Recent results on hadron spectroscopy

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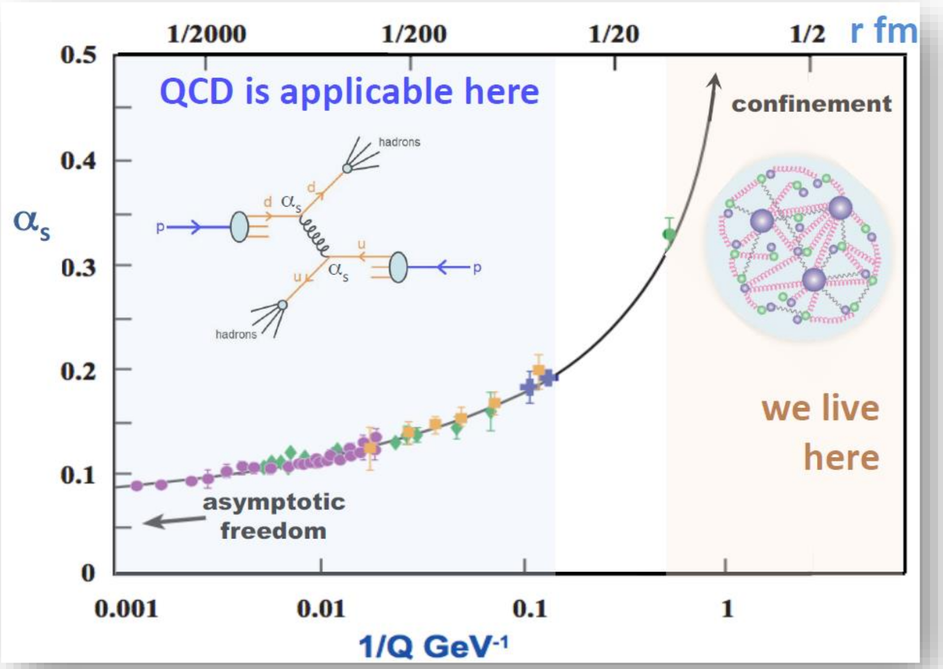
Hadrons

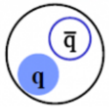
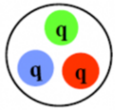
- **Hadrons: complex building block of the visible universe**
- **Emergence of hadron structure**
  - How are hadrons formed from quarks?
  - What is the origin of confinement?
  - How is the mass of hadron generated in QCD?
  - What is the dynamics of effective DoF in hadrons?

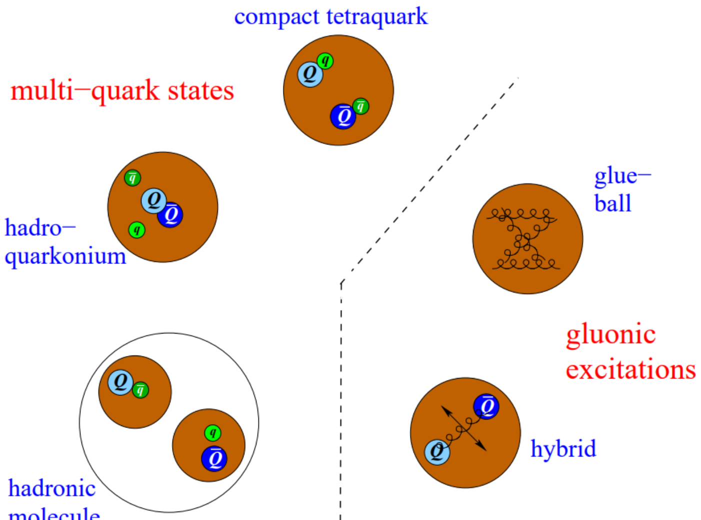


courtesy to S. Olsen

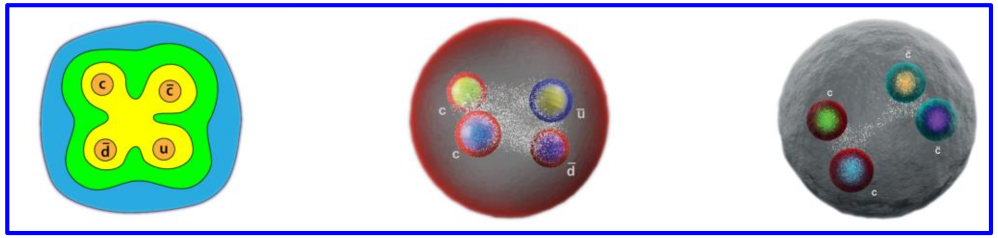
→ **Hadron spectroscopy**

Hadron spectroscopy

- **Quark model: mesons**  **baryons** 
- **Key things to search for: QCD exotics(configurations beyond QM)**
  - Strong evidences for multi-quark in heavy quark sector
    A new "particle zoo": https://qwg.ph.nat.tum.de/exoticshub/
  - **Evidence for gluonic excitations remains sparse**



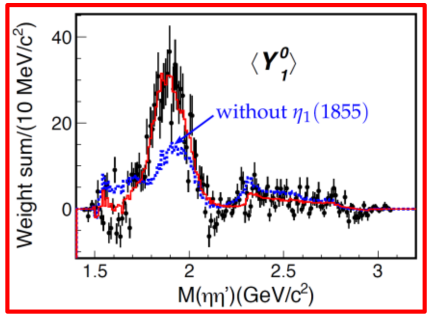
Phys.Rept. 873 (2020) 1

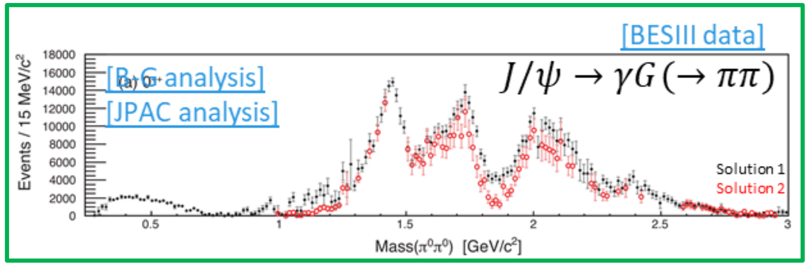


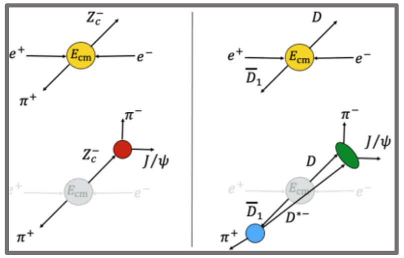
How to identify:

- Manifestly exotic
  - Flavor exotic
  - Spin exotic: $\mathbf{J^{PC} = 0^{--}, even^{+-}, odd^{-+}}$
- Crypto exotic
  - Supernumerary states
  - Abnormal properties

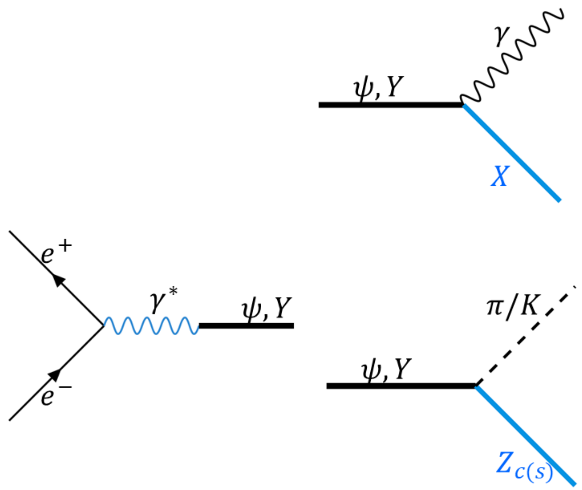
\+ kinematics

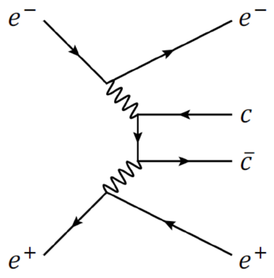




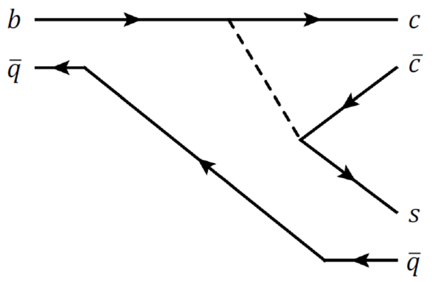


e^+e^- reactions





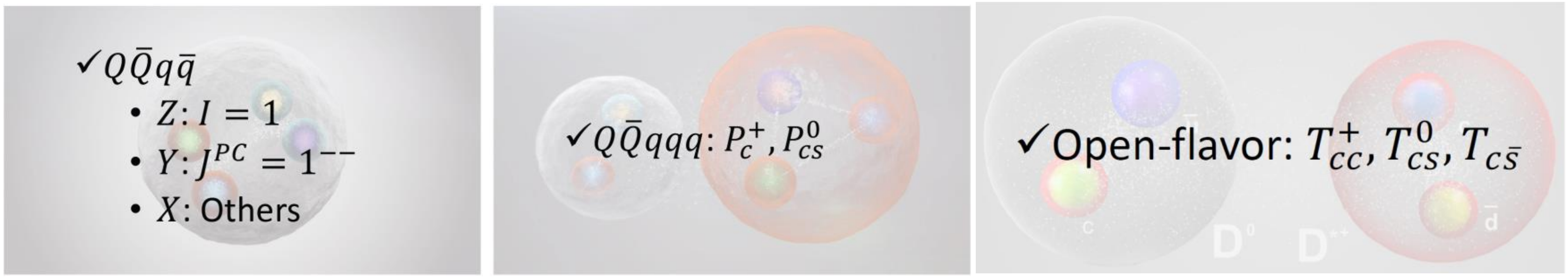
b-hadron decays



prompt production

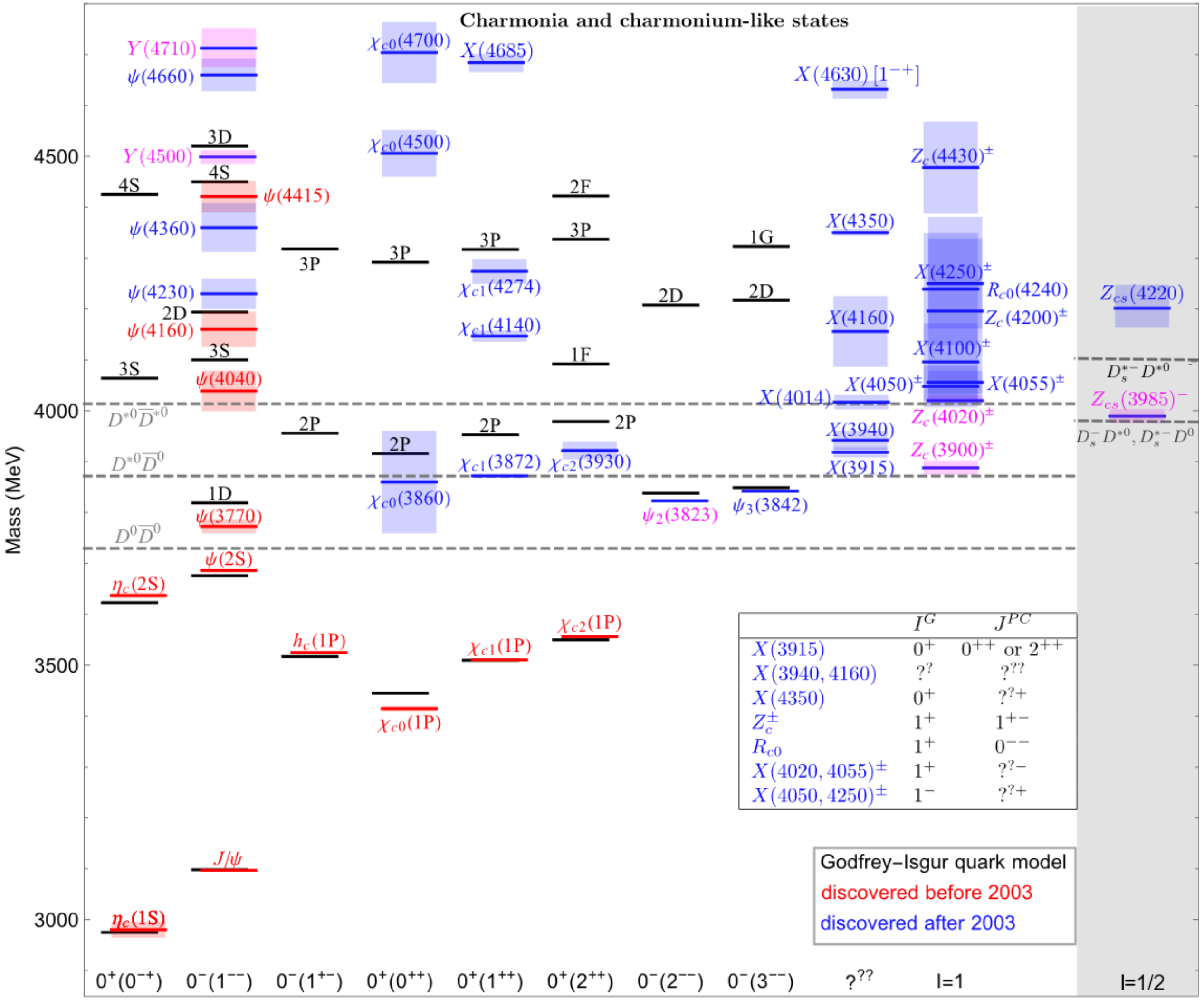
photon-production

Heavy QCD exotics



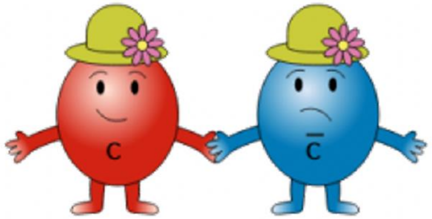
Disclaimer:
not able to cover all results

