Noisy Gates for Quantum Computing

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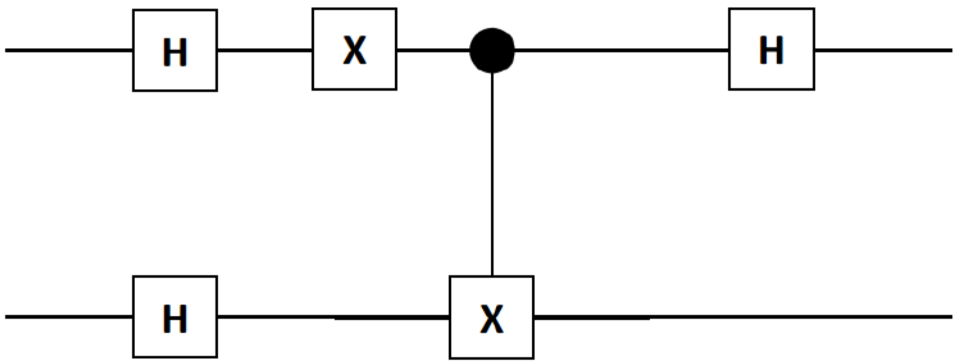
Real Quantum Computers



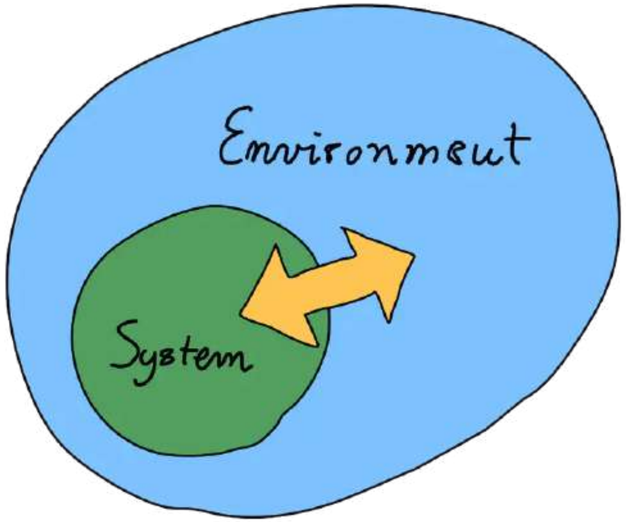
In an ideal (= isolated) world, quantum computers run beautifully

In real life, they are subject to **noise**

- Quantum Error Correction, but even more qubits are needed
- NISQ (Noise Intermediate-Scale Quantum) devices

Study the noise

A **proper theoretical modelling** of the effect of the environment on a quantum systems allows to:



- Have a **physical understanding** of the sources of noise
- Suggest strategies to **mitigate errors**
- Perform **accurate simulations** to predict how the performances scale with the number of qubits/gates.

Georgopoulos, K., Emary, C., & Zuliani, P. (2021). Modeling and simulating the noisy behavior of near-term quantum computers. *Physical Review A*, *104*(6), 062432.
Sun, J., Yuan, X., Tsunoda, T., Vedral, V., Benjamin, S. C., & Endo, S. (2021). Mitigating realistic noise in practical noisy intermediate-scale quantum devices. *Physical Review Applied*, *15*(3), 034026.
Guerreschi, G. G., & Matsuura, A. Y. (2019). QAOA for Max-Cut requires hundreds of qubits for quantum speed-up. *Scientific reports*, *9*(1), 1-7.
Xue, C., Chen, Z. Y., Wu, Y. C., & Guo, G. P. (2021). Effects of quantum noise on quantum approximate optimization algorithm. *Chinese Physics Letters*, *38*(3), 030302.
Resch, S., & Karpuzcu, U. R. (2021). Benchmarking quantum computers and the impact of quantum noise. *ACM Computing Surveys (CSUR)*, *54*(7), 1-35.

