Application of Quantum Computing to High Energy Physics

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Quantum Technologies for High Energy Physics (CERN, Nov. 2, 2022)

Recent Projects in ICEPP

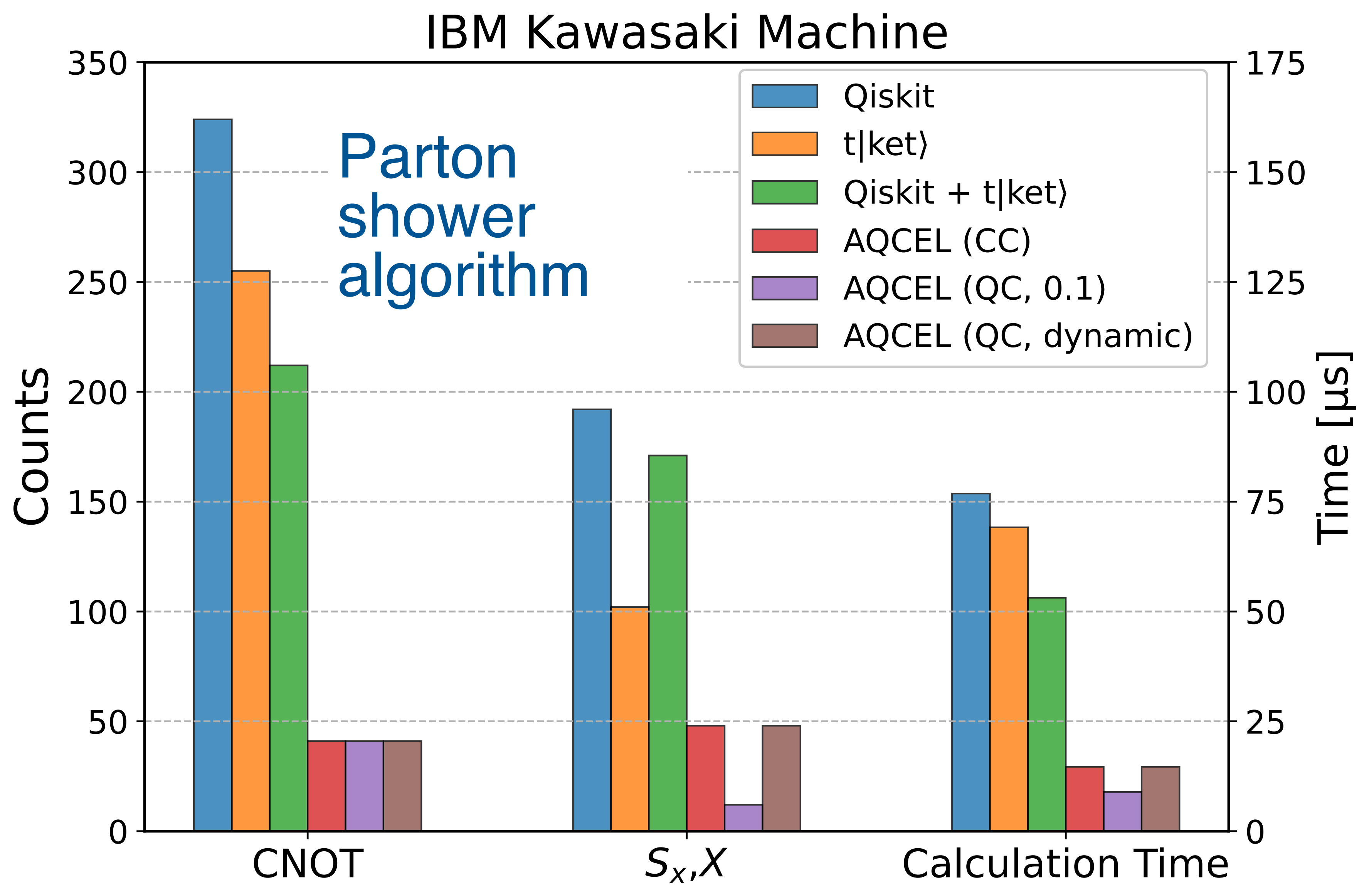
- **software**:
  - quantum simulation *(this talk!)*
  - circuit optimization (AQCEL)

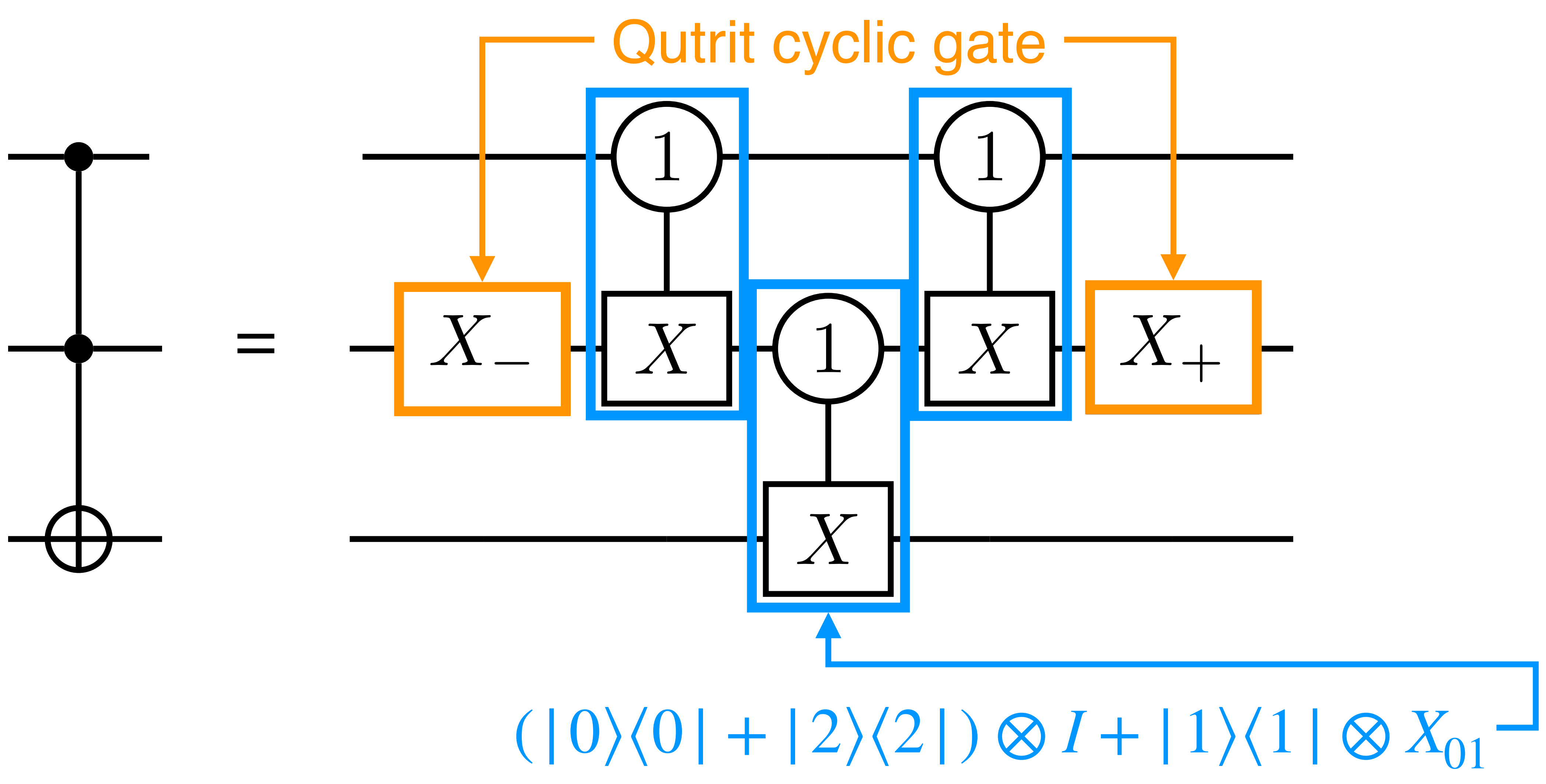
    [Jang, et.al. Quantum 6, 798 (2022), collaboration with LBNL]
  - quantum machine learning
    - classical data: vanishing gradient problem (barren plateau) induced by data-encoding [work in progress, K. Kamisoyama, LN, K. Terashi]
    - quantum data: HEP application

      [work in progress with IBM]
- **middleware**: qutrit-based Toffoli implementation via pulse control

  [Jang, et.al. Poster presented in IEEE Quantum Week 2022, with IBM]
- **hardware**: development of transmon

  [work in progress with IBM]





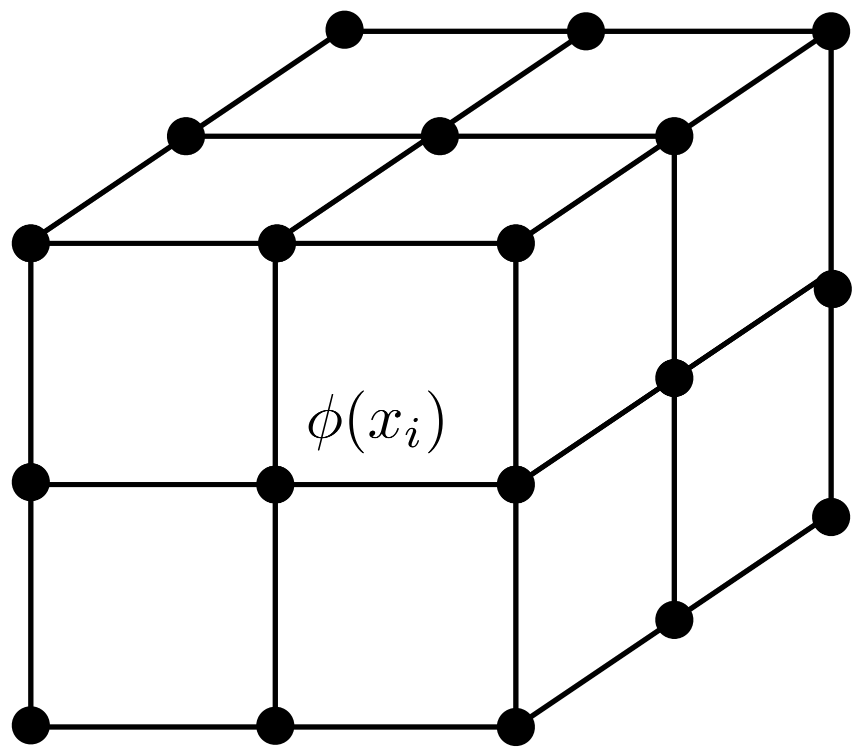
[cf. Inada et al., arXiv:2109.00086]