Experimental observations



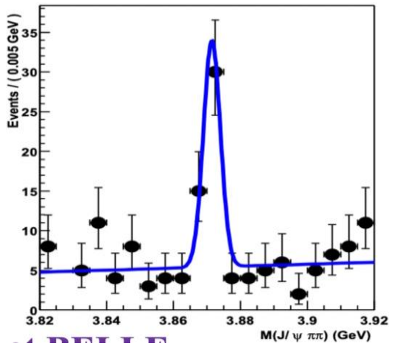
arXiv:2203.08290

- Conventional $c\bar{c}$ mesons fit well with potential model
- Abundance of new states above the open charm threshold

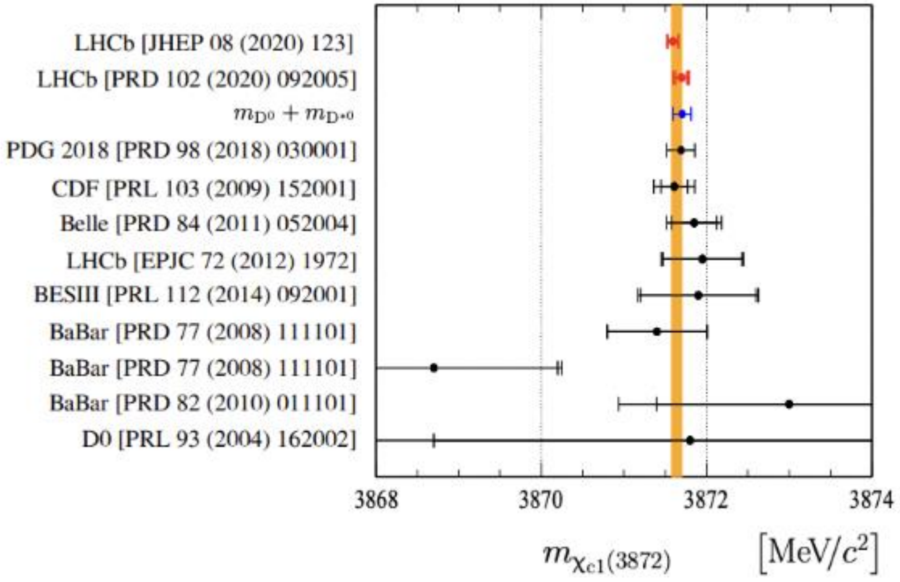


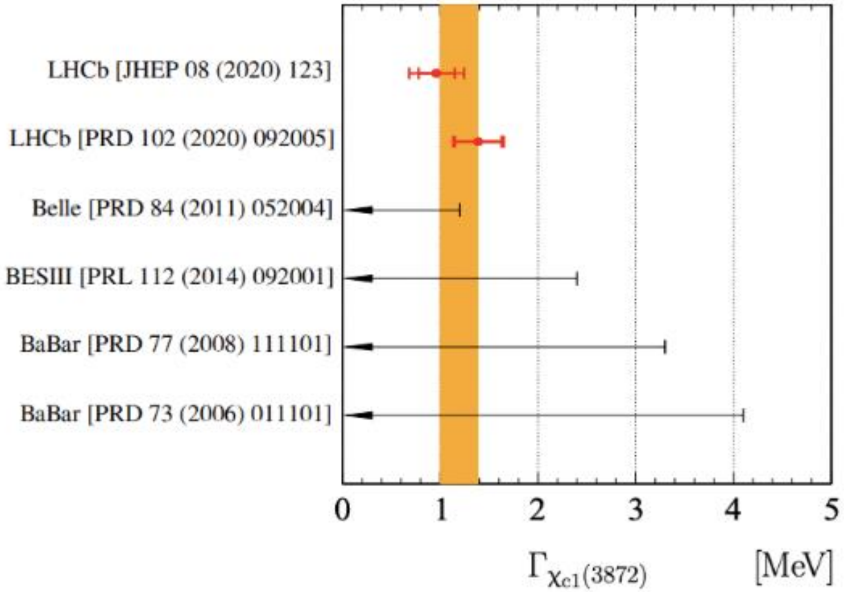
X(3872)

Discovery at Belle



PRL **91**, 262001 (2003)





Many experiments contribute to it

- Spin assignment: $\mathbf{J^{PC} = 1^{++}}$
- Mass is consistent with $\mathbf{m(D^0) + m(D^{*0})}$
- Width is **surprisingly narrow**
- **Prompt production:** X(3872)-$\psi(2S)$ yield ratio from p-p with increasing multiplicities toward p-Pb and Pb-Pb collisions
- **Decay properties:** $\rightarrow \boldsymbol{\omega J/\psi, \rho J/\psi}$; $\rightarrow \boldsymbol{\gamma J/\psi, \gamma\psi(2S)}$

Its nature is still under debate!

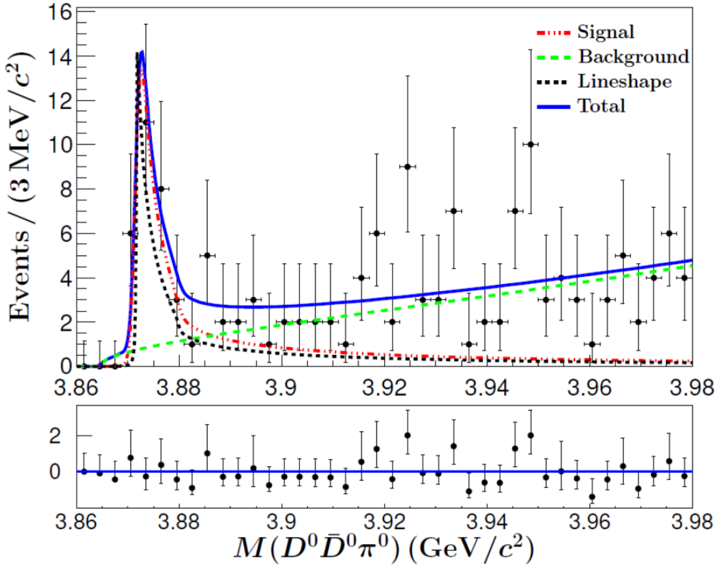
→ conventional $\chi_{c1}(2^3P_1)$, DD^* molecular state, tetraquark, hybrid, vector glueball, or mixed?

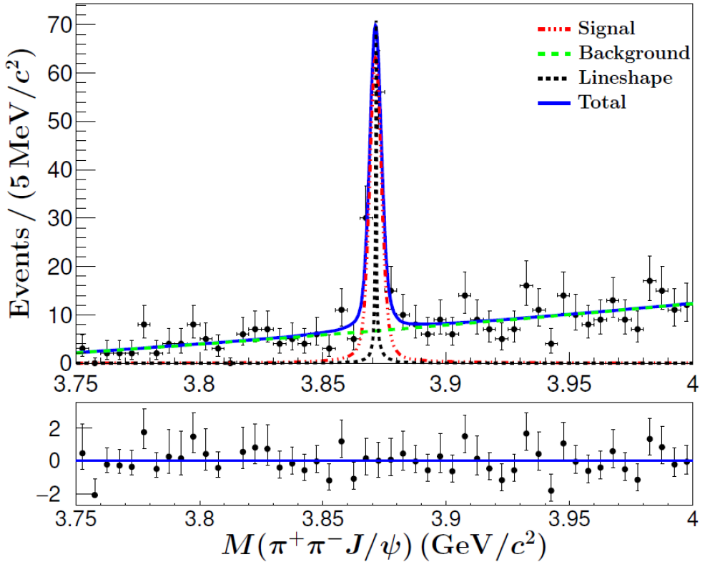
[BESIII PRL 124, 242001 (2020)]

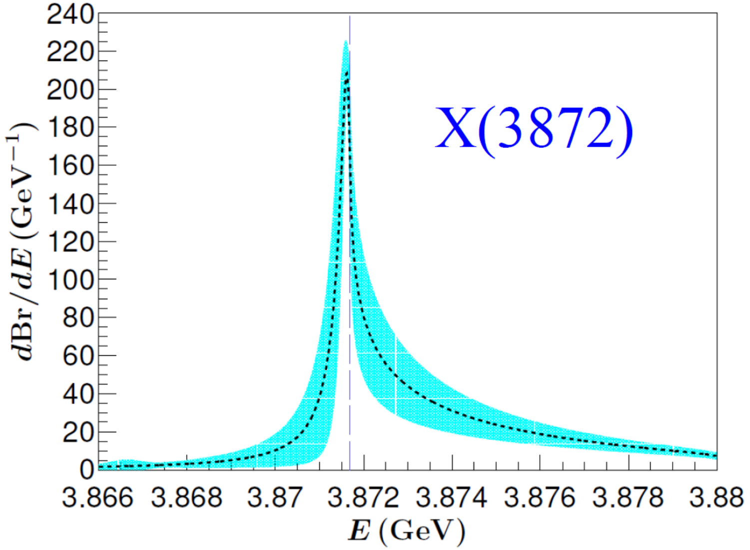
A coupled channel analysis of the X(3872) lineshape

$\mathbf{e^+e^- \to \gamma X(3872), X(3872) \to D^0\bar{D}^0\pi^0}$ **and** $\mathbf{\pi^+\pi^- J/\psi}$

BESIII arXiv:2309.01502







Pole positions

Two sheets with respect to $D^{*0}\bar{D}^0$ branch cut

- Sheet I: $E - E_X - g\sqrt{-2\mu(E - E_R + i\Gamma/2)}$
- Sheet II: $E - E_X + g\sqrt{-2\mu(E - E_R + i\Gamma/2)}$

$E_{\rm I} = (7.04 \pm 0.15^{+0.07}_{-0.08}) + (-0.19 \pm 0.08^{+0.14}_{-0.19})i$ MeV

$E_{\rm II} = \left(0.26 \pm 5.74^{+5.14}_{-38.32}\right) + (-1.71 \pm 0.90^{+0.60}_{-1.96})i$ MeV

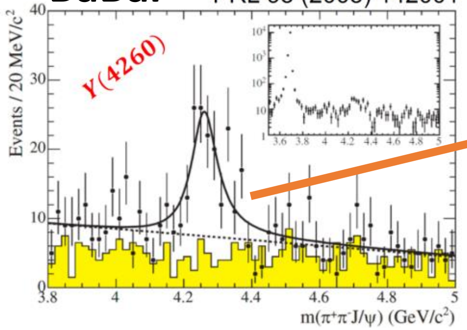
	LHCb	Belle	BESIII
g	$0.108 \pm 0.003^{+0.005}_{-0.006}$	$0.29^{+2.69}_{-0.15}$	$0.16 \pm 0.10^{+1.12}_{-0.11}$
$Re[E_I]$ [MeV]	7.10	7.12	$7.04 \pm 0.15^{+0.07}_{-0.08}$
$Im[E_I]$ [MeV]	-0.13	-0.12	$-0.19 \pm 0.08^{+0.14}_{-0.19}$
$Re[k^+]$ [MeV]	-13.9	-15.3	$-12.6 \pm 5.5^{+6.6}_{-6.2}$
$Im[k^+]$ [MeV]	8.8	7.7	$12.3 \pm 6.8^{+6.0}_{-6.4}$
a (fm)	-27.1	-31.2	$-16.5^{+7.0}_{-27.6}{}^{+5.6}_{-27.7}$
r_e (fm)	-5.3	$-3.0^{+1.3}_{-1.5}$	$-4.1^{+0.9}_{-3.3}{}^{+2.8}_{-4.4}$
$\bar{Z}_A$	0.15 (0.33)	$0.08^{+0.04}_{-0.03}$	$0.18^{+0.06}_{-0.17}{}^{+0.19}_{-0.16}$

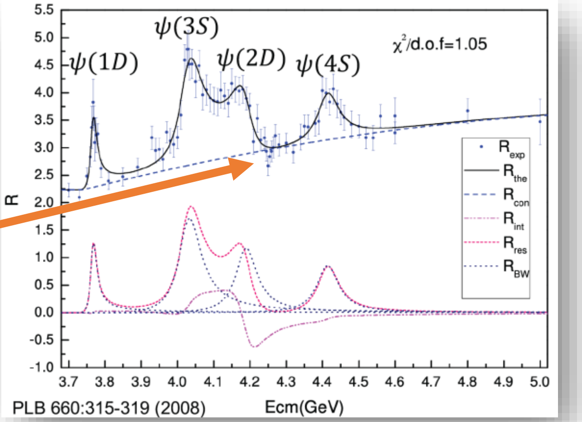
Weinberg's compositeness: Z=1: pure elementary state; Z=0: pure bound (composite) state

Vector states: Y(4260) → Y(4230)

- Y(4260) firstly seen by BaBar
  - Inconsistent with simple $c\bar{c}$ scenario
  - Candidates for exotics:
    - Hybrid /molecule /Tetraquark ?