Standard noise model

Breuer and Petruccione: *The Theory of Open Quantum Systems*, Oxford University Press (2002)

Theory of **open quantum systems**

$$|\psi\rangle \;\rightarrow\; \rho = |\psi\rangle\langle\psi|$$

State vector   Density matrix

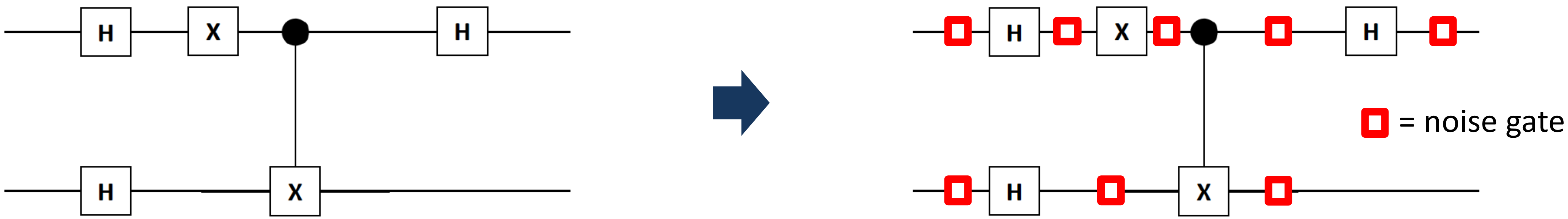
$$\frac{d}{dt}\rho_t = -\frac{i}{\hbar}[H_t, \rho_t] + \sum_k \gamma_k \left[L_k \rho_t L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho_t\} \right]$$

Internal evolution   Effect of the environment

Issues to deal with:

- More complicated dynamics; how to model the environment efficiently
- With the density matrix, the problem scales quadratically with the size of the problem.

How to describe noises



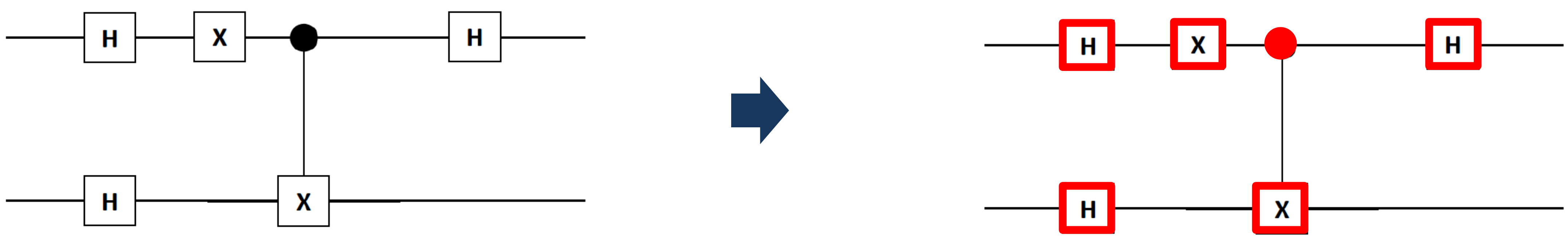
- Gates and noise are formally **decoupled** (a sort of Trotterizzation), because time scales are small (IBM: gate time ~ 10^{-8} s, decoherence times ~ 10^{-4} s)
- Noises (like gates) formally act instantly: Lindblad → Kraus

$$\rho \to \sum_i K_i \rho K_i^\dagger \qquad \sum_i K_i^\dagger K_i = 1$$

- Use the **quantum-jump-like approach** to replace the density matrix with (stochastic) state vector → stochastic dynamics

Noisy Gates

Our approach: provide a more accurate description of the noisy behaviour of a quantum computer



- Noises are **embedded** in the gate → more realistic picture
- State vector (stochastic) description

From Lindblad to stochastic differential equations (SDE)

$$\frac{d}{dt}\rho_t = -\frac{i}{\hbar}[H_t, \rho_t] + \sum_k \gamma_k \left[L_k \rho_t L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho_t\}\right] = \mathfrak{D}(\rho)$$

Gate Noise

$$d|\psi_t\rangle = \left[-\frac{i}{\hbar}H_t dt + \sum_k \left(i\sqrt{\gamma_k} L_k dW_{k,t} - \frac{\gamma_k}{2} L_k^\dagger L_k dt\right)\right]|\psi_t\rangle$$

Stochastic evolution for the state vector (stochastic unravelling)

Formal equivalence: $\rho_t = \mathbb{E}[|\psi_t\rangle\langle\psi_t|]$

Noisy gate

Bassi, A., & Deckert, D. A. (2008). Noise gates for decoherent quantum circuits. *Physical Review A*, *77*(3), 032323.

$$\mathrm{d}\left|\psi_s\right\rangle = \left[-\frac{i}{\hbar}\mathrm{H}_s\mathrm{d}s + \sum_{k=1}^{N^2-1}\left[i\epsilon\mathrm{dW}_{k,s} - \frac{\epsilon^2}{2}\mathrm{d}s\mathrm{L}_k^\dagger\right]\mathrm{L}_k\right]\left|\psi_s\right\rangle$$

The dynamics is **linear**, therefore it can be represented as a gate (noisy gate)

$$\left|\psi_{s=1}(\boldsymbol{\xi})\right\rangle = \bar{\mathrm{N}}(\boldsymbol{\xi})\left|\psi_0\right\rangle$$

Due to the noises **ξ**, the gate is not unitary and norm preserving. But at the statistical level the trace is preserved, and one recovers the standard (Lindblad) behaviour.

