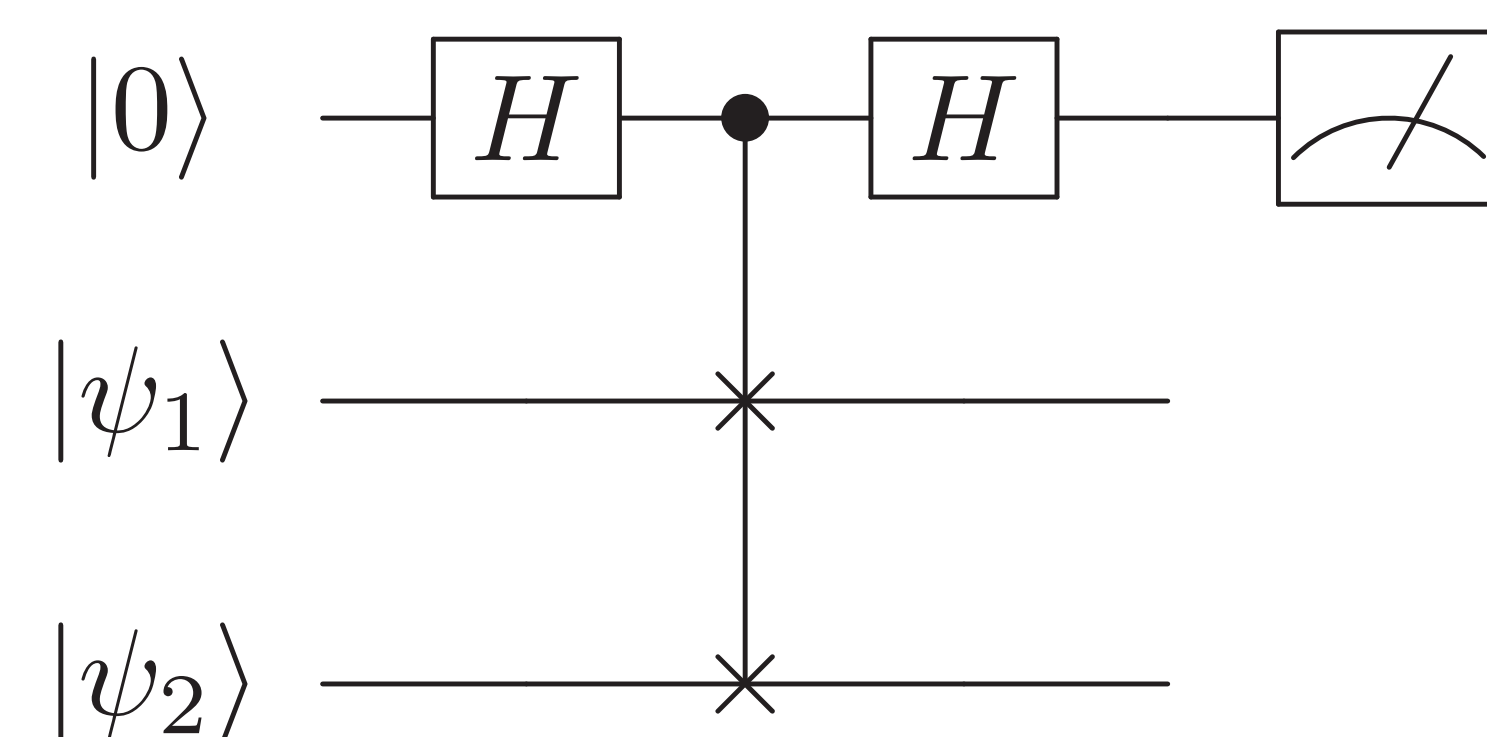


1. Introduction

1. We study the case where **quantum computing** could **speed up jet clustering** of collider data [1].
2. We consider two new quantum algorithms, a quantum subroutine to compute a **Minkowski-based distance** between two data points, and a quantum circuit to track the rough **maximum** into a list of unsorted data.
3. When one or both algorithms are implemented in classical versions of well-known **clustering algorithms** (K-means, Affinity Propagation (AP) and k_T -jet) we obtain **comparable efficiencies** to those of their classical counterparts and potential **speedups** in dimensionality and data length.

