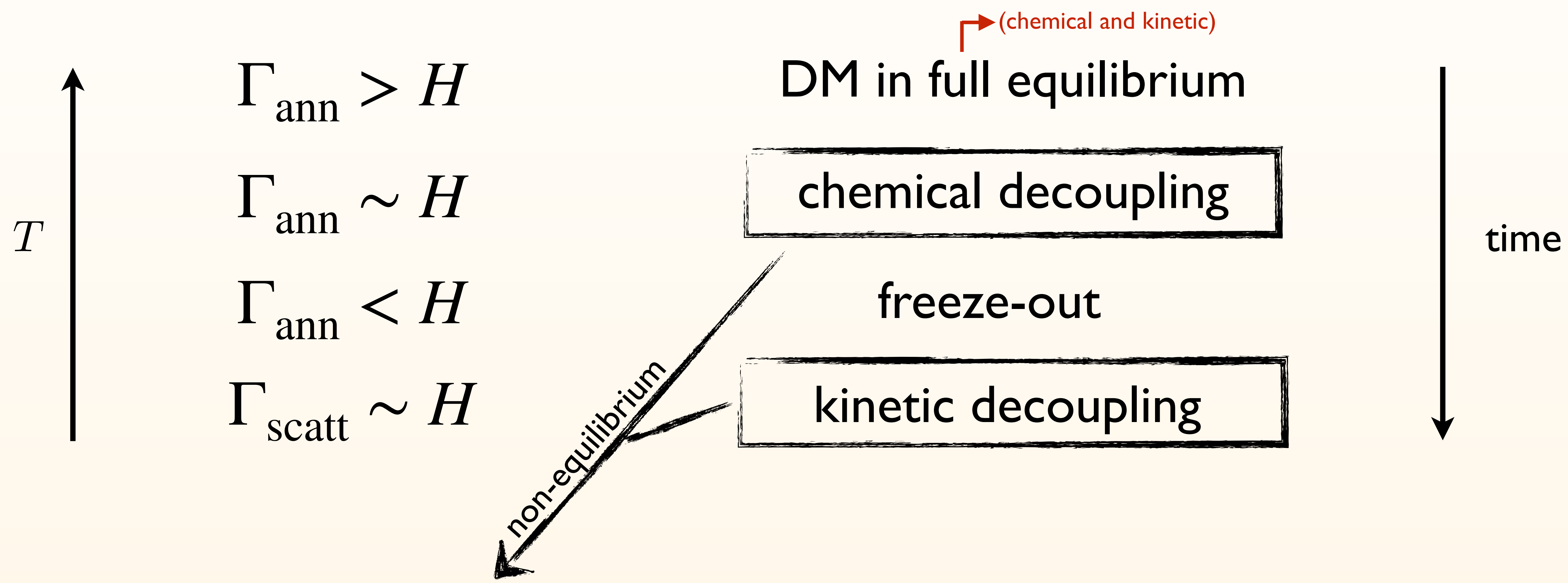
T

$\Gamma_{\text{ann}} > H$	DM in full equilibrium
$\Gamma_{\text{ann}} \sim H$	chemical decoupling
$\Gamma_{\text{ann}} < H$	freeze-out
$\Gamma_{\text{scatt}} \sim H$	kinetic decoupling

time

THERMAL RELIC DENSITY

STANDARD SCENARIO



(chemical and kinetic)

T

$\Gamma_{\rm ann} > H$ DM in full equilibrium

$\Gamma_{\rm ann} \sim H$ chemical decoupling

$\Gamma_{\rm ann} < H$ freeze-out

$\Gamma_{\rm scatt} \sim H$ kinetic decoupling

time

non-equilibrium

time evolution of $f_\chi(p)$ in kinetic theory:

$$\boxed{E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right)} f_\chi = \mathcal{C}[f_\chi]$$

Liouville operator in FRW background

the collision term

THERMAL RELIC DENSITY

STANDARD APPROACH

Boltzmann equation for $f_\chi(p)$:

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

*assumptions for using Boltzmann eq:
classical limit, molecular chaos,...

…for derivation from thermal QFT
see e.g., 1409.3049

THERMAL RELIC DENSITY

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see e.g., 1409.3049

⇓ integrate over *p*
(i.e. take 0th moment)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_{\chi\bar{\chi}\to ij}\sigma_{\rm rel}\rangle^{\rm eq}\left(n_\chi n_{\bar{\chi}} - n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}\right)$$

where the thermally averaged cross section:

$$\langle \sigma_{\chi\bar{\chi}\to ij}v_{\rm rel}\rangle^{\rm eq} = -\frac{h_\chi^2}{n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}}\int \frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar{\chi}}}{(2\pi)^3}\,\sigma_{\chi\bar{\chi}\to ij}v_{\rm rel}\; f_\chi^{\rm eq} f_{\bar{\chi}}^{\rm eq}$$

THERMAL RELIC DENSITY

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$\Downarrow$ integrate over *p*
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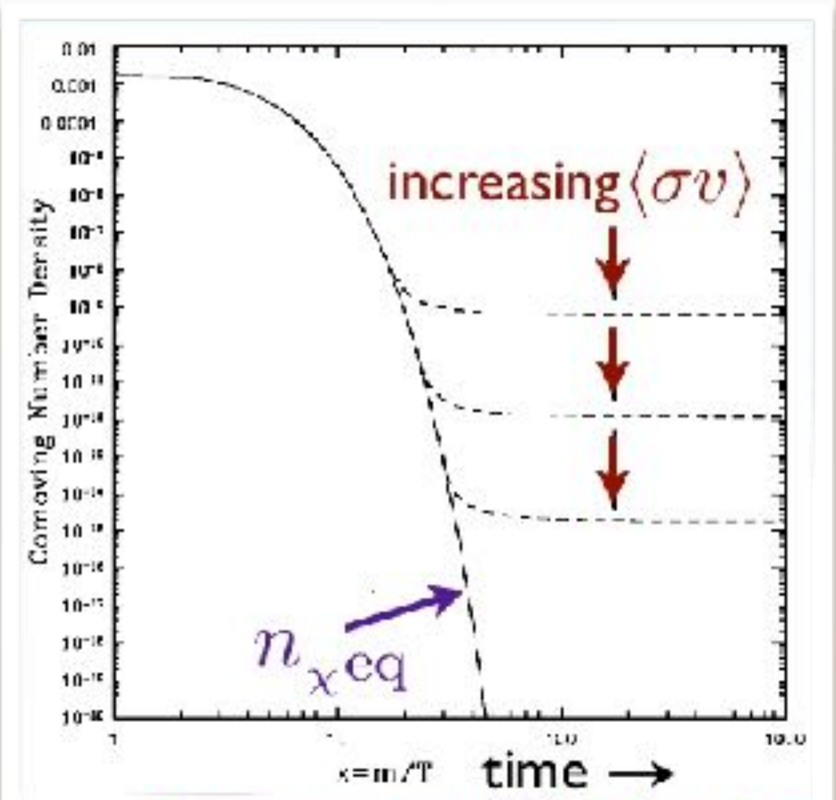