Solution of the SDE

Gardiner, C. W. (1985). *Handbook of stochastic methods* (Vol. 3, pp. 2-20). Berlin: Springer.
Arnold, L. (1974). Stochastic differential equations. *New York:* John Wiley & Sons

$$\bar{\mathrm{N}}(\boldsymbol{\xi}) = \mathrm{U}_g e^{\Lambda} e^{\Xi(\boldsymbol{\xi})}$$

U_g = noiseless gate

$s \in [0, 1]$

$$\Lambda = -\frac{\epsilon^2}{2} \int_0^1 \mathrm{d}s \sum_{k=1}^{N^2-1} [\mathrm{L}_{k,s}^{\dagger} \mathrm{L}_{k,s} - \mathrm{L}_{k,s}^2]$$

Deterministic contribution of the noise, to order O(ϵ^2)

$$\Xi(\boldsymbol{\xi}) = i\epsilon \sum_{k=1}^{N^2-1} \int_0^1 \mathrm{dW_{k,s}} \mathrm{L}_{k,s}$$

Stochastic contribution of the noise, to order O(ϵ^2)

Operators in the interaction picture

IBM single qubit gates

Krantz, P., Kjaergaard, M., Yan, F., Orlando, T. P., Gustavsson, S., & Oliver, W. D. (2019). A quantum engineer's guide to superconducting qubits. *Applied Physics Reviews*, *6*(2), 021318.

$$U(\theta, \phi) = e^{-i\theta R_{xy}(\phi)/2} \qquad R_{xy}(\phi) = \cos(\phi)X + \sin(\phi)Y$$

Generated by the Hamiltonian

$$H(\theta, \phi) = \frac{\theta\hbar}{2} R_{xy}(\phi)$$ for a duration $s \in [0, 1]$

Rotations along the z-axis are implemented as virtual gates.

Note: how to implement the pulse

IBM computers: main single qubit noises

Krantz, P., Kjaergaard, M., Yan, F., Orlando, T. P., Gustavsson, S., & Oliver, W. D. (2019). A quantum engineer's guide to superconducting qubits. *Applied Physics Reviews*, *6*(2), 021318.
Georgopoulos, K., Emary, C., & Zuliani, P. (2021). Modeling and simulating the noisy behavior of near-term quantum computers. *Physical Review A*, *104*(6), 062432.

Depolarization: initial state → maximally mixed state

$$\mathfrak{D}_d(\rho) = \gamma_d \sum_{k=1}^{3} \left[\sigma^k \rho \sigma^k - \rho\right]$$

Relaxation: amplitude damping (initial state → ground state) + phase damping

$$\mathfrak{D}_R(\rho) = \gamma_1 \left[\sigma^+ \rho \sigma^- - \frac{1}{2}\{\mathbb{P}^{(1)}, \rho\}\right] + \gamma_z \left[\mathrm{Z}\rho\mathrm{Z} - \rho\right]$$

$$\lambda_k \sim 10^4 \mathrm{Hz}$$

$$t_g \sim 10^{-8} s$$

$$\epsilon = \sqrt{\lambda t_g} \ll 1$$

Combining the noises and diagonalizing in the canonical form:

$$\mathfrak{D}_\lambda^{(1)}(\rho) = \lambda \sum_{k=1}^{3} \left[\mathrm{L}_k \rho \mathrm{L}_k^\dagger - \frac{1}{2}\{\mathrm{L}_k^\dagger \mathrm{L}_k, \rho\}\right]$$

$$\mathrm{L}_1 = \sqrt{\frac{\lambda_1}{\lambda}}\sigma^-, \quad \mathrm{L}_2 = \sqrt{\frac{\lambda_2}{\lambda}}\sigma^+, \quad \mathrm{L}_3 = \sqrt{\frac{\lambda_3}{\lambda}}\mathrm{Z}$$

$$\lambda_1 = 2\gamma_d, \; \lambda_2 = 2\gamma_d + \gamma_1, \; \lambda_3 = \gamma_d + \gamma_z \qquad \lambda = \lambda_1 + \lambda_2 + \lambda_3$$