2. Quantum distance in Minkowski space

To quantify the **similarity** of two quantum states we rely on the **SwapTest** method [2].



$$|\psi_1\rangle = \frac{1}{\sqrt{2}} (|0, x_i\rangle + |1, x_j\rangle),$$

$$|\psi_2\rangle = \frac{1}{\sqrt{Z_{ij}}} (|\mathbf{x}_i|0\rangle - |\mathbf{x}_j|1\rangle),$$

$$|\varphi_1\rangle = H|0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle),$$

$$|\varphi_2\rangle = \frac{1}{\sqrt{Z_0}} (x_{0,i}|0\rangle + x_{0,j}|1\rangle).$$

The quantum distance is obtained from the **measurement**:

In **Minkowski** space the distance among data points is the invariant mass squared:

$$P(|0\rangle|_{spat}) = \frac{1}{2} + \frac{1}{2} |\langle\psi_1|\psi_2\rangle|^2,$$

$$P(|0\rangle|_{temp}) = \frac{1}{2} + \frac{1}{2} |\langle\varphi_1|\varphi_2\rangle|^2.$$

Finally we obtain:

$$s_{ij}^{(Q)} = 2(Z_0(2P(|0\rangle|_{temp}) - 1) - Z_{ij}(2P(|0\rangle|_{spat}) - 1)).$$

We apply the *SwapTest* twice (**spatial** and **temporal** components):

4. Quantum clustering algorithms

Assuming data has been **loaded** from a quantum Random Access Memory (**qRAM** [3]) we obtain the following speed-ups:

Jet clustering algorithm	Quantum subroutine	Classical version	Quantum version
K-means	Both	$\mathcal{O}(NKd)$	$\mathcal{O}(N \log K \log(d-1))$
AP	Distance	$\mathcal{O}(N^2Td)$	$\mathcal{O}(N^2T \log(d-1))$
k_T jet	Maximum	$\mathcal{O}(N^2)$	$\mathcal{O}(N \log N)$
anti- k_T FastJet	Maximum	$\mathcal{O}(N \log N)$	$\mathcal{O}(N \log N)$

6. Conclusions

- Quantum computing to **speed-up** jet clustering algorithms
- New methods: $\begin{cases} \text{Quantum distance} \rightarrow \text{SwapTest} \\ \text{Quantum maximum search} \rightarrow \text{Amplitude Encoding} \end{cases}$
- Proven achievements in LHC simulated data:
 - Quantum algorithms at least **as good as** classical
- When **QRAM devices exist** one would obtain
 - Quantum K-means \rightarrow From $\mathcal{O}(NKd)$ to $\mathcal{O}(N \log K \log(d-1))$
 - Quantum AP \rightarrow From $\mathcal{O}(N^2Td)$ to $\mathcal{O}(N^2T \log(d-1))$
 - Quantum k_T \rightarrow $\begin{cases} \text{From } \mathcal{O}(N^2) \text{ to } \mathcal{O}(N \log N) \text{ (without Voronoi)} \\ \text{From } \mathcal{O}(N \log N) \text{ to } \mathcal{O}(N \log N) \text{ (with Voronoi)} \end{cases}$
- If **QRAM never exists** \rightarrow other data loading methods
 - Cut-off of Grover-Rudolph \rightarrow From $\mathcal{O}(2^n)$ to $\mathcal{O}(2^{k_0(\epsilon)})$
 - qGANs \rightarrow From $\mathcal{O}(2^n)$ to $\mathcal{O}(\text{poly}(n))$

3. Quantum maximum search

Let $L[0, \dots, N-1]$ be an unsorted list of N items. The quantum algorithm to find the rough **maximum** using **amplitude encoding** is:

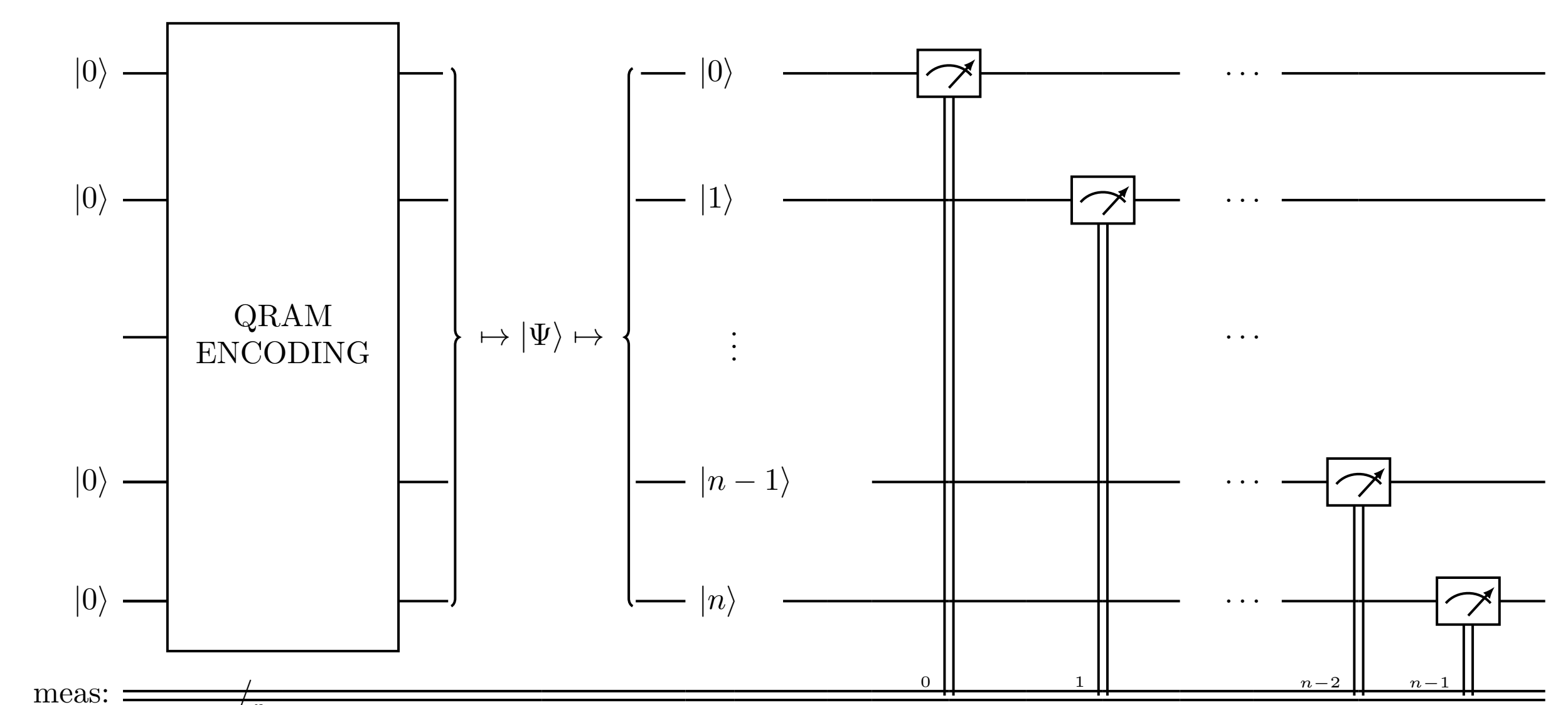
1. The list of N elements is encoded into a $\log_2(N)$ qubits state:

$$|\Psi\rangle = \frac{1}{\sqrt{L_{sum}}} \sum_{j=0}^{N-1} L[j] |j\rangle,$$

where $L_{sum} = \sum_{j=0}^{N-1} L[j]^2$ is a normalization constant.