Fig.: Jungman, Kamionkowski & Griest, PR'96

THERMAL RELIC DENSITY

STANDARD APPROACH

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Critical assumption:

kinetic equilibrium at chemical decoupling

$$f_\chi \sim a(T) f_\chi^{\rm eq}$$

$\Downarrow$

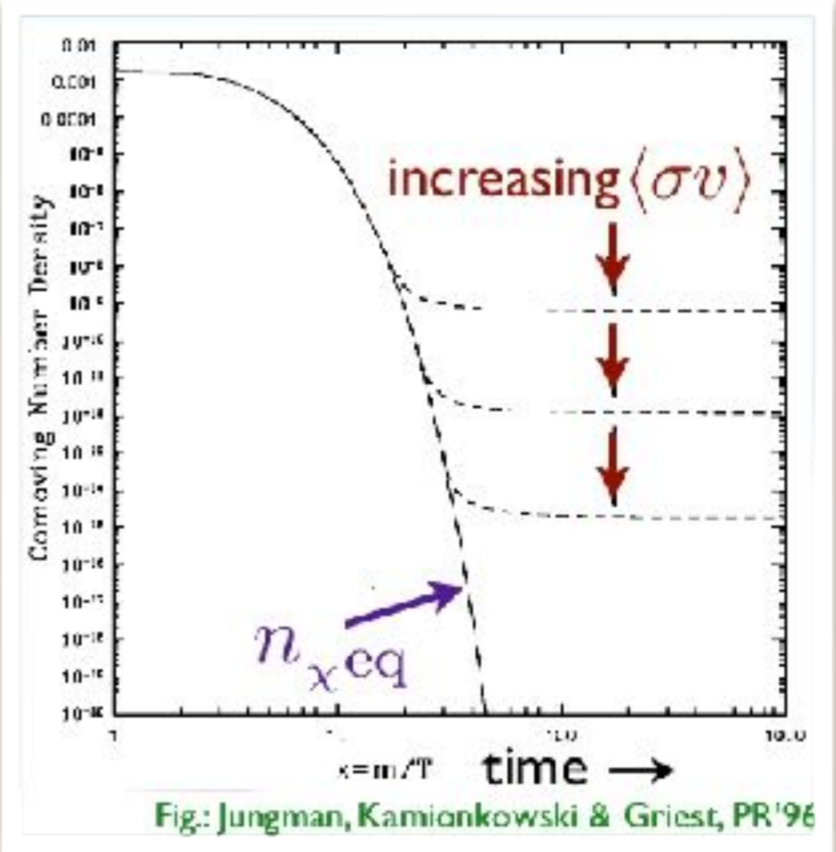
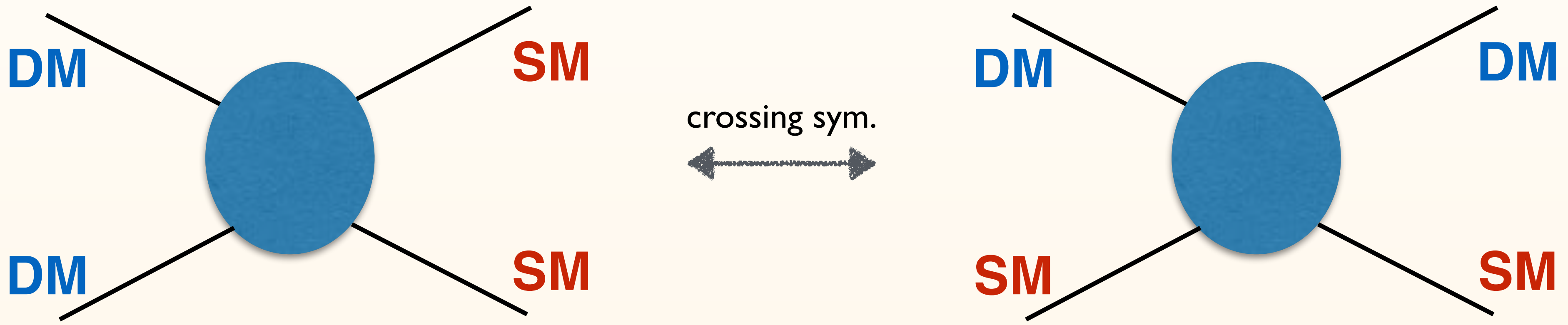


Fig.: Jungman, Kamionkowski & Griest, PR'96

FREEZE-OUT *VS.* DECOUPLING

annihilation

(elastic) scattering



$$\sum_{\text{spins}} \left|\mathcal{M}^{\text{pair}}\right|^2 = F(p_1, p_2, p_1', p_2') \quad \sim \quad \sum_{\text{spins}} \left|\mathcal{M}^{\text{scatt}}\right|^2 = F(k, -k', p', -p)$$

Boltzmann suppression of DM vs. SM $\Rightarrow$ scatterings typically more frequent

dark matter frozen-out but typically
still kinetically coupled to the plasma

Schmid, Schwarz, Widern ’99; Green, Hofmann, Schwarz ’05

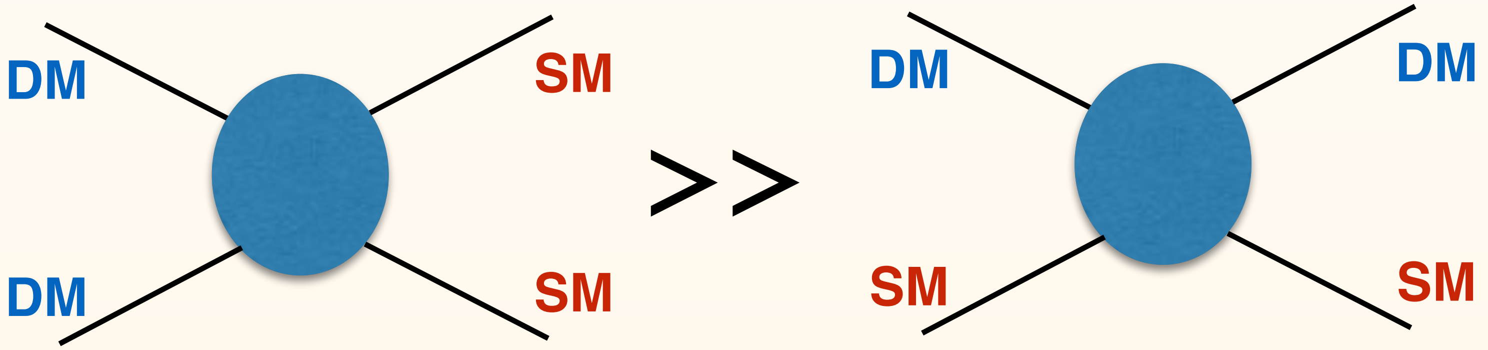
EARLY KINETIC DECOUPLING?

A necessary and sufficient condition: scatterings weaker than annihilation

i.e. rates around freeze-out: $H \sim \Gamma_{\rm ann} \gtrsim \Gamma_{\rm el}$

Possibilities:

A)



e.g., resonant annihilation

B) Boltzmann suppression of SM as strong as for DM

e.g., below threshold annihilation (forbidden-like DM)

C) Scatterings and annihilation have different structure

e.g., semi-annihilation, 3 to 2 models,…

D) Multi-component dark sectors

e.g., additional sources of DM from late decays, …

HOW TO GO BEYOND KINETIC EQUILIBRIUM?

All information is in the full BE:

both about chemical (”normalization”) and kinetic (”shape”) equilibrium/decoupling

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

contains both scatterings and annihilations

Two possible approaches:

