One-loop corrections to the Higgs boson invisible decay in the Dark Doublet phase of the N2HDM

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Motivation

- The Standard Model (SM), although one of the most successful theories in Physics, is incomplete. It does not explain very important experimental observations:
  - Baryon asymmetry
  - Neutrino masses
  - **Dark matter**
- To address these issues, we turn to Beyond the Standard Model (BSM) models. Some of these models are extensions of the SM;
- The discovery of the Higgs boson and the increasingly precise experimental measurements of its properties allow us to probe BSM models with extended Higgs sectors.

Goal: study a SM extension that allows for dark matter candidates and use the experimental measurements of the Higgs boson to constrain its parameter space at NLO.

[D. Azevedo, **P. Gabriel**, M. Mühlleitner, K. Sakurai and R. Santos, "One-loop corrections to the Higgs boson invisible decay in the dark doublet phase of the N2HDM", JHEP 10 (2021), 044. [arXiv:2104.03184]]

Next-to-minimal 2-Higgs doublet model (N2HDM)

- The scalar potential of the N2HDM contains two Higgs doublets and a singlet

$$V_{scalar} = \underbrace{\boxed{m_{11}^2 \Phi_1^\dagger \Phi_1 + \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2}}_{\text{Standard Model}} + m_{22}^2 \Phi_2^\dagger \Phi_2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2 - m_{12}^2\left(\Phi_1^\dagger \Phi_2 + \mathrm{h.c.}\right) + \lambda_3 \Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2$$
$$+\lambda_4 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1 + \frac{\lambda_5}{2}\left(\left(\Phi_1^\dagger \Phi_2\right)^2 + \mathrm{h.c.}\right) + \frac{1}{2} m_S^2 \Phi_S^2 + \frac{\lambda_6}{8}\Phi_S^4 + \frac{\lambda_7}{2}\Phi_1^\dagger \Phi_1 \Phi_S^2 + \frac{\lambda_8}{2}\Phi_2^\dagger \Phi_2 \Phi_S^2$$

- CP conservation and two $\mathbb{Z}_2$ symmetries are imposed on the potential

$$\mathbb{Z}_2^{(1)}: \Phi_1 \to \Phi_1, \Phi_2 \to -\Phi_2, \Phi_S \to \Phi_S \qquad \mathbb{Z}_2^{(2)}: \Phi_1 \to \Phi_1, \Phi_2 \to \Phi_2, \Phi_S \to -\Phi_S$$

- Symmetry $\mathbb{Z}_2^{(1)}$ imposed on the Yukawa sector → fermions couple only to Φ_1
- Four possible charge-conserving and CP-conserving vacuum configurations (phases):
  - Broken phase (BP): $\langle\Phi_1\rangle \neq 0, \langle\Phi_2\rangle \neq 0, \langle\Phi_S\rangle \neq 0$
  - **Dark doublet phase (DDP):** $\langle\Phi_1\rangle \neq 0, \langle\Phi_2\rangle = 0, \langle\Phi_S\rangle \neq 0$
  - Dark singlet phase (DSP): $\langle\Phi_1\rangle \neq 0, \langle\Phi_2\rangle \neq 0, \langle\Phi_S\rangle = 0$
  - Fully dark phase (FDP): $\langle\Phi_1\rangle \neq 0, \langle\Phi_2\rangle = 0, \langle\Phi_S\rangle = 0$

[I. Engeln, P. Ferreira, M.M. Mühlleitner, R. Santos and J. Wittbrodt, JHEP 08 (2020) 085]

Dark Doublet phase (DDP)

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + \boxed{\rho_1} + iG^0) \end{pmatrix}$$

$$\boxed{\Phi_2 = \begin{pmatrix} H_D^+ \\ \frac{1}{\sqrt{2}}(H_D + iA_D) \end{pmatrix}}$$

Dark doublet

$$\Phi_S = v_S + \boxed{\rho_S}$$

Minimum conditions

$$m_{11}^2 = -\frac{1}{2}(v^2\lambda_1 + v_S^2\lambda_7)$$

$$m_S^2 = -\frac{1}{2}(v^2\lambda_7 + v_S^2\lambda_6)$$

$$m_{12}^2 = 0$$

$$\boxed{\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = R(\alpha)^T \begin{pmatrix} \rho_1 \\ \rho_S \end{pmatrix}}$$

$$m_{H_1} \leq m_{H_2}$$

$$R(\alpha) = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{pmatrix}$$

Mass eigenstates:

Visible Sector
- Higgs bosons: H_1, H_2
- Goldstone bosons: G^0, G^+, G^-

Dark Sector
- Charged Higgs: $H_D^+, \mathrm{H}_\mathrm{D}^-$
- Dark neutral scalars: $\boxed{\boldsymbol{H_D}, A_D}$

Dark matter candidates

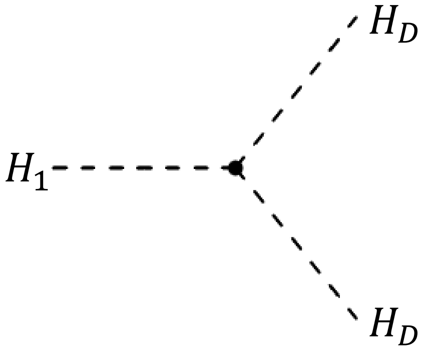
$\mathbb{Z}_2^{(1)}$ conserved $\longrightarrow$ "darkness" quantum number

$\mathbb{Z}_2^{(2)}$ spontaneously broken due to $v_S \neq 0$

$$\{m_{H_1}^2, m_{H_2}^2, m_{H_D}^2, m_{A_D}^2, m_{H^\pm}^2, v, v_S, \alpha, m_{22}^2, \lambda_2, \lambda_8\}$$

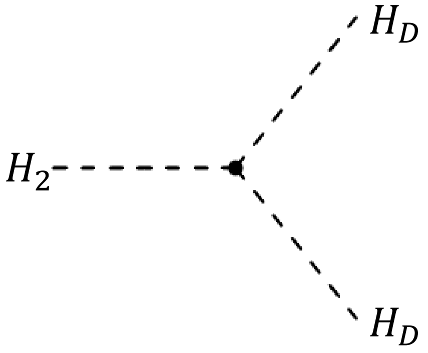
Higgs decays to dark matter

Light Higgs ($h_{SM} = H_1$)



$$\lambda_{H_1 H_D H_D} = \frac{2\cos\alpha}{v}\left(\frac{\lambda_8 v_S^2}{2} + m_{22}^2 - m_{H_D}^2\right) - \sin\alpha\,\lambda_8 v_S$$

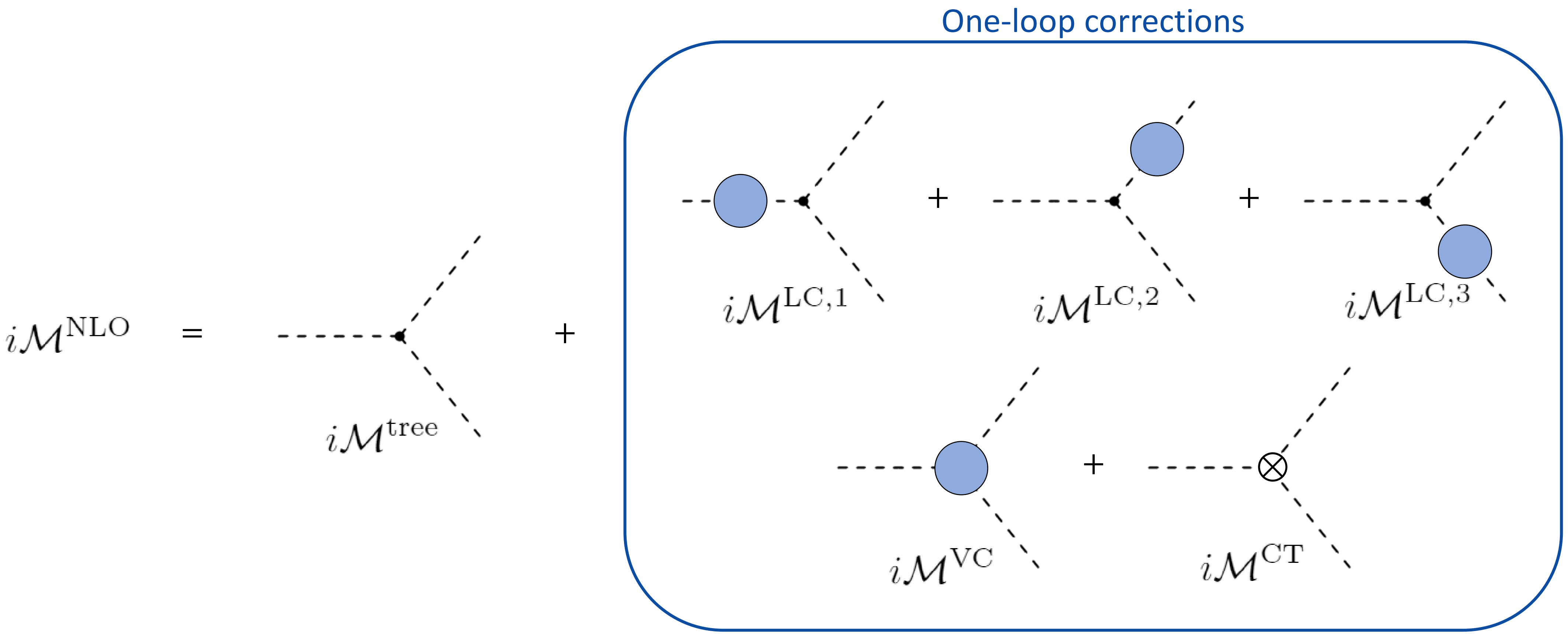
Heavy Higgs ($h_{SM} = H_2$)



$$\lambda_{H_2 H_D H_D} = -\frac{2\sin\alpha}{v}\left(\frac{\lambda_8 v_S^2}{2} + m_{22}^2 - m_{H_D}^2\right) - \cos\alpha\,\lambda_8 v_S$$

- In the Heavy Higgs scenario, the parameter space is more constrained due to the upper bound on the mass of the non-SM Higgs boson H_1.
- Also in the Heavy Higgs scenario, the SM Higgs has an additional decay channel due to the kinematically allowed process $H_2 \to H_1 H_1$. This affects the values of the branching ratios.

