DRalgo: Taming the dragon

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BSM^2 - Beyond the Standard Model BrainStorming Meeting

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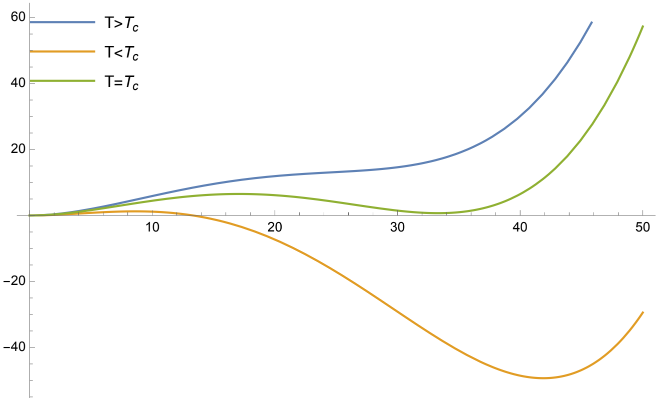
Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

DESY.

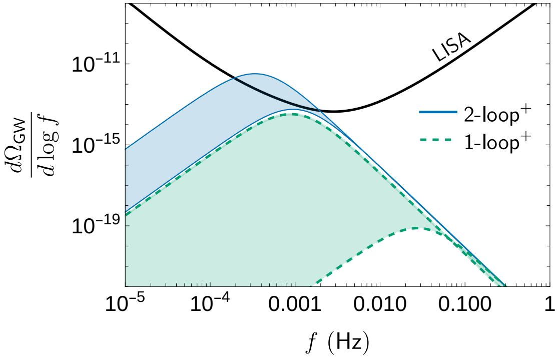
The Electroweak phase transitions in a nutshell



Typical calculations

- Effective potential: $V_{\text{eff}}(\phi, T)$
- Latent heat: $\alpha \sim \frac{d}{d \log T} V_{\text{eff}}(\phi, T_c)$
- Nucleation rate: $\Gamma \sim e^{-S_3/T}$
- Inverse duration: $\frac{\beta}{H} \sim \frac{d}{d \log T} S_3/T$
- Wall speed: $\dot{\phi} \sim v_w \phi$

Electroweak phase transitions in a nutshell



Gould & Tenkanen 2104.04399

Why care about phase transitions?

- Electroweak Baryogenesis
- Gravitational waves

Theoretical Inputs

- Latent heat $\rightarrow T_c, \alpha$
- Nucleation rate $\rightarrow T_N, \beta$
- Wall speed $\rightarrow v_w$

Getting uncertainties under control reveals wonderful physics

Standard method—1-loop effective potential + thermal masses

Example potential

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

Effective potential

$$V_T = \sum_{i\in\text{particles}} J(m_i^2)$$

Field-dependent masses

$$m_S^2 = m^2 + \lambda\phi^2$$
$$m_F^2 = y_t^2\phi^2$$
$$m_V^2 = g^2\phi^2$$

Mass resummation: $m^2 \to m_{\text{eff}} \equiv m^2 + \sum_i g_i^2 T^2 \sim m^2 + T^2\left[a\lambda + by_t^2 + cg^2\right]$.

$$J^{d=4-2\varepsilon}(m_i^2) = -\frac{1}{64\pi^2\varepsilon} - \frac{\pi^2}{90}T^4 + \cancel{\frac{T^2 m_i^2}{24}} - \frac{T(m_{\text{i,eff}})^{3/2}}{12\pi} - \frac{m_i^4}{64\pi^2}\log\left[\frac{e^{2\gamma_E}\mu^2}{16\pi^2T^2}\right] + \ldots$$

Why the large uncertainties?

A natural fine-tuning

$$m_{\text{eff}}^2 = (m^2 + \underbrace{aT^2}_{\text{1-loop Thermal}}) \ll m^2$$

$$\implies \mu \frac{d}{d\log\mu} m_{\text{eff}}^2 \approx m_{\text{eff}}^2$$

Fine-tuning $\implies \underbrace{bT^2}_{\text{2-loop Thermal}} \approx m_{\text{eff}}^2$

Logarithms $\implies \log T^2/m_{\text{eff}}^2 \gg 1$

Higher-order corrections are huge

Solution: Use an effective field theory

(9508379,2104.04399)

Why an EFT? $\log T^2/m_{\text{eff}}^2 \to \underbrace{\log T^2/\mu^2}_{\text{Match at } \mu\sim T} + \underbrace{\log \mu^2/m_{\text{eff}}^2}_{\text{RG-evolution in the EFT}}$ ✓

Integrating out heavy "particles"

In equilibrium we can view temperature effects through Matsubara modes:

$$\partial_\mu \phi(x) \partial^\mu \phi(x) \to \vec{\nabla}\phi(\vec{x}) \cdot \vec{\nabla}\phi(\vec{x}) + \sum_{n=-\infty}^{\infty} (2\pi n T)^2 \phi(\vec{x})^2$$

In essence an infinite tower of heavy particles$\sim T \gg m$

What do we do with heavy particles? $\to$ Integrate them out

In practice: Write down the most general 3d-spatial Lagrangian
$\to$ match the coefficients

$$\frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda \phi^4 \to \frac{1}{2} m_{3\text{d}}^2 \phi^2 + \frac{1}{4} \lambda_{3\text{d}} \phi^4$$

What changes in the effective theory?