Single qubit noisy gate

$$\bar{\mathrm{N}}(\boldsymbol{\xi}) = \mathrm{U}_g e^{\Lambda} e^{\bar{\Xi}(\boldsymbol{\xi})}$$

$$\Lambda(\theta,\phi) = -\frac{\epsilon_1^2+\epsilon_2^2}{4}\mathbb{1} - \frac{\epsilon_1^2-\epsilon_2^2}{4}\frac{\sin(\theta/2)}{\theta/2}\mathrm{R}(\theta,\bar{\phi})$$

$$\bar{\phi} = \phi + \pi/2$$

$$\Xi(\theta,\phi|\boldsymbol{\xi}) = if_0\mathrm{Z} + if_1\mathrm{R}_{xy}(\phi) + if_2\mathrm{R}_{xy}(\bar{\phi})$$

$$f_0 = \epsilon_3\xi_{3,+} - i\frac{e^{i\phi}\epsilon_2\xi_{2,-} - e^{-i\phi}\epsilon_1\xi_{1,-}}{2}$$

$$f_1 = \frac{e^{i\phi}\epsilon_2\xi_{2,w} + e^{-i\phi}\epsilon_1\xi_{1,w}}{2},$$

$$f_2 = \epsilon_3\xi_{3,-} + i\frac{e^{i\phi}\epsilon_2\xi_{2,+} - e^{-i\phi}\epsilon_1\xi_{1,+}}{2}$$

Stochastic properties of the noise

$$\mathbb{E}[\xi_{k,\pm}^2] = \frac{1}{2}[1 \pm \frac{\sin(2\theta)}{2\theta}]$$

$$\mathbb{E}[\xi_{k,+}\xi_{j,-}] = \frac{1-\cos(2\theta)}{4\theta}\delta_{kj}$$

$$\mathbb{E}[\xi_{k,w}^2] = 1,\ \mathbb{E}[\xi_{k,+}\xi_{k,w}] = \sin(\theta)/\theta$$

$$\mathbb{E}[\xi_{k,-}\xi_{k,w}] = [1-\cos(\theta)]/\theta$$

IBM two qubit gates and noises

Krantz, P., Kjaergaard, M., Yan, F., Orlando, T. P., Gustavsson, S., & Oliver, W. D. (2019). A quantum engineer's guide to superconducting qubits. Applied Physics Reviews, 6(2), 021318.
McKay, D. C., Wood, C. J., Sheldon, S., Chow, J. M., & Gambetta, J. M. (2017). Efficient Z gates for quantum computing. Physical Review A, 96(2), 022330.
Rigetti, C., & Devoret, M. (2010). Fully microwave-tunable universal gates in superconducting qubits with linear couplings and fixed transition frequencies. Physical Review B, 81(13), 134507.

$$\mathrm{U}^{(1,2)}(\theta,\phi) \;=\; e^{-i\theta \mathrm{Z}^{(1)}\otimes \mathrm{R}_{xy}^{(2)}(\phi)/2}$$

$$R_{xy}(\phi) = \cos(\phi)X + \sin(\phi)Y$$

Generated by the **Hamiltonian**

$$\mathrm{H}^{(1,2)}(\theta,\phi) = \frac{\hbar\theta}{2}\mathrm{Z}^{(1)} \otimes \mathrm{R}_{xy}^{(2)}$$