BaBar PRL 95 (2005) 142001

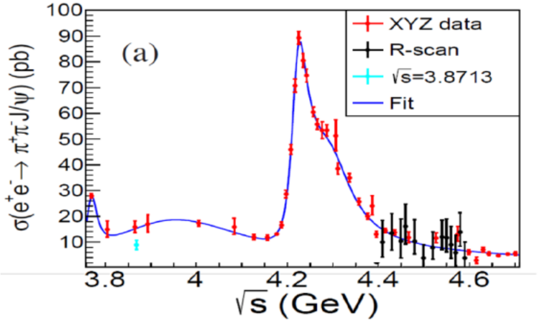




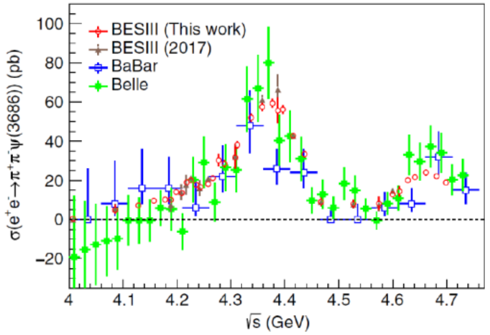
BESIII

Y(4230) appears in $\pi\pi J/\psi, \omega\chi_{c0}, \pi\pi h_c, \eta_c 3\pi, \ KKJ/\psi, D^0 D^{*-}\pi^+, D^{*0} D^{*-}\pi^+$

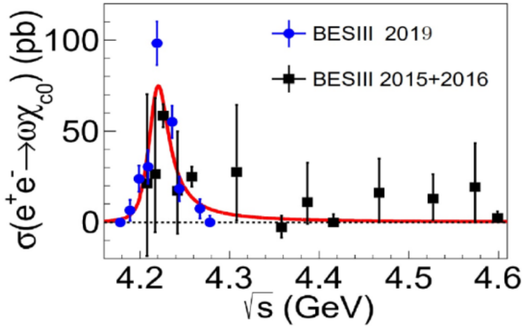
PRD106, 072001 (2022)



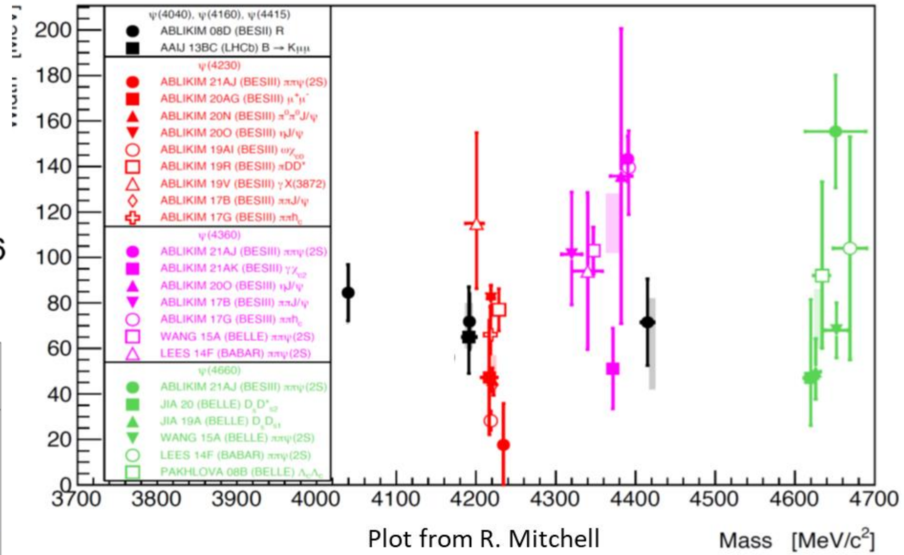
PRD104, 052102 (2021)



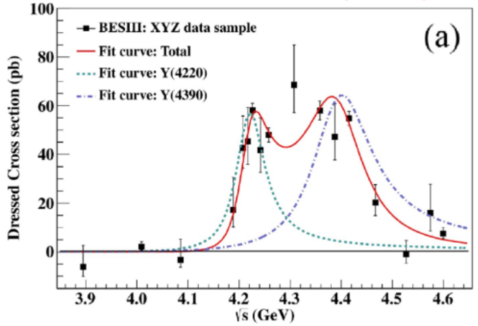
PRD99, 091103 (2019)



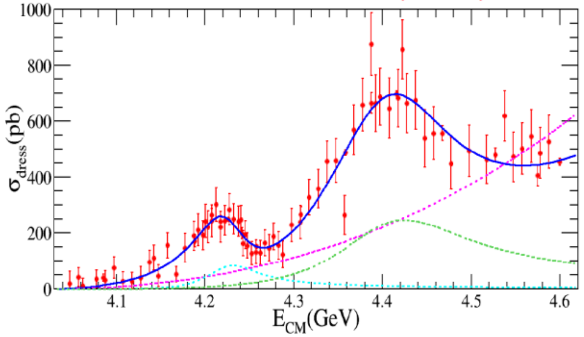
PDG 2022 ψ States



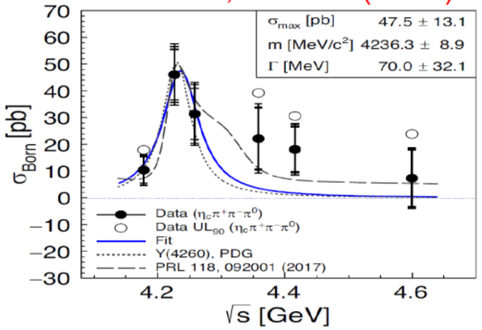
PRL118, 092002 (2017)



PRL122, 102002 (2019)



PRD 103, 032006 (2021)



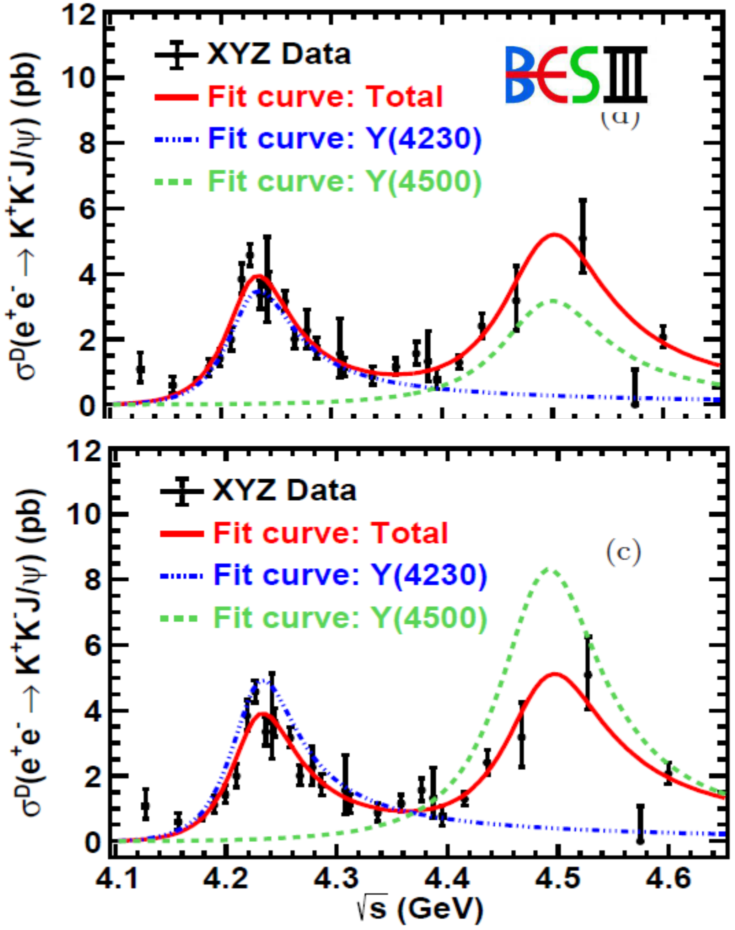
Mass~4220 MeV, width~ 50 MeV

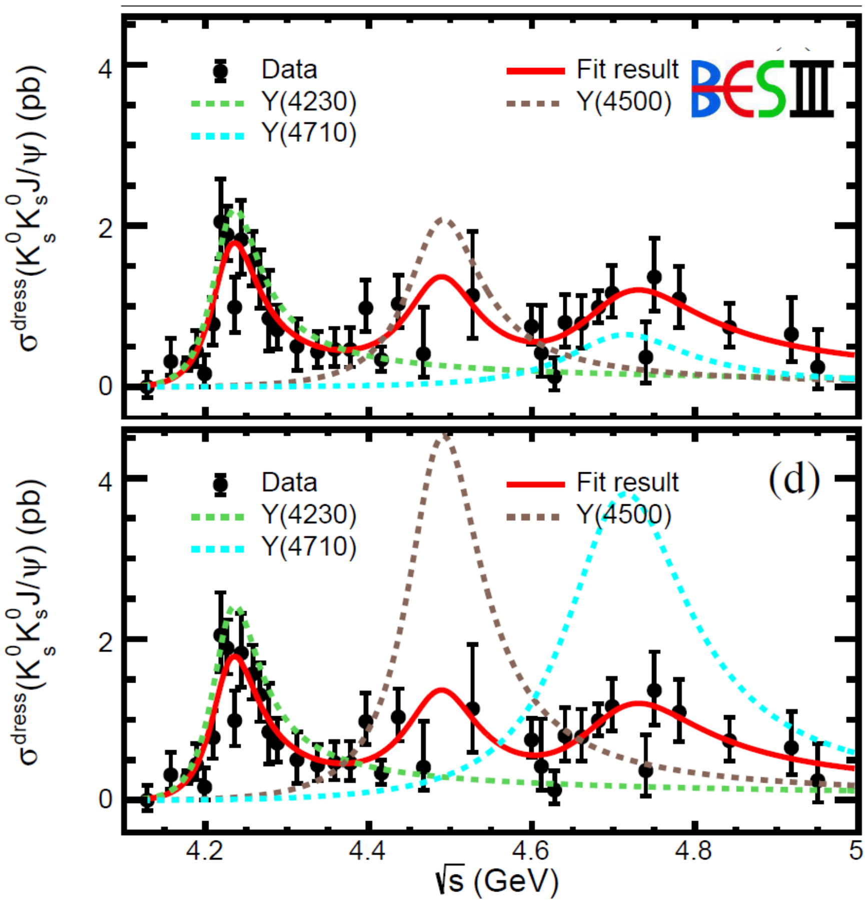
$e^+e^- \to K^+K^-J/\psi$ CPC 46, 111002 (2022)

$e^+e^- \to K_S K_S J/\psi$ PRD 107, 092005 (2023)

Data samples from 4.13 to 4.60 GeV(15.6 fb^{-1})

Data samples from 4.13 to 4.95 GeV(21.2 fb^{-1})



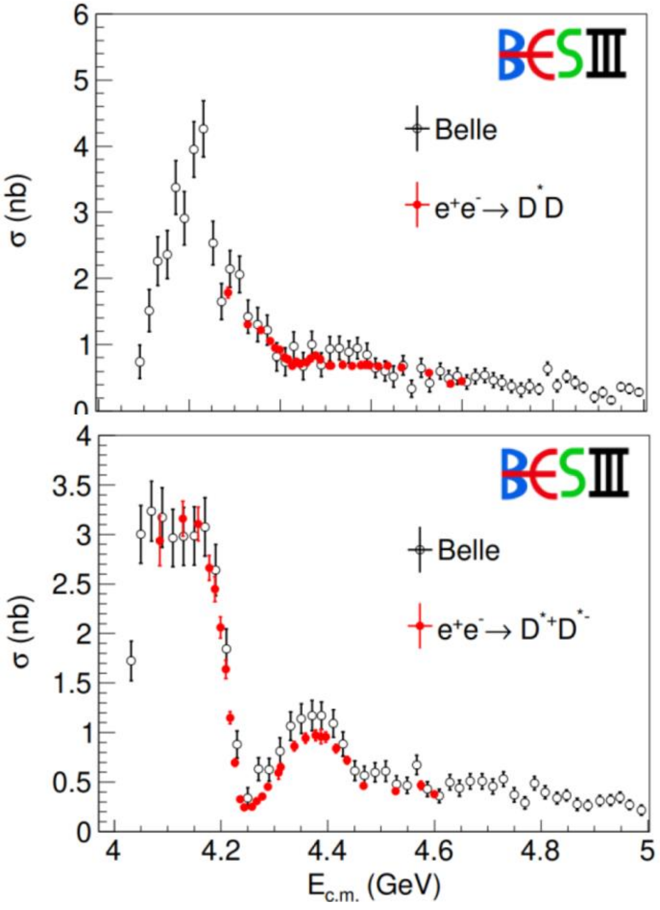


Y(4230) and Y(4500) observed (29σ / 8σ)