Numerical study of Quantum Field Theory

- (conventional) lattice QFT
  - discretize **spacetime**
    →using Monte Carlo method

$$Z = \int [d\phi] e^{-S[\phi]}$$

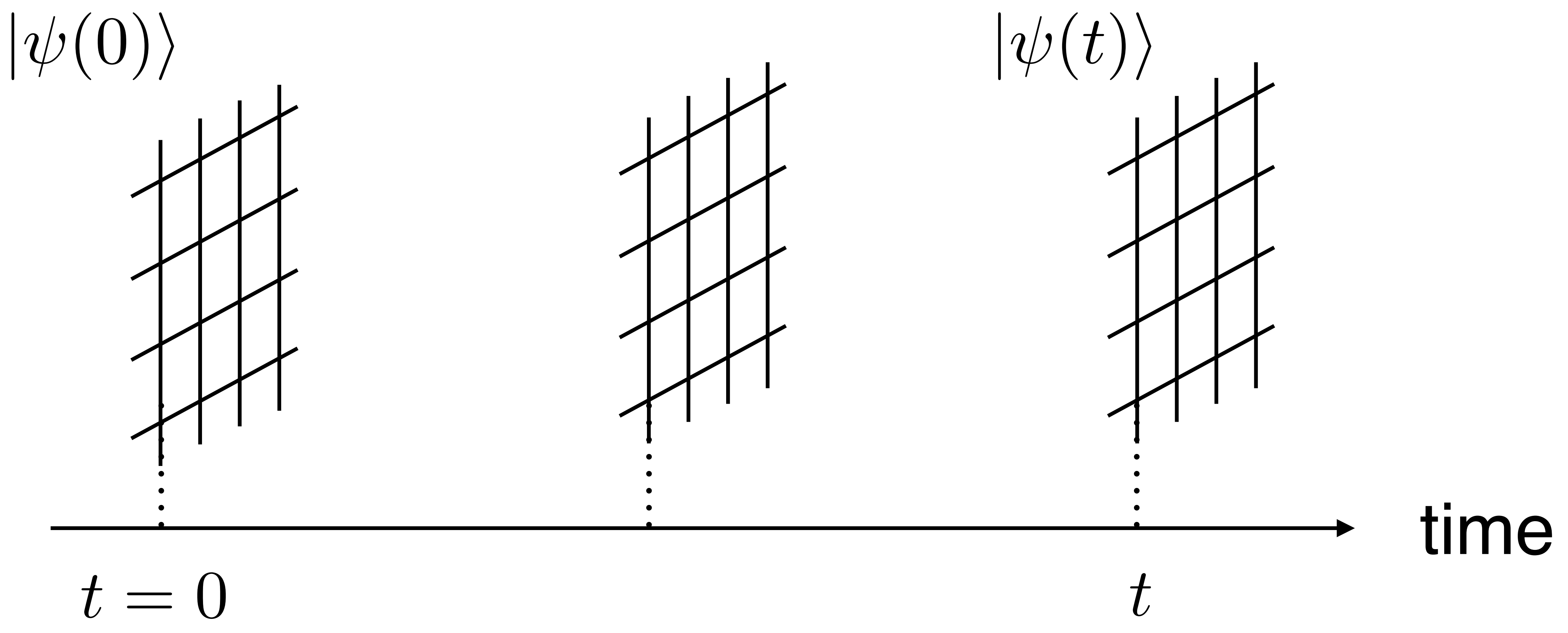
  - infamous sign problem
    - topological term
    - real-time dynamics, etc.



- Hamiltonian simulation
  - discretize **space**
  - no sign problem!

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

  - need exponential resources…

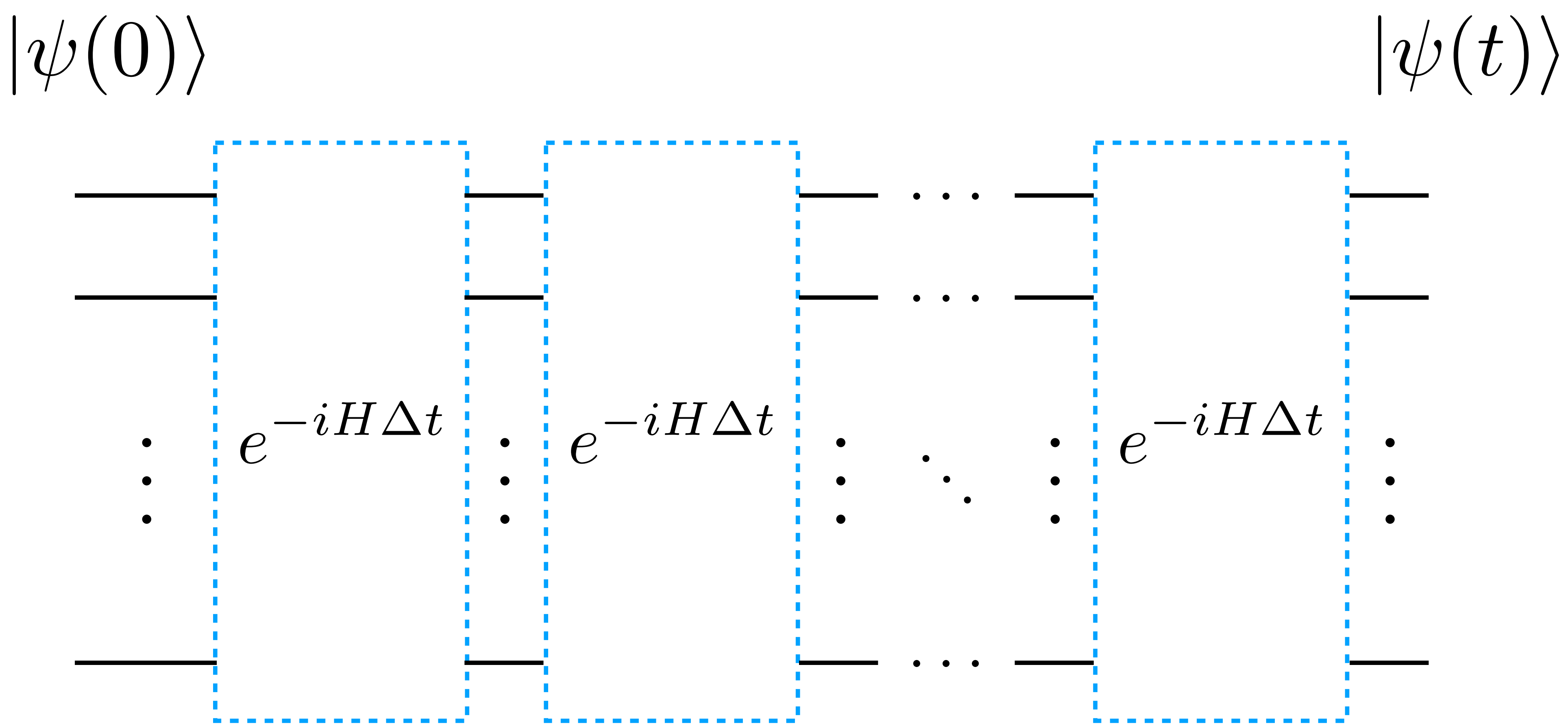


Quantum Simulation

- quantum computer = natural Hamiltonian simulator!
- application to HEP (ex. scattering) [e.g. Jordan-Lee-Preskill]
  - state preparation (vacuum/wave packet)
  - time evolution (scattering)
  - measurement
- exponential speedup!
  - but still need many resources… can we reduce them?
  - in general, when do we have an advantage?
  - NISQ simulation? interesting and feasible problems?

$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$
$$= \left(e^{-iH\Delta t}\right)^{s}|\psi(0)\rangle$$

→Suzuki-Trotter decomposition



Schwinger model

- ultimate goal: 3+1d non-Abelian gauge theory
- simple toy model: 1+1d U(1) gauge theory = **Schwinger model** [Schwinger]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \boxed{\frac{g\theta}{4\pi}\epsilon^{\mu\nu}F_{\mu\nu}} + \mathrm{i}\bar{\psi}\gamma^{\mu}\left(\partial_{\mu} + \mathrm{i}gA_{\mu}\right)\psi - m\bar{\psi}\psi$$

- exactly solvable when $m = 0$
- simple but still non-trivial
  - screening/confinement transition
  - we can include topological term (cannot be treated in MC method)

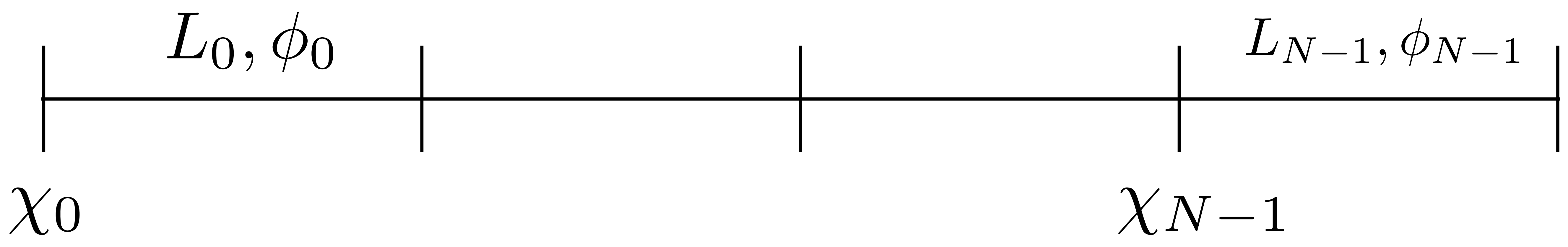
Contents

- Introduction
- Quantum simulation of Schwinger model
    - state preparation in the presence of probe charges
      [Honda-Itou-Kikuchi-LN-Okuda, Phys. Rev. D 105, 014504]
    - real-time dynamics via variational method [work in progress with A. Bapat and C. W. Bauer]
    - gauge invariance in variational ansatz [work in progress with A. Bapat]
- Conclusion

