International Conference on Neutrinos and Dark Matter (NuDM-2022)
25-28 September 2022
Sharm El-Sheikh, Egypt- Online

Particle dispersion in classical vector DM background

Based on 2205.03617 with S. Yun

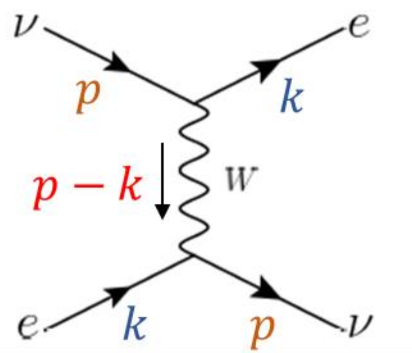
Eung Jin Chun

KIAS KOREA INSTITUTE FOR ADVANCED STUDY

Neutrino oscillations in matter

Wolfenstein 1978

- "The effect of coherent forward scattering (leaving the medium unchanged) must be taken into account when considering the oscillations of neutrinos traveling through matter."



$$\mathcal{H}_{eff} = 2\sqrt{2}\, G_F\, \overline{\nu_{eL}}\gamma^\mu e_L\, \overline{e_L}\gamma_\mu \nu_{eL}$$

$$\Rightarrow \sqrt{2} G_F N_e\, \overline{\nu_{eL}}\gamma^0 \nu_{eL}$$

$$\rightarrow E \approx \mathrm{p} + \frac{m_\nu^2}{2\mathrm{p}} + \sqrt{2} G_F N_e$$

"Wolfenstein potential"

Classical field=coherent state

Glauber, Sudarshan, 1963

- Eigenstate of annihilation operator:

$$|A_c\rangle \propto e^{\int_{\mathrm{k}} \phi_{\mathrm{k}}^r a_{\mathrm{k}}^{r\dagger}}|0\rangle$$

$$\langle A_c|a_{\mathrm{k}}^r|A_c\rangle = \phi_{\mathrm{k}}^r$$

- Energy density:

$$\rho_\phi V = \langle A_c|\int_k E_{\mathrm{k}} a_{\mathrm{k}}^{r\dagger} a_{\mathrm{k}}^r|\phi_c\rangle = \int_{\mathrm{k}} E_{\mathrm{k}}|\phi_{\mathrm{k}}^r|^2$$

$$\hat{A}_\mu(x) = \int \frac{d^3\mathrm{k}}{(2\pi)^3 2E_{\mathrm{k}}}\left[\epsilon_\mu^r a_{\mathrm{k}}^r e^{-ik\cdot x} + \epsilon_\mu^{r*} a_{\mathrm{k}}^{r\dagger} e^{ik\cdot x}\right]$$

$$A_\mu(x) \equiv \langle A_c|\hat{A}_\mu(x)|A_c\rangle = \int \frac{d^3\mathrm{k}}{(2\pi)^3 2E_{\mathrm{k}}}\left[\epsilon_\mu^r \phi_{\mathrm{k}}^r e^{-ik\cdot x} + \epsilon_\mu^{r*} \phi_{\mathrm{k}}^{r*} e^{ik\cdot x}\right]$$

Monochromatic + non-relativistic + isotropic

$$\phi_k^r = (2\pi)^3 2E_0 \delta^3(\mathrm{k}-\mathrm{k}_0)\phi_0/\sqrt{3}, \; k_0 \approx (m_\phi, m_\phi \vec{v})$$

$$\rho_\phi \approx 2E_0^2|\phi_0|^2 \approx 2m_\phi^2|\phi_0|^2$$

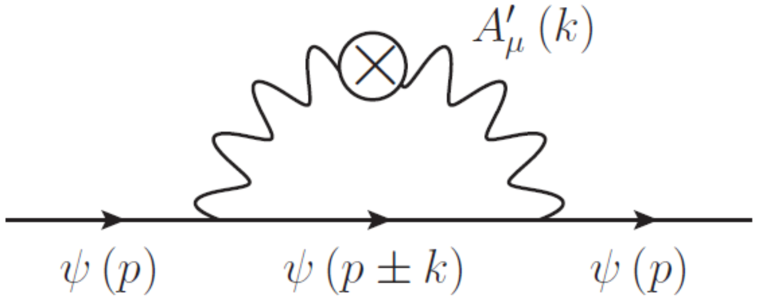
$$|\phi_0| \approx 10^8\mathrm{GeV}\left(\frac{10^{-20}\mathrm{eV}}{m_\phi}\right)$$