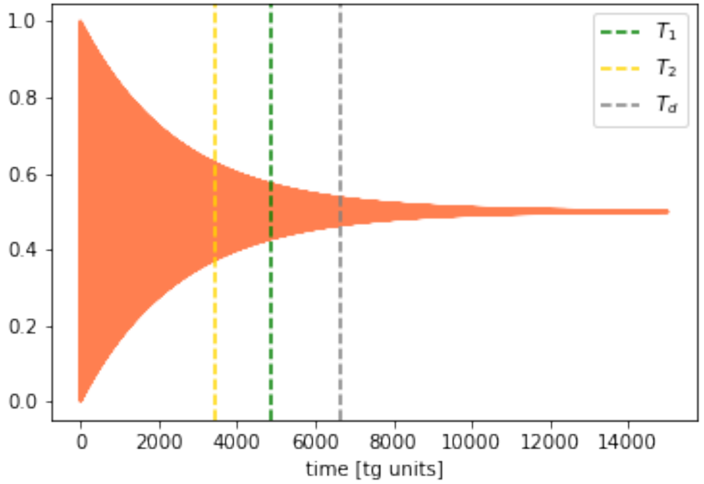
for a duration $s \in [0,1]$

Single qubit **noises**

$$\mathfrak{D}_{\epsilon^2}^{(1,2)}(\rho) = \lambda \sum_{i\in\{0,1\}} \sum_{k=1}^{3} \left[\mathrm{L}_k^{(i)}\rho \mathrm{L}_k^{(i)\dagger} - \frac{1}{2}\{\mathrm{L}_k^{(i)\dagger}\mathrm{L}_k^{(i)}, \rho\}\right]$$

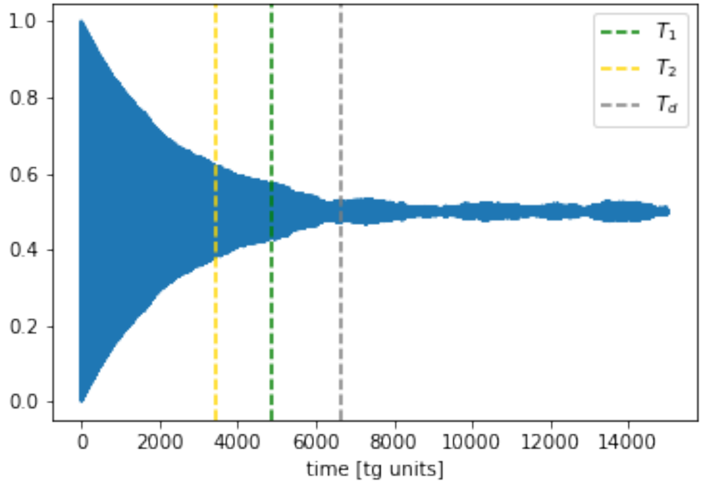
Simulation of the (single qubit) noisy X gate