State preparation
in Schwinger model

Lattice Hamiltonian of Schwinger model

- χ_n: staggered fermion [Susskind, Kogut-Susskind]
- L_n, ϕ_n: link variables (gauge field)



$$H_{\rm lat} = J \sum_{n=0}^{N-2} \left(L_n + \frac{\theta}{2\pi} \right)^2 - {\rm i} w \sum_{n=0}^{N-2} (\chi_n^\dagger e^{{\rm i}\phi_n} \chi_{n+1} - {\rm c.c.}) + m \sum_{n=0}^{N-1} (-)^n \chi_n^\dagger \chi_n$$

- gauge invariance: Gauss's law constraint

$$L_n - L_{n-1} = \chi_n^\dagger \chi_n - \frac{1-(-)^n}{2}$$

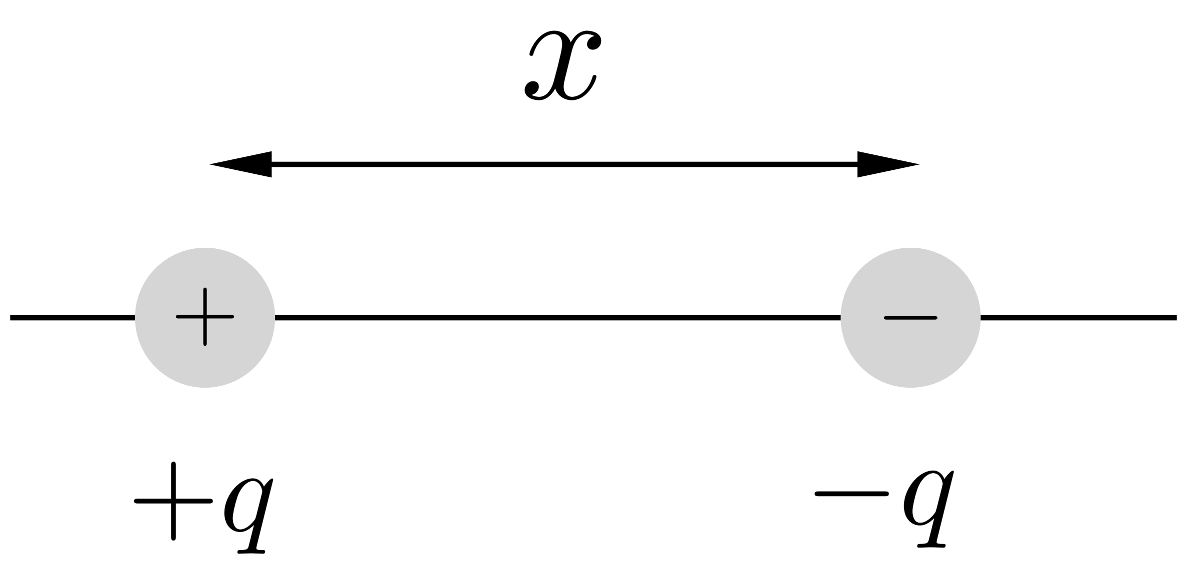
- we can **eliminate** gauge fields!
  - automatically gauge invariant, no boson fields
  - cannot be used in higher dimension

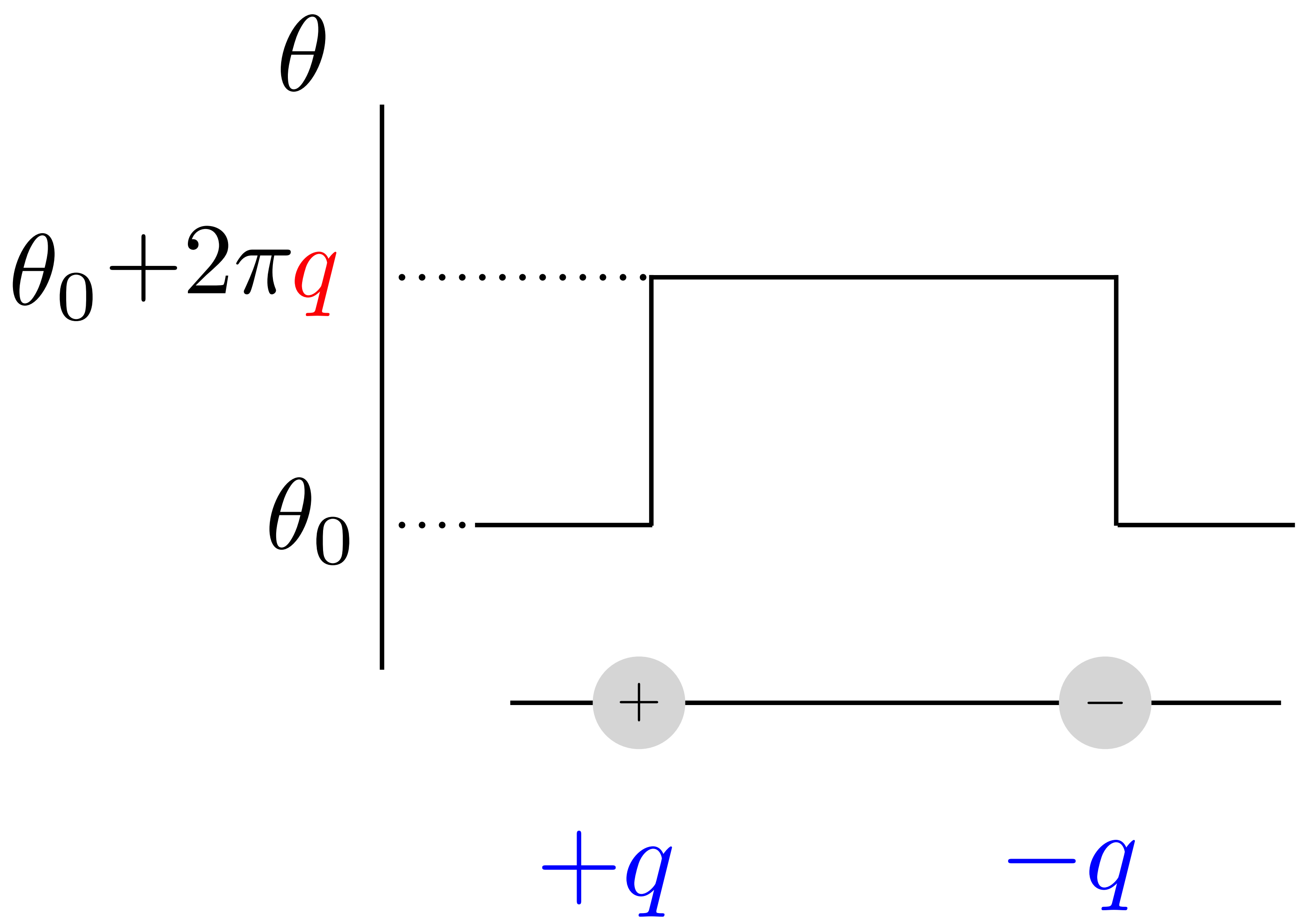
Spin Hamiltonian of Schwinger model

- fermion formalism→spin system (Jordan-Wigner transformation)

$$H_{\rm spin} = J\sum_{n=0}^{N-2}\left(\sum_{k=0}^{n}\frac{Z_k+(-)^k}{2}+\frac{\theta_n}{2\pi}\right)^2+\frac{w}{2}\sum_{n=0}^{N-2}(X_nX_{n+1}+Y_nY_{n+1})+\frac{m}{2}\sum_{n=0}^{N-1}(-)^nZ_n$$

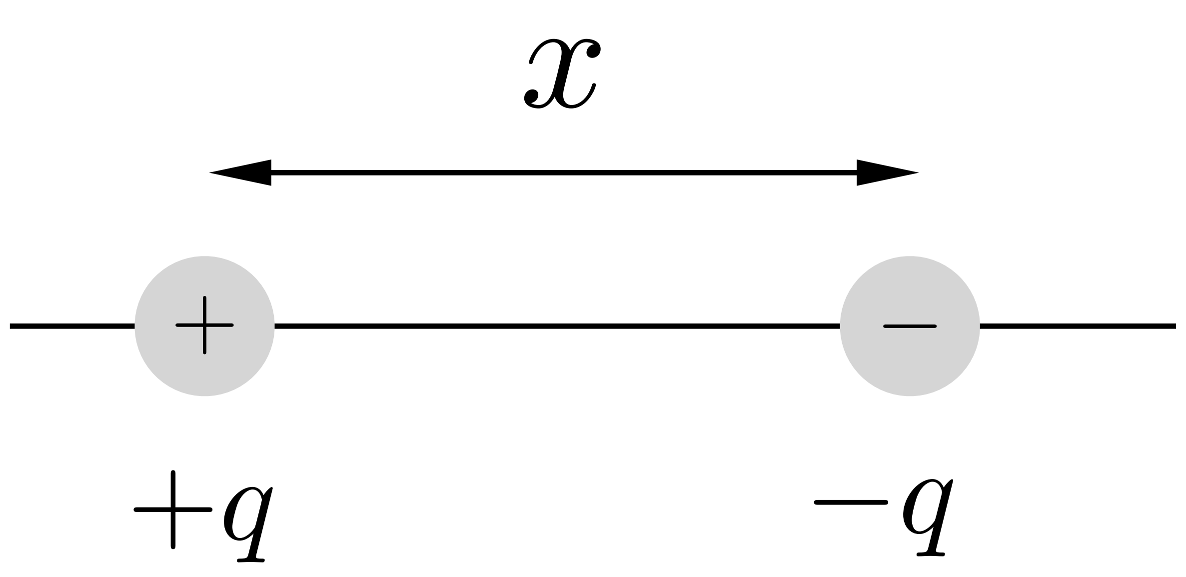
- constant background electric field↔ $\theta_n = \theta$ (constant)
- introducing probe charges↔position-dependent θ_n