Resummation of all terms

$$J^{d=4-2\varepsilon}(m_i^2) = \underbrace{-\frac{1}{64\pi^2\varepsilon} - \frac{\pi^2}{90}T^4 + \frac{T^2 m_i^2}{24} - \frac{m_i^4}{64\pi^2}\log\left[\frac{e^{2\gamma_E}\mu^2}{16\pi^2 T^2}\right] + \ldots}_{\to V_{\text{tree}}^{3d}(\phi)} \underbrace{- \frac{T(m_{i,eff})^{3/2}}{12\pi}}_{\to \frac{(m_{i,3d}^2(T))^{3/2}}{12\pi}}$$

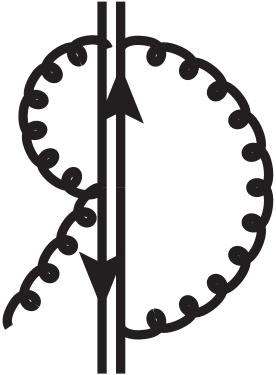
1-loop correction in the effective theory

$$J^{d=3-2\varepsilon}(m_i^2) = -\frac{(m_{i,3d}^2(T))^{3/2}}{12\pi}$$

We trade explicit temperature dependence for implicit dependence in the effective couplings. Bonus: No fermions

Get the high-temperature EFT in Mathematica within seconds!

`https://github.com/DR-algo/DRalgo` (2205.08815)



DRalgo : Automatic matching to two loops

- → Two-loop thermal masses ✓
- → Two-loop Debye masses ✓
- → One-loop thermal couplings ✓
- → Two-loop effective potential ✓
- → Beta functions ✓

This tutorial

How DRalgo works

How to calculate the latent heat

How to calculate the nucleation rate

The temporal component and Debye masses

Temporal vectors can be massive $A^\mu = (A^0, A^i)$

$$\frac{1}{2}\mathrm{D}^i A_0 \mathrm{D}^i A_0 + \frac{1}{2} m_D^2 A_0^2 + \frac{1}{4}\lambda_K \phi^2 A_0^2 + \frac{1}{4!}\lambda_A A_0^4$$

In practice we can ignore A_0

Temporal scalars are often heavy $m_D \gg m_{3\mathrm{d}} \to$ Integrate them out!

$\to$ Corrections to $m_{3\mathrm{d}},\ \lambda_{3\mathrm{d}},\ g_{3\mathrm{d}}$

This step is automatic in DRalgo

Conventions for the EFT

Dimensionless action

Our original partition function is $Z = \mathrm{Tr} e^{-S_3/T}$

We can make S_3 dimensionless by shifting our fields: $\phi \to \sqrt{T}\phi$.

$$\frac{1}{2}m_{3\mathrm{d}}^2\phi^2 + \frac{1}{4}\lambda_{3\mathrm{d}}\phi^4 \to T\left[\frac{1}{2}m_{3\mathrm{d}}^2\phi^2 + \frac{1}{4}\underbrace{T\lambda_{3\mathrm{d}}}_{\to\lambda_{3\mathrm{d}}}\phi^4\right]$$

So effective couplings contain a factor of T

Just a convention, nothing deep

Renormalization-scale evolution

Example for a quartic coupling measured at $\mu_0 \sim m_Z$: $\lambda_{4\text{d}}(\mu_0)$

Effective couplings:

$$\lambda_{3\text{d}}(\mu) = T\left[\lambda_{4\text{d}}(\mu) + \lambda_{4\text{d}}^2(\mu)\left(a \log\frac{\mu}{T} + b\right)\right]$$

RG-invariant: $T^{-1}\frac{d}{d\log\mu}\lambda_{3\text{d}}(\mu) = \underbrace{\beta_\lambda + a\lambda_{4\text{d}}^2}_{=0} + \mathcal{O}(\lambda_{4\text{d}}^4) = \mathcal{O}(\lambda_{4\text{d}}^4)$

How it works in practice

Evolve $\lambda_{4\text{d}}$ from $\mu = \mu_0$ to $\mu = T$

Plug the result into $\lambda_{3\text{d}}$ with $\mu = T$ (makes the logarithm small)

Calculate the effective potential

Change T until the two minima coincide at T_c

Calculate observables at T_c

The nucleation rate in the effective theory

How it works in practice

Evolve λ_{4d} from $\mu = \mu_0$ to $\mu = T$.

Plug the result into λ_{3d} with $\mu = T$.

Calculate the three-dimensional bounce action S_3

Change T until $e^{-S_3} \sim e^{-140}$

Calculate the inverse-duration: $\beta/H = \frac{d}{d \log T} S_3$

Optional trick: $\frac{d}{d \log T} S_3 = \frac{\partial \lambda_3}{\partial \log T} \frac{\partial}{\partial \lambda_{3d}} S_3$