Fermions propagating in a vector medium

- Consider fermions propagating in a classical vector DM background

$$\mathcal{L}' = \frac{1}{2} g \hat{A}'_\mu \bar{\psi} \gamma^\mu \psi \qquad \hat{A}'_\mu(x) \to A'_\mu(x) \sim \epsilon^r_\mu \, \phi^r_{\mathrm{k}} e^{-ik\cdot x} + \epsilon^{r*}_\mu \phi^{r*}_{\mathrm{k}} e^{ik\cdot x}$$

- Coherent forward scattering → medium contribution to the propagator
  → modified dispersion & field normalization



$$\mathcal{L}^{\mathrm{eff}}_\psi = \bar{\psi} i \not{\partial} \psi - m_\psi \bar{\psi}\psi - \bar{\psi} \not{\Sigma} \psi$$

$$-i\not{\Sigma} = (-ig_f^2) \int \frac{d^4k}{(2\pi)^4} \Gamma^\mu \frac{(\not{p} + \not{k} + m_\psi)}{(p+k)^2 - m_\psi^2} \Gamma^\nu \Delta^{\gamma'}_{\mu\nu}$$

$$\Delta^{\gamma'}_{\mu\nu}\Big|_{\mathrm{DM}} = \sum_a \epsilon^{a*}_\mu \epsilon^a_\nu \Big(2\pi\delta(k^2 - m^2_{\gamma'}) f^a_{\gamma'} \Big)$$

$$f^a_{\gamma'} = (2\pi)^3 \, \delta^{(3)}[\vec{k} - \vec{k}_{\gamma'}] \Big(\Theta(k_0) n_{\gamma'} + \Theta(-k_0) n_{\gamma'} \Big) \xi^a \qquad \xi^a = 1/3$$

Dispersion relation

- The modified propagator & its poles:

$$-i\not{\Sigma} \simeq -i\left(\not{p}\Sigma_p + \not{k}\Sigma_k \mp m_\psi \Sigma_m\right)$$

$$\Sigma_p = \frac{\delta m_\psi^2}{3}\frac{\Delta + m_{\gamma'}^2}{\left(\Delta + m_{\gamma'}^2\right)^2 - 4m_{\gamma'}^2 E^2}\,, \quad \delta m_\psi^2 = g_\psi^2 \frac{\rho_{\gamma'}}{m_{\gamma'}^2}$$

$$\Sigma_k = \frac{2\delta m_\psi^2}{3}\frac{\Delta - 2m_{\gamma'}^2}{\left(\Delta + m_{\gamma'}^2\right)^2 - 4m_{\gamma'}^2 E^2}\left(\frac{E}{m_{\gamma'}}\right),$$

$$\Sigma_m = 3\Sigma_p\,,$$

$$\not{p} - m_\psi - \not{\Sigma} \simeq \gamma^0\Big(E\left(1-\Sigma_p\right) - m_{\gamma'}\Sigma_k\Big) - \vec{\gamma}\cdot\vec{p}\Big(1-\Sigma_p\Big) - m_\psi\Big(1 \mp \Sigma_m\Big) = 0\,.$$

$$\Big(E\left(1-\Sigma_p\right) - m_{\gamma'}\Sigma_k\Big)^2 = |\vec{p}|^2\Big(1-\Sigma_p\Big)^2 + m_\psi^2\Big(1 - \alpha_m\Sigma_p\Big)^2,$$

- Solutions for $E = E\left(\mathrm{p}; k, m_{\gamma'}^2, \delta m_\psi^2\right)$, or $\Delta \equiv E^2 - \mathrm{p}^2 - m_\psi^2$.