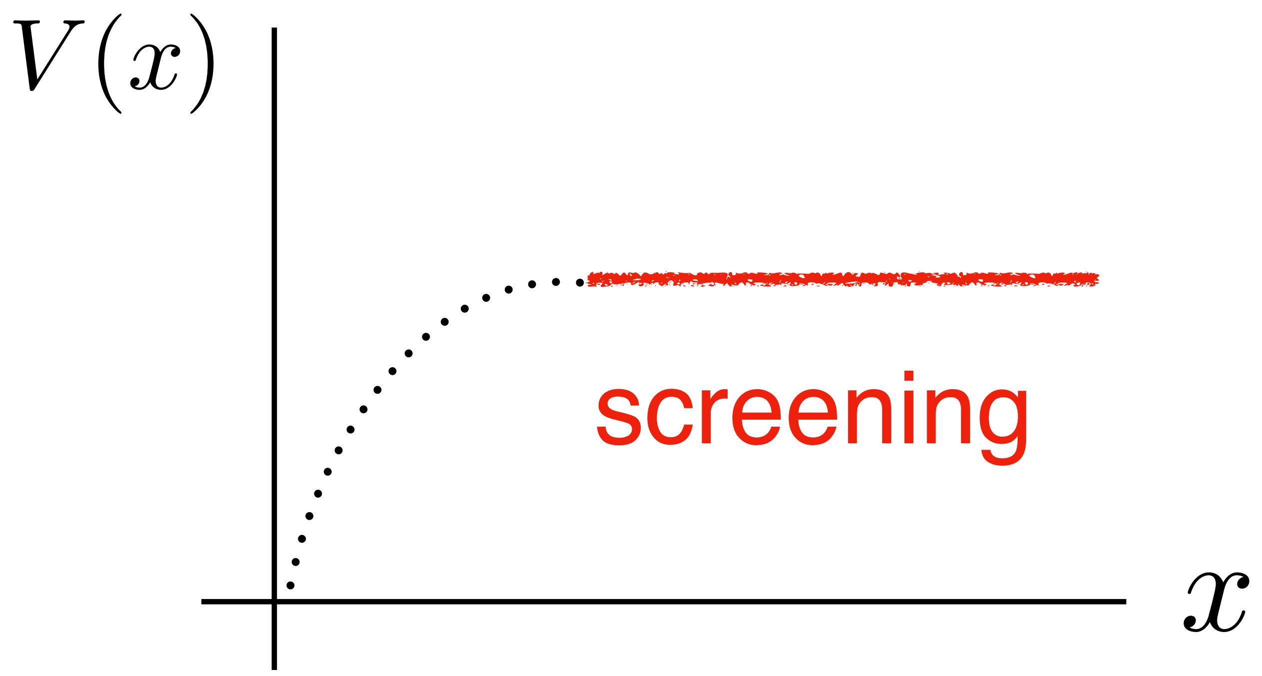


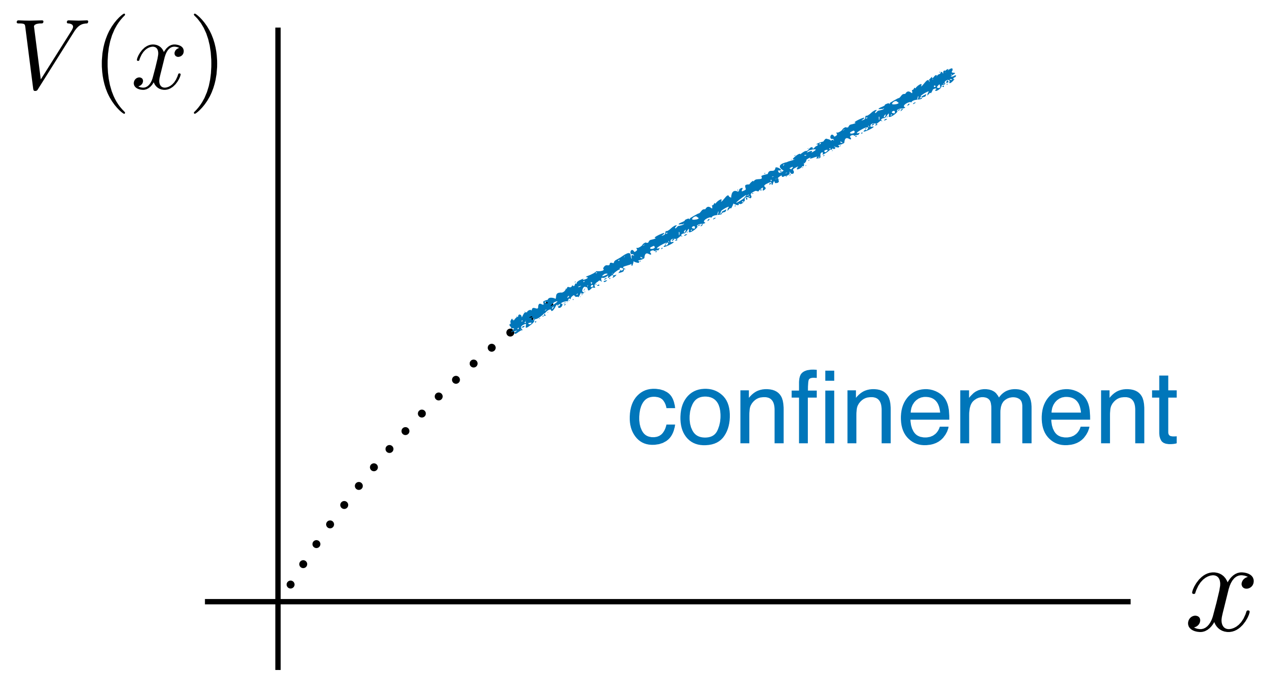


Screening vs Confinement in Schwinger model

[Schwinger]
[Gross-Klebanov-Matytsin-Smilga, Iso-Murayama]







- $m = 0$ (exactly solvable): $$V(x) = \frac{q^2 g^2}{2\mu}(1 - e^{-\mu x})$$ screening

- $m \neq 0$ (mass pert.): $$V(x) \sim m\Sigma\left[1 - \cos(2\pi q)\right] x$$ screening, $q \in \mathbb{Z}$
  confinement, $q \notin \mathbb{Z}$

Adiabatic state preparation

Goal: obtain ground state in the presence of probe charges (position dep. θ_n)

$$H = J \sum_{n=0}^{N-2} \left(\sum_{k=0}^{n} \frac{Z_k + (-)^k}{2} + \frac{\theta_n}{2\pi} \right)^2 + \frac{w}{2} \sum_{n=0}^{N-2} (X_n X_{n+1} + Y_n Y_{n+1}) + \frac{m}{2} \sum_{n=0}^{N-1} (-)^n Z_n$$

$H_A(0) = H_0$ $\qquad$ $H_A(t)$ $\qquad$ $H_A(T) = H$ $\qquad$ t

$|\Omega_0\rangle$

$$|\Omega\rangle = \lim_{T \to \infty} \mathrm{T} \exp \left(-\mathrm{i} \int_0^T \mathrm{d}t H_A(t) \right) |\Omega_0\rangle$$