fBE

cBE

0-th moment: n_χ
2-nd moment: T_χ
$\ldots$

solve numerically for full $f_\chi(p)$

have insight on the distribution

no constraining assumptions

numerically challenging

often an overkill

consider system of equations for moments of $f_\chi(p)$

partially analytic/much easier numerically

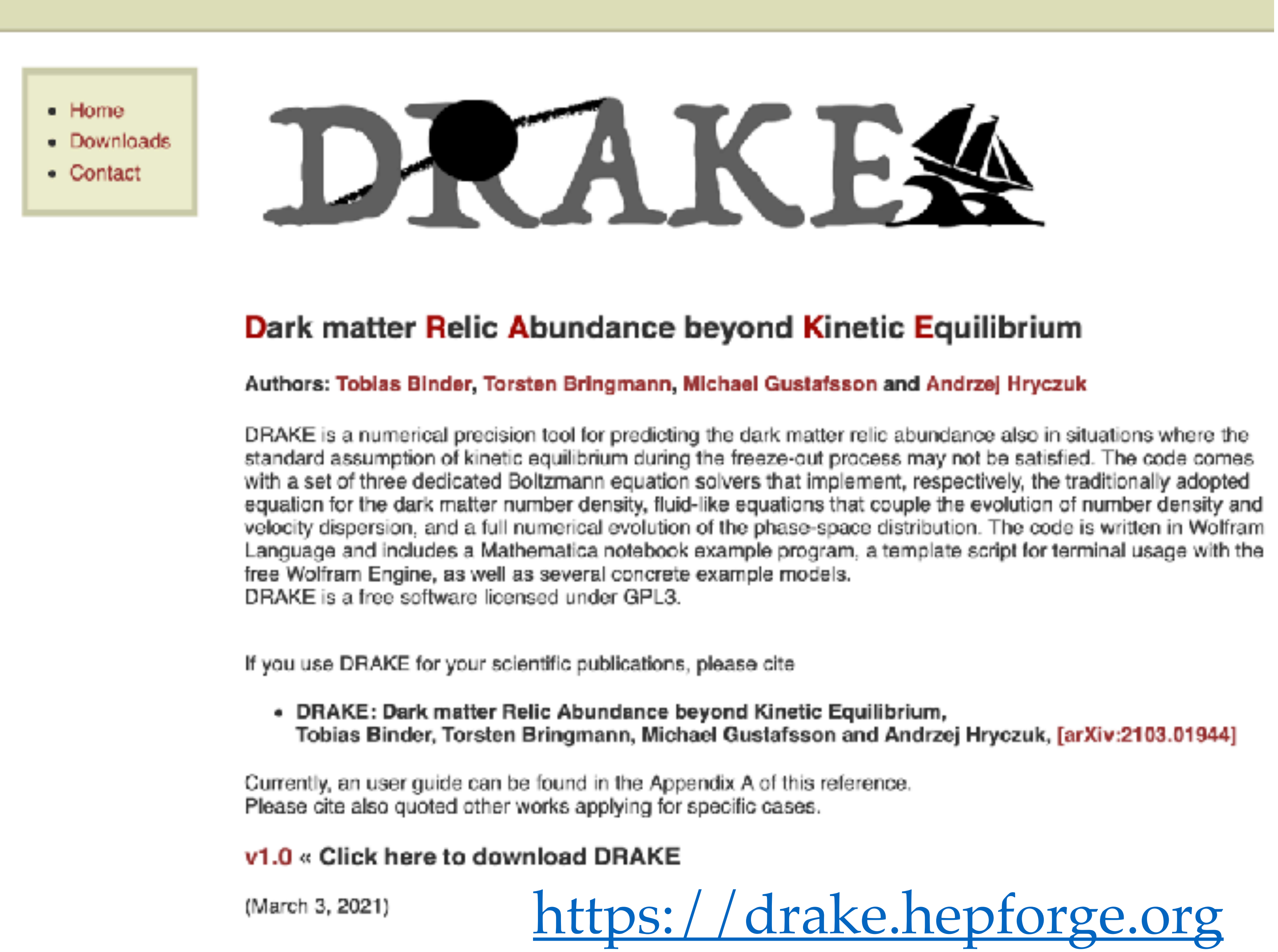
manifestly captures all of the relevant physics

finite range of validity

no insight on the distribution

NEW TOOL!

GOING BEYOND THE STANDARD APPROACH



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DRAKE

Dark matter Relic Abundance beyond Kinetic Equilibrium

Authors: Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk

DRAKE is a numerical precision tool for predicting the dark matter relic abundance also in situations where the standard assumption of kinetic equilibrium during the freeze-out process may not be satisfied. The code comes with a set of three dedicated Boltzmann equation solvers that implement, respectively, the traditionally adopted equation for the dark matter number density, fluid-like equations that couple the evolution of number density and velocity dispersion, and a full numerical evolution of the phase-space distribution. The code is written in Wolfram Language and includes a Mathematica notebook example program, a template script for terminal usage with the free Wolfram Engine, as well as several concrete example models.
DRAKE is a free software licensed under GPL3.

If you use DRAKE for your scientific publications, please cite

- **DRAKE: Dark matter Relic Abundance beyond Kinetic Equilibrium,**
  Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk, [arXiv:2103.01944]

Currently, an user guide can be found in the Appendix A of this reference.
Please cite also quoted other works applying for specific cases.

v1.0 « Click here to download DRAKE

(March 3, 2021)

https://drake.hepforge.org

Applications:

DM relic density for
any (user defined) model*

Interplay between chemical and
kinetic decoupling

Prediction for the DM
phase space distribution

Late kinetic decoupling
and impact on cosmology

see e.g., 1202.5456

…

(only) prerequisite:
Wolfram Language (or *Mathematica*)

*at the moment for a single DM species and w/o co-annihlations… but stay tuned for extensions!

EXAMPLE D:

WHEN ADDITIONAL INFLUX OF DM ARRIVES

D) Multi-component dark sectors

Sudden injection of more DM particles distorts $f_\chi(p)$
(e.g. from a decay or annihilation of other states)

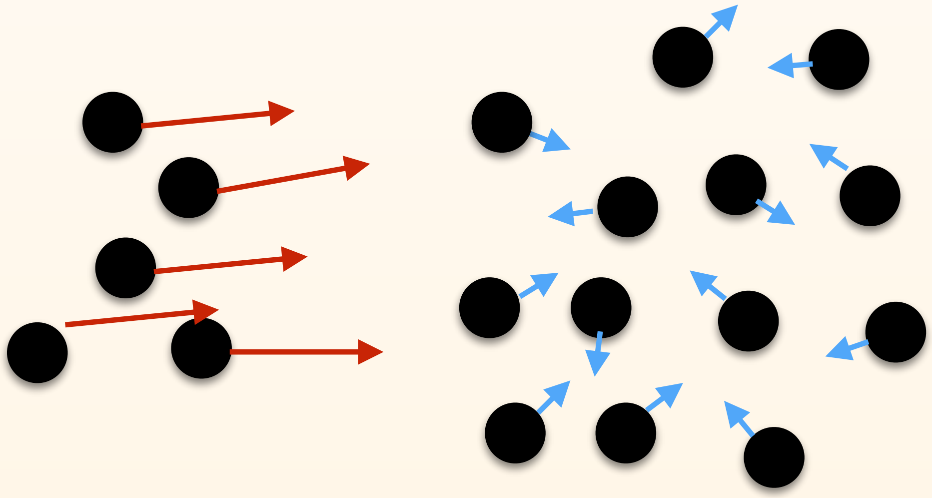
- this can modify the annihilation rate (if still active)
- how does the thermalization due to elastic scatterings happen?

1) DM produced via:

- 1st component from thermal freeze-out
- 2nd component from a decay $\phi \to \bar{\chi}\chi$

2) DM annihilation has a threshold

e.g. $\chi\bar{\chi} \to f\bar{f}$ with $m_\chi \lesssim m_f$



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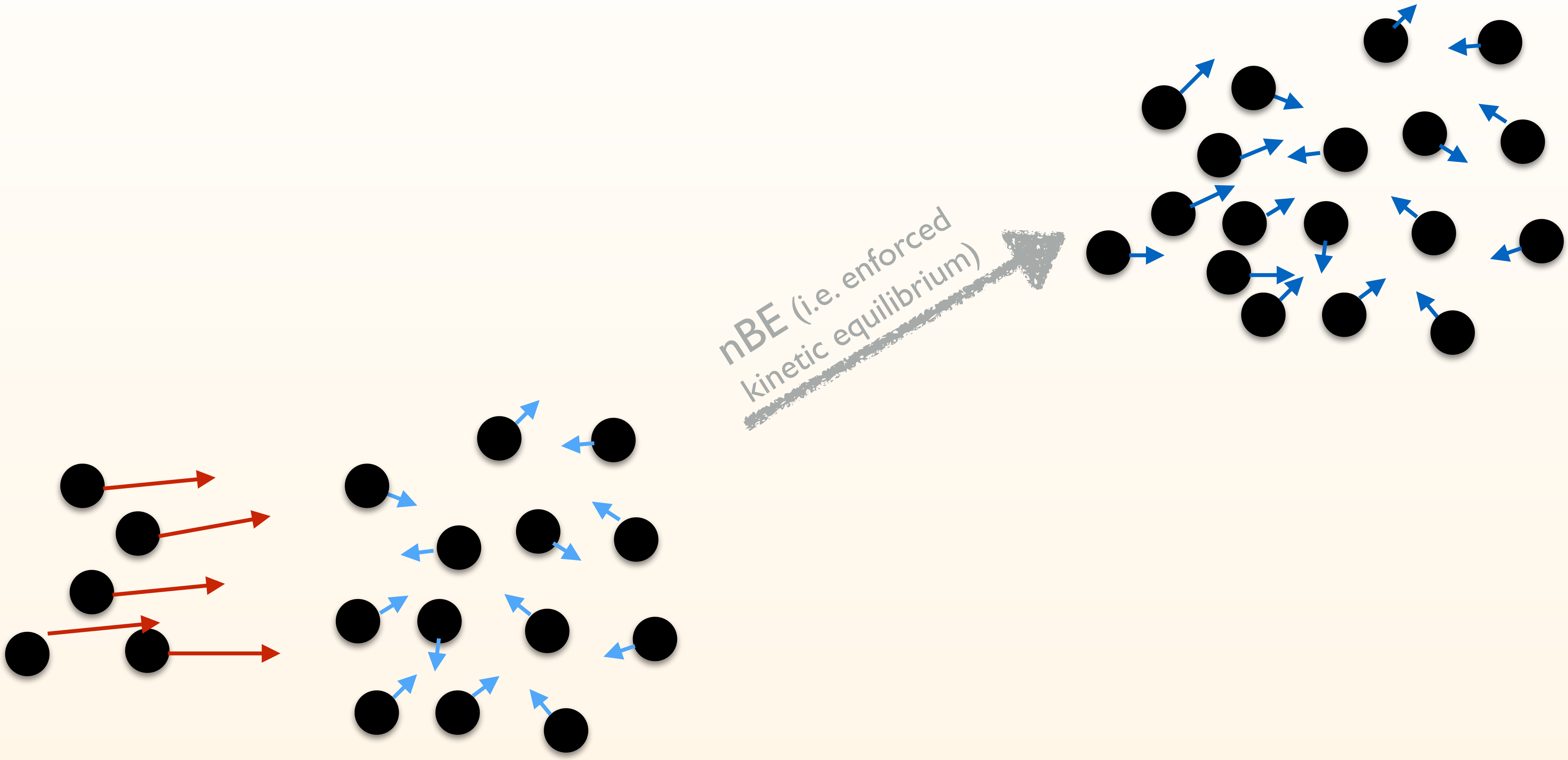