One-loop corrections



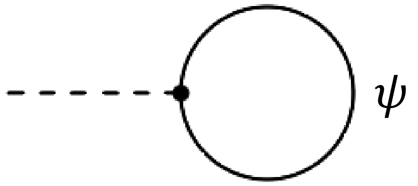
$$\Gamma^{\text{NLO}}_{H_i \to H_D H_D} = \Gamma^{\text{LO}}_{H_i \to H_D H_D} + \frac{\sqrt{m_{H_i}^2 - 4m_{H_D}^2}}{16\pi m_{H_i}^2} \text{Re}\left[(\mathcal{M}^{\text{tree}}_{H_i \to H_D H_D})^* \mathcal{M}^{\text{1loop}}_{H_i \to H_D H_D} \right]$$

$$BR^{\text{NLO}}(H_i \to H_D H_D) = \frac{\Gamma^{\text{NLO}}_{H_i \to H_D H_D}}{\Gamma^{\text{NLO}}_{H_i}}$$

Renormalization

- Calculation of the amplitudes of loop diagrams often leads to divergent integrals

$$\int_0^\infty \frac{\mathrm{d}^4 \mathrm{p}_\psi}{(2\pi)^4} \frac{1}{p_\psi^2 - m_\psi^2} \xrightarrow{\quad p_\psi \to \infty \quad} \text{UV divergence}$$



- Divergent parts are isolated through dimensional regularization;
- We assume that parameters and field wave-functions can be decomposed into a renormalized quantity (finite) and a counter-term (infinite)

$$\rho_0 = \rho + \delta\rho \qquad\qquad \phi_0 = \left(1 + \frac{\delta Z}{2}\right)\phi$$

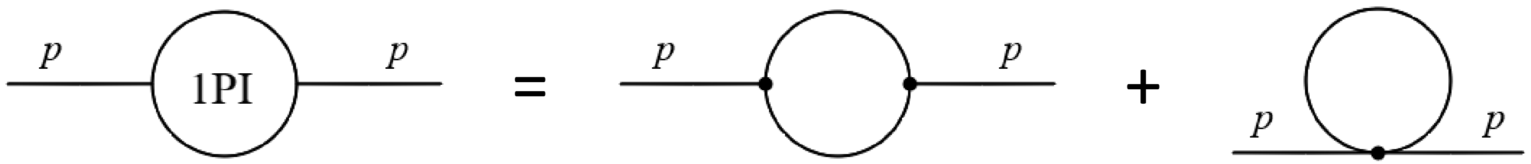
- All counter-terms must be fixed in such way that they cancel out all the divergent terms, leaving the renormalized amplitudes finite.

On-Shell scheme

$$G(p^2) = \frac{i}{p^2 - m_\phi^2 + \hat{\Sigma}_\phi(p^2)}$$

$$\hat{\Sigma}_\phi(p^2) = \Sigma_\phi(p^2) - \delta m_\phi^2 + \frac{\delta Z_\phi^*}{2}(p^2 - m_\phi^2) + (p^2 - m_\phi^2)\frac{\delta Z_\phi}{2}$$

$i\Sigma_\phi(p^2)$ =



Renormalization conditions:

1. Pole of the propagator must be at the mass ($p^2 = m_\phi^2$)

$$\hat{\Sigma}_\phi\left(m_\phi^2\right) = 0$$

2. Residue of the propagator must be fixed at i

$$\left.\frac{\partial \hat{\Sigma}_\phi(p^2)}{\partial p^2}\right|_{p^2 = m_\phi^2} = 0$$

3. No mixing at the propagator pole (only for mixing fields)

$$\hat{\Sigma}_{\phi_i \phi_j}\left(m_{\phi_j}^2\right) = 0, \qquad (i \neq j)$$

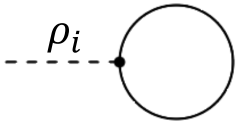
$$\delta m_\phi^2 = \Sigma_\phi^{Tad}(m_\phi^2)$$

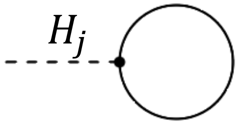
$$\delta Z_\phi = -Re\left[\left.\frac{\partial \Sigma_\phi^{Tad}(p^2)}{\partial p^2}\right|_{p^2 = m_\phi^2}\right]$$

$$\delta Z_{\phi_i \phi_j} = \frac{2}{m_i^2 - m_j^2} Re\left[\Sigma_{\phi_i \phi_j}^{Tad}\left(m_{\phi_j}^2\right)\right], (i \neq j)$$

Tadpole renormalization

$$T_i = \left\langle \frac{\partial V}{\partial \rho_i} \right\rangle \qquad T_{H_j} = \sum_i R_{ji} T_i \qquad i \in \{1, S\} \quad j \in \{1, 2\}$$

$iT_i =$ 

$iT_{H_j} =$ 

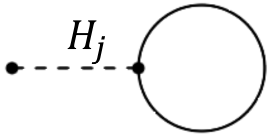
- The N2HDM scalar potential contains terms linear in the CP-even fields of the visible sector.
- These linear terms can be represented by tadpole Feynman diagrams.
- The vacuum is fixed at the proper value by requiring that the tadpole terms vanish at the vacuum state (minimum conditions)

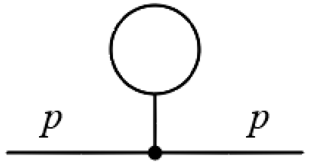
Renormalization condition: $T_i + \delta T_i = 0$

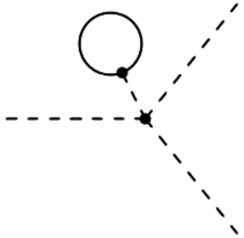
$$\underbrace{v_0 = v + \delta v \qquad v_{S_0} = v_S + \delta v_S}_{\text{Alternative tadpole scheme}}$$

$$\delta v = c_\alpha \frac{i}{m_{H_1}^2} iT_{H_1} + s_\alpha \boxed{\frac{i}{m_{H_2}^2} iT_{H_2}}$$

$$\delta v_s = -s_\alpha \frac{i}{m_{H_1}^2} iT_{H_1} + c_\alpha \frac{i}{m_{H_2}^2} iT_{H_2}$$

$$\frac{i}{m_{H_j}^2} iT_{H_j} =$$ 

$i\Sigma_\phi^{Tad}(p^2) = i\Sigma_\phi(p^2) +$ 



[J. Fleischer and F. Jegerlehner, Phys. Rev. D 23 (1981) 2001]

[M. Krause, D. Lopez-Val, M. Mühlleitner and R. Santos, JHEP 12 (2017) 077]

Mixing angle renormalization

- The renormalization of the mixing angle is done using the KOSY scheme

$$\alpha_0 = \alpha + \delta\alpha \qquad\qquad R(\alpha+\delta\alpha) = \begin{pmatrix} \cos(\alpha+\delta\alpha) & \sin(\alpha+\delta\alpha) \\ -\sin(\alpha+\delta\alpha) & \cos(\alpha+\delta\alpha) \end{pmatrix} = R(\alpha)R(\delta\alpha)$$

- Using the rotation matrix to perform the rotation between the bare gauge and mass eigenstates, we get a relation between the mixing angle counter-term and the off-diagonal WFRCs.

$$\boxed{\delta\alpha = \frac{\delta Z_{H_1 H_2} - \delta Z_{H_2 H_1}}{4}} \longrightarrow$$

Gauge-dependent terms do not vanish in the final amplitude

[S. Kanemura, Y. Okada, E. Senaha and C.-P. Yuan, Physical Review D 70 (2004)]

- Pinch technique
  - Based on the fact that any physical process must be gauge-independent;
  - It can be demonstrated that the gauge-dependences cancel-out at the self-energy level;
  - We use a scattering process to obtain new gauge-independent self-energies:

$$\Sigma^{\mathrm{PT}}(\mathrm{p}^2) = \Sigma^{Tad}(p^2)\Big|_{\xi=1} + \Sigma^{Add}(p^2)$$

$$\boxed{\delta\alpha = \frac{1}{2\left(m_{H_1}^2 - m_{H_2}^2\right)} Re\left(\left[\Sigma_{H_1H_2}^{Tad}\left(m_{H_1}^2\right) + \Sigma_{H_1H_2}^{Tad}\left(m_{H_2}^2\right)\right]_{\xi=1} + \Sigma_{H_1H_2}^{Add}\left(m_{H_1}^2\right) + \Sigma_{H_1H_2}^{Add}\left(m_{H_2}^2\right)\right)} \qquad \delta Z_{\phi_i\phi_j} = \frac{2}{m_i^2 - m_j^2} Re\left[\Sigma_{\phi_i\phi_j}^{Tad}\left(m_{\phi_j}^2\right)\right]$$

[J. M. Cornwall and J. Papavassiliou, Phys.Rev.D 40 (1989) 3474]

Electric charge renormalization

- The physical value of the electric charge is defined as the $ee\gamma$ coupling for on-shell external particles in the Thomson limit;
- The renormalization of the electrical charge is done assuming the condition that all corrections to the $ee\gamma$ vertex must vanish when the external particles are on their mass-shell ($p_\gamma^2 = 0,\ p_e^2 = m_e^2$);
- Due to a Ward identity, the charge counter-term can be expressed simply as a function of the photon and Z boson self-energies.

$$e_0 = (1 + \delta Z_e)e \qquad \alpha = \frac{e^2}{4\pi} \qquad \delta Z_e^{\alpha(0)} = \underbrace{\boxed{\frac{1}{2}\frac{\partial \Sigma_{\gamma\gamma}^T(p^2)}{\partial p^2}\bigg|_{p^2=0}}}_{\text{large log corrections}} + \frac{s_W}{c_W}\frac{\Sigma_{\gamma Z}^{\mathrm{T}}(0)}{m_Z^2}$$

- To minimize the effect of the log contributions, we use the G_μ scheme in which the fine-structure constant is written as a function of the very well measured Fermi constant G_μ;
- A large part of the corrections is included at LO and must be subtracted from the explicit corrections to avoid double counting.

$$\alpha_{G_\mu} = \frac{\sqrt{2}G_\mu m_W^2}{\pi}\left(1 - \frac{m_W^2}{m_Z^2}\right) \qquad \boxed{\delta Z_e\Big|_{G_\mu} = \delta Z_e^{\alpha(0)} - \underbrace{\boxed{\frac{1}{2}(\Delta r)_{1\mathrm{L}}}}_{\text{Corrections to muon decay } f(\Sigma_{\gamma\gamma}^T,\, \Sigma_{ZZ}^{Tad,T},\, \Sigma_{\gamma Z}^T,\, \Sigma_W^{Tad,T})}}$$