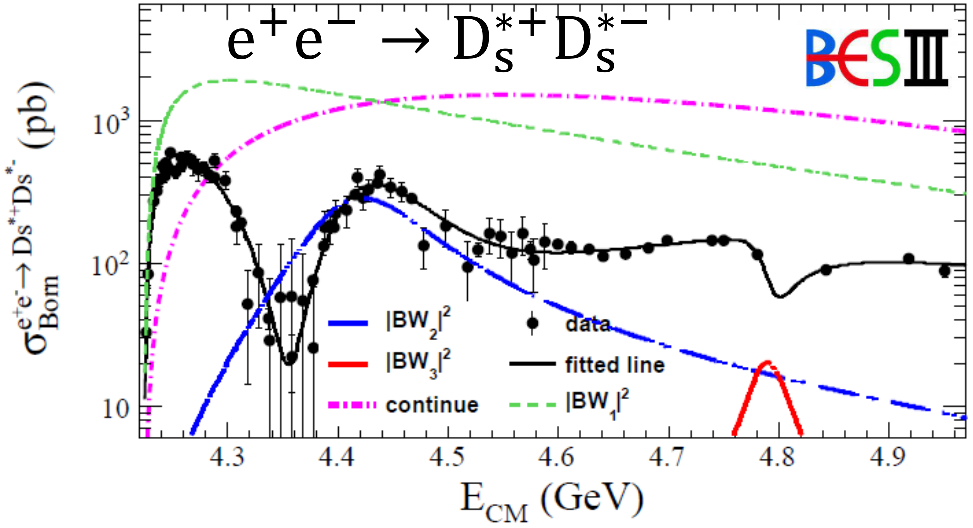
Evidence for Y(4710) (4.0σ)

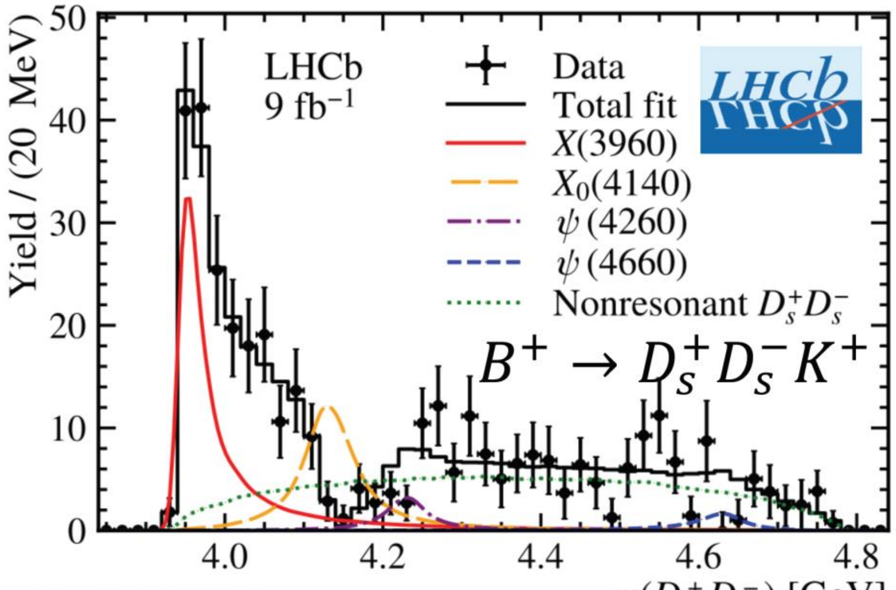
Open charm final states

JHEP 05 155 (2022)

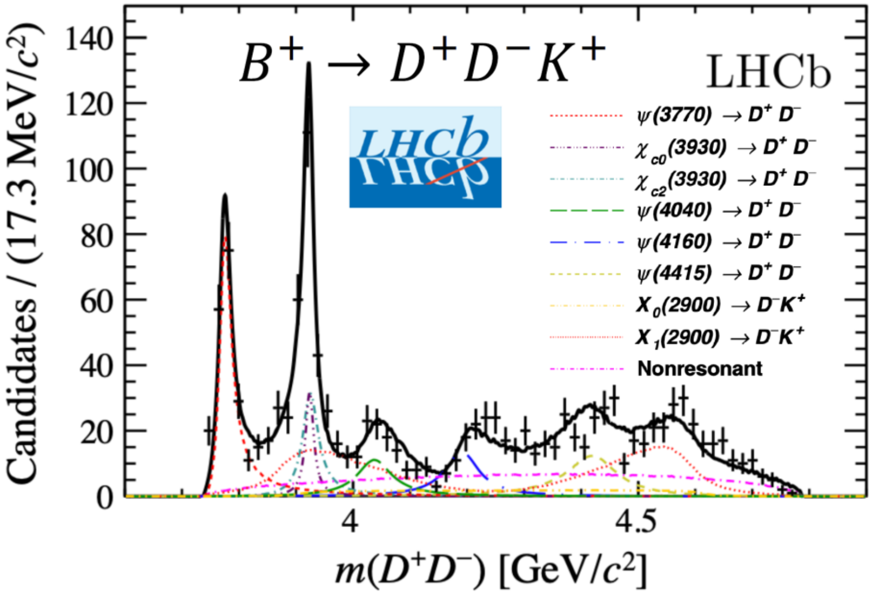


arXiv:2305.10789



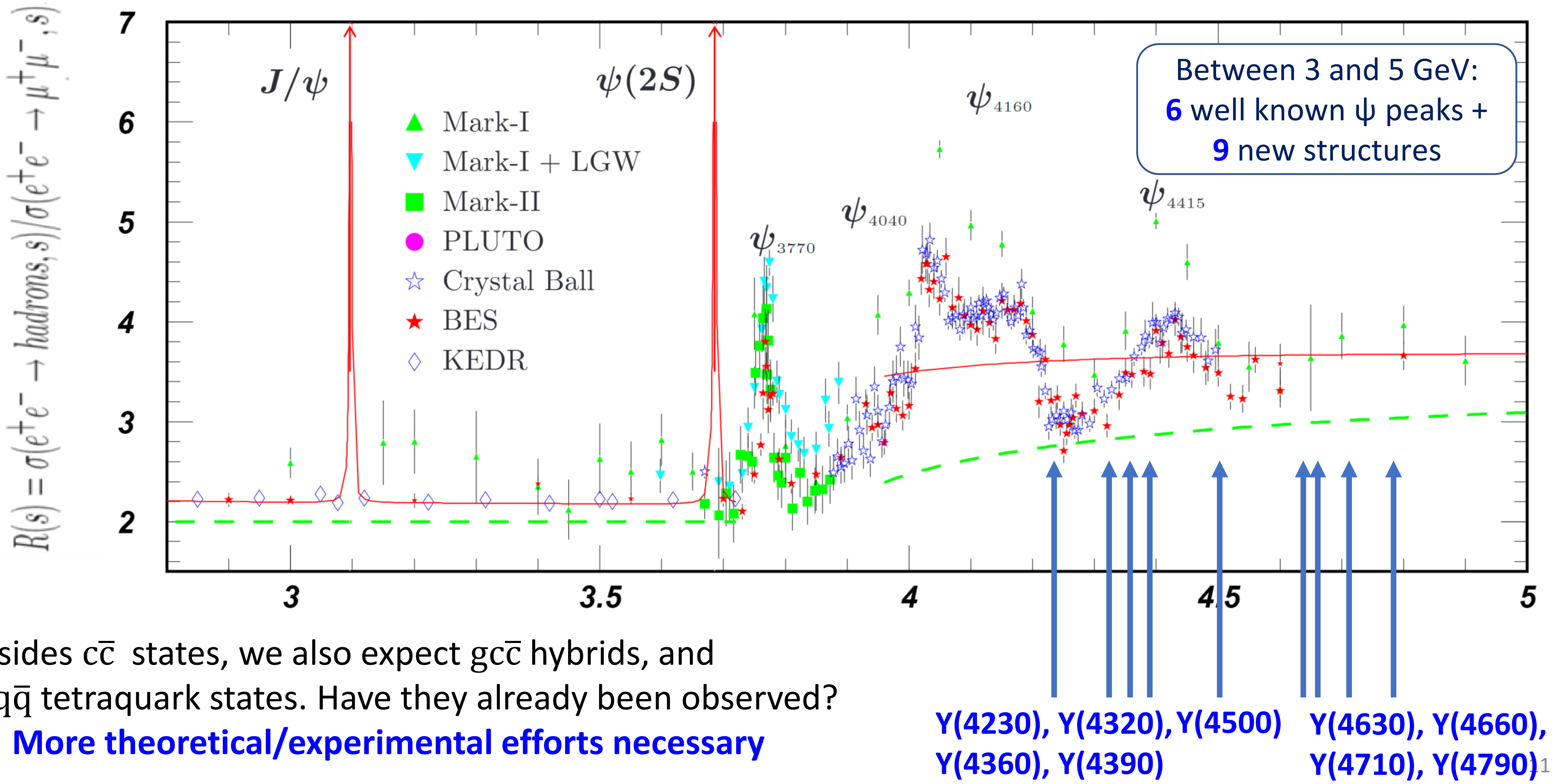


arXiv:2210.15153



PRD 102, 112003 (2020)

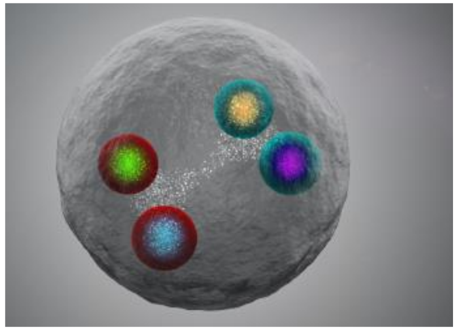
How many vectors in charmonium energy region?



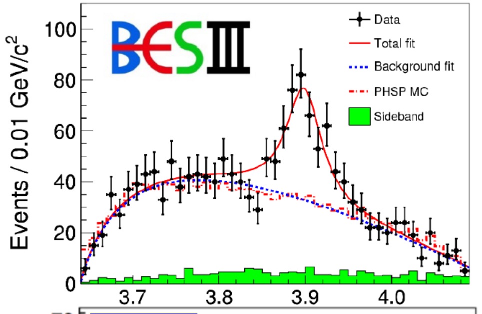
Besides $c\bar{c}$ states, we also expect $gc\bar{c}$ hybrids, and $c\bar{c}q\bar{q}$ tetraquark states. Have they already been observed?

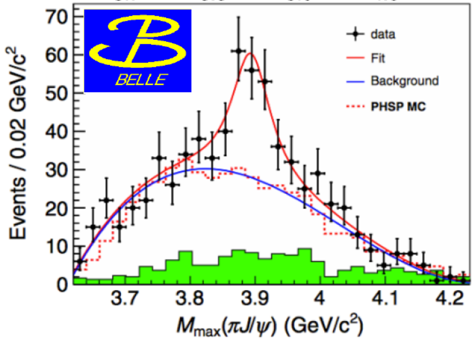
→ **More theoretical/experimental efforts necessary**