no enhanced annihilation, <u>more DM in the end</u>

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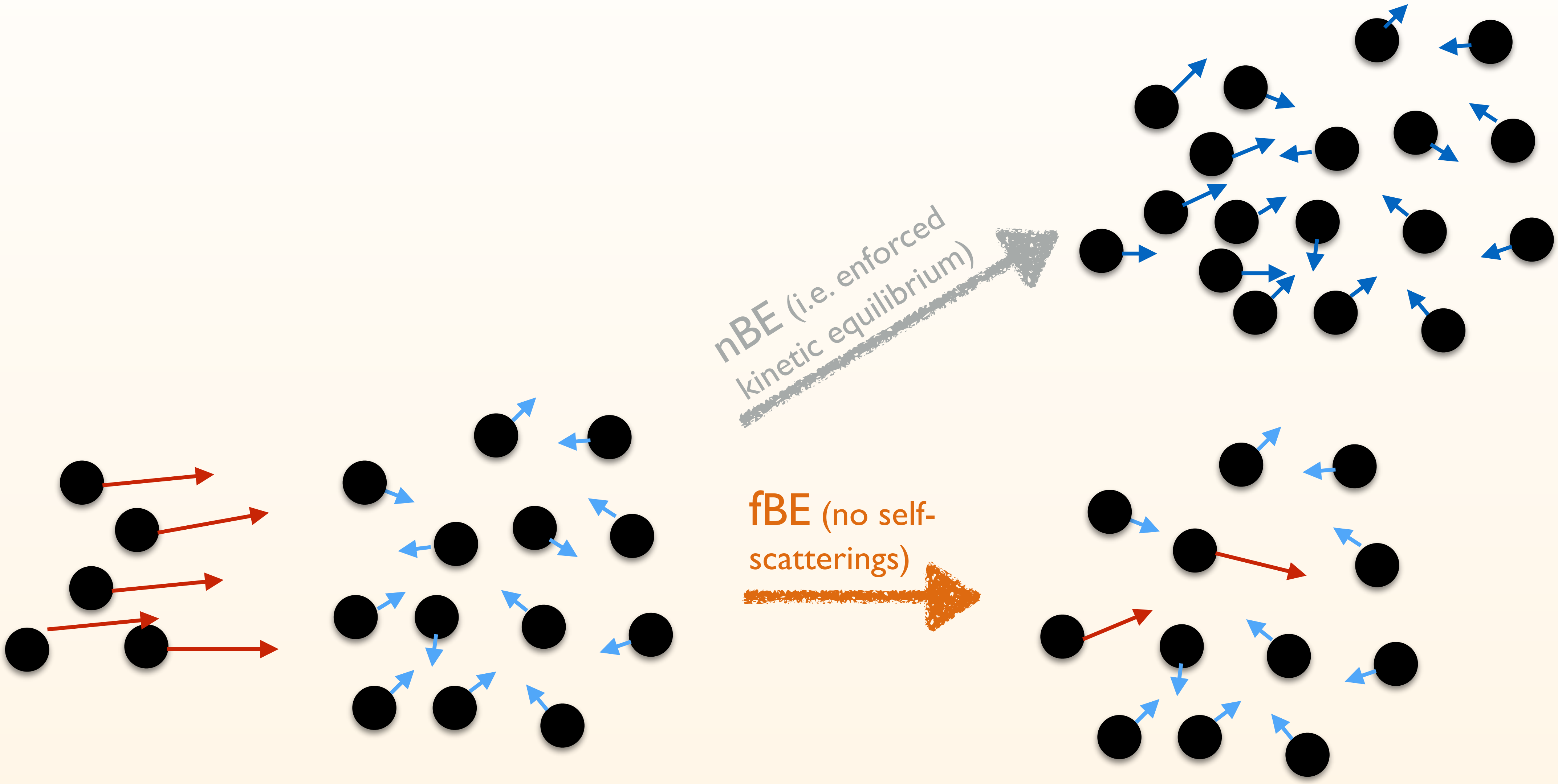

nBE (i.e. enforced kinetic equilibrium)

no enhanced annihilation, more DM in the end

fBE (no self-scatterings)

some injected particles will annihilate together with themselves and cold component

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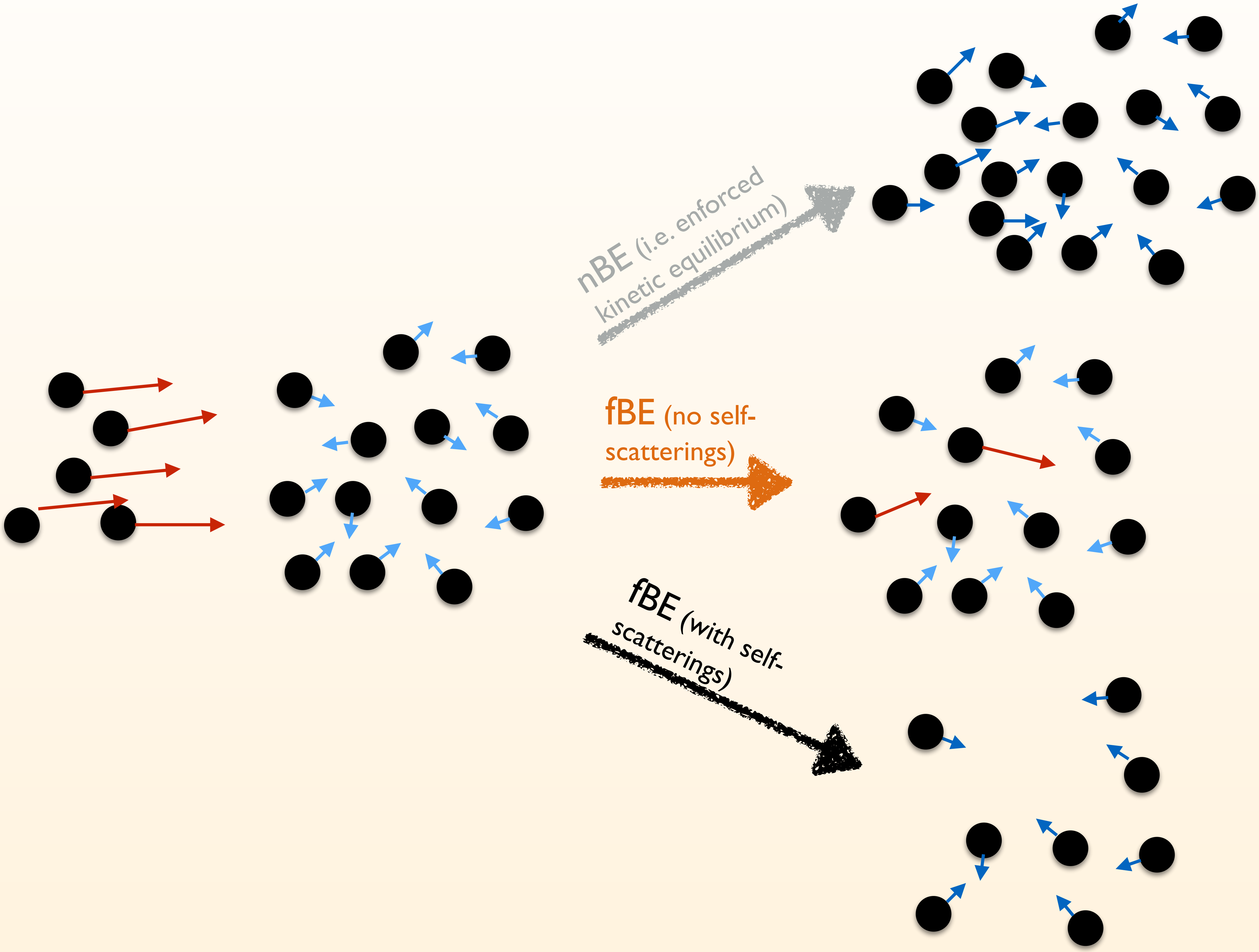