[A. Denner, Fortsch. Phys. 41 (1993) 307]
[A. Bredenstein, A. Denner, S. Dittmaier and M.M. Weber, Phys. Rev. D 74 (2006) 013004]

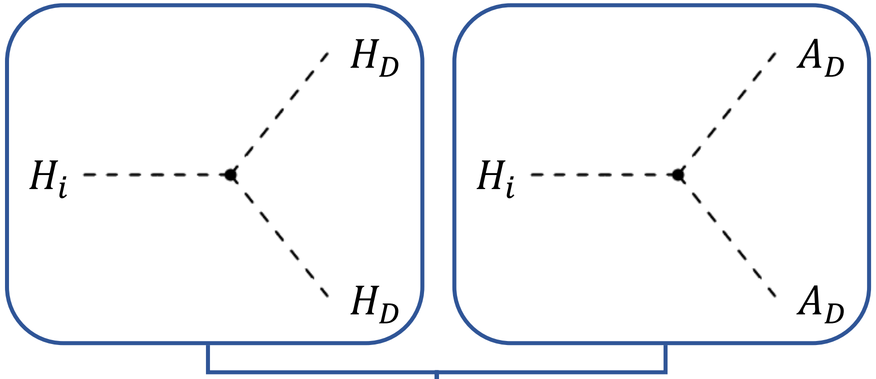
Renormalization of the dark parameters

- Counter-terms for m_{22}^2 and λ_8 fixed using three different renormalization schemes separately
  - $\overline{\text{MS}}$ scheme: the counter-terms are fixed in such way that they exactly cancel the remaining divergent terms
  - Process-dependent scheme: consists in using a set of auxiliary processes that have couplings that depend on the parameters we wish to renormalize and requiring that they respect the renormalization condition:

$$\Gamma_{\text{Aux}}^{\text{NLO}} = \Gamma_{\text{Aux}}^{\text{LO}} \Rightarrow Re\left(\mathcal{M}_{Aux}^{1loop}\right) = 0$$

Main process

Auxiliary process



Same dependence on δm_{22}^2 and $\delta\lambda_8$

$$\mathcal{M}_{H_i\to A_D A_D}^{1loop} = \mathcal{M}_{H_i\to A_D A_D}^{VC} + \mathcal{M}_{H_i\to A_D A_D}^{CT}\Big|_{\delta m_{22}^2,\delta\lambda_8=0} + \boxed{\mathcal{M}_{H_i\to A_D A_D}^{\delta m_{22}^2,\delta\lambda_8}}$$

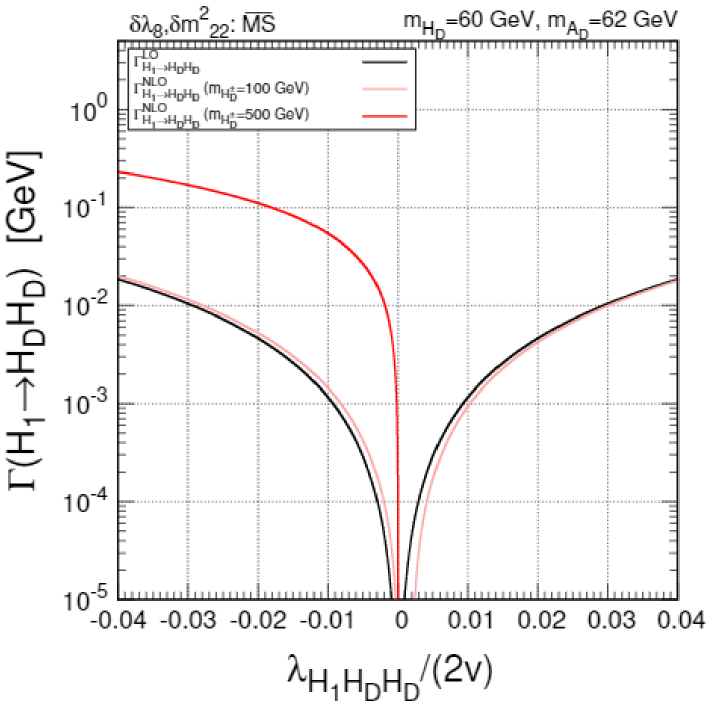
$$\mathcal{M}_{H_i\to A_D A_D}^{\delta m_{22}^2,\delta\lambda_8} = \mathcal{M}_{H_i\to H_D H_D}^{\delta m_{22}^2,\delta\lambda_8}$$

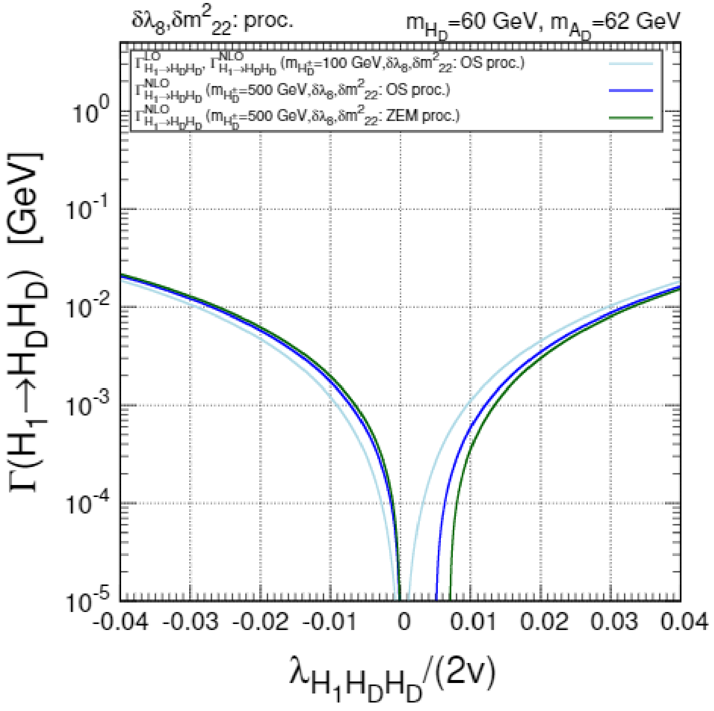
$$\boxed{\begin{aligned}\mathcal{M}_{H_i\to H_D H_D}^{1loop} &= \mathcal{M}_{H_i\to H_D H_D}^{VC} + \mathcal{M}_{H_i\to H_D H_D}^{CT}\Big|_{\delta m_{22}^2,\delta\lambda_8=0} \\ &-\mathcal{M}_{H_i\to A_D A_D}^{VC} - \mathcal{M}_{H_i\to A_D A_D}^{CT}\Big|_{\delta m_{22}^2,\delta\lambda_8=0}\end{aligned}}$$

On-Shell (OS)	$p_{H_i}^2 = m_{H_i}^2,\, p_{A_D}^2 = m_{A_D}^2$	$m_{A_D} \leq \frac{m_{H_i}}{2}$
Zero External Momentum (ZEM)	$p_{H_i}^2 = p_{A_D}^2 = 0$	No constraints on the masses

Effect of the renormalization scheme on the decay rate

Decay rate vs. dark coupling (in IDM limit)





Inert Doublet Model limit

$$\lambda_8 = 0,\ \alpha = 0,\ v_S \to \infty$$

$$\lambda_L = \frac{\lambda^{IDM}_{H_1 H_D H_D}}{2v} = \frac{m_{22}^2 - m_{H_D}^2}{v^2}$$

$\boldsymbol{m_{H_1}} = 125.09\ \text{GeV}$ $\quad$ $\boldsymbol{m_{H_2}} = 500\ \text{GeV}$

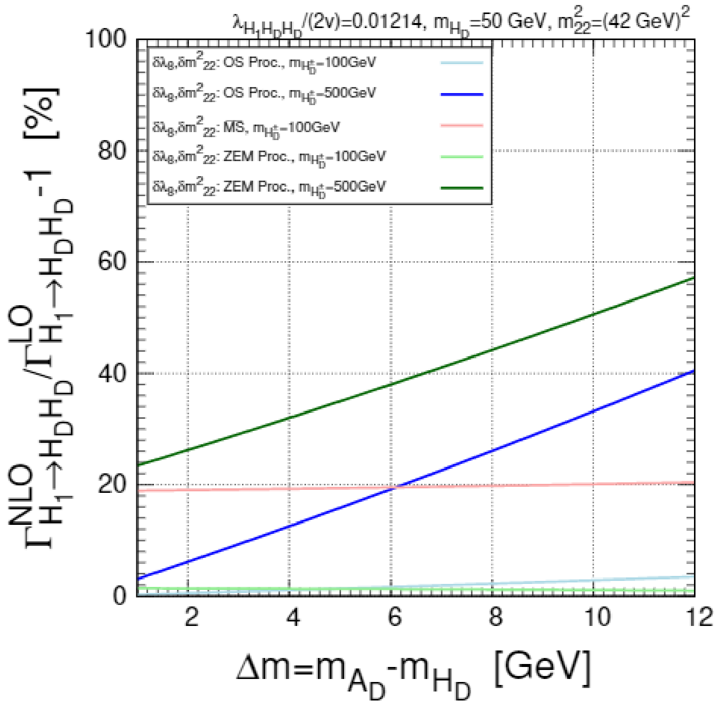
$\boldsymbol{m_{H_D}} = 60\ \text{GeV}$ $\quad$ $\boldsymbol{m_{H_D^\pm}} = 100\ \text{GeV or } 500\ \text{GeV}$

$\boldsymbol{m_{A_D}} = 62\ \text{GeV}$ $\quad$ $\boldsymbol{\lambda_2} = 0.12$

- In the $\overline{\text{MS}}$ scheme the partial decay width depends strongly on the mass of the charged Higgs; very large one-loop corrections for larger values of $m_{H_D^\pm}$.
- In the process-dependent schemes the one-loop corrections remain much more reasonable, even for large masses of the charged Higgs

Effect of the renormalization scheme on the decay rate

Size of corrections vs. dark neutral mass difference



Inert Doublet Model limit

$$\lambda_8 = 0,\ \alpha = 0,\ v_S \to \infty$$

$$\lambda_L = \frac{\lambda^{IDM}_{H_1H_DH_D}}{2v} = \frac{m_{22}^2 - m_{H_D}^2}{v^2}$$