Lindblad equation



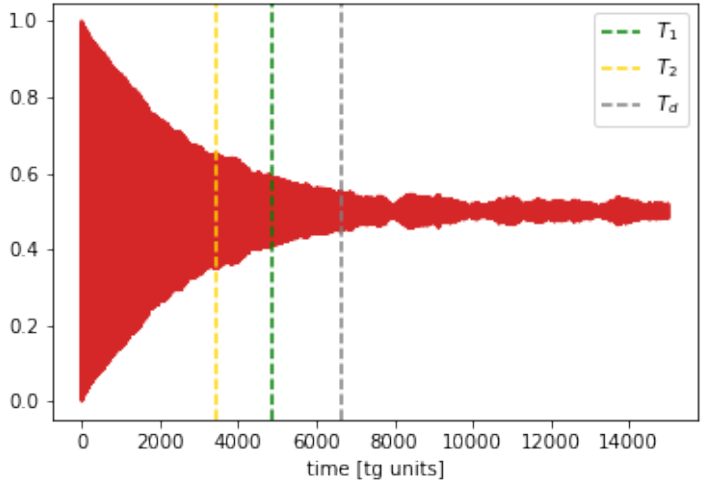
Numerical solution

Noisy gates



Average over 1000 realizations

Qiskit simulator

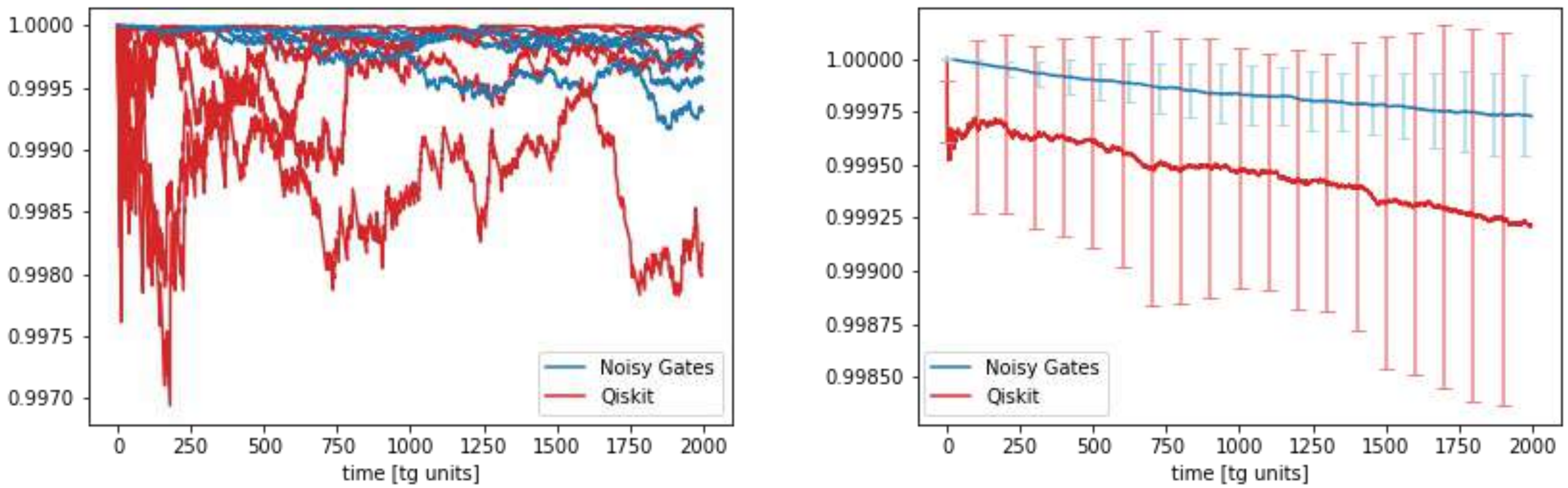


Average over 1000 realizations

Initial state = $|0\rangle$

Shown: $\langle 0|\rho|0\rangle$

Simulation of the (single qubit) noisy X gate



Fidelity

Lindblad and noisy gates

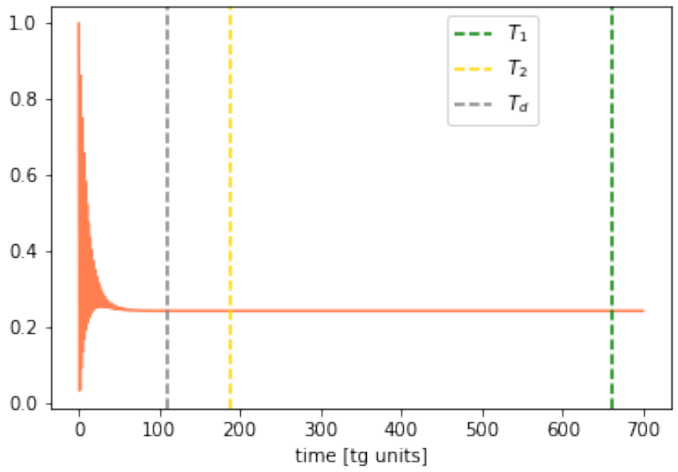
Lindblad and Qiskit simulator

100 independent simulations each including 1000 runs

(5 shown on the left)

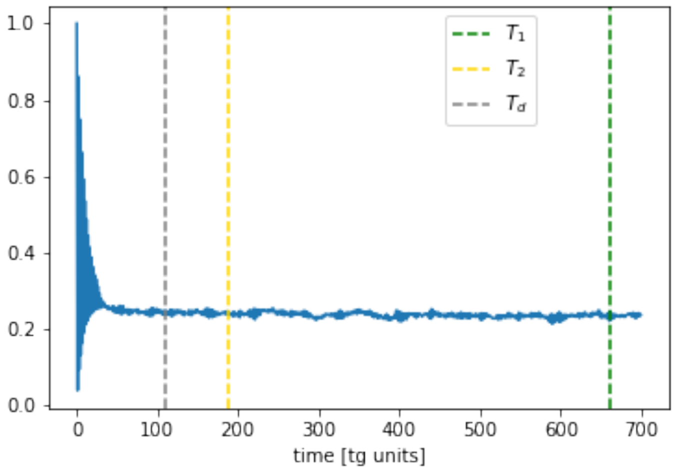
Simulation of the (two qubit) noisy CR gate