ground state of H_0

no charges, $w = 0$

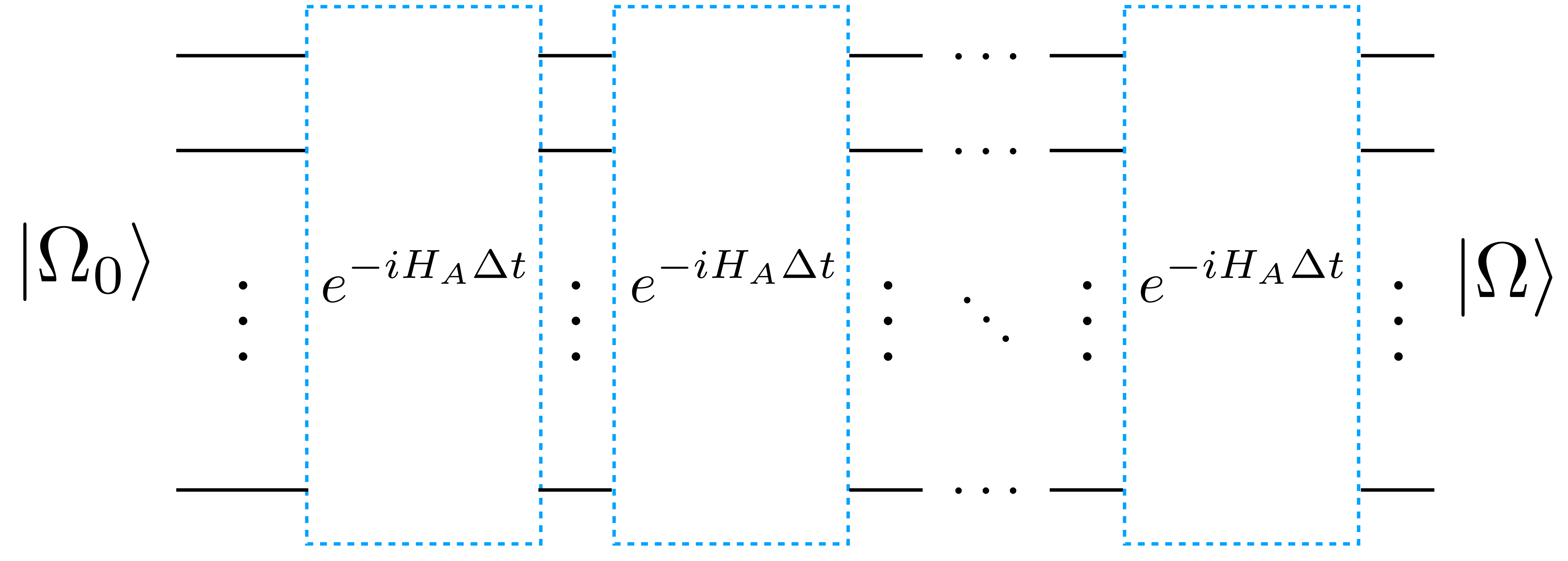
ground state of H

∃probe charges

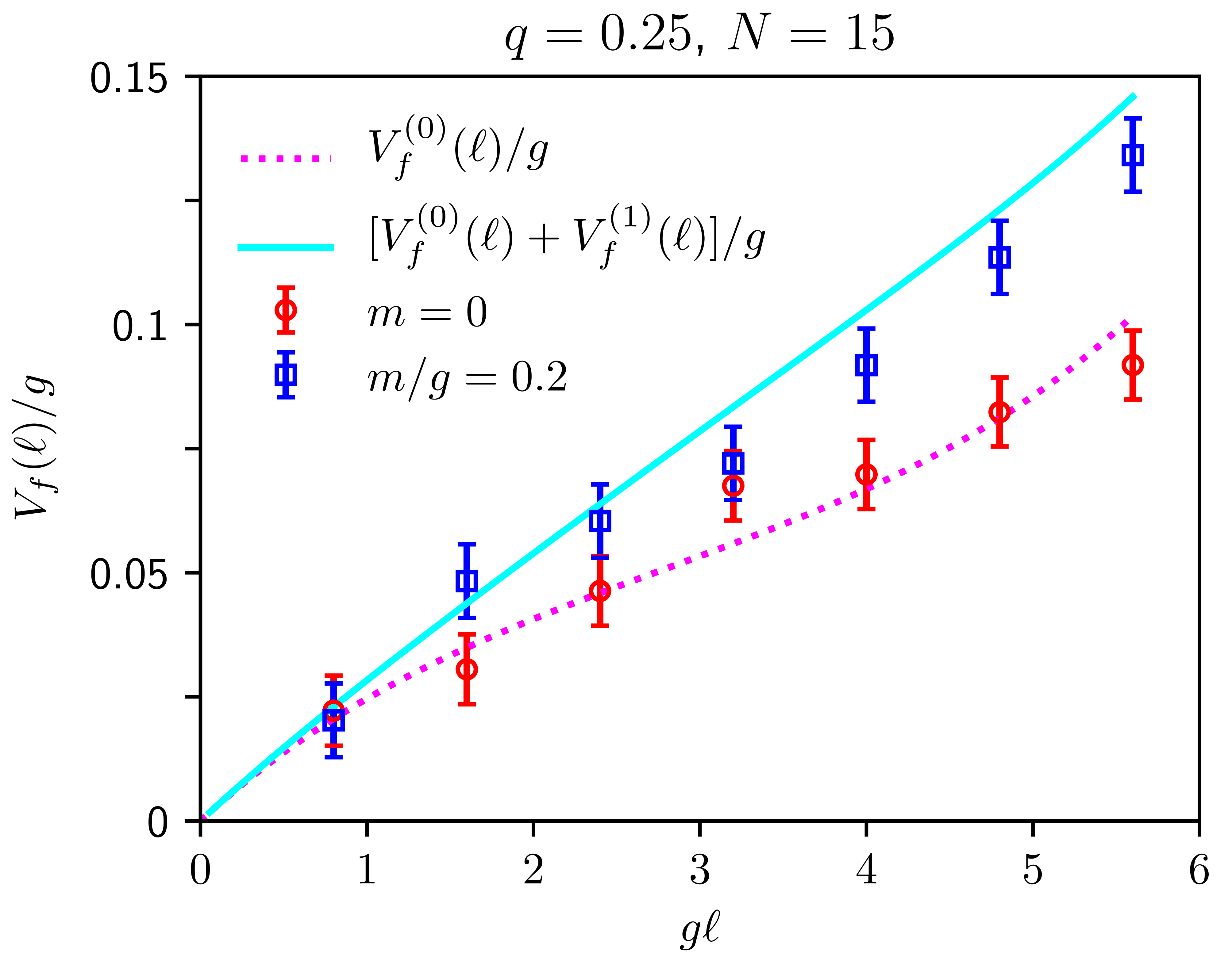
Results for $q \notin \mathbb{Z}$

[Honda-Itou-Kikuchi-LN-Okuda, Phys. Rev. D 105, 014504]

- digitized adiabatic state preparation→ compute energy $\langle \Omega | H | \Omega \rangle$
- expect confinement for massive case (in infinite volume and continuum limit)
  screening massless
- linear behavior for massive case!



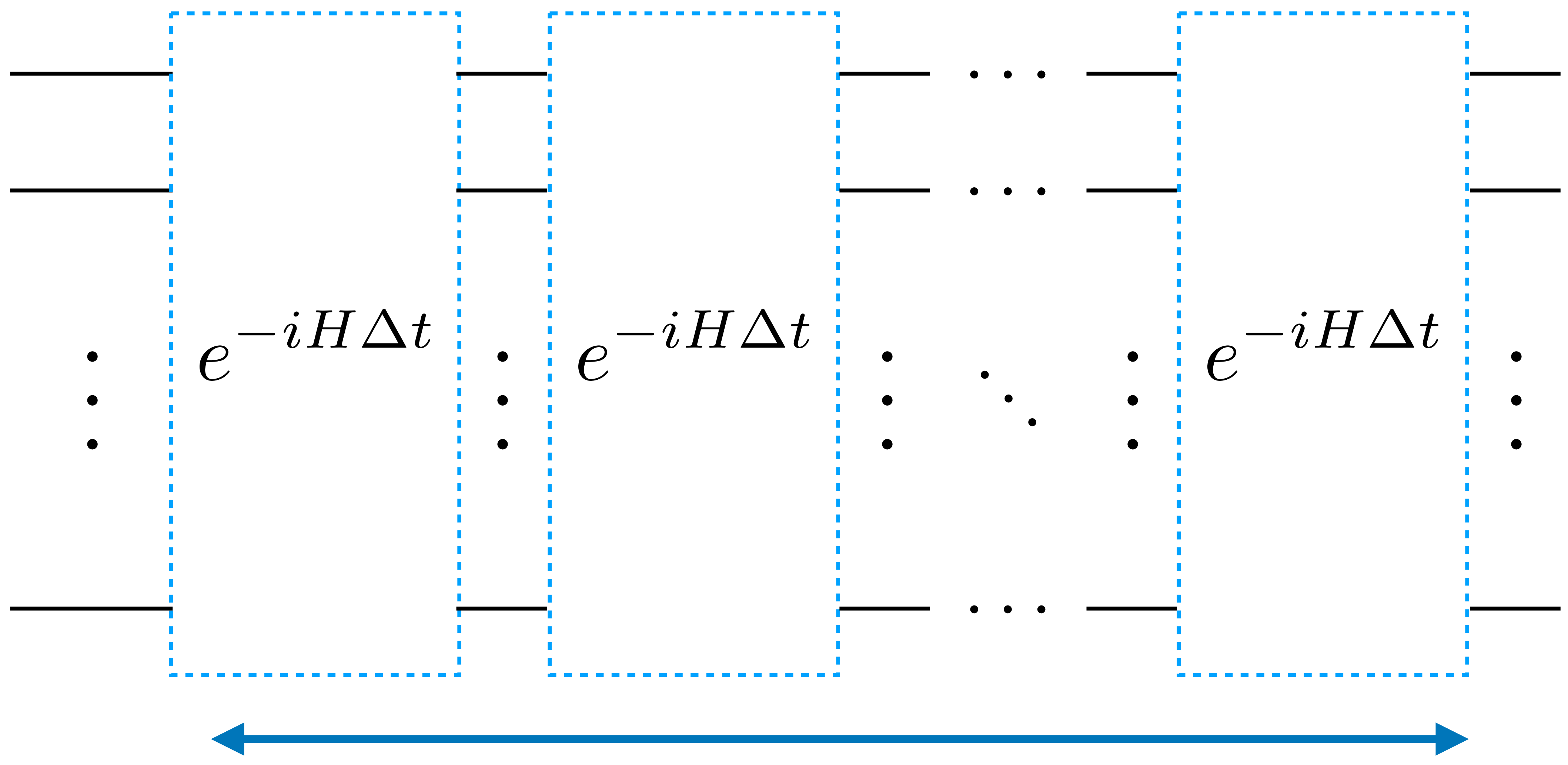
$$\mathrm{T}\exp\left(-\mathrm{i}\int_0^T \mathrm{d}t H_A(t)\right)$$