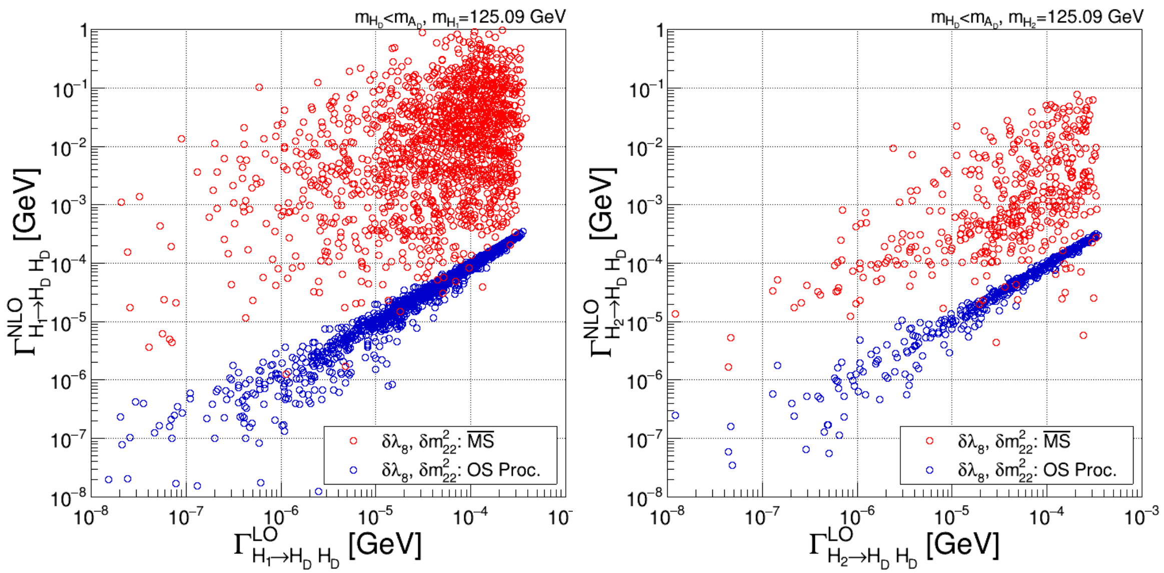
$\boldsymbol{m_{H_1}} = 125.09$ GeV $\quad \boldsymbol{m_{H_2}} = 500$ GeV

$\boldsymbol{m_{H_D}} = 50$ GeV $\quad \boldsymbol{m_{H_D^\pm}} = 100$ GeV or 500 GeV

$\boldsymbol{m_{22}^2} = (42\text{ GeV})^2 \quad \boldsymbol{\lambda_2} = 0.12$

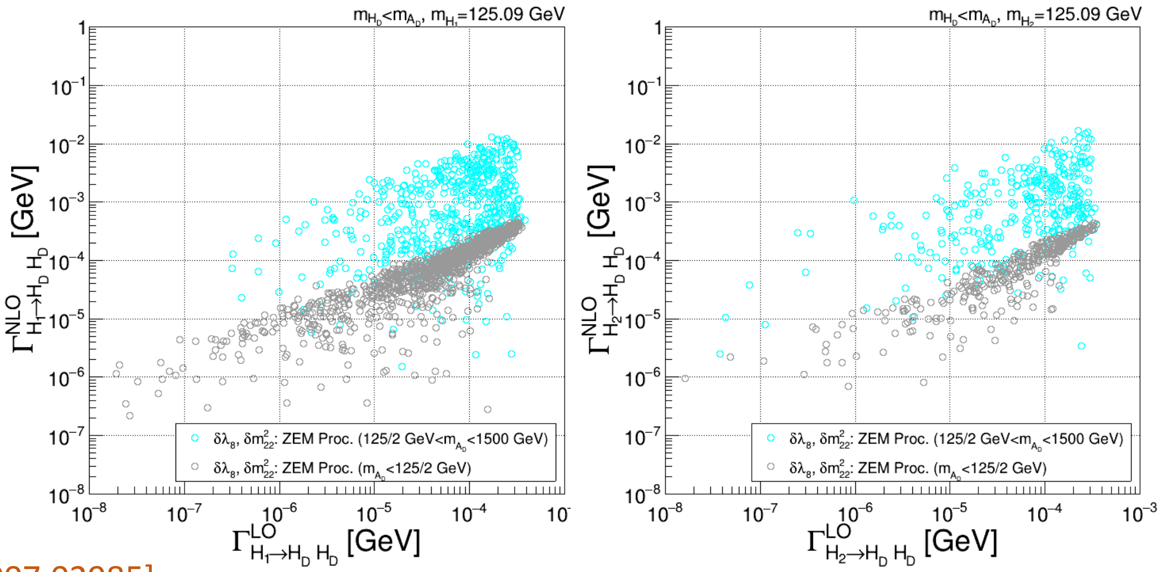
- Size of the one-loop corrections remains constant with respect to the mass difference in the $\overline{\text{MS}}$ scheme;
- In the process dependent schemes, the one-loop corrections become larger as the mass difference increases;
- Same behavior with respect to $m_{H_D^\pm}$ is observed.

Parameter space scan analysis



- LO decay width limited by experimental constraints on the Higgs couplings to SM particles;
- In the $\overline{\text{MS}}$ scheme, the NLO decay widths of most points are several orders of magnitude higher than at LO (very large corrections);
- In the OS process-dependent scheme, the decay widths are more “well-behaved” due to small Δm ($|\Delta m| \lesssim 6$ GeV).

- In the ZEM process-dependent scheme, the points with $m_{A_D} \leq 125/2$ GeV have one-loop corrections similar to the OS process-dependent scheme.
- For points with $m_{A_D} > 125/2$ GeV, the corrections in the ZEM process-dependent scheme become very large due to high Δm.

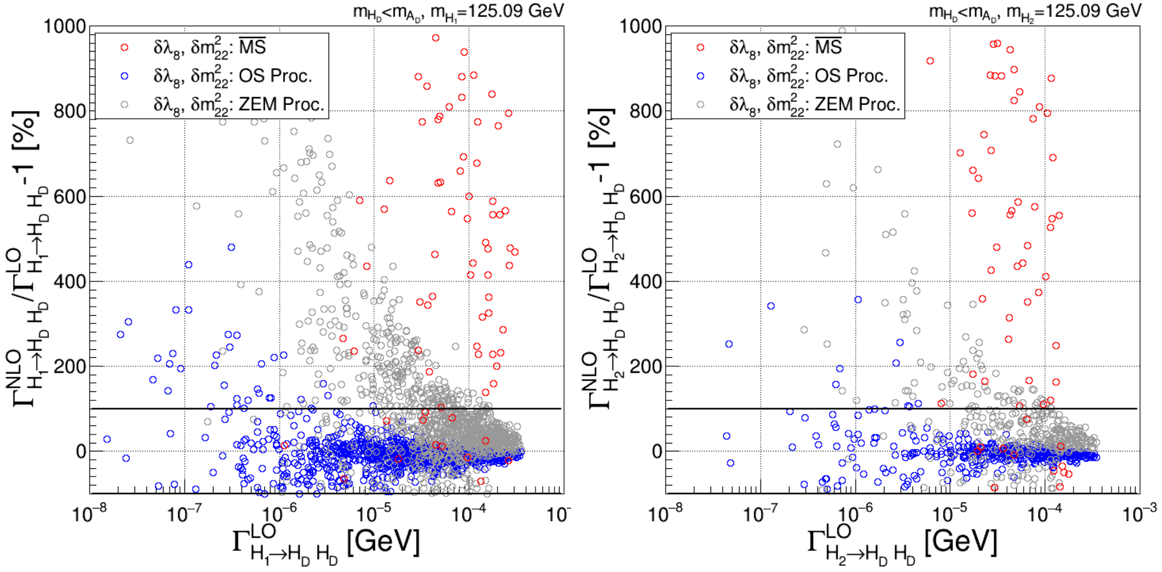


[points generated with ScannerS, containing the most relevant theoretical and experimental constraints]

[R. Coimbra, M.O.P. Sampaio and R. Santos, Eur. Phys. J. C 73 (2013) 2428]

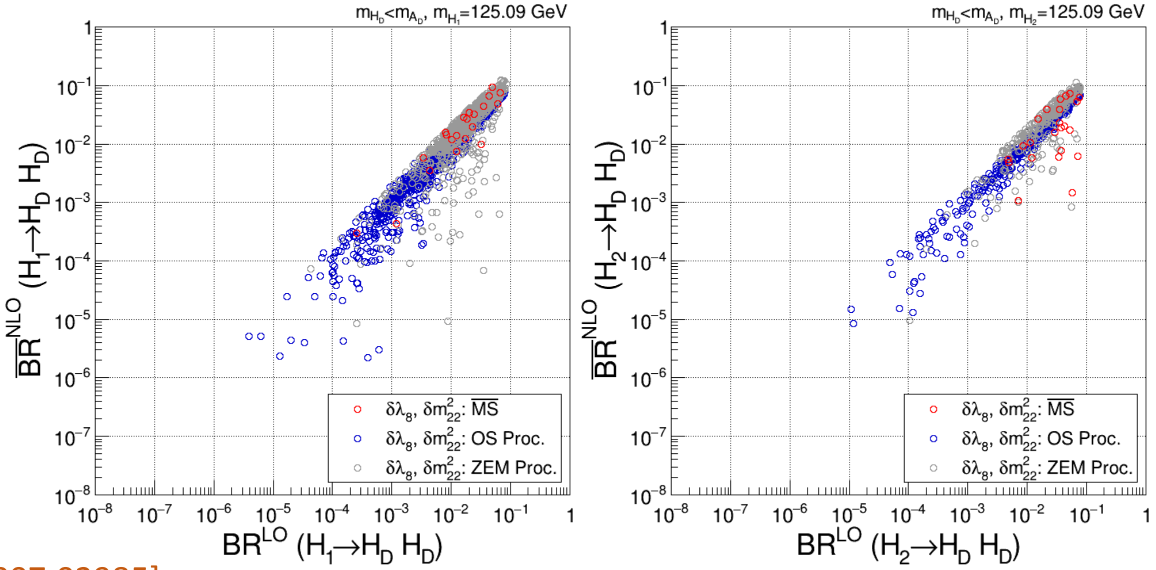
[M. Mühlleitner, M. O. P. Sampaio, R. Santos, J. Wittbrodt, (Jul 6, 2020) e-Print: 2007.02985]

Parameter space scan analysis



$$m_{A_D} \leq \frac{125}{2}\ \text{GeV}$$

- Corrections in the OS process-dependent scheme mostly between ±100% (most stable);
- Considerable number of points with corrections larger than 100% in the ZEM process-dependent scheme (less stable than OS);
- $\overline{\text{MS}}$ scheme contains very large corrections with very few points having corrections between ±100% (least stable).