$$\sum_{i=0}^{5} \Upsilon_i \Delta^i = 0$$

$$\begin{aligned}
\Upsilon_0 &= \frac{1}{9} m_{\gamma'}^4 \delta m_\psi^2 \Big[\delta m_\psi^2 \Big(8|\vec{p}|^2 + (9-\alpha_m^2)\, m_\psi^2 \Big) \\
&\quad + 6 \Big(m_{\gamma'}^2 - 4 m_\psi^2 - 4|\vec{p}|^2 \Big) \Big(4|\vec{p}|^2 + (3+\alpha_m)\, m_\psi^2 \Big) \Big] , \\
\Upsilon_1 &= m_{\gamma'}^2 \Big[-\frac{2}{9} \delta m_\psi^2 \Big(\delta m_\psi^2 \left(10|\vec{p}|^2 + (9+\alpha_m^2)\, m_\psi^2 \right) \\
&\quad - 12 \left(m_\psi^2 + |\vec{p}|^2 \right) \left(2|\vec{p}|^2 + (3-\alpha_m)\, m_\psi^2 \right) \Big) \\
&\quad + \frac{1}{3} m_{\gamma'}^2 \Big(48 \left(m_\psi^2 + |\vec{p}|^2 \right)^2 \\
&\quad - 2\delta m_\psi^2 \left(22|\vec{p}|^2 + (21+\alpha_m)\, m_\psi^2 \right) + 3\delta m_\psi^4 \Big) \\
&\quad + 2 m_{\gamma'}^4 \Big(-4 \left(m_\psi^2 + |\vec{p}|^2 \right) + \delta m_\psi^2 \Big) + m_{\gamma'}^6 \Big] , \\
\Upsilon_2 &= \frac{8}{9} \delta m_\psi^4 \Big(|\vec{p}|^2 + \frac{9-\alpha_m^2}{8} m_\psi^2 \Big) \\
&\quad - \delta m_\psi^2 m_{\gamma'}^2 \Big(2\delta m_\psi^2 - \frac{42-2\alpha_m}{3} m_\psi^2 - \frac{40}{3} |\vec{p}|^2 \Big) \\
&\quad + 2 m_{\gamma'}^4 \Big(8 \left(m_\psi^2 + |\vec{p}|^2 \right) - 3\delta m_\psi^2 \Big) - 4 m_{\gamma'}^6 , \\
\Upsilon_3 &= \delta m_\psi^4 - \frac{2}{3} \delta m_\psi^2 \Big(2|\vec{p}|^2 + (3-\alpha_m)\, m_\psi^2 \Big) \\
&\quad + m_{\gamma'}^2 \Big(6\delta m^2 - 8 \left(m_\psi^2 + |\vec{p}|^2 \right) \Big) + 6 m_{\gamma'}^4 , \\
\Upsilon_4 &= -2\delta m_\psi^2 - 4 m_{\gamma'}^2 , \qquad \Upsilon_5 = 1 .
\end{aligned}$$

Field normalization in medium

- Inverse propagator in vacuum:

$$S_F = \frac{1}{\not p - m_\psi} = \frac{\not p + m_\psi}{p^2 - m_\psi^2} \Longrightarrow \left[\frac{\not p + m_\psi}{2E}\right]_{E=\sqrt{\mathrm{p}^2+m_\psi^2}}$$

- Normalization of field in medium: $\psi \to z^{1/2}\,\psi$:

$$S_F = \frac{1}{\not p - \not\Sigma - m_\psi} \equiv \frac{1}{\not V - M_\psi} = \frac{\not V + M_\psi}{V^2 - M_\psi^2} \Longrightarrow \left[\frac{\not V + M_\psi}{2V_0\left(V_0 - \sqrt{\mathrm{V}_\mathrm{p}^2 + M_\psi^2}\right)}\right]_{E=E_\mathrm{p}}$$

$$V_0 \equiv E\left(1-\Sigma_p\right) - m_{\gamma'}\Sigma_k$$
$$\mathrm{V}_\mathrm{p} \equiv \mathrm{p}\left(1-\Sigma_p\right)$$
$$M_\psi \equiv m_\psi\left(1 \mp 3\Sigma_p\right)$$

$$\Rightarrow Z = \left[\frac{\partial}{\partial E}\left(V_0 - \sqrt{V_\mathrm{p}^2 + M_\psi^2}\right)\right]_{\mathrm{pole}}^{-1}$$

Limiting analytical solutions (γ_μ)

- $\delta m_\psi^2 < m_\psi m_{\gamma'}$:

$$\Delta \approx \begin{cases} \dfrac{2\delta m_\psi^2 m_{\gamma'}^2}{\delta m_\psi^2 + 3m_{\gamma'}^2} \\ \delta m_\psi^2 \end{cases} \qquad Z \approx \begin{cases} \dfrac{3m_{\gamma'}^2}{\delta m_\psi^2 + 3m_{\gamma'}^2} & \text{for} \quad |\vec{p}| \gg m_\psi \\ 1 & \text{for} \quad |\vec{p}| \ll m_\psi \end{cases}$$