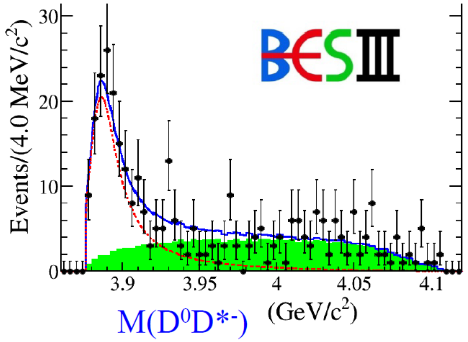
Z_c states



$Z_c(3900)$, 2013

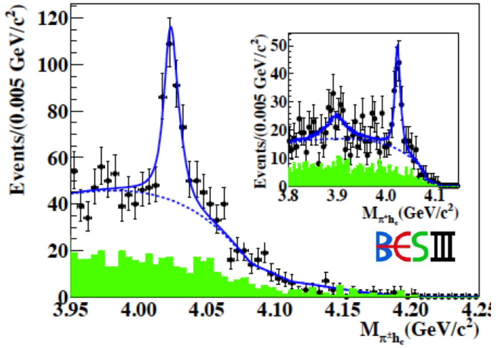


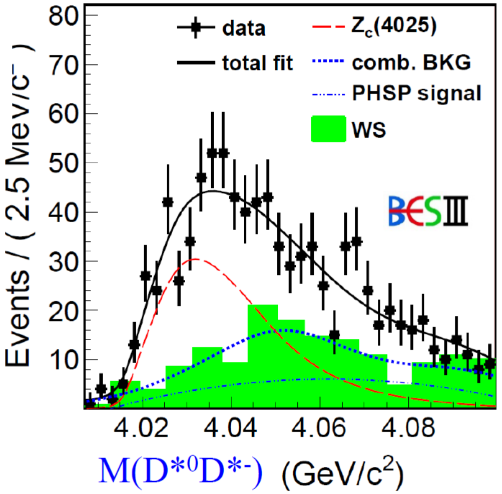




$I = 1\ Z_c(3900)$ near $D\overline{D^*}$ threshold

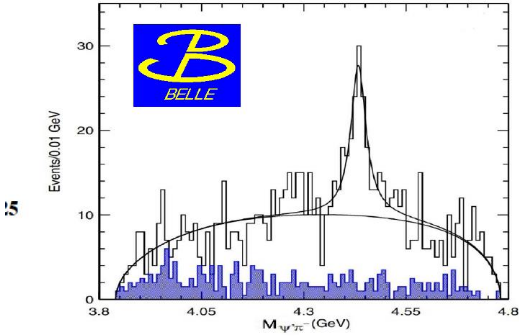
$Z_c(4020)$, 2013

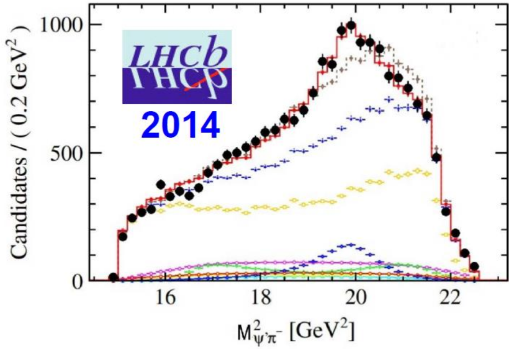




$I = 1\ Z_c(4020)$ near $D^*\overline{D^*}$ threshold

$Z_c(4430)$, 2008

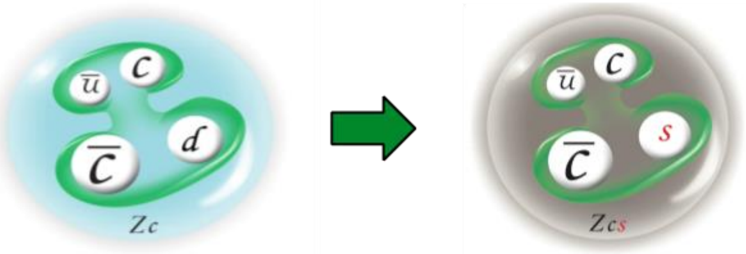




All are observed in π+charmonium

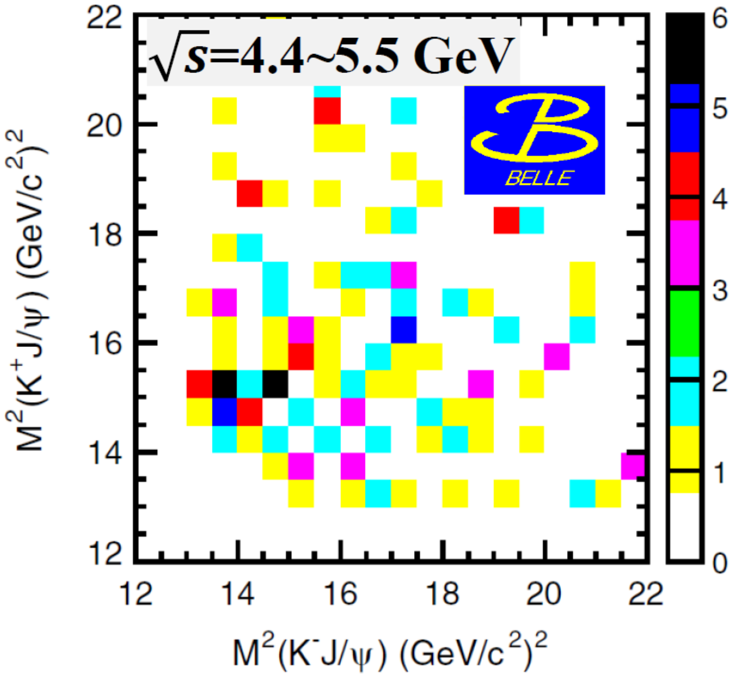
$c\bar{c}u\bar{d}$

Existence of states with $d \to s$?



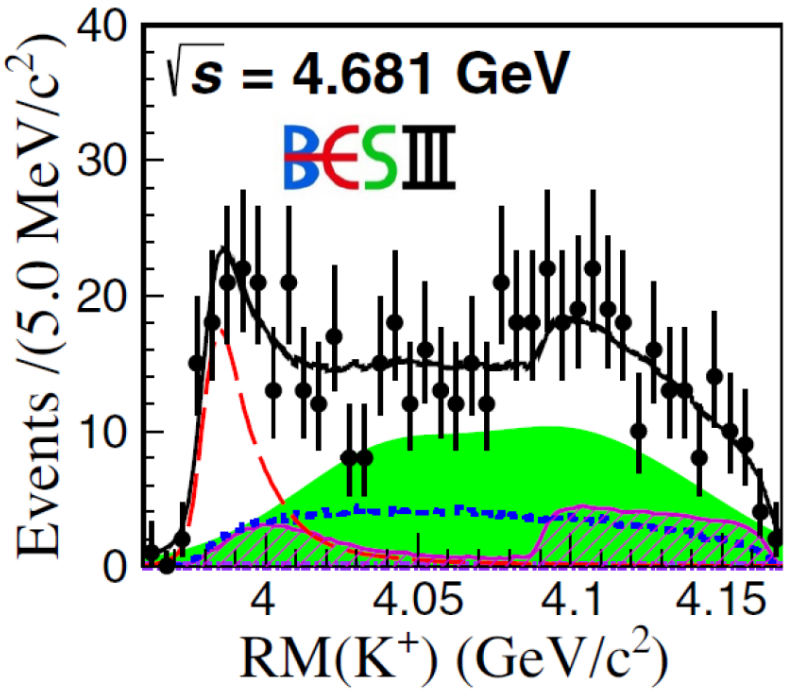
Search for states decay into $KJ/\psi, D_s^*\overline{D}, D_s\,\overline{D^*}$

$e^+e^- \to K^+K^-J/\psi$



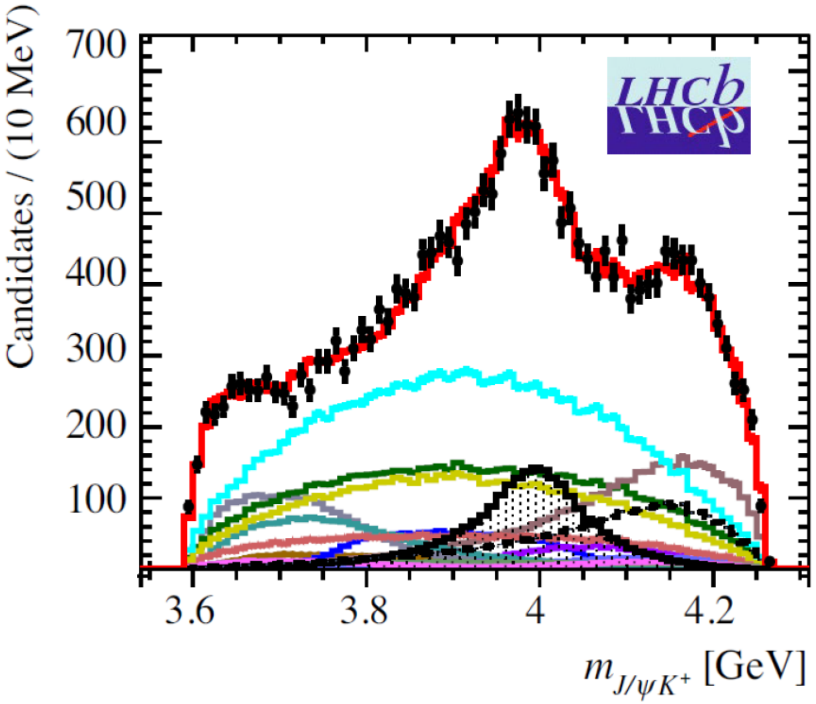
PRD 89, 072015 (2014)

$e^+e^- \to K^+(D_s^- D^{*0} + D_s^{*-} D^0)$



PRL 126, 102001 (2021)

$B^+ \to J/\psi \phi K^+$



PRL 127, 082001 (2021)

No significant signal in $K^\pm J/\psi$ decay mode!

(statistics low!)

$Z_{cs}(3985)$ in $\bar{D}^*D_s + \bar{D}D^*_s$ mode!

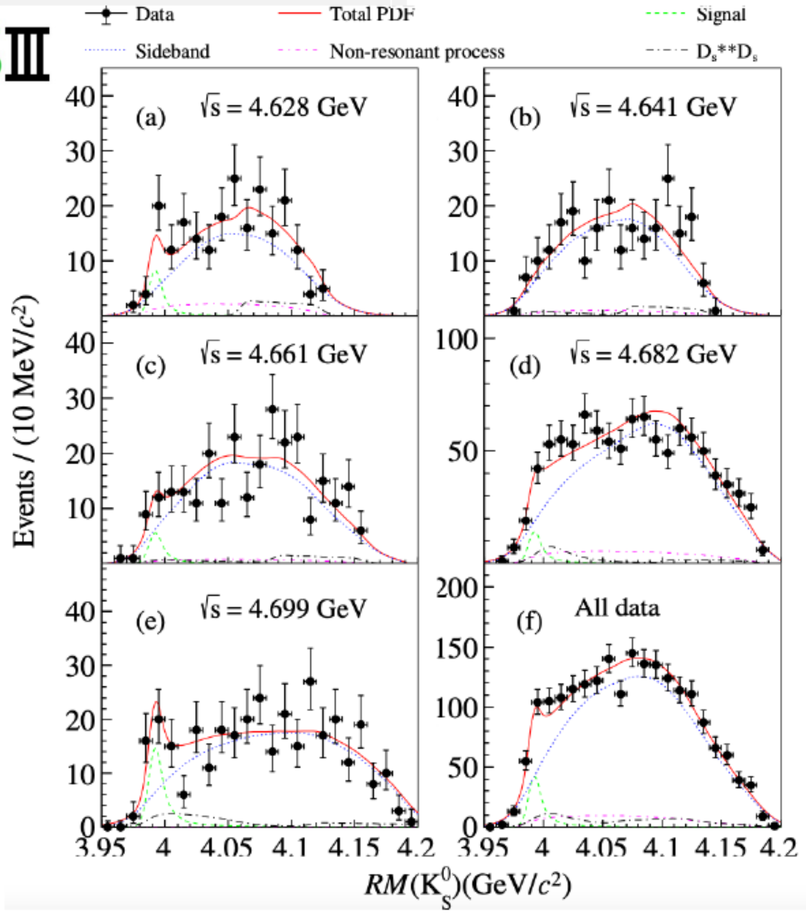
$Z_{cs}(4000)$ and $Z_{cs}(4220)$ in $K^\pm J/\psi$ decay mode!

State	Signif.	JP	Mass (MeV)	Width (MeV)
$Z_{cs}(3985)$	5.3σ	??	$3982.5^{+1.8}_{-2.6} \pm 2.1$	$12.8^{+5.3}_{-4.4} \pm 3.0$
$Z_{cs}(4000)$	15σ	1+	$4003 \pm 6^{+4}_{-14}$	131±15±26
$Z_{cs}(4220)$	5.9σ	1+	$4216 \pm 24^{+43}_{-30}$	$233 \pm 52^{+97}_{-73}$

Widths very different, not the same state!

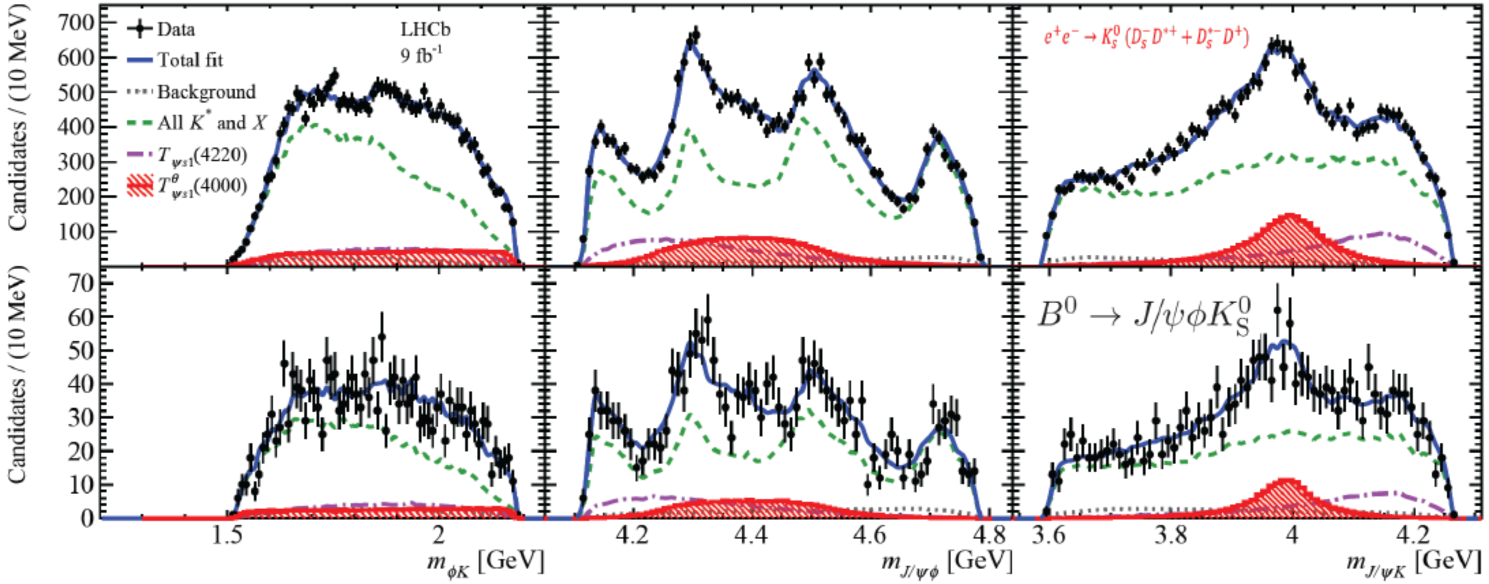
Evidence for the neutral $Z_{cs}(3985), Z_{cs}(4000)$