- Considering only corrections between ±100% (rough perturbative limit), all surviving points have NLO branching ratios at or bellow the experimental limit ($BR(h_{125} \to inv.) = 0.11$)
- No constraints can be extracted for the parameter space yet.

[points generated with ScannerS, containing the most relevant theoretical and experimental constraints]

[R. Coimbra, M.O.P. Sampaio and R. Santos, Eur. Phys. J. C 73 (2013) 2428]

[M. Mühlleitner, M. O. P. Sampaio, R. Santos, J. Wittbrodt, (Jul 6, 2020) e-Print: 2007.02985]

Conclusions

- Two possible scenarios for the decay of the SM Higgs boson into a pair of dark matter particles, within the dark doublet phase (DDP) of the N2HDM: the Light Higgs and the Heavy Higgs scenarios.
- Three different schemes were used to renormalize the dark parameters m_{22}^2 and λ_8: the $\overline{\text{MS}}$ scheme and two process-dependent schemes (OS and ZEM). It was shown that with regards to the size of the NLO corrections the $\overline{\text{MS}}$ scheme is too unstable to be trusted in comparison with the process-dependent schemes.
- We scanned over the parameter space and calculated the one-loop corrected partial decay widths of the possible Higgs decays to dark matter, concluding that if we require that the corrections are not unphysically large ($< 100\%$), all the points remain at or below the experimental limit for the Higgs-to-invisible branching ratio ($BR(h_{125} \to inv.) = 0.11$).
- No constraints on the parameter space could be obtained from our results. However, our results are very close to the experimental limit and as the measurements on the couplings and invisible decays of the Higgs boson improve, we are certain that these results will lead to constraints on the parameters space of the DDP in the future.
- In future work, we want to include in these calculations the one-loop corrections to all possible Higgs decays within the DDP. This is needed if we want our results to be reliable as this would reduce the error of the calculated branching ratios to the choice of renormalization scheme.

Backup

CP-even mixing

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + \rho_1 + iG^0) \end{pmatrix} \quad \Phi_2 = \begin{pmatrix} H_D^+ \\ \frac{1}{\sqrt{2}}(H_D + iA_D) \end{pmatrix} \quad \Phi_S = v_S + \rho_S$$

$$M_{ij}^\rho = \frac{\partial^2 V_{scalar}}{\partial\rho_i \partial\rho_j}$$

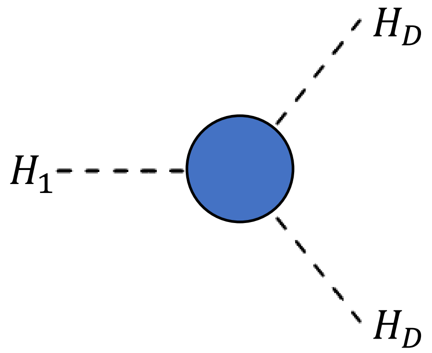
$$M^\rho = \begin{pmatrix} v^2\lambda_1 & v v_S \lambda_7 \\ v v_S \lambda_7 & v_S^2 \lambda_6 \end{pmatrix} \longrightarrow$$ ρ_1 and ρ_S are not mass eigenstates

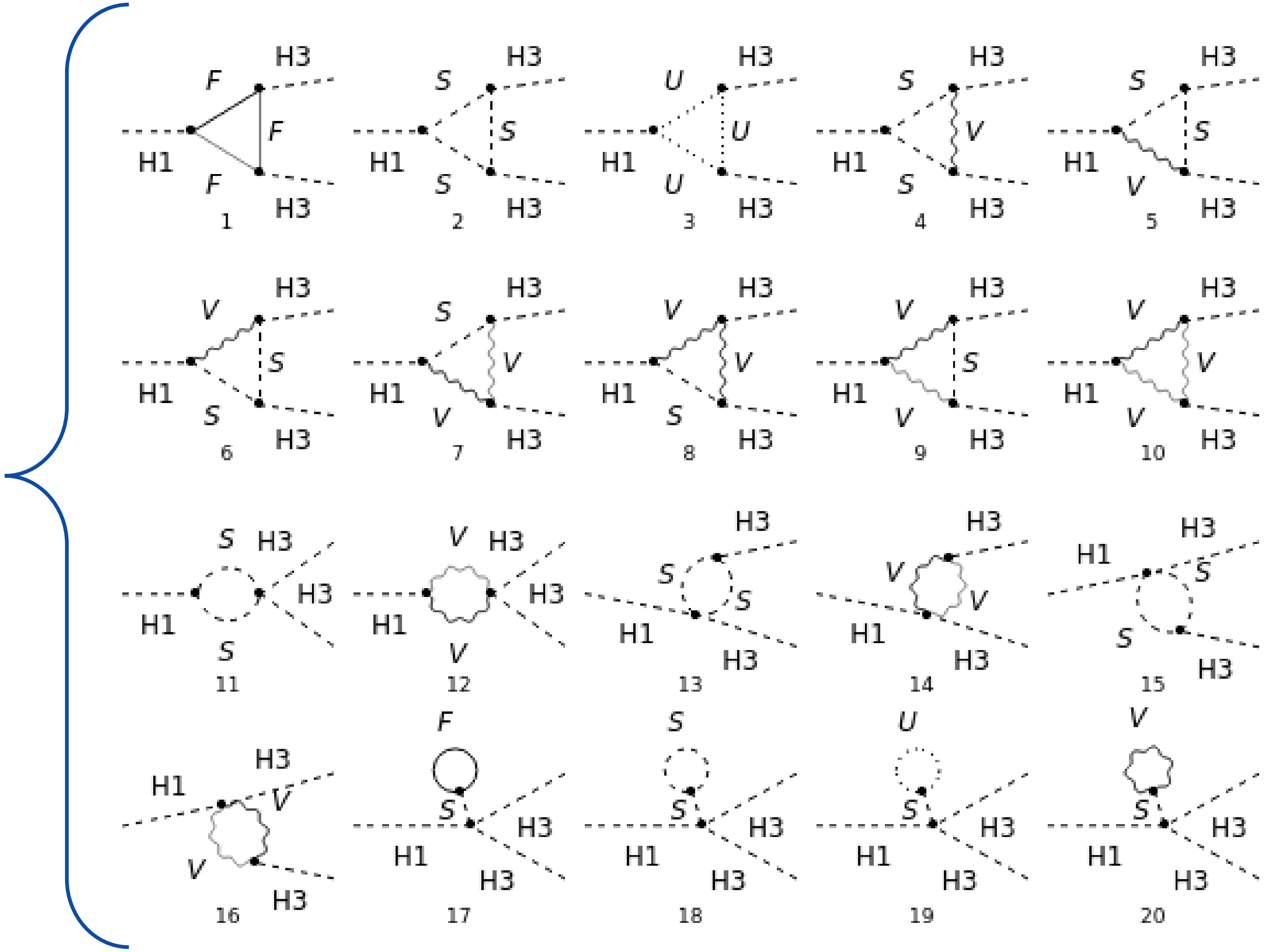
$$\longrightarrow R(\alpha) M^\rho R(\alpha)^T = \begin{pmatrix} m_{H_1}^2 & 0 \\ 0 & m_{H_2}^2 \end{pmatrix} \qquad m_{H_1} \leq m_{H_2}$$

$$R(\alpha) = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{pmatrix}$$

$$\boxed{\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = R(\alpha)^T \begin{pmatrix} \rho_1 \\ \rho_S \end{pmatrix}} \qquad \begin{cases} H_1 = \rho_1 \cos\alpha - \rho_S \sin\alpha \\ H_2 = \rho_1 \sin\alpha + \rho_S \cos\alpha \end{cases}$$

Vertex corrections





Renormalization (tadpoles)

$$T_1 = \frac{1}{2}\left(v^2\lambda_1 + v_S^2\lambda_7\right) + m_{11}^2$$

$$T_S = \frac{1}{2}\left(v^2\lambda_7 + v_S^2\lambda_6\right) + m_S^2$$

$$m^2 \to m^2 + \delta m^2 + \boxed{\Delta m^2}$$

Induced by the VEV renormalization

$$\lambda_{H_iH_DH_D} \to \lambda_{H_iH_DH_D} + \boxed{\Delta\lambda_{H_iH_DH_D}}$$

$$T_i = -\delta T_i$$

$$\begin{pmatrix}\delta v\\ \delta v_S\end{pmatrix} = R(\alpha)\underbrace{\boxed{R(\alpha)^T (M^\rho)^{-1} R(\alpha)}}_{(\mathrm{D}^2)^{-1}}\underbrace{\boxed{R(\alpha)^T\begin{pmatrix}\delta T_1\\ \delta T_S\end{pmatrix}}}_{\begin{pmatrix}\delta \mathrm{T}_{H_1}\\ \delta \mathrm{T}_{H_2}\end{pmatrix}}$$

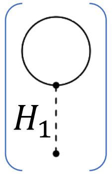
$$\begin{pmatrix}\delta v\\ \delta v_S\end{pmatrix} = \begin{pmatrix} c_\alpha \frac{i}{m_{H_1}^2} iT_{H_1} + s_\alpha \frac{i}{m_{H_2}^2} iT_{H_2} \\ -s_\alpha \frac{i}{m_{H_1}^2} iT_{H_1} + c_\alpha \frac{i}{m_{H_2}^2} iT_{H_2}\end{pmatrix}$$

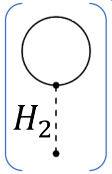
$$\underbrace{v_0 = v + \delta v \qquad v_{S0} = v_S + \delta v_S}$$

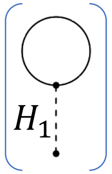
Alternative tadpole scheme

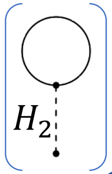
$$M^\rho = \begin{pmatrix} v^2\lambda_1 & v v_S \lambda_7 \\ v v_S \lambda_7 & v_S^2 \lambda_6 \end{pmatrix}$$

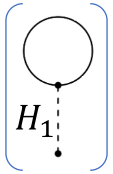
$$\begin{pmatrix}\delta T_1\\ \delta T_S\end{pmatrix} = M^\rho \begin{pmatrix}\delta v\\ \delta v_S\end{pmatrix}$$