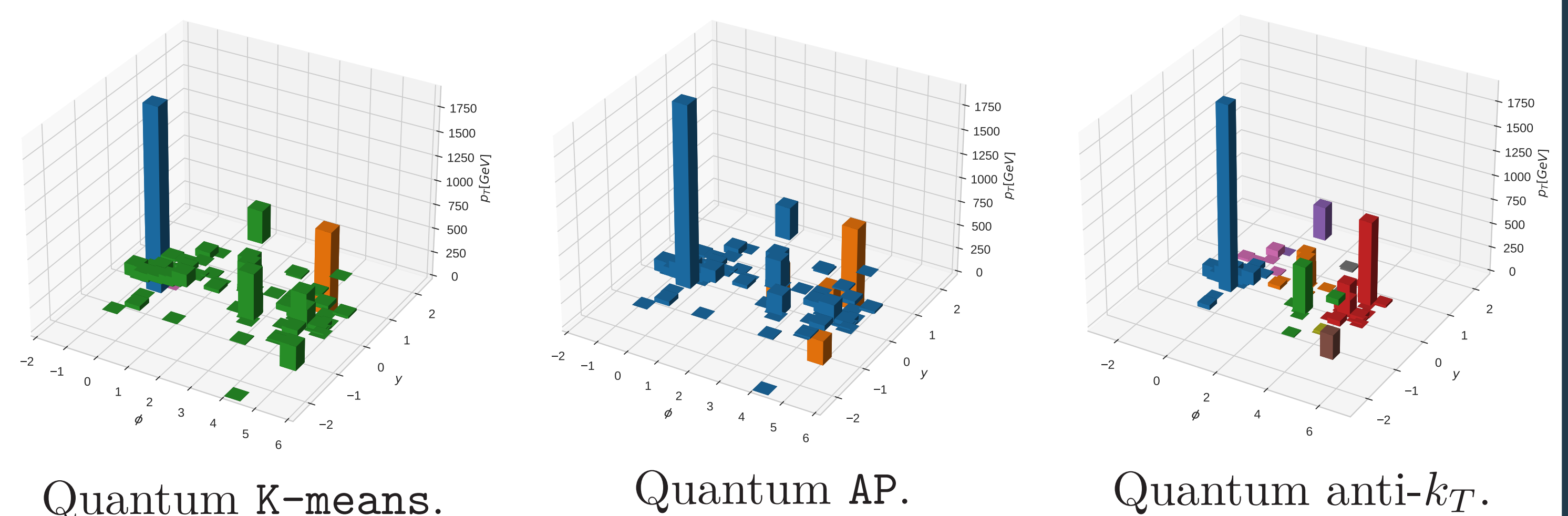
2. The final state is measured.

This procedure is **repeated several** times to reduce the statistical uncertainty. The quantum circuit of this algorithm is as follows:



5. Quantum simulations

We tested our quantum clustering algorithms with a **simulated** physical N -particle **LHC event**, and we obtain these classifications:



The performances of the **quantum** versions in **comparison** with their classical counterparts are shown below.

	Quantum K-means	Quantum AP	Quantum k_T	Quantum anti- k_T	Quantum Cam/Aachen
ε_c	0.94	1.00	0.98	0.99	0.98

7. References

- [1] J.J.M. de Lejarza, L. Cieri and G. Rodrigo, *Quantum clustering and jet reconstruction at the LHC*, *Phys. Rev. D* **106** (2022) 036021.
- [2] H. Buhrman, R. Cleve, J. Watrous and R. de Wolf, *Quantum fingerprinting*, *Phys. Rev. Lett.* **87** (2001) 167902.
- [3] V. Giovannetti, S. Lloyd and L. Maccone, *Quantum random access memory*, *Physical Review Letters* **100** (2008) 160501.