PRL129, 112003 (2022) $e^+e^- \to K_S^0 \, (D_s^- D^{*+} + D_s^{*-} D^+)$



$B^0 \to J/\psi \phi K_S^0$

arXiv:2301.04899v2



Significance >4.0σ after including systematic uncertainties

Significance 5.4σ with isospin symmetry imposed

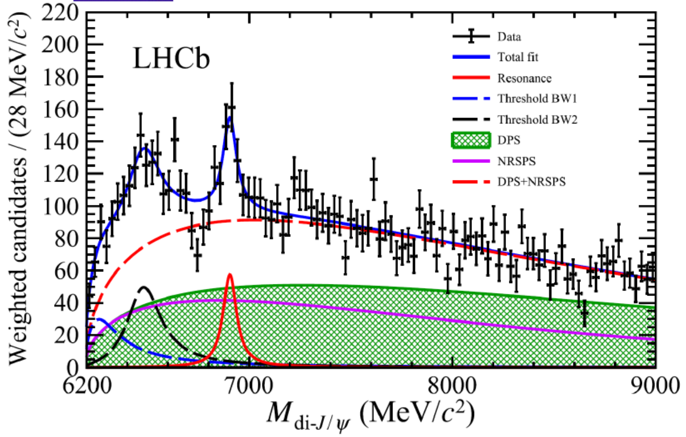
State	Mass (MeV/c^2)	Width (MeV)	Significance
$Z_{cs}(3985)^+$	$3985.2^{+2.1}_{-2.0} \pm 1.7$	$13.8^{+8.1}_{-5.2} \pm 4.9$	5.3σ
$Z_{cs}(3985)^0$	$3992.2 \pm 1.7 \pm 1.6$	$7.7^{+4.1}_{-3.8} \pm 4.3$	4.6σ

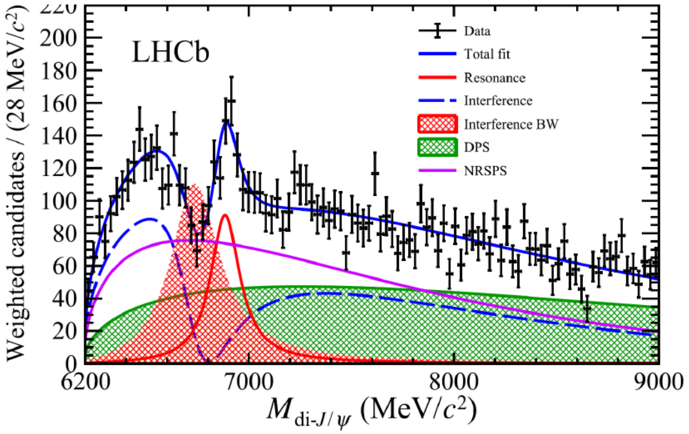
$\mathrm{I} = \frac{1}{2} \; Z_{cs}(3985)$

Mass (MeV)	Width (MeV)	Fit fraction (%)	ΔM (MeV)
$3991\,^{+12}_{-10}\,^{+9}_{-17}$	$105\,^{+29}_{-25}\,^{+17}_{-23}$	$7.9 \pm 2.5\,^{+3.0}_{-2.8}$	$-12\,^{+11}_{-10}\,^{+6}_{-4}$

Fully charmed $T_{cc\bar{c}\bar{c}}$

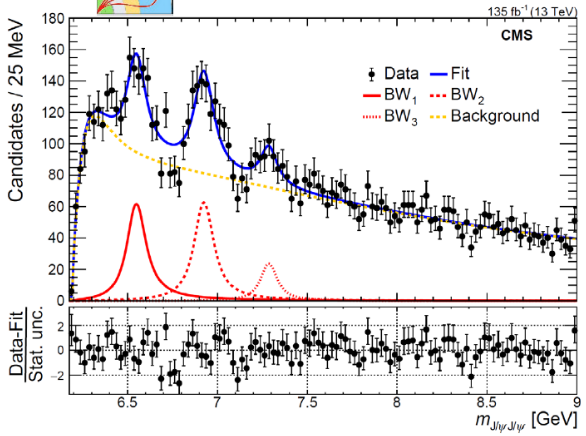
LHCb Sci.Bull. 65 (2020) 1983

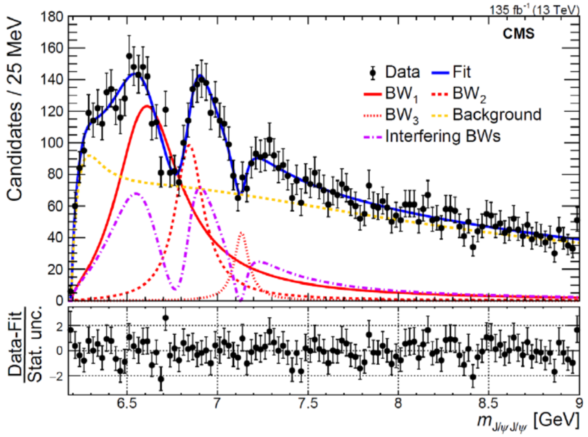




- Narrow structure at 6.9 GeV, >5σ
- X(6900): M~6900 MeV, Γ~100 MeV
- Structure just above J/ψJ/ψ threshold

CMS arXiv:2306.07164

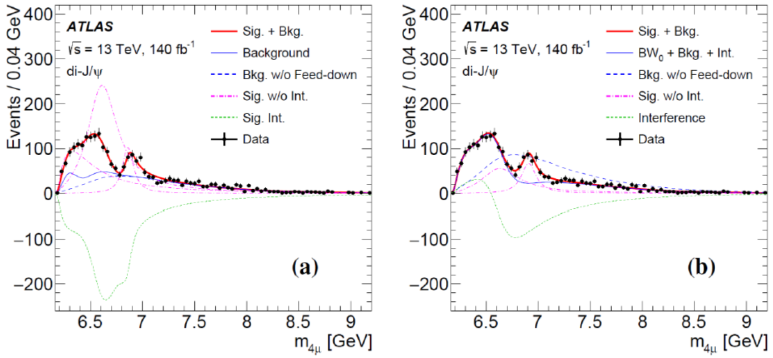




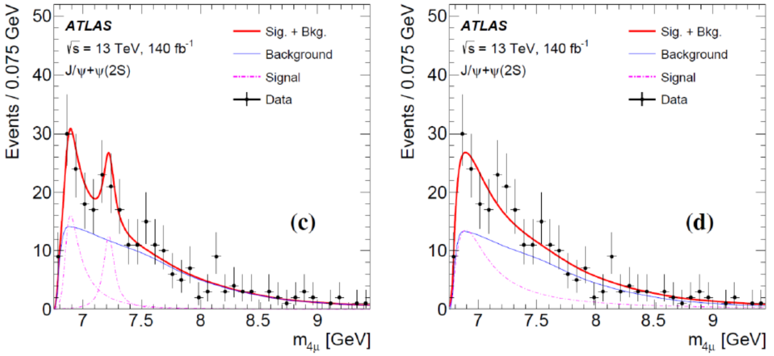
- X(6900) consistent with LHCb
- New state X(6600) with 6.5σ
- Evidence for X(7300) with 4.1σ

ATLAS EXPERIMENT arXiv:2304.08962

X→J/ψJ/ψ

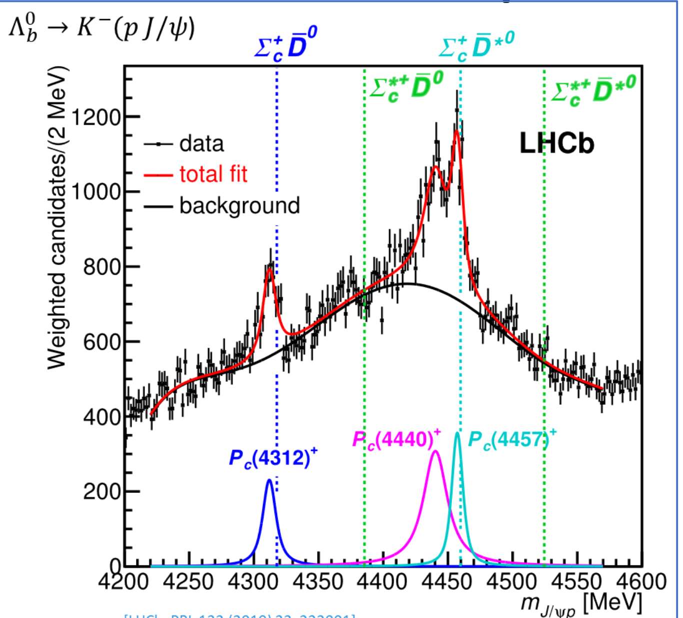


X→J/ψ ψ(2S)



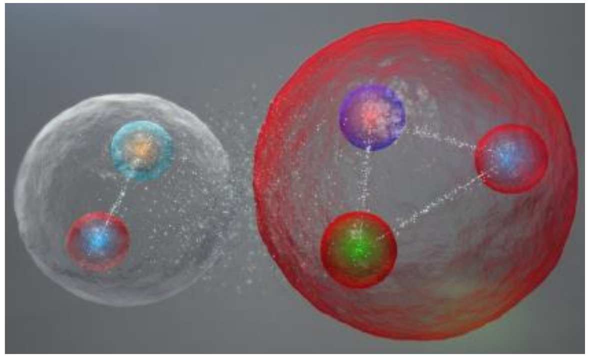
- X(6900) consistent with LHCb
- New state X(6600) with 6.5σ
- Evidence for states in M(J/ψψ(2S))

Pentaquarks



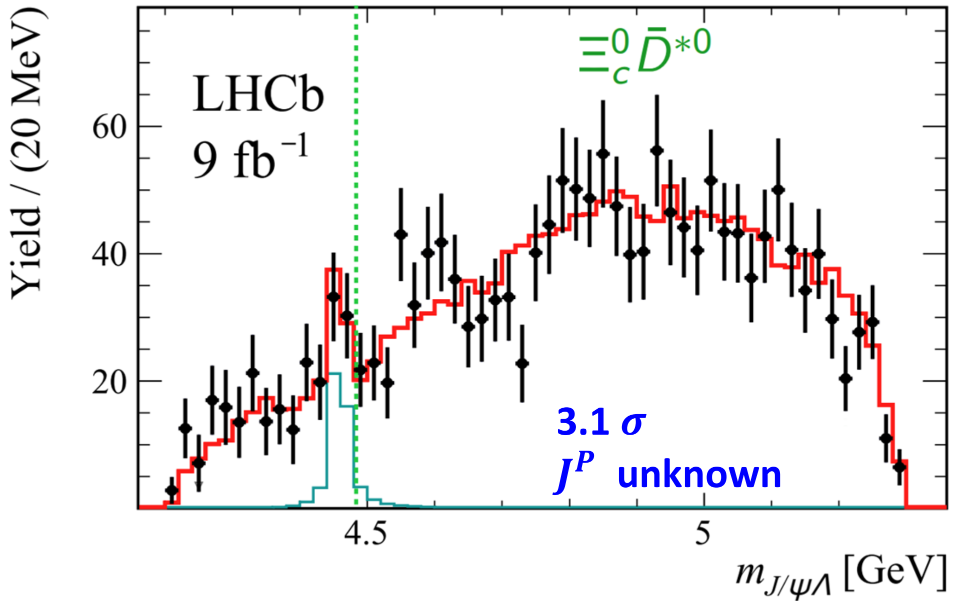
[LHCb, PRL 122 (2019) 22, 222001]

[LHCb, PRL 115 (2015), 072001]

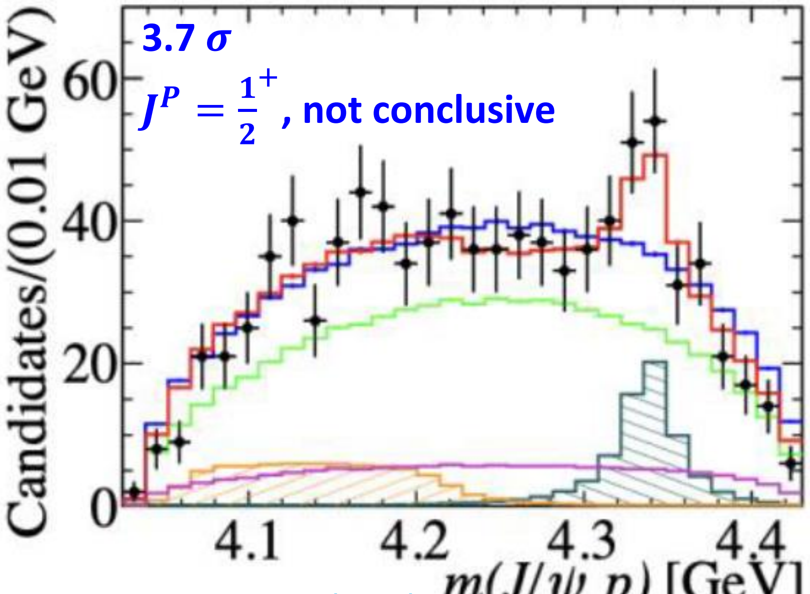


Evidence of P_{cs}^0 in $\Xi_b^- \to J/\psi \Lambda K^-$

[LHCb, Sci.Bull. 66 (2021) 1278-1287]