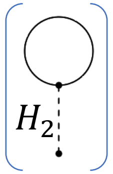
$\delta v = c_\alpha$ 

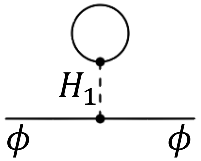
$+\ s_\alpha$ 

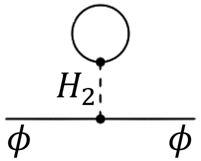
$\delta v_S = -s_\alpha$ 

$+\ c_\alpha$ 

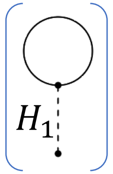
$\Delta m_\phi^2 = i\Big[\lambda_{H_1\phi\phi}$ 

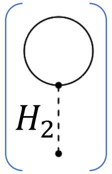
$+\ \lambda_{H_2\phi\phi}$ 

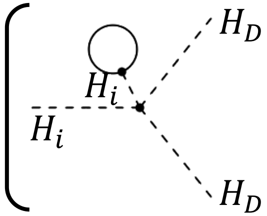
$\Big] = i\Big[$ 

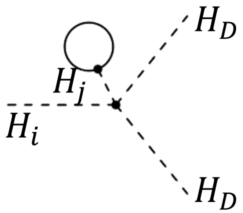
$+$ 

$\Big]$

$\Delta\lambda_{H_iH_DH_D} = \Big[\lambda_{H_iH_iH_DH_D}$ 

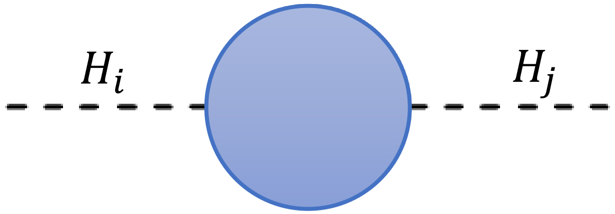
$+\ \lambda_{H_iH_jH_DH_D}$ 

$\Big] = \Big[$ 

$+$ 

$\Big]$

Renormalization (mixing fields)



$$\Phi = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \longrightarrow \Phi_0 = \begin{pmatrix} H_{1,0} \\ H_{2,0} \end{pmatrix} \approx \left(\mathbb{I}_{2\times 2} + \frac{\delta Z_H}{2} \right) \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$$

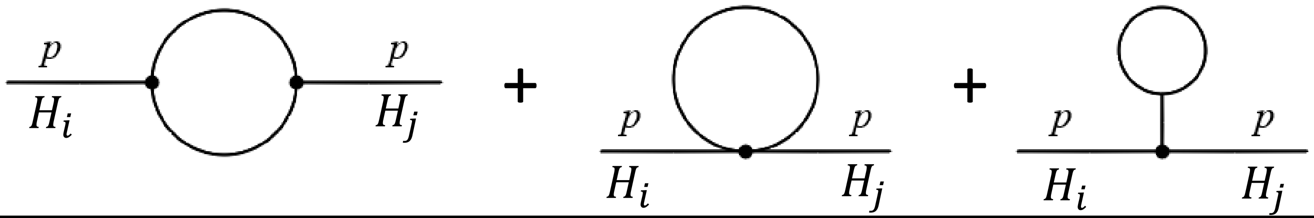
$$G^{-1}(p^2) = -i \left(p^2 \mathbb{I}_{2\times 2} - D^2 + \hat{\Sigma}_H(p^2) \right)$$

$$\hat{\Sigma}_{H_iH_j}(p^2) = \Sigma_{H_iH_j}(p^2) - \delta M_{ij}^2 + \frac{\delta Z_{H_jH_i}^*}{2} \left(p^2 \mathbb{I}_{2\times 2} - D_{jj}^2 \right) + \left(p^2 \mathbb{I}_{2\times 2} - D_{ii}^2 \right) \frac{\delta Z_{H_iH_j}}{2}$$

$$\delta Z_H = \begin{pmatrix} \delta Z_{H_1H_1} & \delta Z_{H_1H_2} \\ \delta Z_{H_2H_1} & \delta Z_{H_2H_2} \end{pmatrix}$$

$$D^2 = \begin{pmatrix} m_{H_1}^2 & 0 \\ 0 & m_{H_2}^2 \end{pmatrix}$$

$i\Sigma_{H_iH_J}(p^2)$ =



Renormalization (mixing angle)

$$\alpha_0 = \alpha + \delta\alpha \qquad R(\alpha+\delta\alpha) = \begin{pmatrix} \cos(\alpha+\delta\alpha) & \sin(\alpha+\delta\alpha) \\ -\sin(\alpha+\delta\alpha) & \cos(\alpha+\delta\alpha) \end{pmatrix} = R(\alpha)R(\delta\alpha)$$

$$\boxed{\begin{aligned} \cos\delta\alpha &\approx 1 \\ \sin\delta\alpha &\approx \delta\alpha \end{aligned}}$$

KOSY scheme

$$\begin{pmatrix} H_{1,0} \\ H_{2,0} \end{pmatrix} = R(\alpha_0)^T \begin{pmatrix} \rho_{1,0} \\ \rho_{S,0} \end{pmatrix} = R(\delta\alpha)^T R(\alpha)^T \sqrt{Z_\rho} \begin{pmatrix} \rho_1 \\ \rho_S \end{pmatrix} = \underbrace{R(\delta\alpha)^T R(\alpha)^T \sqrt{Z_\rho} R(\alpha)}_{\sqrt{Z_H}} \underbrace{R(\alpha)^T \begin{pmatrix} \rho_1 \\ \rho_S \end{pmatrix}}_{\begin{pmatrix} H_1 \\ H_2 \end{pmatrix}}$$

OS scheme

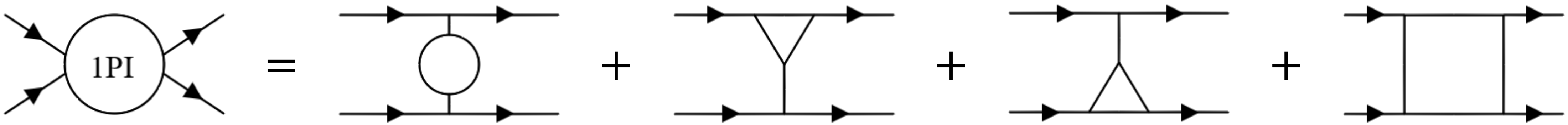
$$\begin{pmatrix} H_{1,0} \\ H_{2,0} \end{pmatrix} \approx \sqrt{Z_H}^{OS} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$$

$$\begin{cases} \dfrac{\delta Z^{OS}_{H_1H_2}}{2} = \delta C + \delta\alpha \\ \dfrac{\delta Z^{OS}_{H_2H_1}}{2} = \delta C - \delta\alpha \end{cases} \longrightarrow \quad \delta\alpha = \frac{\delta Z^{OS}_{H_1H_2} - \delta Z^{OS}_{H_2H_1}}{4}$$

$$\delta\alpha = \frac{1}{2\left(m^2_{H_1} - m^2_{H_2}\right)} Re\left(\Sigma^{Tad}_{H_1H_2}\left(m^2_{H_1}\right) + \Sigma^{Tad}_{H_2H_1}\left(m^2_{H_2}\right)\right)$$

$$\sqrt{Z_H} \approx \begin{pmatrix} 1 + \dfrac{\delta Z_{H_1H_1}}{2} & \delta C + \delta\alpha \\ \delta C - \delta\alpha & 1 + \dfrac{\delta Z_{H_2H_2}}{2} \end{pmatrix} \qquad \sqrt{Z_H}^{OS} \approx \begin{pmatrix} 1 + \dfrac{\delta Z_{H_1H_1}}{2} & \dfrac{\delta Z_{H_1H_2}}{2} \\ \dfrac{\delta Z_{H_2H_1}}{2} & 1 + \dfrac{\delta Z_{H_2H_2}}{2} \end{pmatrix}$$

Renormalization (pinch technique)



$$\mathcal{M}(s,t,m_i) = \mathcal{M}_{\text{self}}(t,\xi) + \mathcal{M}_{\text{tri}}(t,m_i,\xi) + \mathcal{M}_{\text{box}}(t,s,m_i,\xi)$$

$$\frac{\partial^2 \mathcal{M}_{\text{box}}}{\partial\xi\partial s} = 0 \longrightarrow \mathcal{M}_{\text{box}} = \hat{\mathcal{M}}_{\text{box}}(t,s,m_i) + h(t,m_i,\xi) \longrightarrow \tilde{\mathcal{M}}_{\text{tri}}(t,m_i,\xi) = \mathcal{M}_{\text{tri}}(t,m_i,\xi) + h(t,m_i,\xi)$$

$$\frac{\partial^2 \tilde{\mathcal{M}}_{\text{tri}}}{\partial\xi\partial m_i} = 0 \longrightarrow \tilde{\mathcal{M}}_{\text{tri}} = \hat{\mathcal{M}}_{\text{tri}}(t,m_i) + f(t,\xi) \longrightarrow \hat{\mathcal{M}}_{\text{self}}(t,\xi) = \mathcal{M}_{\text{self}}(t,\xi) + f(t,\xi)$$

Gauge-dependence cancels at the self-energy level

Renormalization of the dark parameters ($\overline{\text{MS}}$ scheme)

- The $\overline{\text{MS}}$ scheme consists in fixing the counter-terms in such way that they exactly cancel the remaining divergent terms;
- We use the parameters' β functions, which measure the dependence of the parameter on the renormalization scale μ, to obtain the expressions for the counter-terms δm_{22}^2 and $\delta\lambda_8$;

$$\beta_\rho^{(1)} = 32\pi^2 \frac{\partial\rho}{\partial \ln\mu} \qquad \boxed{\delta\rho = \frac{1}{32\pi^2}\boxed{\beta_\rho^{(1)}}\Delta} \qquad \Delta = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi$$

$$\beta_{m_{22}^2}^{(1)} = 2\lambda_4 m_{11}^2 + 4\lambda_3 m_{11}^2 + 6\lambda_2 m_{22}^2 - \frac{45}{30} g_Y^2 m_{22}^2 - \frac{9}{2} g_L^2 m_{22}^2 + \lambda_8 m_S^2$$

$$\beta_{\lambda_8}^{(1)} = 2\lambda_4\lambda_7 + 4\lambda_3\lambda_7 + \frac{\lambda_8}{10}\left(30\lambda_6 + 40\lambda_8 - 45 g_L^2 + 60\lambda_2 - \frac{45}{3} g_Y^2\right)$$