Lindblad equation



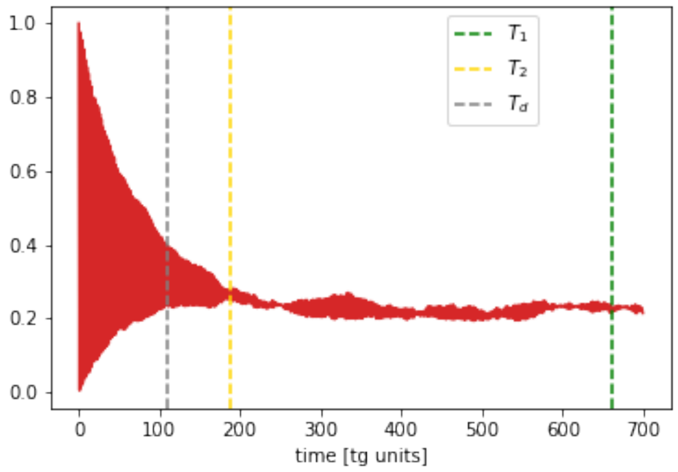
Numerical solution

Noisy gates



Average over 1000 realizations

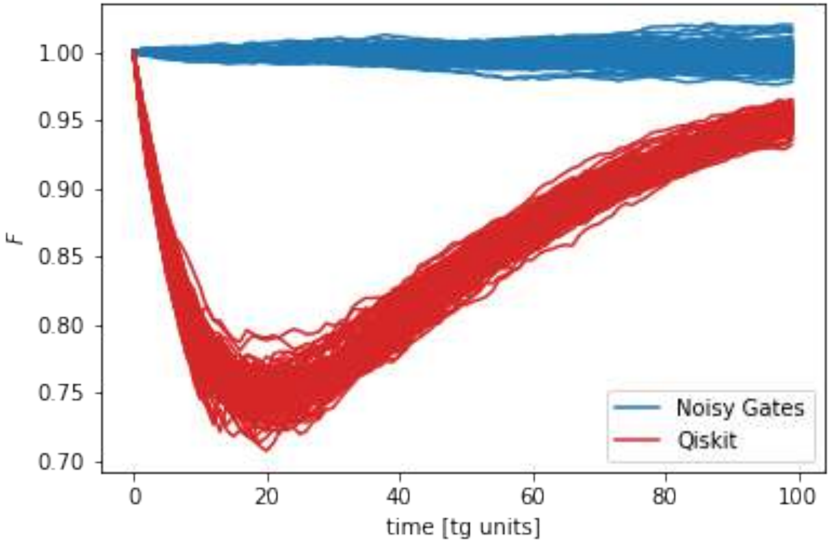
Qiskit simulator

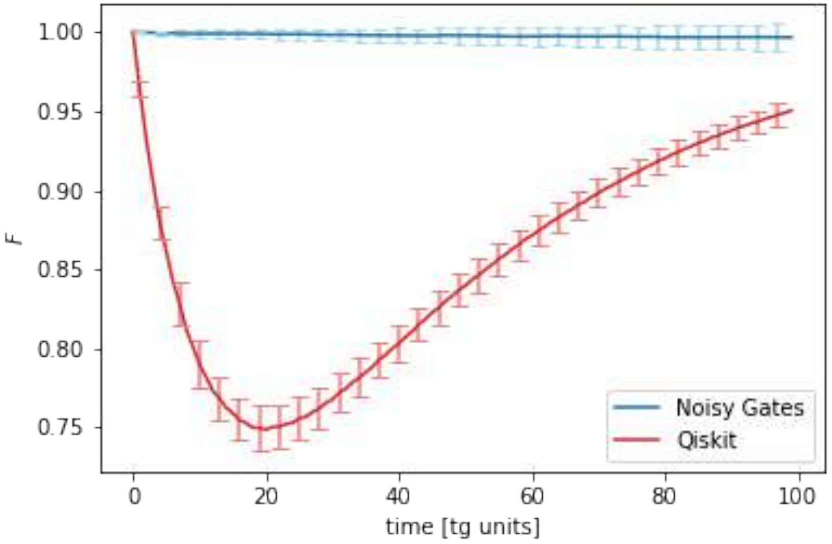


Average over 1000 realizations

Initial state = $|10\rangle$ Shown: $\langle 10|\rho|10\rangle$

Simulation of the (single qubit) noisy X gate





Fidelity

Lindblad and noisy gates

Lindblad and Qiskit simulator

100 independent simulations each including 1000 runs

Future work

Better analysis of noises, especially for two qubit gates

Comparison with real quantum computer

Application to algorithm of interest, to extract “long-time” behaviour

Thank you

Check the **poster** of **Giovanni di Bartolomeo** and **Michele Vischi** for further details