nBE (i.e. enforced kinetic equilibrium)

no enhanced annihilation, <u>more DM in the end</u>

fBE (no self-scatterings)

some injected particles will annihilate together <u>with themselves and cold component</u>

fBE (with self-scatterings)

energy redistribution will allow more DM particles <u>to reach energies over annihilation threshold</u>

AH, Laletin 2204.07078

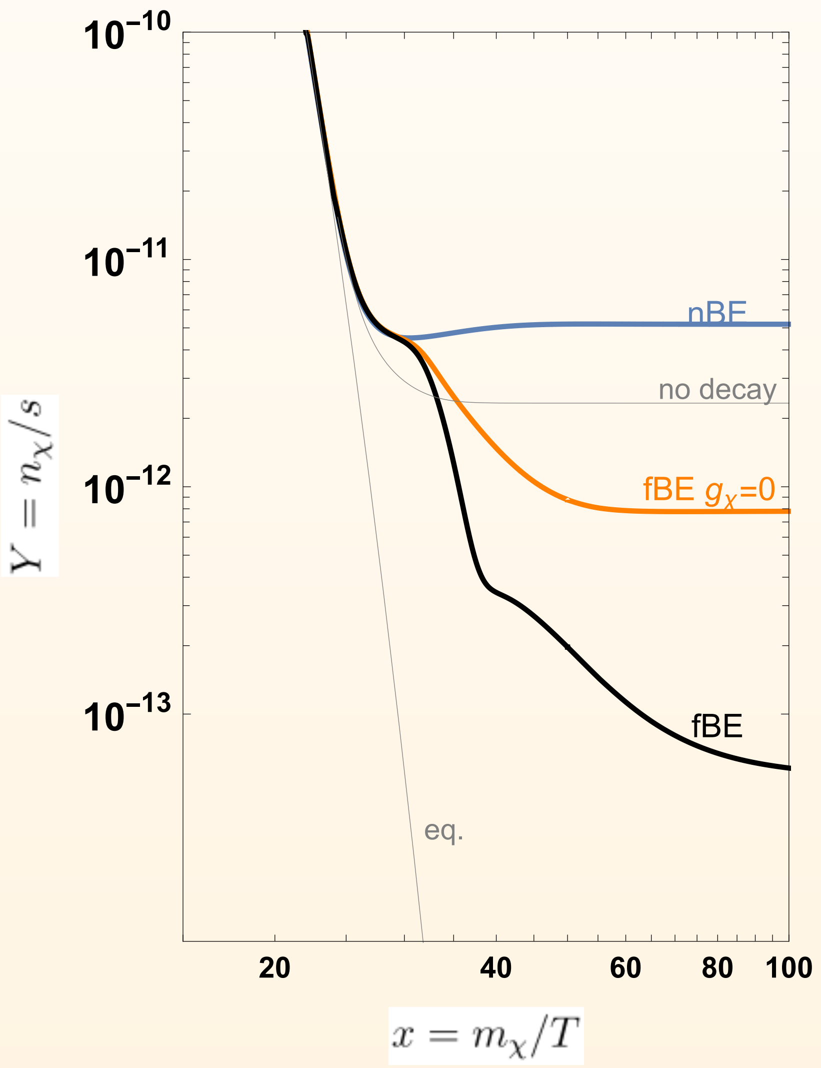
EXAMPLE EVOLUTION

1) DM produced via:
- 1st component from thermal freeze-out
- 2nd component from a decay $\phi \to \bar{\chi}\chi$