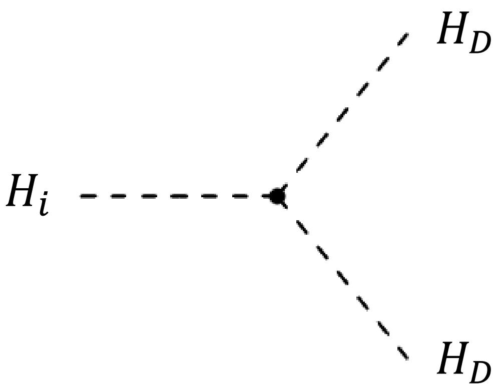
Obtained with SARAH

- An additional counter-term is needed in order to obtain finite amplitudes
  - We can define the counter-term Δv_S such that it cancels the remaining divergence

$$\boxed{\boxed{\Delta v_S} = -\left(s_\alpha \frac{T_{H_1}}{m_{H_1}^2} + c_\alpha \frac{T_{H_2}}{m_{H_2}^2}\right)_{div}}$$

Additional counter-term resulting from tadpole renormalization

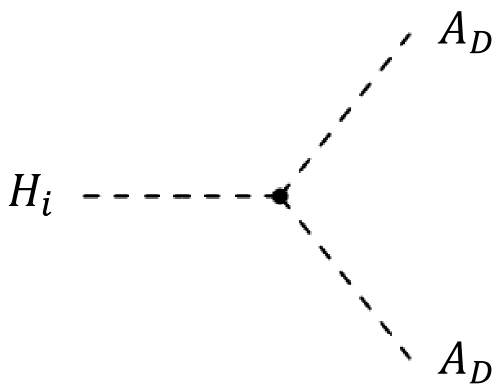
Dependence on the dark parameters



$$\lambda_{H_i H_D H_D} = \frac{2R_{i1}}{v}\left(\frac{\lambda_8 v_S^2}{2} + m_{22}^2 - m_{H_D}^2\right) - R_{i2}\lambda_8 v_S$$

$$\mathcal{M}_{H_i \to H_D H_D}^{\delta m_{22}^2, \delta\lambda_8} = \frac{\partial \lambda_{H_i H_D H_D}}{\partial m_{22}^2}\delta m_{22}^2 + \frac{\partial \lambda_{H_i H_D H_D}}{\partial \lambda_8}\delta\lambda_8$$

$$= \frac{2R_{i1}}{v}\delta m_{22}^2 + \left(\frac{R_{i1}}{v}v_S^2 - R_{i2}v_S\right)\delta\lambda_8$$



$$\lambda_{H_i A_D A_D} = \frac{2R_{i1}}{v}\left(\frac{\lambda_8 v_S^2}{2} + m_{22}^2 - m_{A_D}^2\right) - R_{i2}\lambda_8 v_S$$

$$\mathcal{M}_{H_i \to A_D A_D}^{\delta m_{22}^2, \delta\lambda_8} = \frac{\partial \lambda_{H_i A_D A_D}}{\partial m_{22}^2}\delta m_{22}^2 + \frac{\partial \lambda_{H_i A_D A_D}}{\partial \lambda_8}\delta\lambda_8$$

$$= \frac{2R_{i1}}{v}\delta m_{22}^2 + \left(\frac{R_{i1}}{v}v_S^2 - R_{i2}v_S\right)\delta\lambda_8$$

Observables

$$|\mathcal{M}^{NLO}|^2 = (\mathcal{M}^{NLO})^* \mathcal{M}^{NLO} = (\mathcal{M}^{LO} + \mathcal{M}^{1L})^*(\mathcal{M}^{LO} + \mathcal{M}^{1L}) = |\mathcal{M}^{LO}|^2 + 2Re\Big((\mathcal{M}^{LO})^*\mathcal{M}^{1L}\Big) + \mathcal{O}(NNLO)$$

Partial decay width

$$\Gamma_{H_iH_DH_D} = \frac{\sqrt{m_{H_i}^2 - 4m_{H_D}^2}}{32\pi m_{H_i}^2}\left|\mathcal{M}_{H_iH_DH_D}\right|^2 \longrightarrow \Gamma_{H_iH_DH_D}^{NLO} = \frac{\sqrt{m_{H_i}^2 - 4m_{H_D}^2}}{32\pi m_{H_i}^2}\left[|\mathcal{M}^{LO}|^2 + 2Re\Big((\mathcal{M}^{LO})^*\mathcal{M}^{1L}\Big)\right]$$

Branching ratio

$$BR(H_i \to H_DH_D) = \frac{\Gamma_{H_iH_DH_D}}{\boxed{\Gamma_{H_i}}} \longrightarrow \overline{BR}^{NLO}(H_i \to H_DH_D) = \mathcal{R}_{H_i}BR^{NLO}(H_i \to H_DH_D)$$

$$\Gamma_{H_i} = \sum_X \Gamma_{H_i \to X}$$

NLO approximation

$$\mathcal{R}_{H_i} = \frac{\Gamma_{H_i}^{NLO}}{\overline{\Gamma}_{H_i}^{NLO}} = \frac{\Gamma_{H_i}^{NLO}}{\Gamma_{H_i} - \Gamma_{H_i \to H_DH_D}^{LO} + \Gamma_{H_i \to H_DH_D}^{NLO}}$$

$$\overline{\Gamma}_{H_i}^{NLO} = \Gamma_{H_i \to SM}^{LO} + \Gamma_{H_i \to A_DA_D}^{LO} + \Gamma_{H_i \to H_D^+H_D^-}^{LO} + \delta_{i2}\Gamma_{H_2 \to H_1H_1}^{LO} + \Gamma_{H_i \to H_DH_D}^{NLO}$$

Software

- Mathematica packages
  - **FeynRules 2.3.35** – given the implementation of the model, outputs its Feynman rules.
    [N.D. Christensen and C. Duhr, Comput. Phys. Commun. 180 (2009) 1614]
    [C. Degrande, C. Duhr, B. Fuks, D. Grellscheid, O. Mattelaer and T. Reiter, Comput. Phys. Commun. 183 (2012) 1201]
    [A. Alloul, N.D. Christensen, C. Degrande, C. Duhr and B. Fuks, Comput. Phys. Commun. 185 (2014) 2250]
  - **FeynArts 3.11** – generates Feynman diagrams and their corresponding amplitudes
    [J. Kublbeck, M. Bohm and A. Denner, Comput. Phys. Commun. 60 (1990) 165]
    [T. Hahn, Comput. Phys. Commun. 140 (2001) 418]
  - **FeynCalc 9.2.1** – handles QFT algebraic simplifications (contraction of Lorentz indices, calculation of color factors, Dirac algebra, etc.)
    [R. Mertig, M. Bohm and A. Denner, Comput. Phys. Commun. 64 (1991) 345]
    [V. Shtabovenko, R. Mertig and F. Orellana, Comput. Phys. Commun. 207 (2016) 432]
  - **LoopTools 2.14** – outputs numerical values for the finite parts of loop integrals
    [T. Hahn and M. Perez-Victoria, Comput. Phys. Commun. 118 (1999) 153]
    [G.J. van Oldenborgh and J.A.M. Vermaseren, Z. Phys. C 46 (1990) 425]
  - **SARAH 4.14.2** – tool for SUSY and non-SUSY models implementation and analysis.
    [F. Staub, Comput. Phys. Commun. 181 (2010) 1077]
    [F. Staub, Adv. High Energy Phys. 2015 (2015) 840780]
- **N2HDECAY** – outputs tree-level total decay widths and branching ratios for the N2HDM, including state-of-the-art QCD corrections.
  [I. Engeln, M. Mühlleitner and J. Wittbrodt, Comput. Phys. Commun. 234 (2019) 256]
- **ScannerS** – generates parameter space points, taking into consideration the most relevant theoretical and experimental constraints.
  [R. Coimbra, M.O.P. Sampaio and R. Santos, Eur. Phys. J. C 73 (2013) 2428]
  [M. Mühlleitner, M. O. P. Sampaio, R. Santos, J. Wittbrodt, (Jul 6, 2020) e-Print: 2007.02985]

ScannerS

- Theoretical constraints
  - Perturbative unitarity
  - Boundedness from below
  - Vacuum stability
- Experimental constraints
  - Electroweak precision data
  - Higgs couplings measurements
  - Scalar exclusion limits
  - Dark matter constraints
    - Relic abundance (Planck experiment)
    - Nucleon-DM cross section for direct detection (XENON1T experiment)