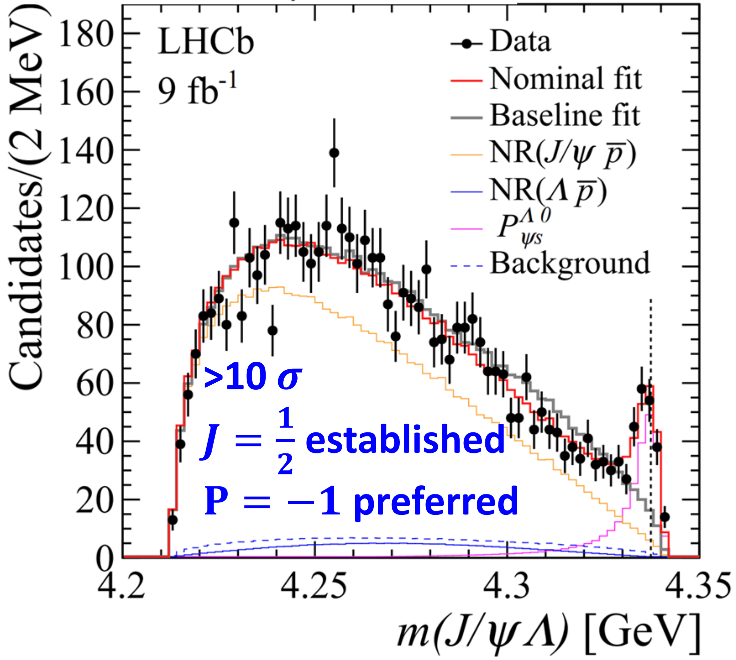
$B_s^0 \to J/\psi p\bar{p}$ - evidence of new P_c



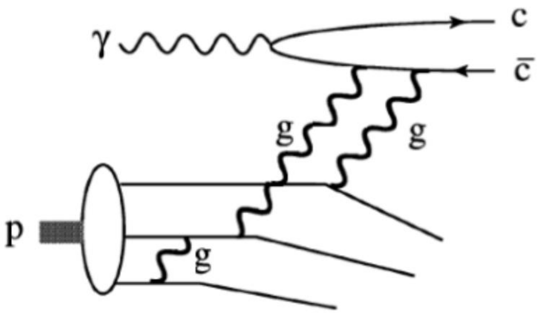
PRL 128, 062001 (2022)

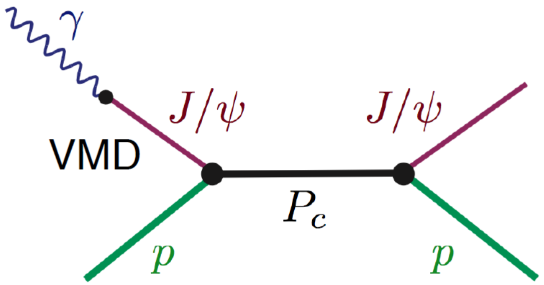
Observation of $P_{\psi s}^{\Lambda} \to J/\psi \Lambda$

[LHCb, 2210.10346]

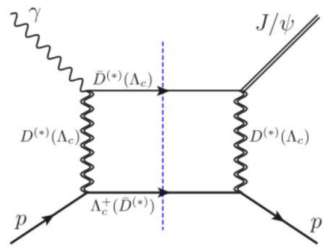


Search for P_c in photon production

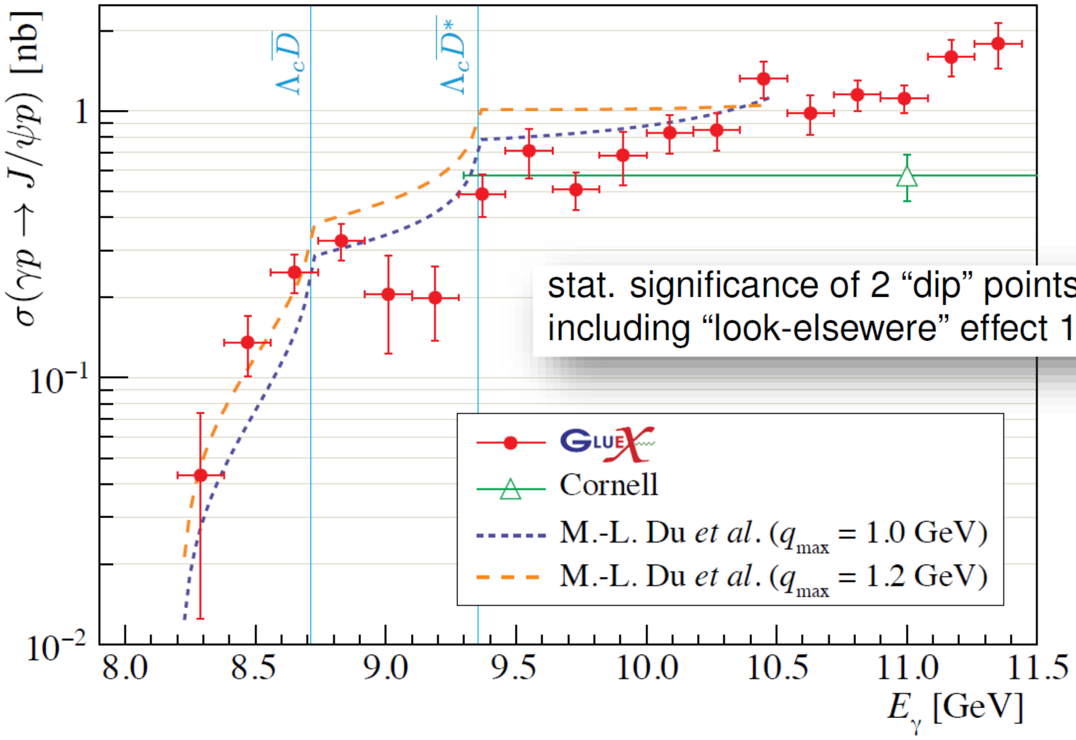




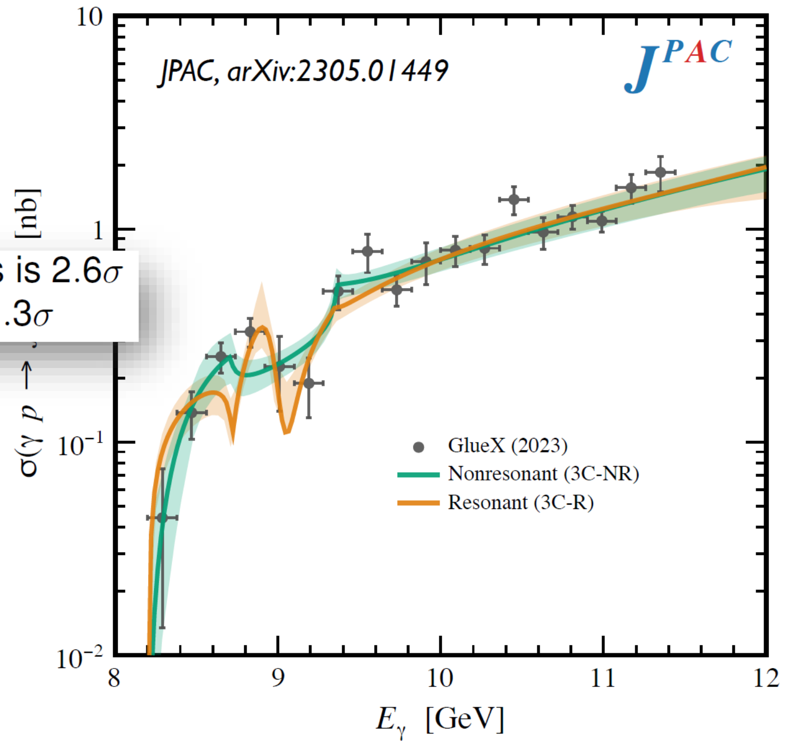
$\gamma p \to \Lambda_c \overline{D}^{(*)} \to J/\psi p$



PRC 108 (2023) 025201



stat. significance of 2 “dip” points is 2.6σ
including “look-elsewere” effect 1.3σ



- Gluon exchange as a probe of proton structure
- Conspicuous structure at open charm thresholds
- Production mechanism must be understood to interpret data

Beyond hidden-charm: Doubly open-charm tetraquark T_{cc}

- Peak in $D^0D^0\pi^+$ just below $D^{*+}D^0$ threshold, extremely narrow
- $I = 0, J^{PC} = 1^{++}$

$cc\bar{u}\bar{d}$

Coupled channel model

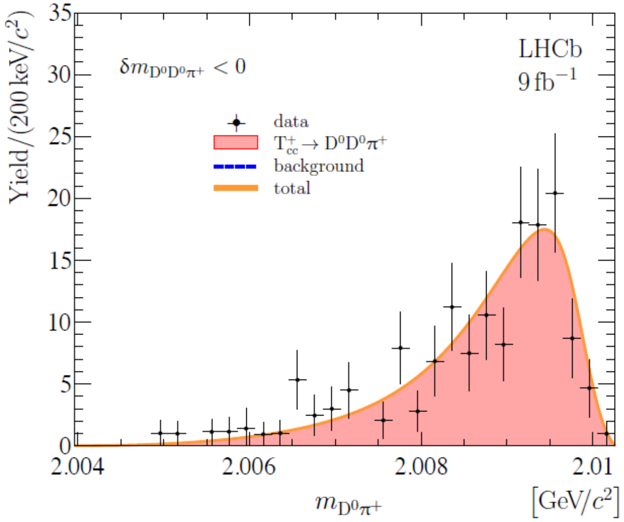
$D^{*+}D^0 + D^{*0}D^+$

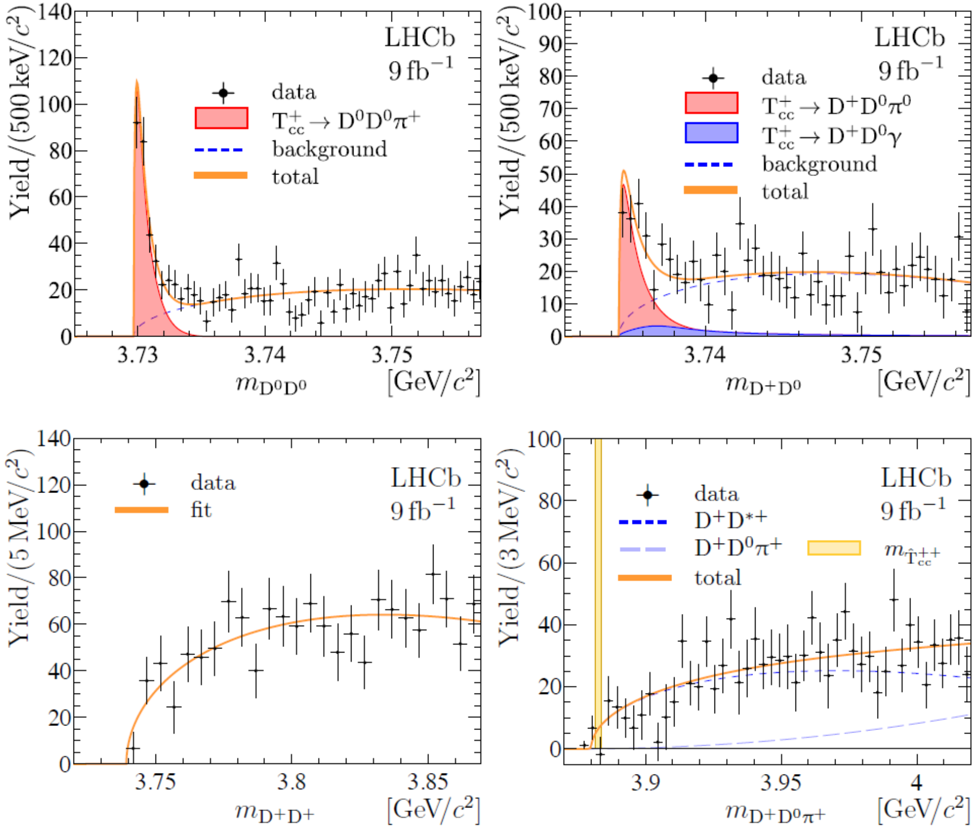
$\rightarrow \{D^0D^0\pi^+, D^0D^+\pi^0, D^0D^+\gamma\}$

Yields pole parameters:

- **Binding energy**: $-360 \pm 40^{+4}_{-0}$ keV
- **Width**: $48 \pm 2^{+0}_{-14}$ keV

[arXiv: 2109.01038] (Nature Physics)
[arXiv: 2109.01056] (Nature Communications)