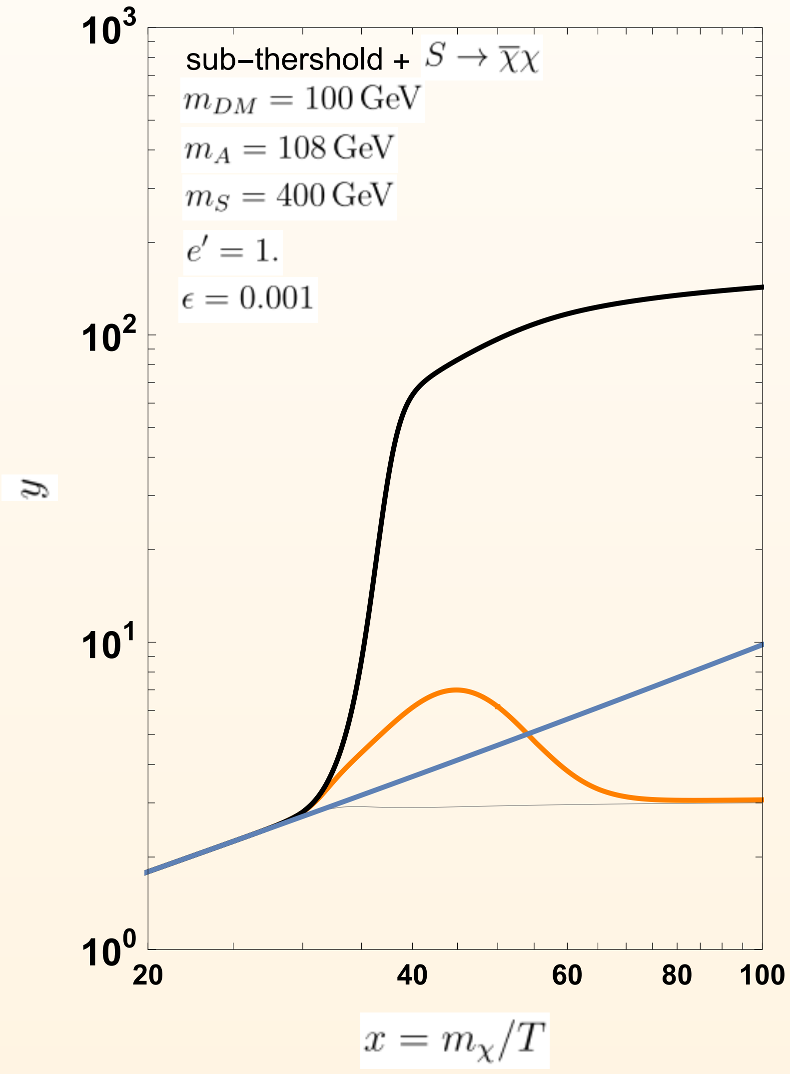
2) DM annihilation has a threshold

e.g. $\chi\bar{\chi} \to f\bar{f}$ with $m_\chi \lesssim m_f$

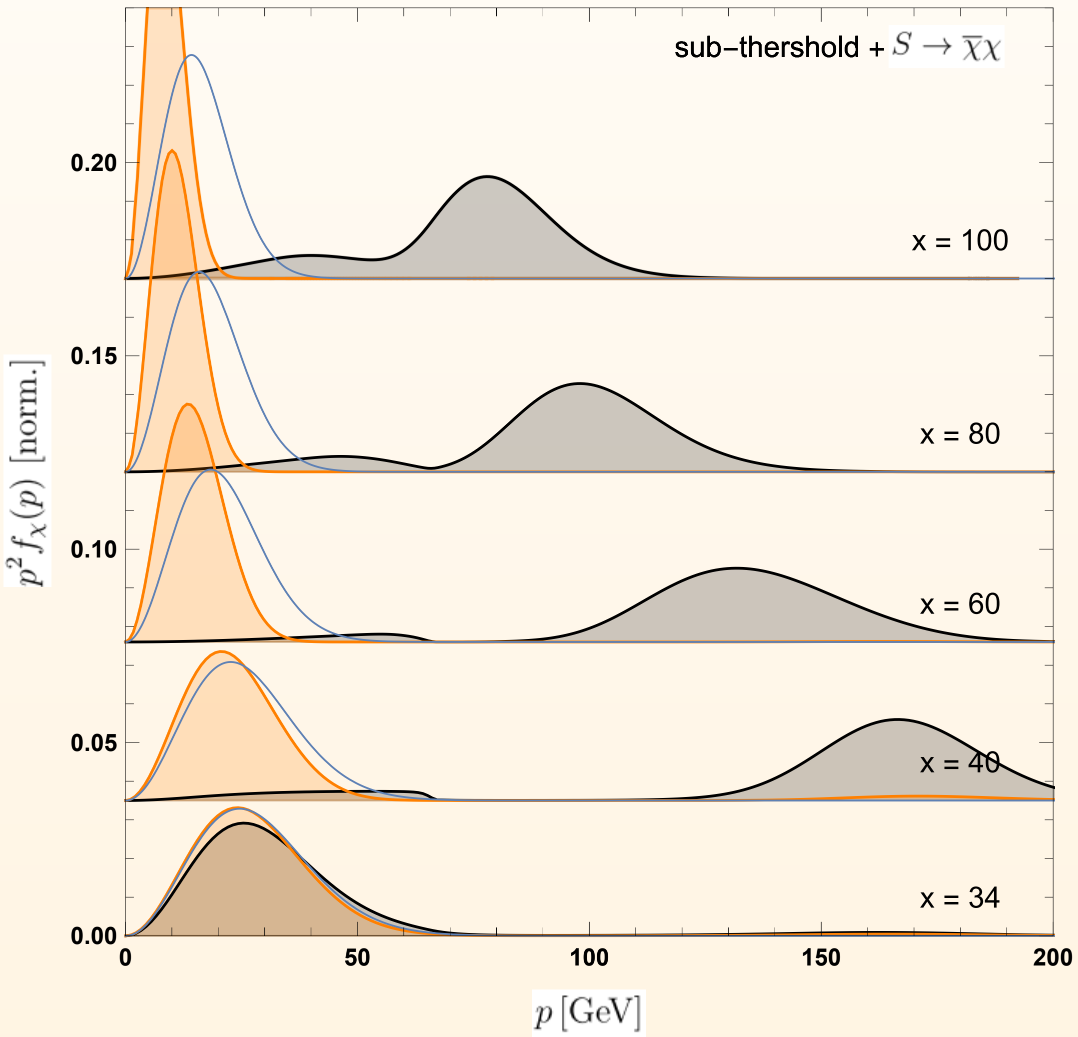
$Y \sim$ number density



$y \sim$ temperature



$p^2 f(p) \sim$ momentum distribution



FREEZE-IN:

C) with semi-annihilation process

HOW ABOUT SEMI-PRODUCTION?

AH, Laletin 2104.05684
(see also Bringmann et al. 2103.16572)

Consider process of production that is the inverse of semi-annihilation:

$$\chi\phi \to \chi\chi$$

DM

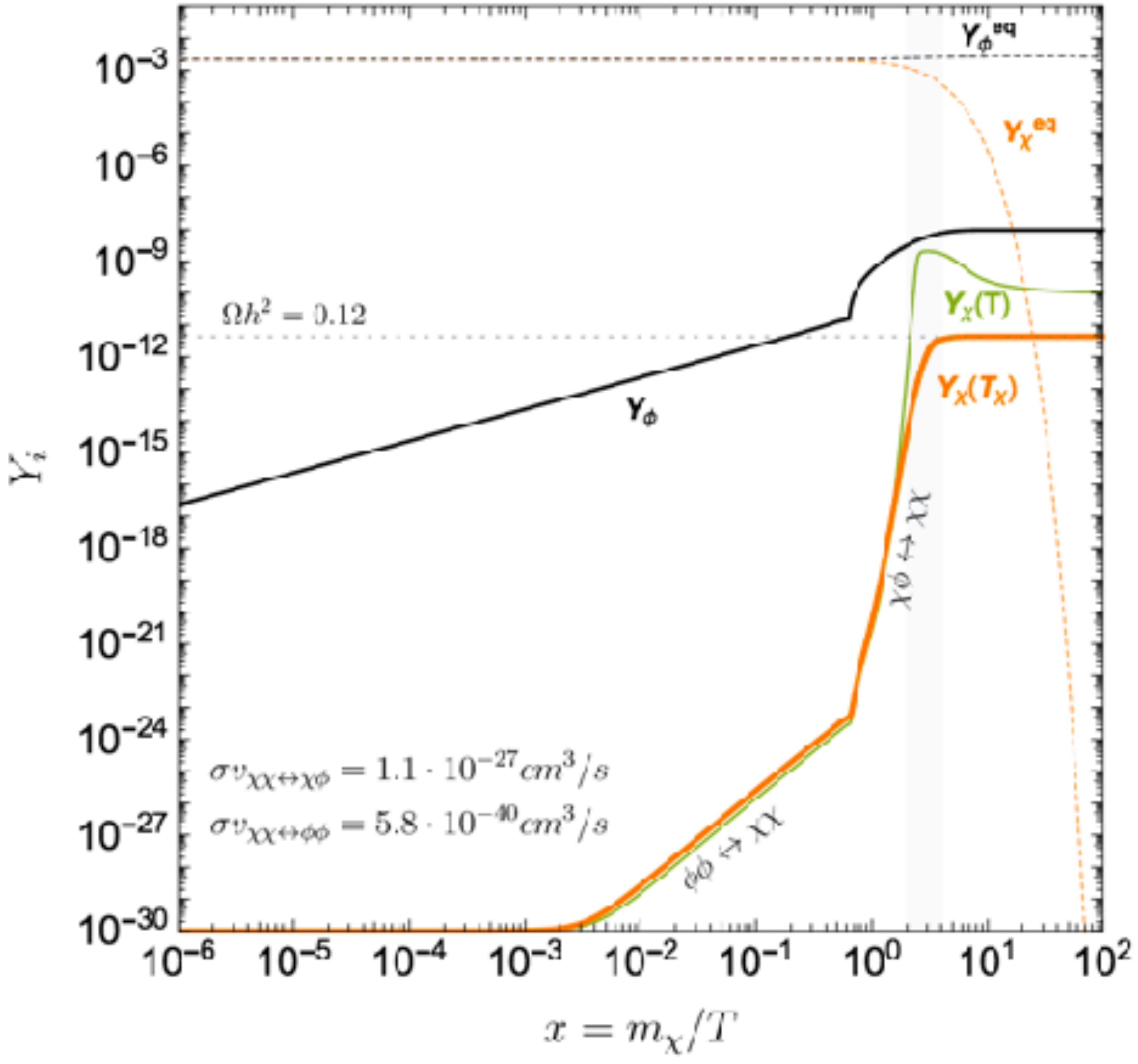
mediator or a SM state

What is different (from the decay/pair-annihilation freeze-in)?