Variational quantum simulation in Schwinger model

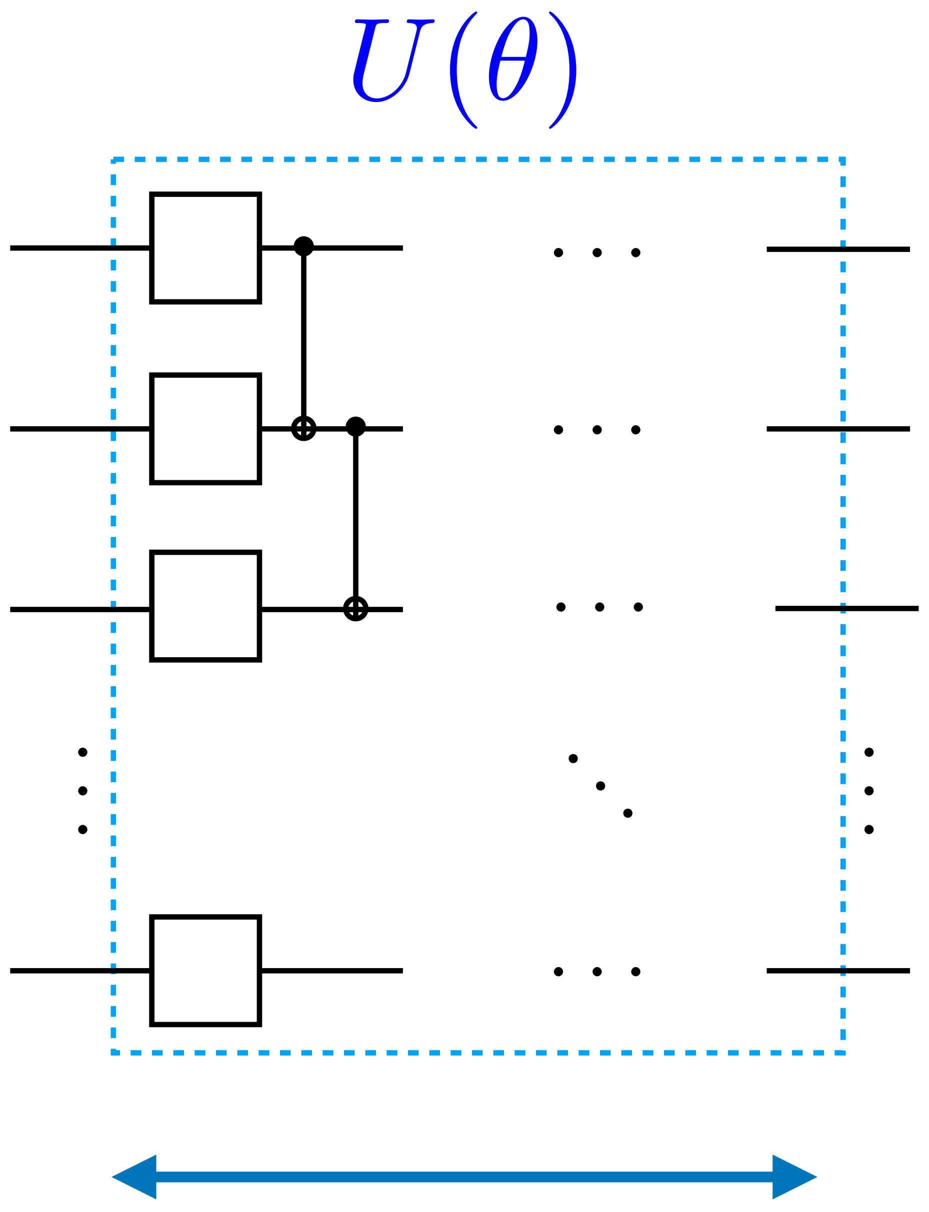
Problem in Trotterized time evolution

- Trotterized time evolution
  - depth ∝ #(time steps)
  - severe decoherence in long-time evolution!



depth∝ (#step)

- one way to avoid problem: **variational quantum simulation (VQS)**
  - fixed depth→NISQ friendly!



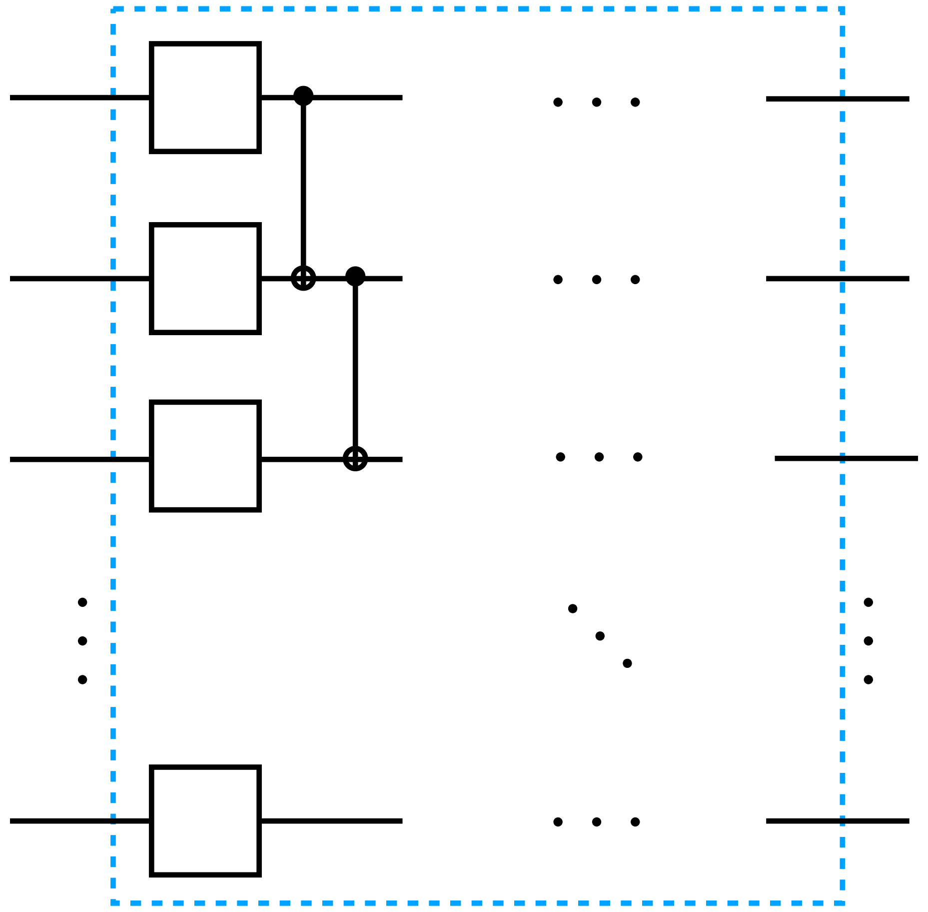
$$|\psi(\theta(t))\rangle \approx e^{-iHt}|\psi(0)\rangle$$

depth fixed

Variational Quantum Simulation

[Li-Benjamin, Phys. Rev. X **7**, 021050
Yuan-Endo, et.al., Quantum 3, 191]

- GOAL: compute $|\Psi(t)\rangle \propto e^{-iHt}|\Psi(0)\rangle$ from a given initial state
- parametrize family of states along time evolution by $\theta(t)$: $|\psi(\theta(t))\rangle \approx |\Psi(t)\rangle$
- evolution of $|\Psi(t)\rangle \rightarrow$ evolution of $\theta(t)$



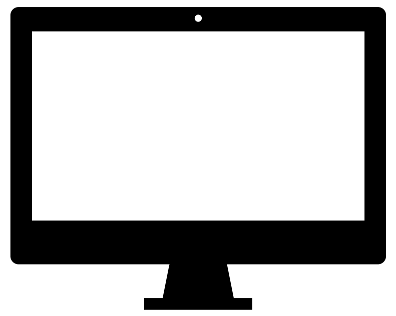
$|\psi(\theta(t))\rangle$

$$M_{ij} = \mathrm{Re}\frac{\partial\langle\psi(\theta)|}{\partial\theta_i}\frac{\partial|\psi(\theta)\rangle}{\partial\theta_j}$$

$$V_{ij} = \mathrm{Im}\frac{\partial\langle\psi(\theta)|}{\partial\theta_i}H|\psi(\theta)\rangle$$

→

solve classically



$$\sum_j M_{ij}\dot{\theta}_j = V_i$$

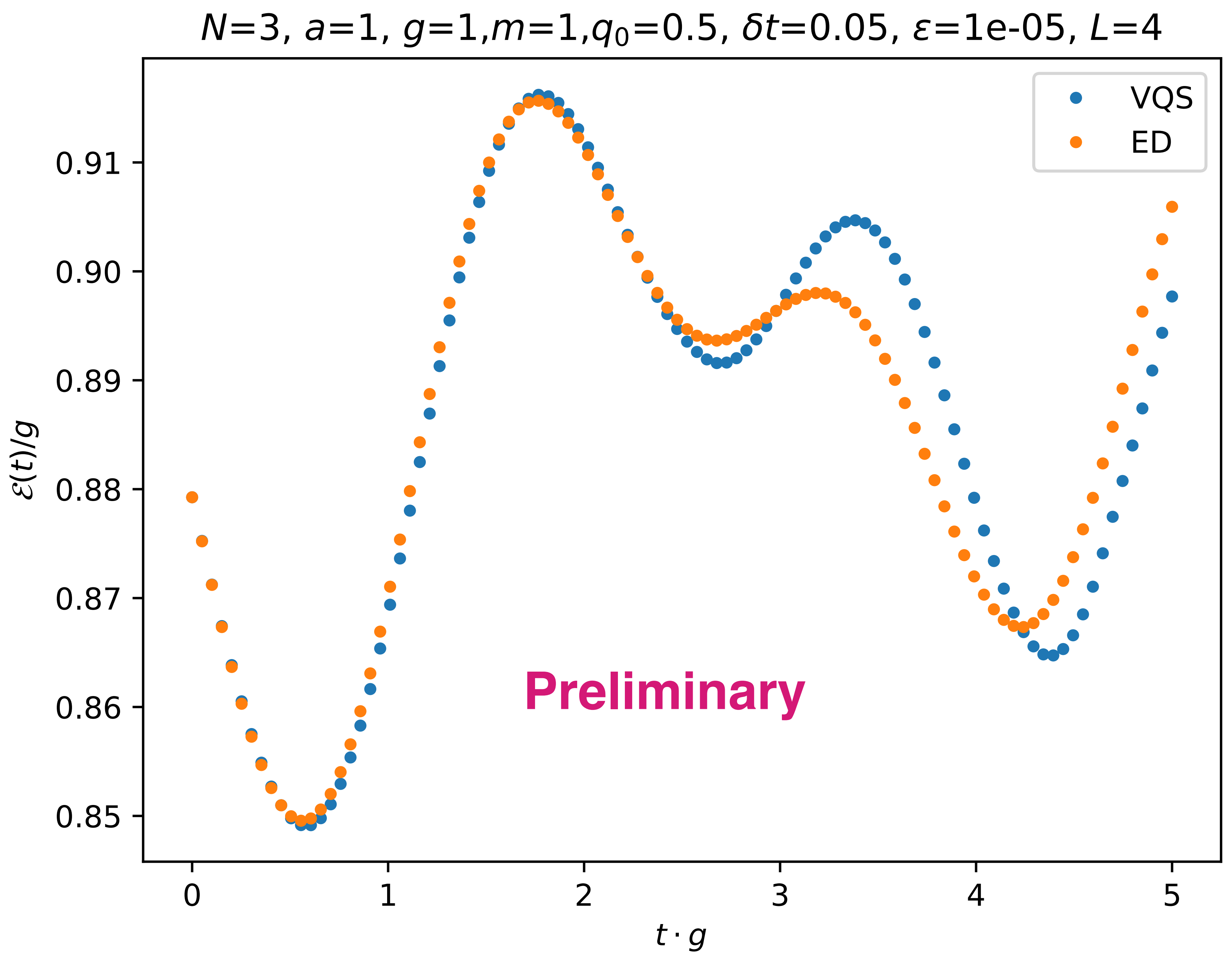
VQS in Schwinger model

[work in progress with A. Bapat and C. W. Bauer]