- $\delta m_\psi^2 > m_\psi m_{\gamma'}$:

$$\Delta \approx \begin{cases} \dfrac{2}{\sqrt{3}}\delta m_\psi |\vec{p}| \\ \delta m_\psi^2 \end{cases} \qquad Z \approx \begin{cases} 1/2 & \text{for} \quad |\vec{p}| \gg \delta m_\psi \\ \dfrac{\delta m_\psi^2/2}{2m_\psi^2 + \delta m_\psi^2} & \text{for} \quad |\vec{p}| \ll \delta m_\psi \end{cases}$$

Limiting analytical solutions ($\gamma_\mu \gamma_5$)

- $\delta m_\psi^2 < m_{\gamma'}^2$, $m_{\gamma'}^2 < \delta m_\psi^2$:

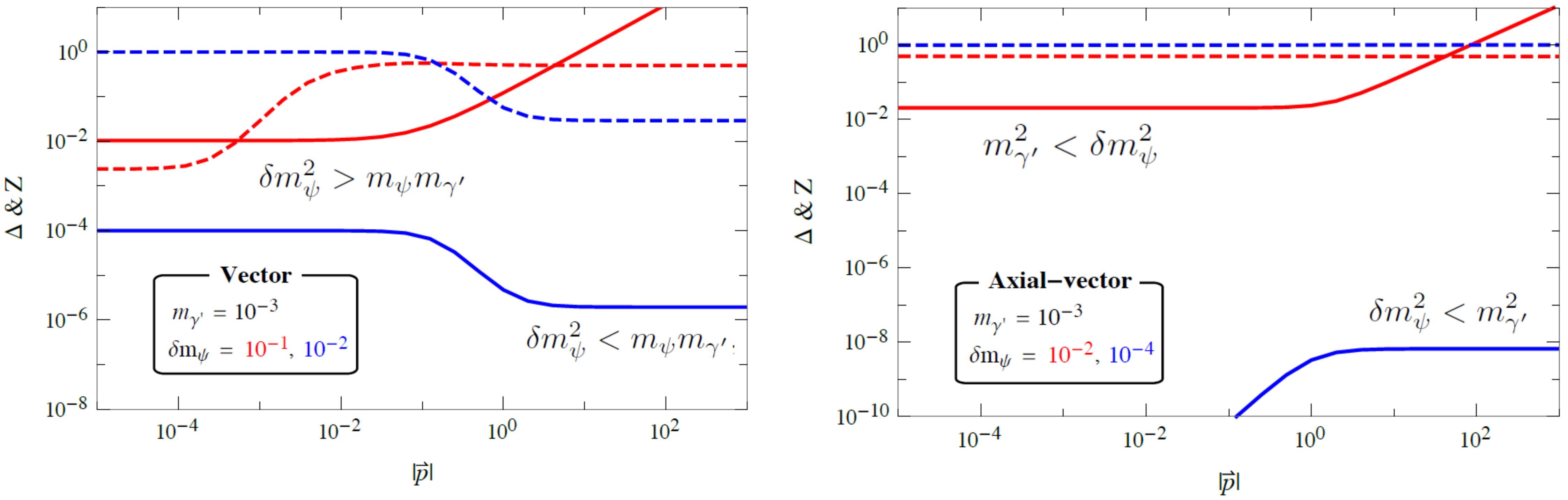
$$\Delta \approx \begin{cases} \dfrac{2\delta m_\psi^2 m_{\gamma'}^2}{\delta m_\psi^2 + 3m_{\gamma'}^2} & \text{for } \ |\vec{p}| \gg m_\psi \\ \dfrac{2}{3}\dfrac{\delta m_\psi^2 m_{\gamma'}^2}{\delta m_\psi^2 + m_{\gamma'}^2}\dfrac{|\vec{p}|^2}{m_\psi^2} & \text{for } \ |\vec{p}| \ll m_\psi \end{cases}$$

$$Z \approx 1 \qquad \text{for } \ \delta m_\psi^2 < m_{\gamma'}^2$$
$$Z \approx 1/2 \qquad \text{for } \ m_{\gamma'}^2 < \delta m_\psi^2$$