Parameter scan ranges

Light Higgs

$$m_{H_1} = 125.09\ GeV$$

$$130\ GeV < m_{H_2} < 1500\ GeV$$

Heavy Higgs

$$m_{H_2} = 125.09\ GeV$$

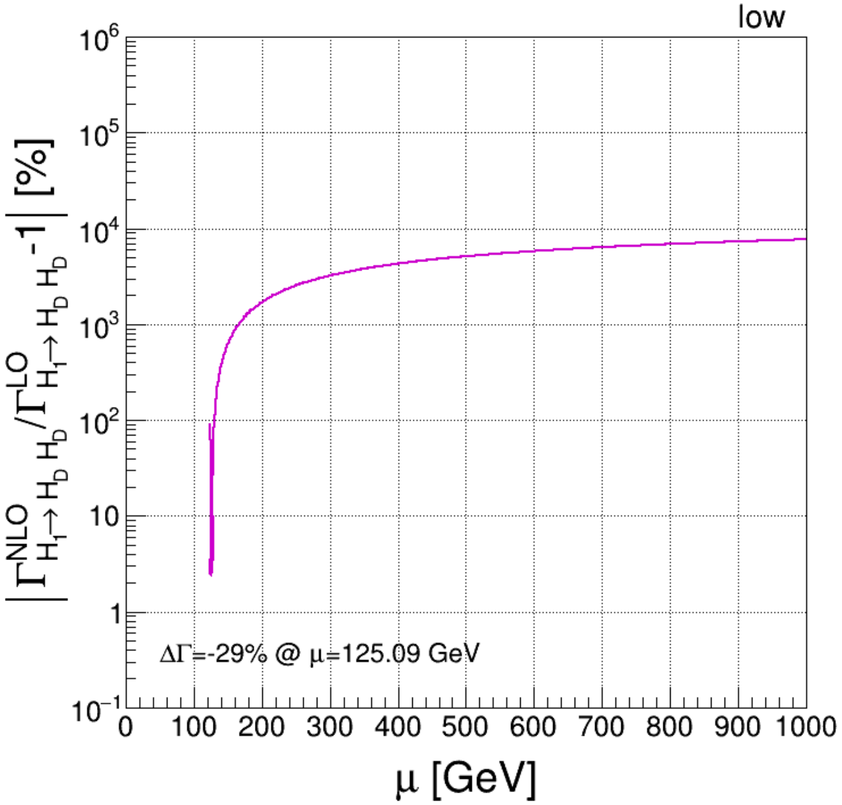
$$62\ GeV < m_{H_1} < 120\ GeV$$

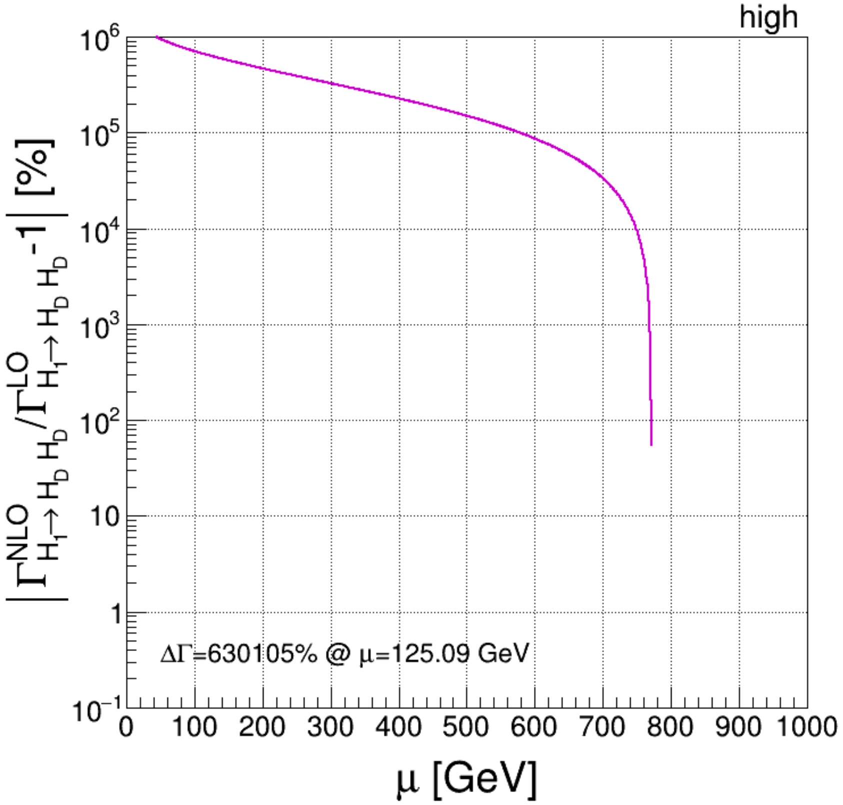
$$1\ GeV < m_{H_D} < 62\ GeV \qquad 1\ GeV < m_{A_D} < 1400\ GeV \qquad 65\ GeV < m_{H_D^\pm} < 1400\ GeV$$

$$10^{-3} GeV^2 < m_{22}^2 < 10^6\ GeV^2 \qquad 0 < \lambda_2 < 4\pi \qquad -4\pi < \lambda_8 < 4\pi$$

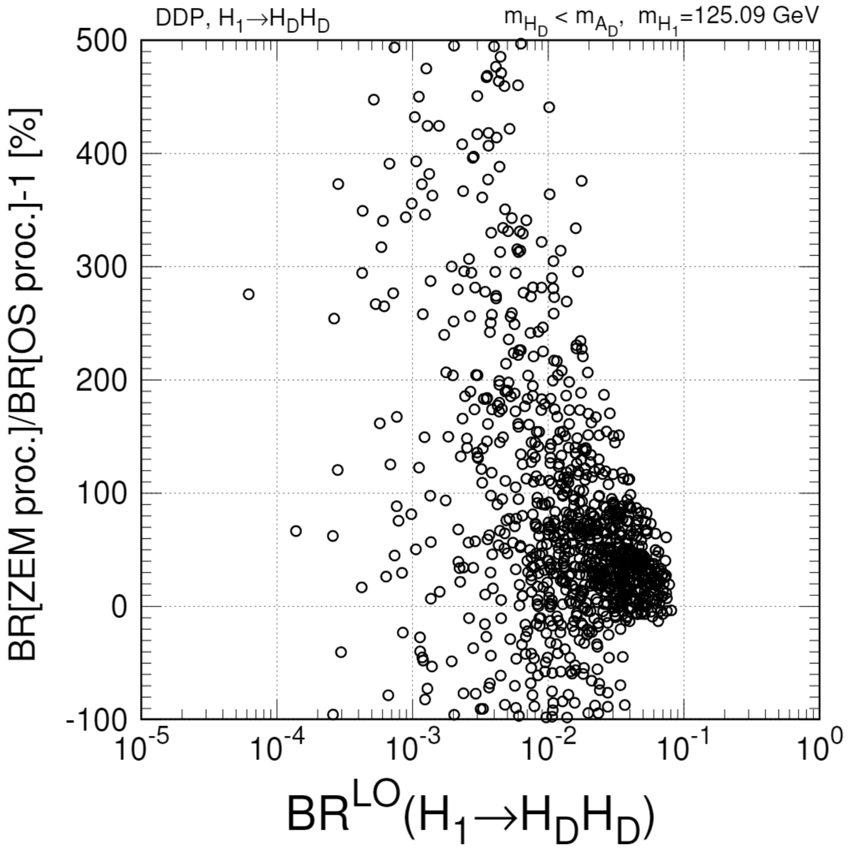
$$1\ GeV < v_S < 5000\ GeV \qquad -\frac{\pi}{2} < \alpha < \frac{\pi}{2}$$

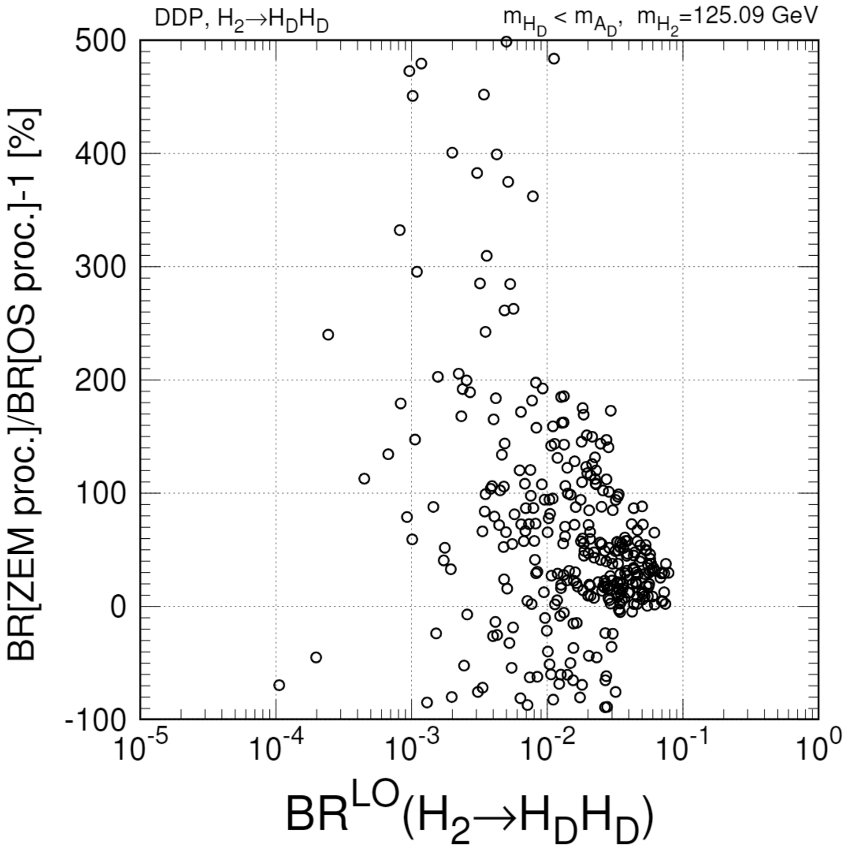
$\overline{\text{MS}}$ scheme and renormalization scale



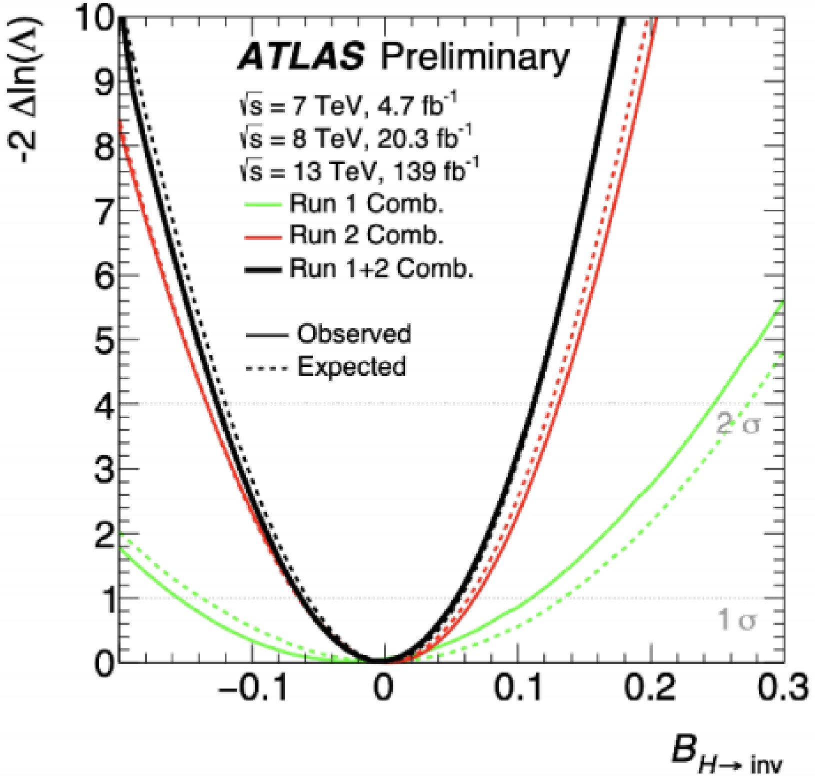


OS process-dependent vs ZEM process-dependent





Experimental limits



[ATLAS-CONF-2020-052]

Future limits:

HL-LHC: $BR_{inv} < 0.019$

ILC: $BR_{inv} < 0.019$

FCC: $BR_{inv} < 0.019$

[J. de Blas et al., JHEP 01, 139 (2020)]