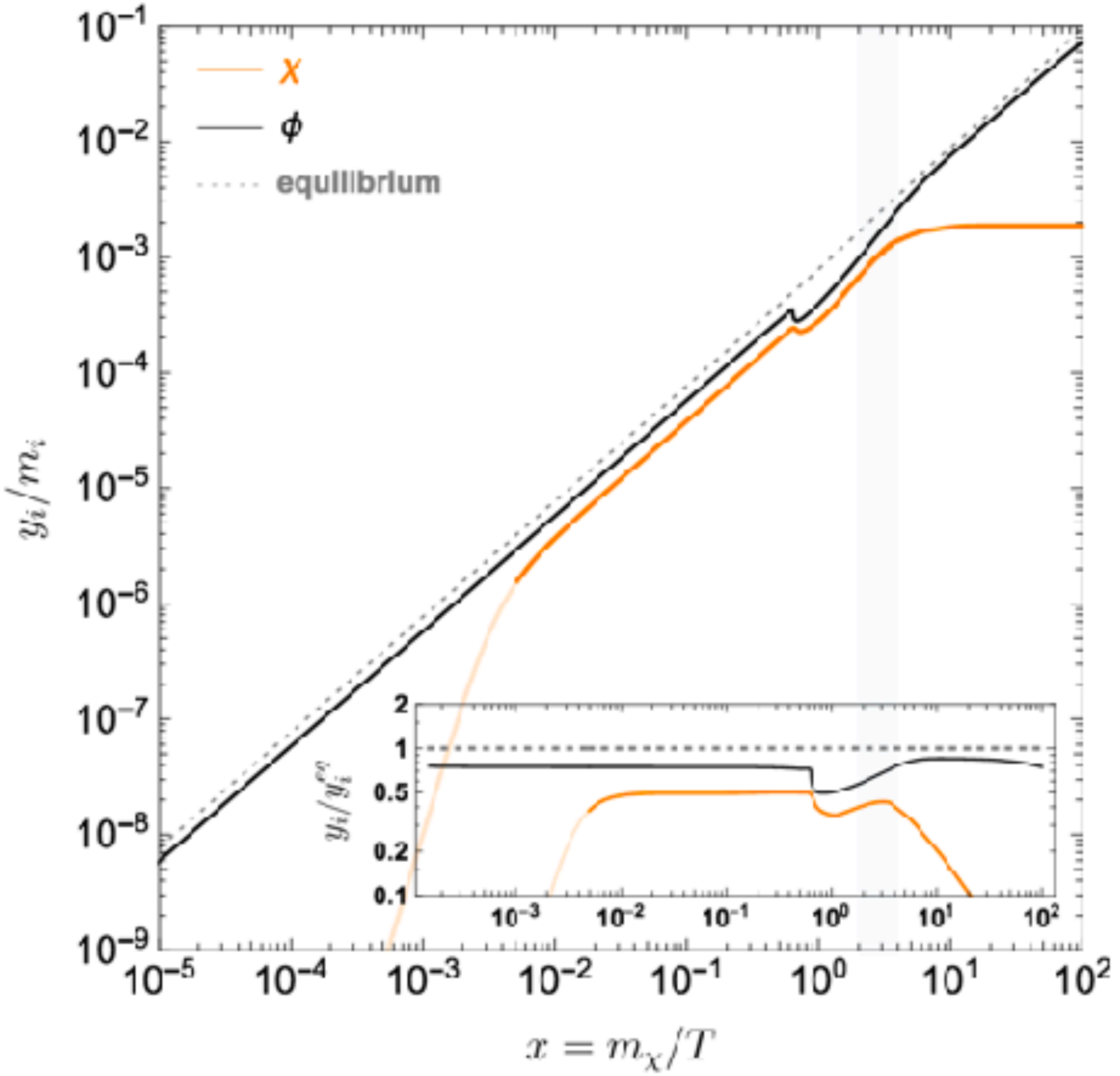
- The production rate is proportional to the DM density. (Smaller initial abundance → larger cross section…)
- Semi-production modifies the energy of DM particles in a non-trivial way, so the temperature evolution can affect the relic density

EVOLUTION

co-moving number density



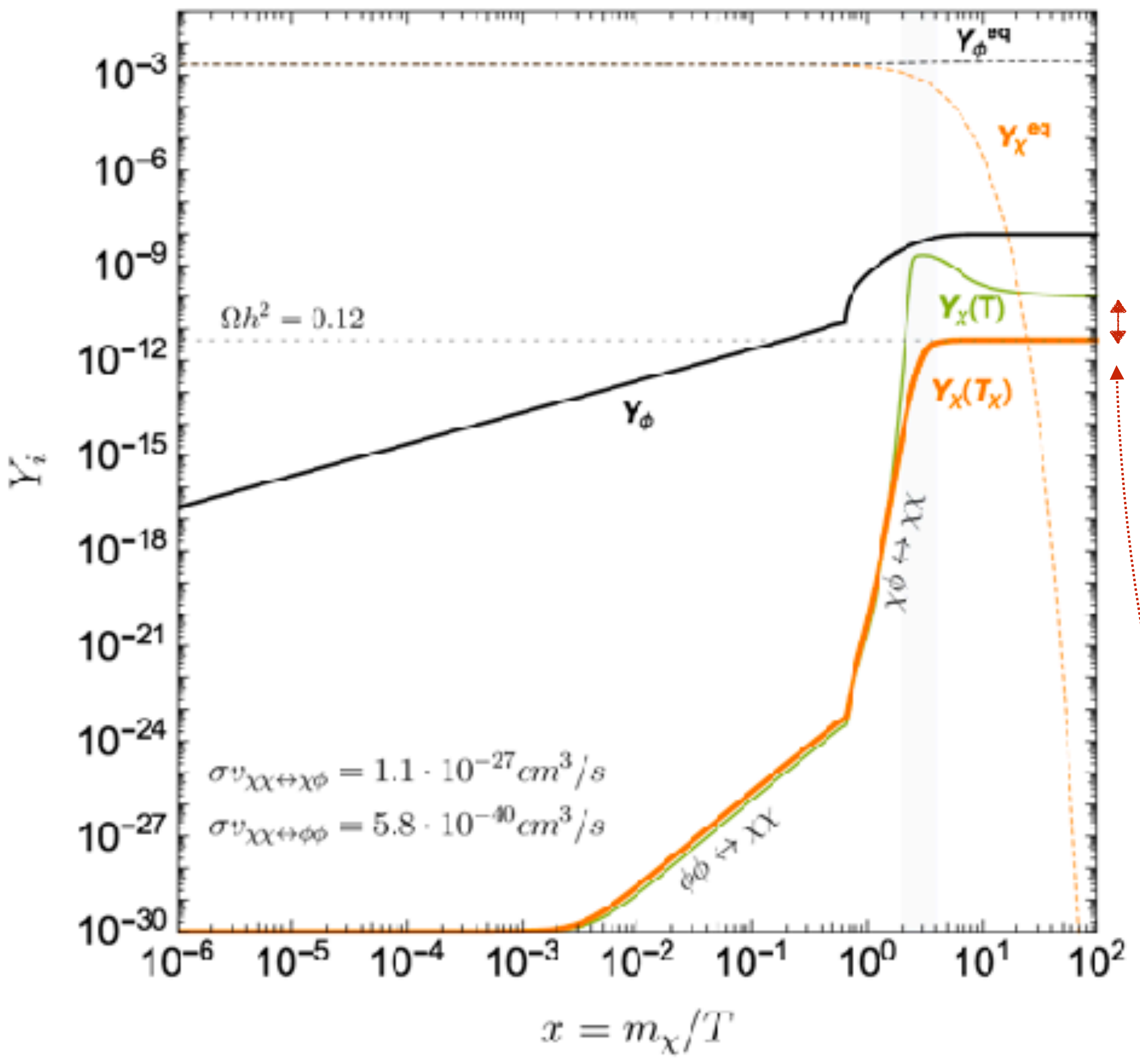
'temperature'