- $\delta m_\psi^2 > m_{\gamma'}^2$:

$$\Delta \approx \begin{cases} \dfrac{2}{\sqrt{3}}\delta m_\psi |\vec{p}| & \text{for } \ |\vec{p}| \gg m_\psi \\ 2\delta m_\psi m_\psi + \delta m_\psi^2 & \text{for } \ |\vec{p}| \ll m_\psi \end{cases}$$

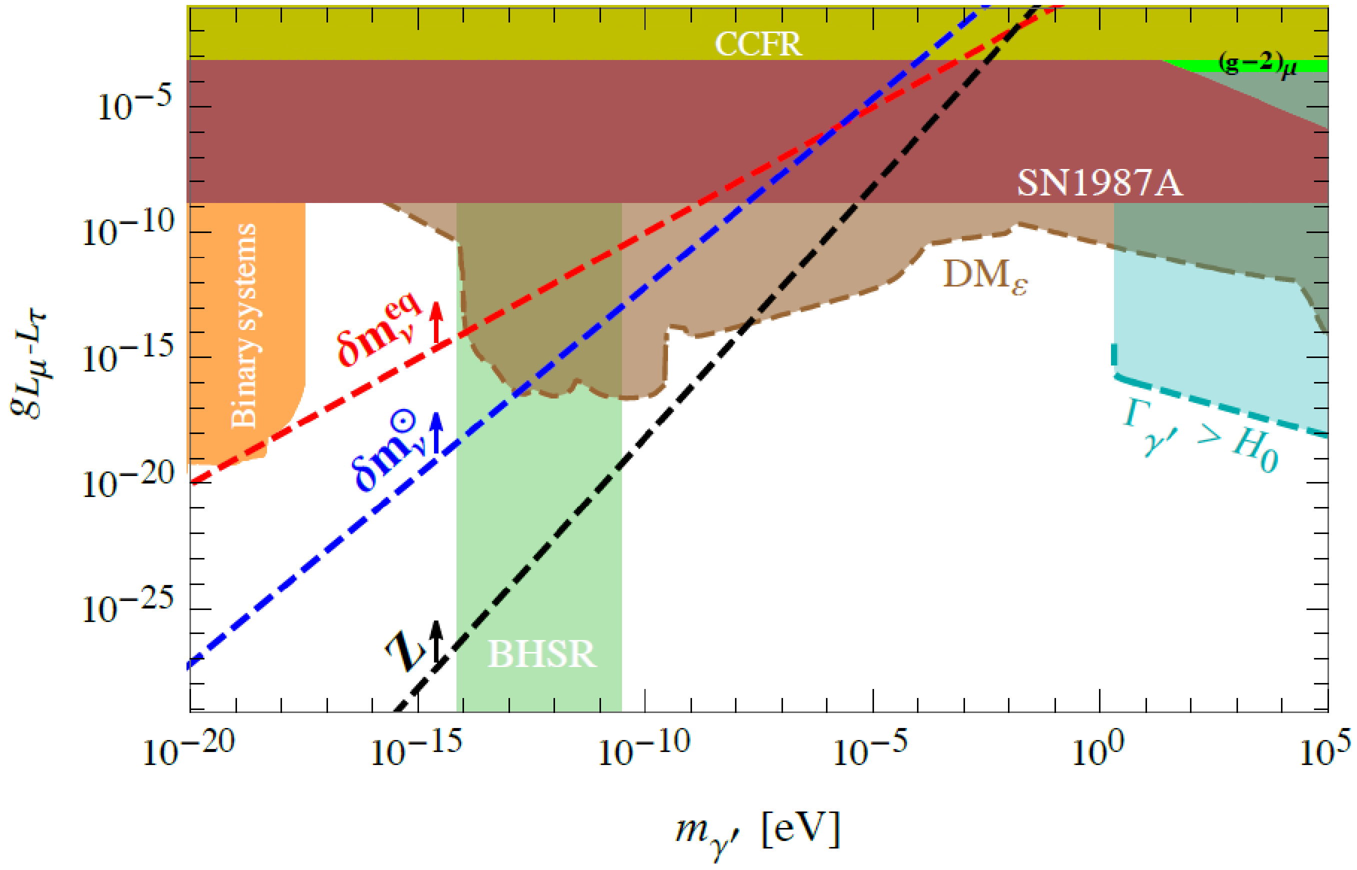
Numerical solutions for Δ & Z



Constraining VDM couplings

- The rest mass correction, $E \sim \sqrt{m_\psi^2 + \delta m_\psi^2}$, may be in conflict with the observations $\left(m_\psi\right)_{\text{obs.}}$: m_e and m_ν^{eq}.
- In high-momentum limit, $\Delta \propto \mathrm{p}\, \delta m_\nu$ amounts to add an constant potential $\delta E_\nu \propto \delta m_\nu$ spoiling the MSW effect if VDM is flavor-dependent.
- The normalization $Z = 1/2$ in the relativistic limit contradict to various SM precision measurements such as lepton-flavor universality and so on.