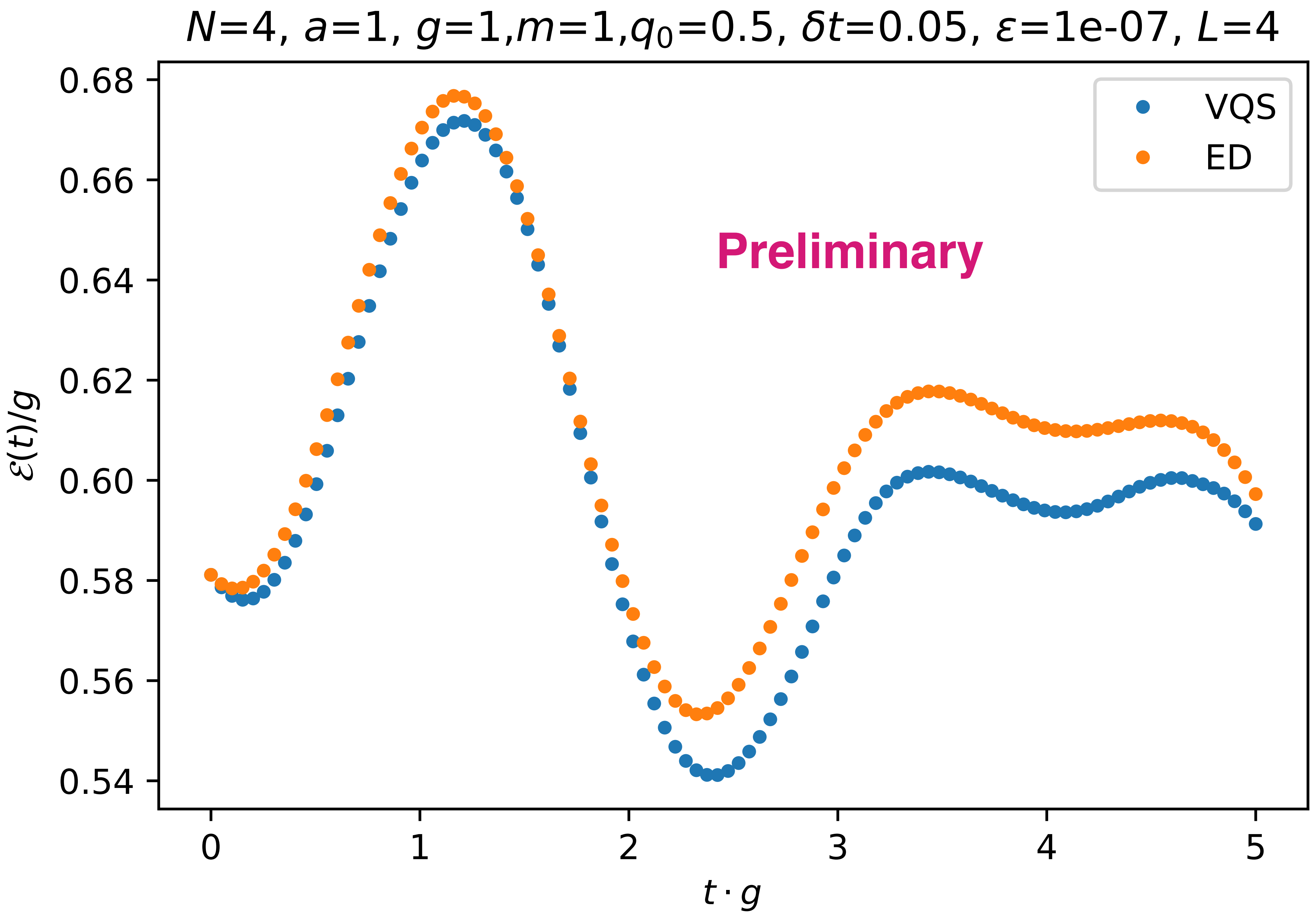
$$H = J \sum_{n=0}^{N-2} \left(\sum_{k=0}^{n} \frac{Z_k + (-)^k}{2} + \frac{\theta}{2\pi} \right)^2 + \frac{w}{2} \sum_{n=0}^{N-2} (X_n X_{n+1} + Y_n Y_{n+1}) + \frac{m}{2} \sum_{n=0}^{N-1} (-)^n Z_n$$

- real-time dynamics under external electric field $\leftrightarrow$ nonzero θ

electric field



electric field



Gauge invariant ansatz in Schwinger model

Gauge invariance in Schwinger model

$$L_n - L_{n-1} = \chi_n^\dagger \chi_n - \frac{1-(-)^n}{2}$$

- fermion model
  - gauge fields are eliminated by Gauss's law →no truncation
  - automatically gauge invariant!
  - cannot be used in other dimensions…
- original model
  - gauge fields remain: need truncation
  - have to impose Gauss's law
  - can be used in other dimensions

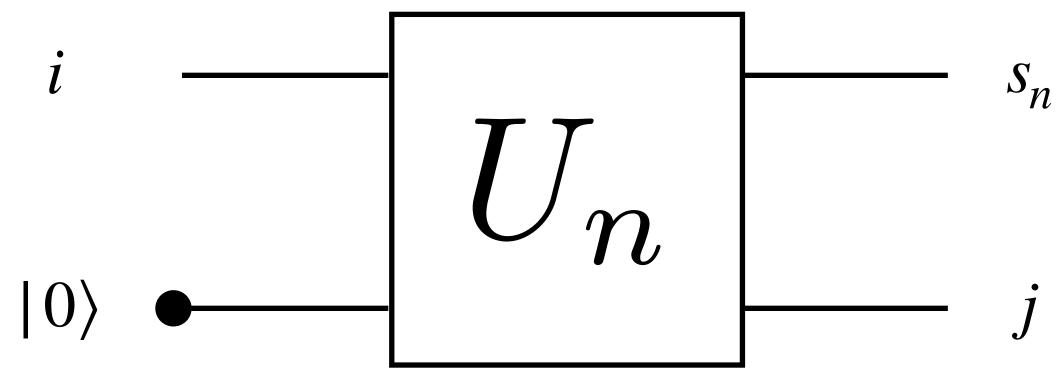
MPS-based approach

- matrix product states (MPS): class of quantum states

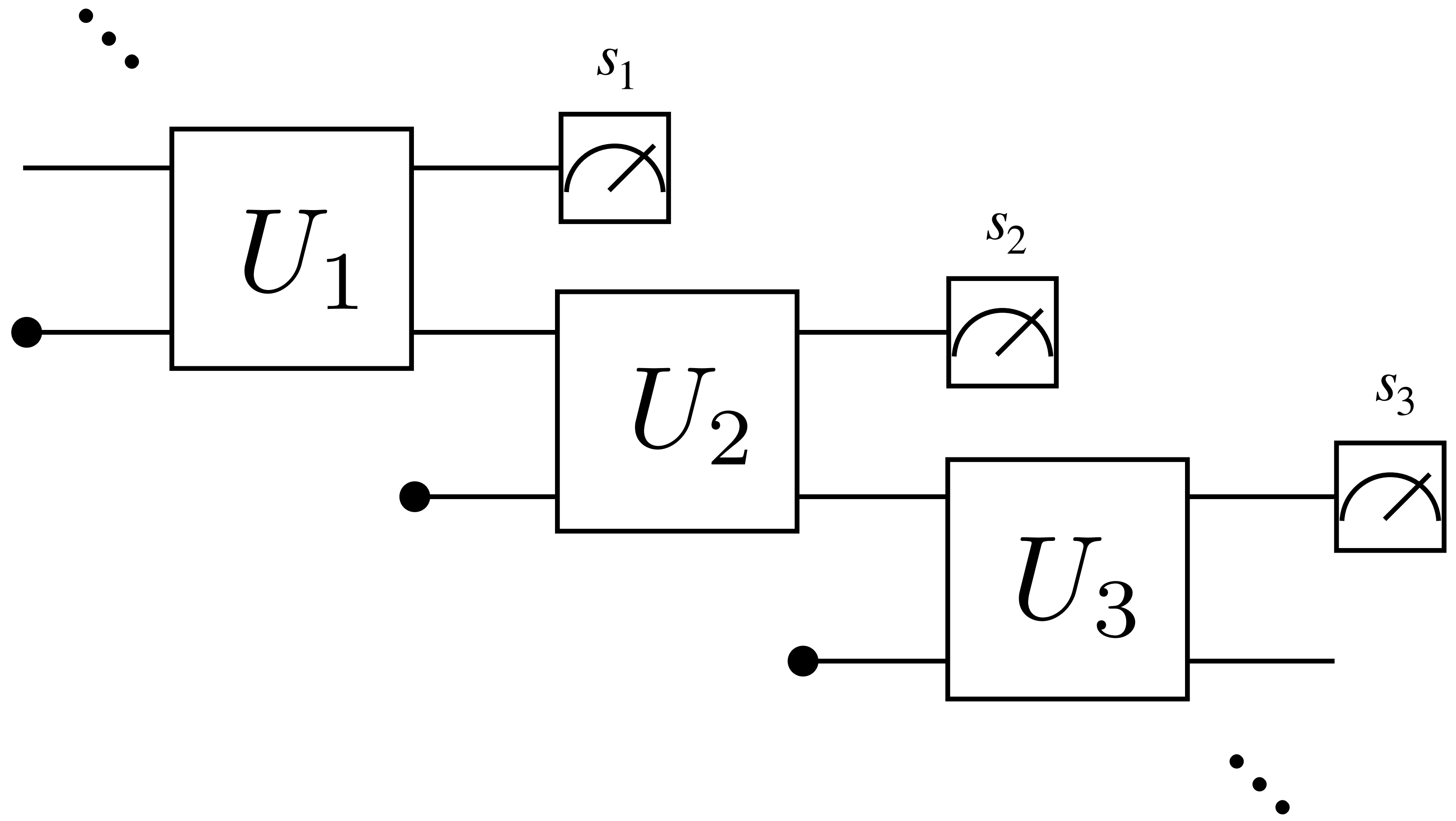
$$|\psi\rangle = \sum_{s_1 \cdots s_N} \left(A_1^{s_1} \cdots A_N^{s_N}\right) |s_1 \cdots s_N\rangle$$