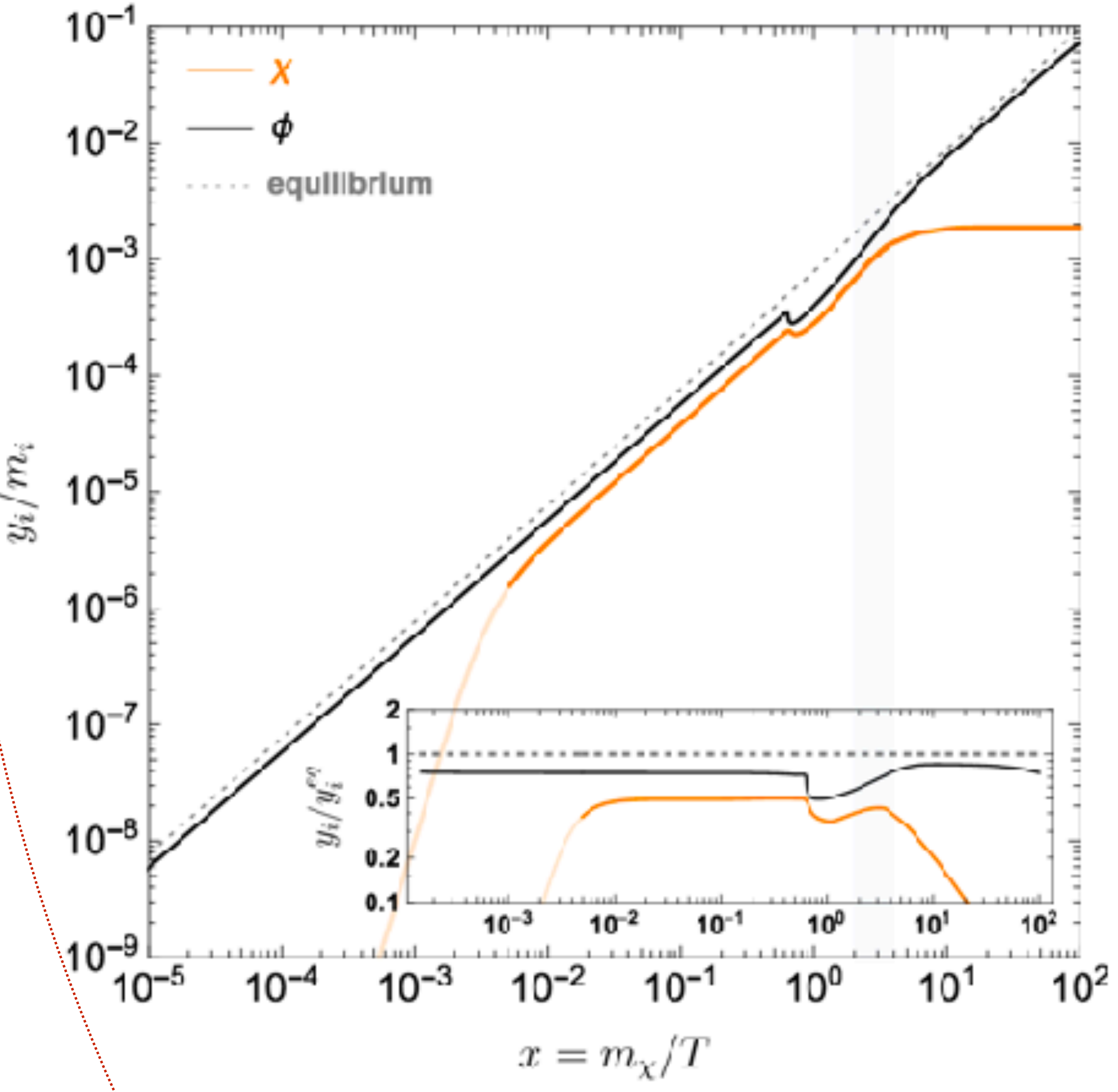
$m_\chi = 100$ GeV, $\mu_\phi = 1$ GeV, $\lambda_1 = 1.1 \times 10^{-2}$, $\lambda_2 = 10^{-8}$, $\lambda_{h\phi} = 6 \times 10^{-11}$

EVOLUTION

co-moving number density



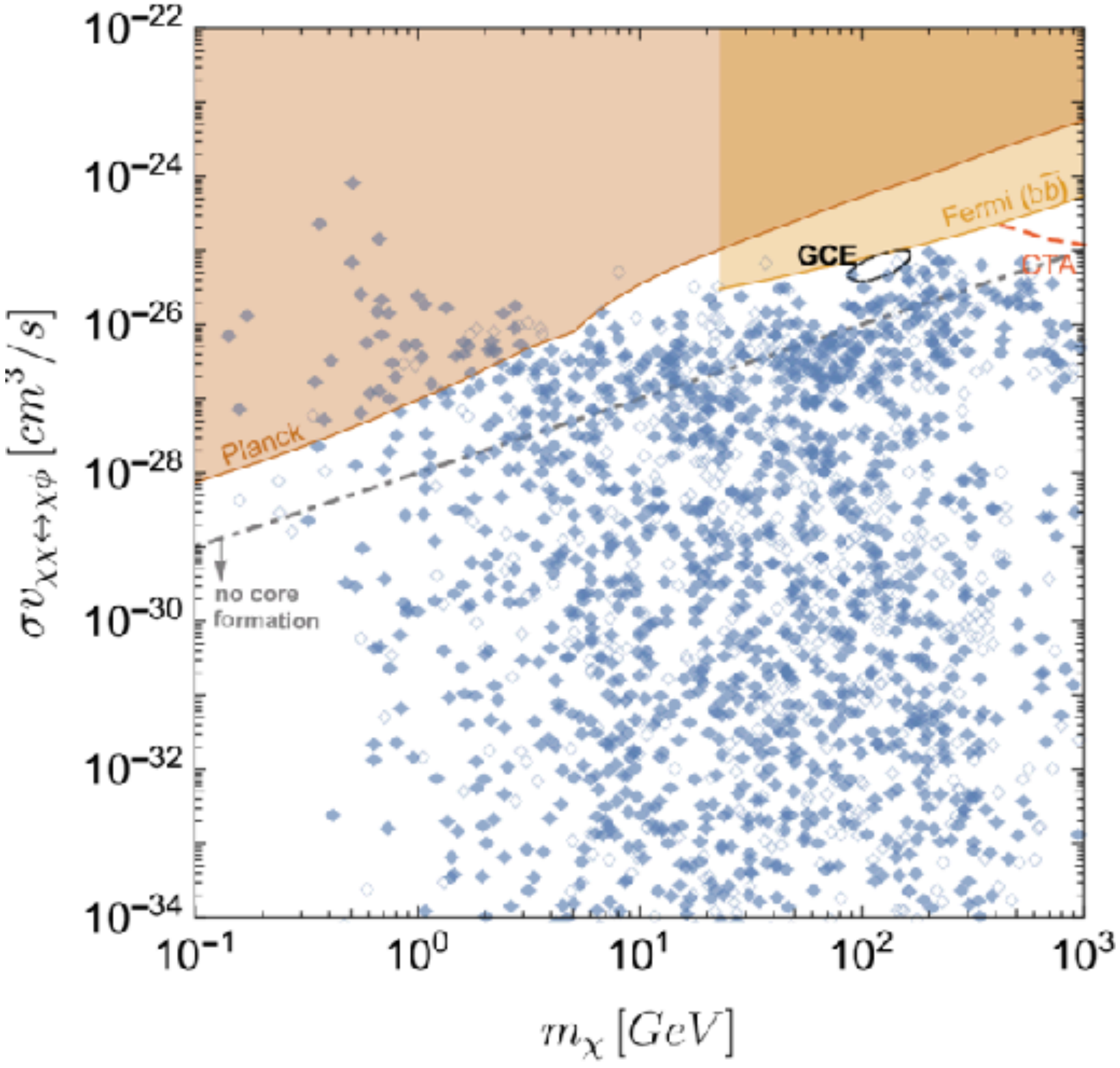
’temperature’



$m_\chi = 100$ GeV, $\mu_\phi = 1$ GeV, $\lambda_1 = 1.1 \times 10^{-3}$, $\lambda_2 = 10^{-8}$, $\lambda_{h\phi} = 6 \times 10^{-11}$

The full calculation compared to one assuming $T_\chi = T$
can differ by more than order of magnitude!

INDIRECT DETECTION



- The results of the scan in the parameter space for the DM production dominated by the semi-annihilation processes.

- The coloured squares indicate the points, which are within the reach of the future searches for the mediator ϕ and the empty ones are beyond these prospects.

- The points above the grey dot-dashed line can potentially explain the core formation in dSph [1803.09762]

SUMMARY

1. Kinetic equilibrium is a necessary (often implicit) assumption for standard relic density calculations in all the numerical tools…
…while it is not always warranted!

2. Much more accurate treatment comes from solving the full phase space Boltzmann equation (fBE) to obtain result for $f_{\rm DM}(p)$ where one can study also self-thermalization from self-scatterings

3. Introduced DRAKE : a new tool to extend the current capabilities to the regimes beyond kinetic equilibrium

4. Multi-component sectors, when studied at the fBE level, can reveal quite unexpected behavior