Constraints on $L_\mu - L_\tau$

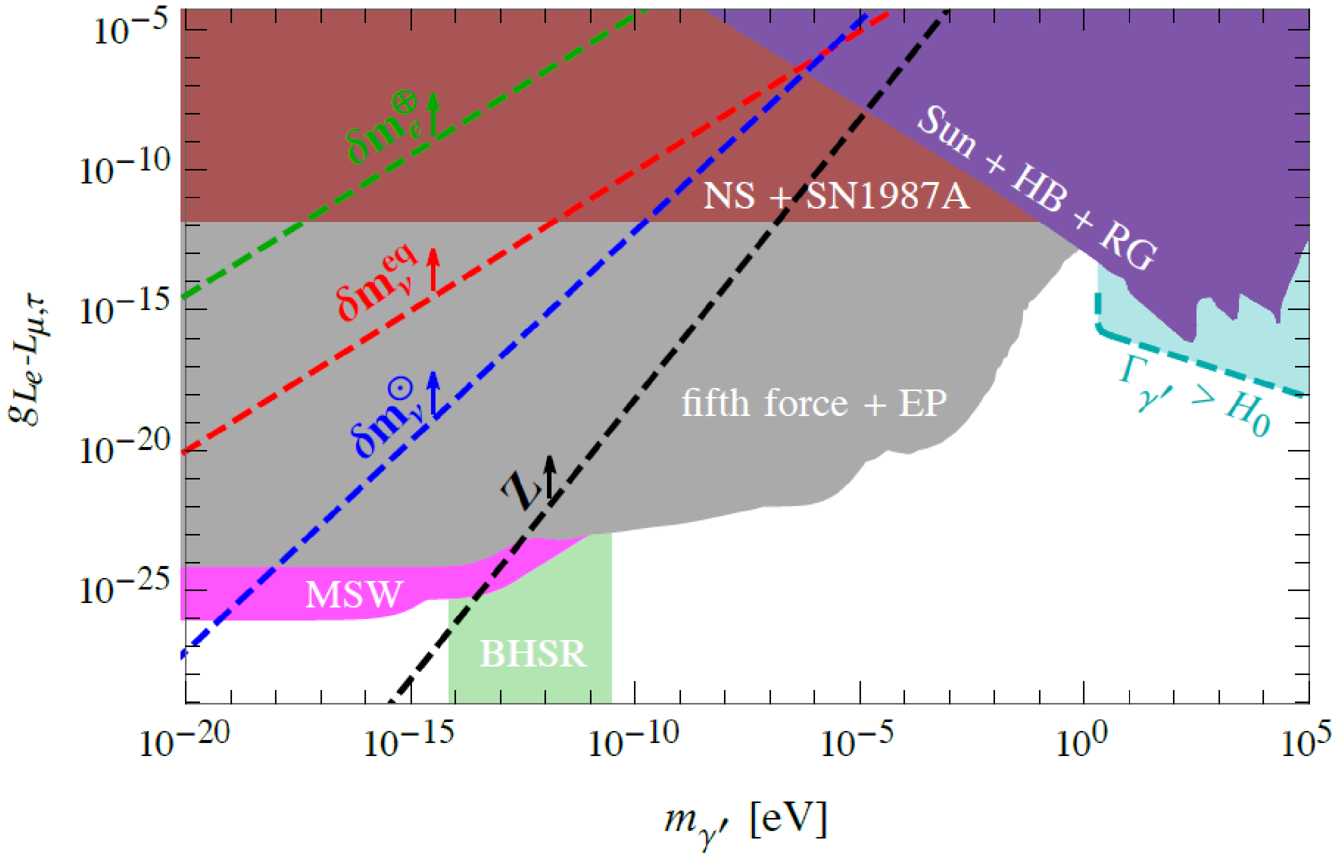


$\delta m_\nu^{eq} \sim T_{eq}^{3/2}$

$\delta E_\nu \sim \delta m_\nu$

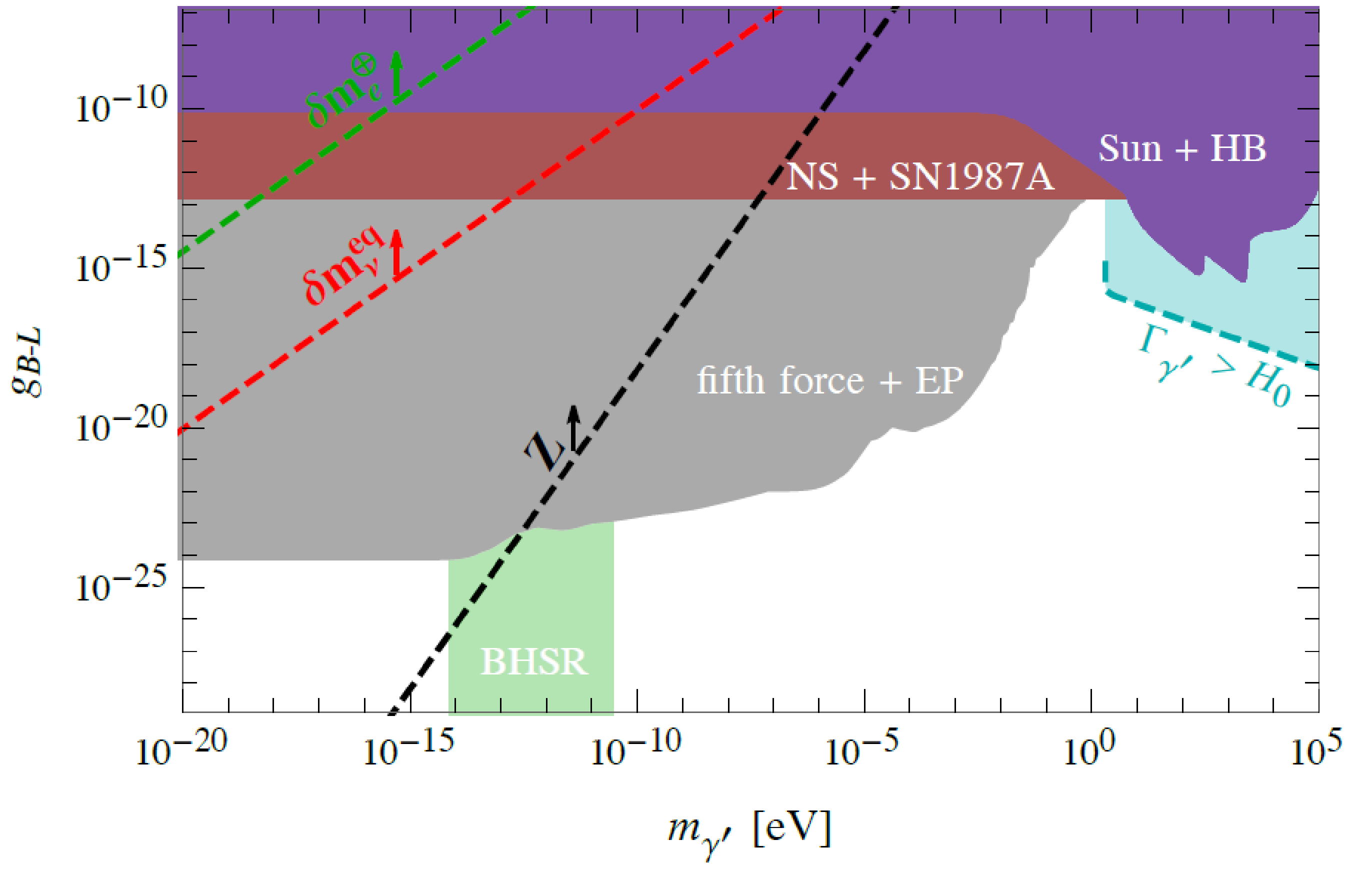
$Z_{\mu,\tau} \neq 1$

Constraints on $L_e - L_{\mu,\tau}$



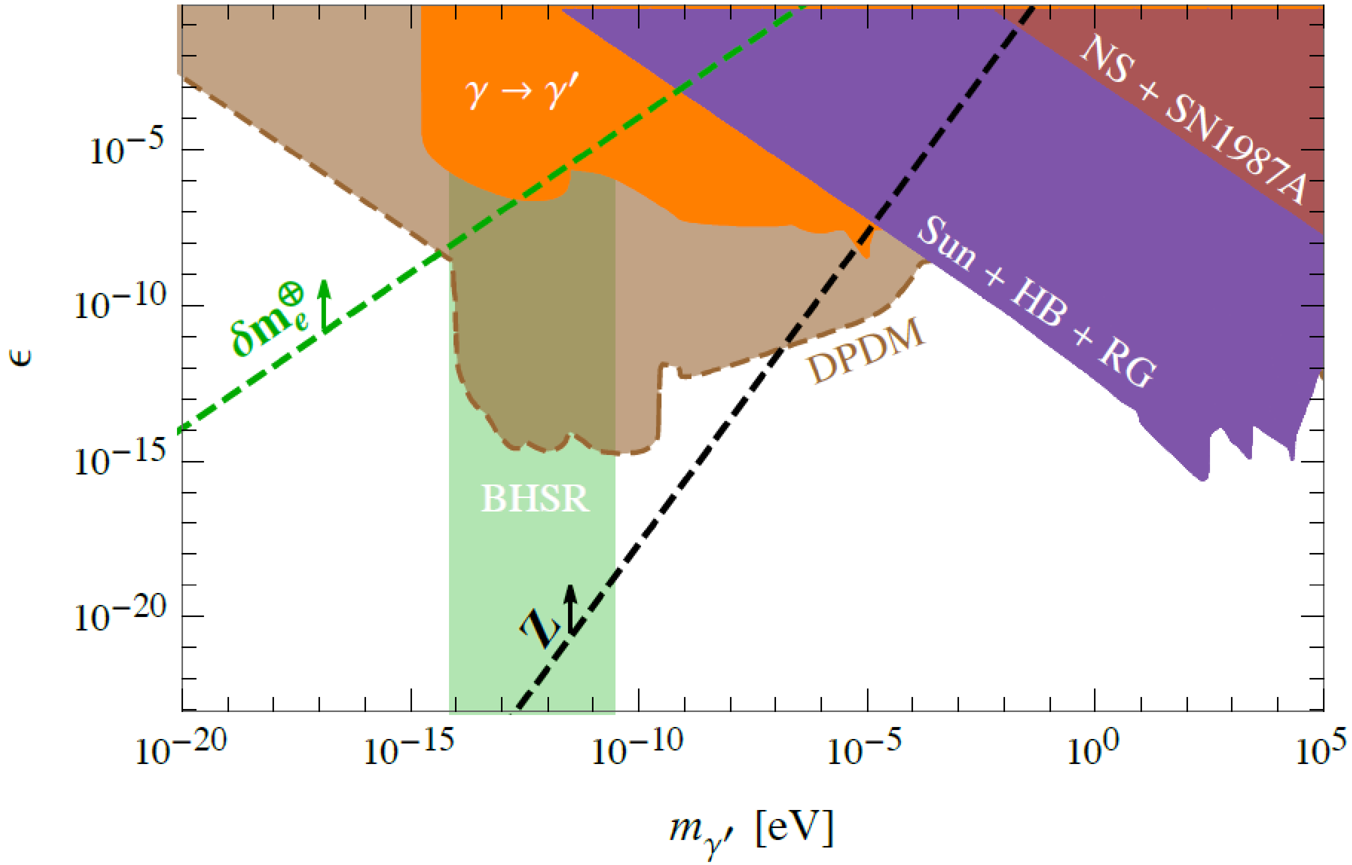
$Z_l \neq 1$

Constraints on $B - L$



$Z_{q,l} \neq 1$

Constraints on the kinetic mixing



$Z_l \neq Z_\nu = 1$

Summary

- When considering an ultra-light (scalar or vector) DM, its impacts on the particle dispersion and normalization have to be taken into account.
- There can be a medium-induced rest mass, or a potential term in the dispersion which may be sizable and thus constrained by the observations.
- For VDM, a peculiar field normalization $Z = 1/2$ appears in the relativistic limit which highly constrains its coupling.