- can be mapped to parametrized quantum circuit

$A_{1,ij}^{s_n} =$



→



[e.g. Schoen, et.al. Phys. Rev. Lett. 95, 110503]

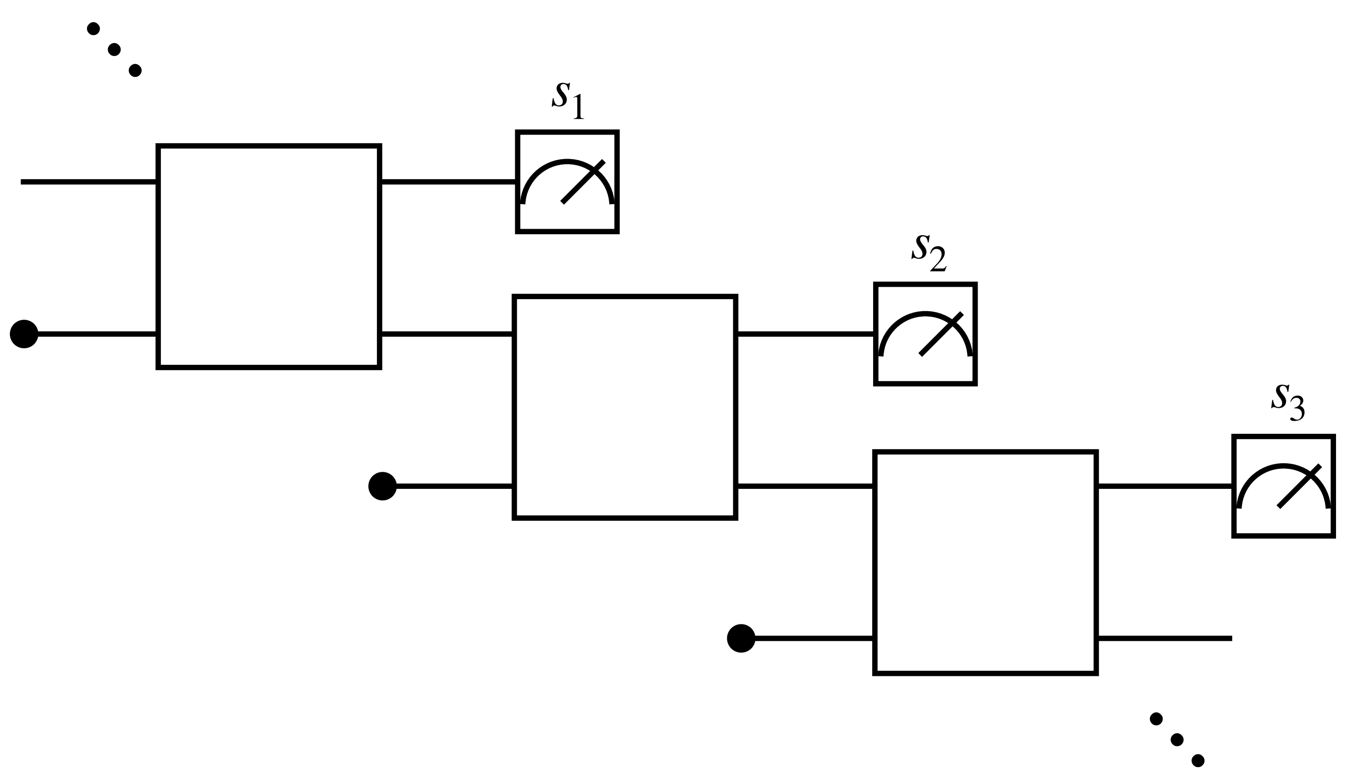
- Gauss's law→constraints on MPS block A [Buyens, et. al. Phys. Rev. Lett. **113**, 091601]
  →constraints on unitary gates U

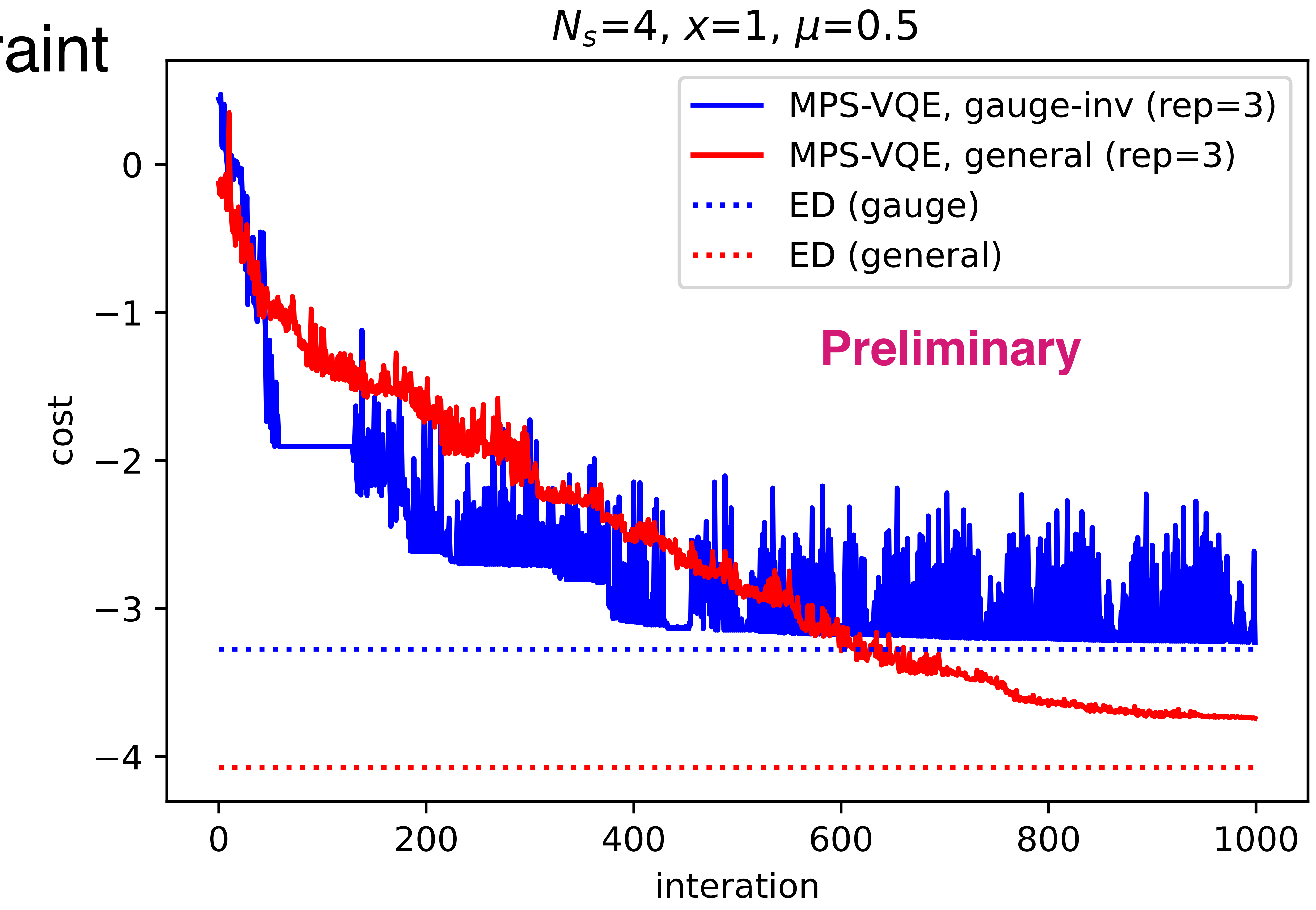
Gauge invariant ansatz in Schwinger model

[work in progress with A. Bapat]

- Schwinger model with two-level truncation ($\mathbb{Z}_2$ gauge theory)
- construct MPS ansatz $|\psi_{\rm MPS}(\theta)\rangle$
- minimize $\langle\psi_{\rm MPS}(\theta)|H|\psi_{\rm MPS}(\theta)\rangle$ (variational quantum eigensolver: VQE)
- compare and gauge inv. MPS and general MPS
  - gauge inv. MPS: **with** constraint→energy with constraint
  - general MPS: **without** constraint→energy without constraint

$$|\psi_{\rm MPS}(\theta)\rangle \quad =$$





Summary and Future direction

- quantum simulation is a natural direction for HEP
- Schwinger model is a simple but still interesting model!
  - screening/confinement via adiabatic preparation
  - time evolution by a circuit with fixed depth: variational quantum dynamics
- extension to higher dimensions? $\rightarrow$keeping gauge invariance is important!
  - MPS-based gauge invariant ansatz
- future direction
  - noise robust problem/implementation/encoding?
  - towards full-fledged models (non-Abelian, higher dim., boson, etc.)