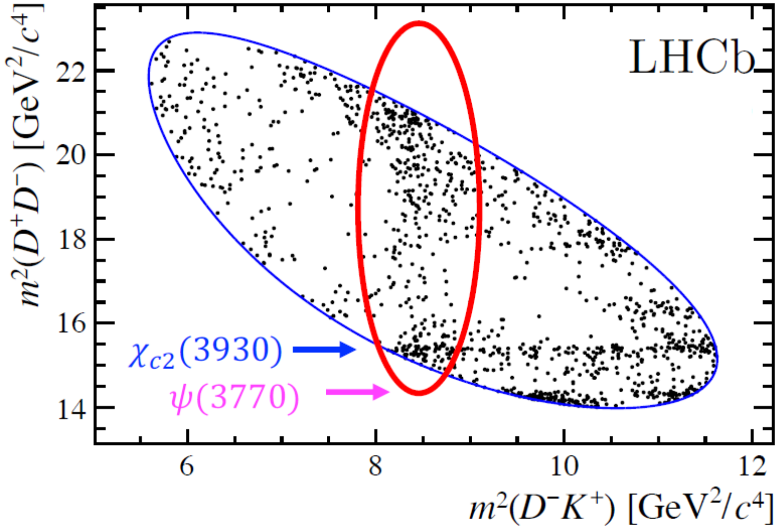


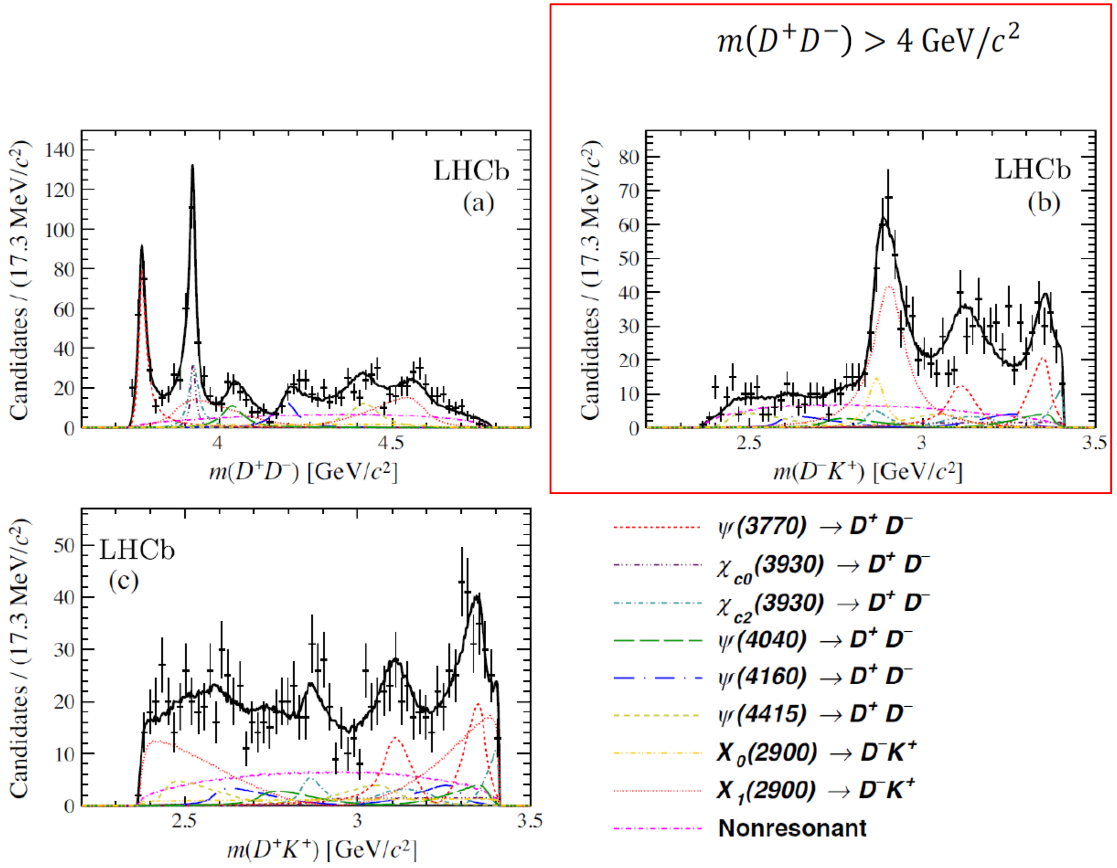


Open-charm tetra quarks

Evidence for T_{cs} states decay into D^-K^+

$B^+ \to D^+D^-K^+$ [PRD 102 (2020) 112003, PRL 125 (2020) 242001]





$$X_0(2900): \quad M = 2.866 \pm 0.007 \pm 0.002\,\mathrm{GeV}/c^2, \quad \Gamma = 57 \pm 12 \pm 4\,\mathrm{MeV}$$
$$X_1(2900): \quad M = 2.904 \pm 0.005 \pm 0.001\,\mathrm{GeV}/c^2, \quad \Gamma = 110 \pm 11 \pm 4\,\mathrm{MeV}$$

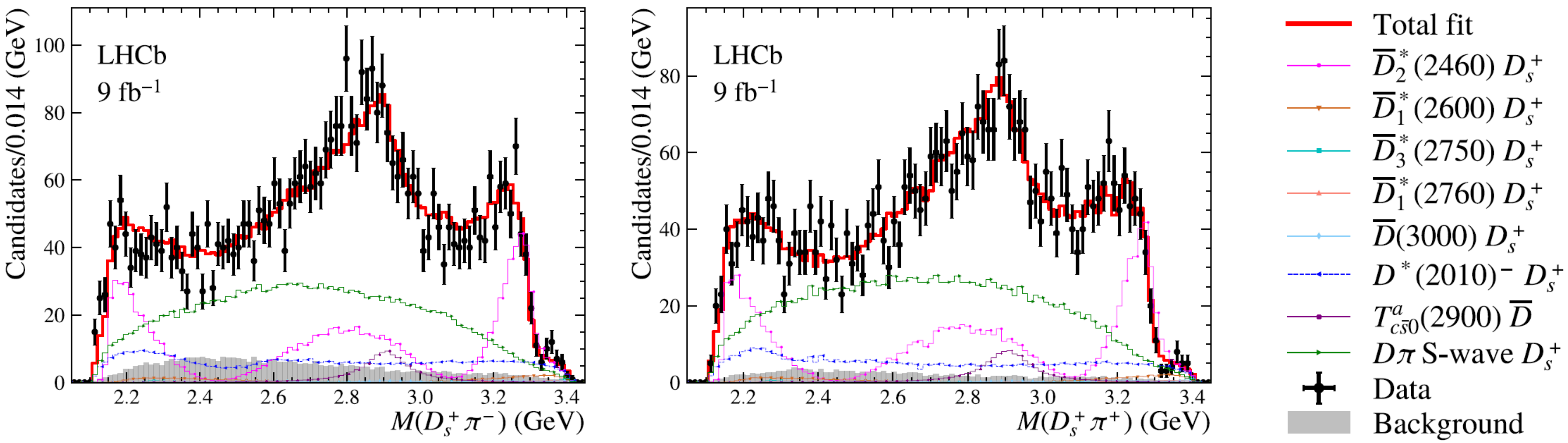
Discovery of open-charm tetraquarks with four different flavors [$cs\bar{u}\bar{d}$]

Open-charm tetra quarks

Observation of $T_{c\bar{s}}$ states decay into $D_s^+\pi^\pm$

PRL 131 (2023) 041902
PRD 108 (2023) 012017

$B^0 \to \bar{D}^0 D_s^+ \pi^- \quad B^+ \to D^- D_s^+ \pi^+$



$T^a_{c\bar{s}0}(2900)^0 \to D_s^+\pi^-$ & $T^a_{c\bar{s}0}(2900)^{++} \to D_s^+\pi^+$ significance $> 9\sigma$

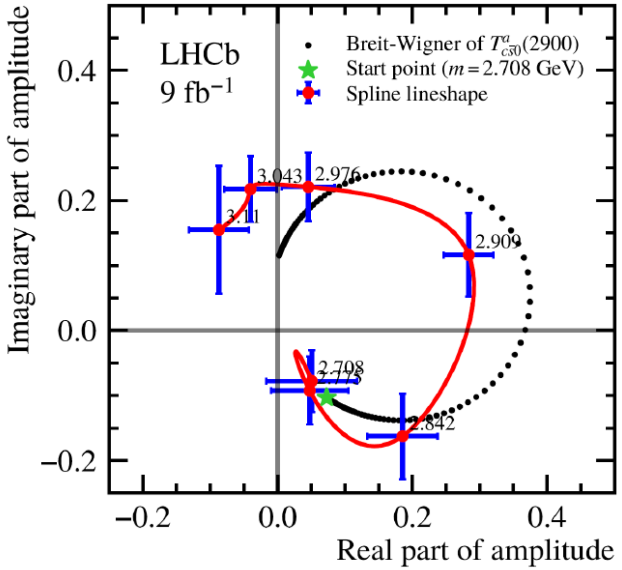
$J^P = 0^+$ favored over other spin-parity by more than 7.5σ

$$M = 2.908 \pm 0.011 \pm 0.020 \text{ GeV}$$

$$\Gamma = 0.136 \pm 0.023 \pm 0.011 \text{ GeV}$$

Flavor partner of $T_{cs0}(2900)$? Multiplets to be revealed in the future

Assuming isospin symmetry
➔joint amplitude analysis



Isospin triplet?

$T^a_{c\bar{s}0}(2900)^0$
$T^a_{cs0}(2900)^+$?
$T^a_{c\bar{s}0}(2900)^{++}$

Light QCD exotics

Light QCD exotics

- **Strong evidences for multi-quark in heavy quark sector**
- **However, evidence for gluonic excitations remains sparse**
  - Light Flavor-exotic hard to establish
  - Assignment of some $SU(3)_{flavor}$ $|\, q\bar{q}\, \rangle$ nonets difficult
- **Role of gluons:**
  - Gluons mediate the strong force
  - **Hadron constituent: Mass? Quantum numbers? …**
  - Gluons' unique self-interacting property

    → **New form of matter: glueballs, hybrids**
  - **Gluonic Excitations provide measurements of the QCD potential**

Critical to confinement and mass dynamical generation

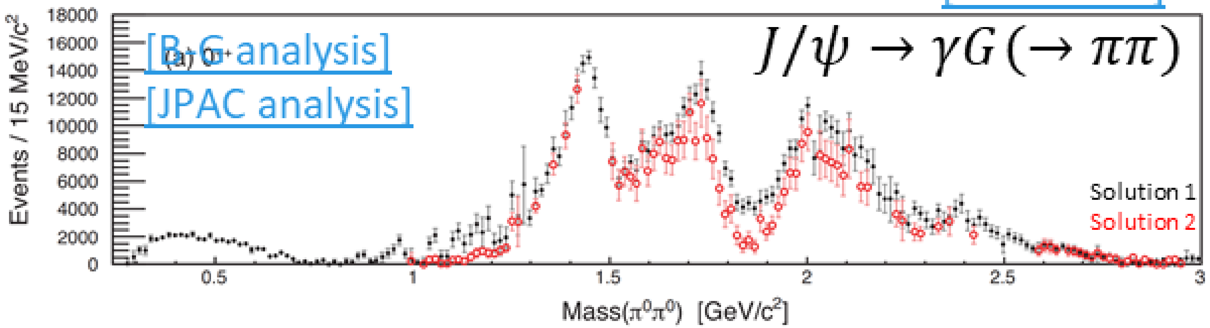
Glueballs

- **Low-lying glueballs with ordinary J^{PC} → mixing with $q\bar{q}$**
  - ➢ ~~**Observe a new peak**~~
  - ➢ Challenge: reveal the exotic admixture
- **Scalar glueball is expected to have a large production in J/ψ radiative decays**: $B(J/\psi \to \gamma G_{0+}) = 3.8(9) \times 10^{-3}$ by LQCD
  - Observed $B(J/\psi \to \gamma f_0(1710))$ is **x10** larger than $f_0(1500)$
  - ➢ **BESIII: $f_0(1710)$ largely overlapped with scalar glueball**

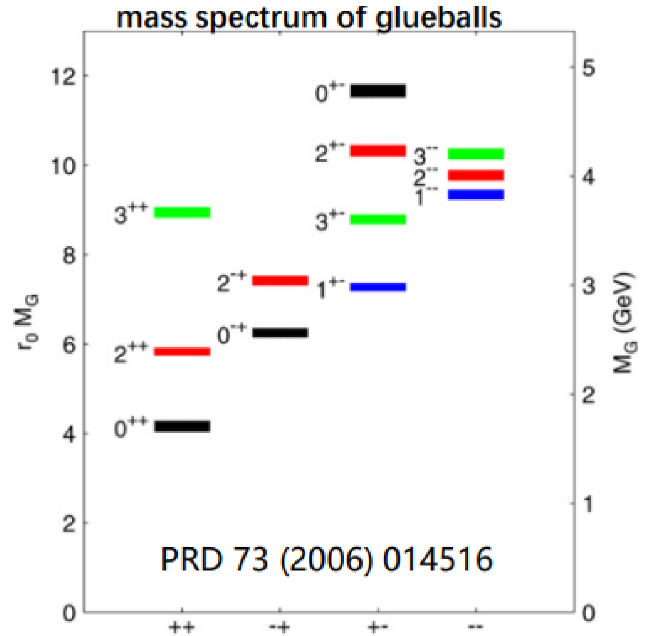
BESIII PRD 87 092009 (2013), PRD 92 052003 (2015), PRD 98 072003 (2018)

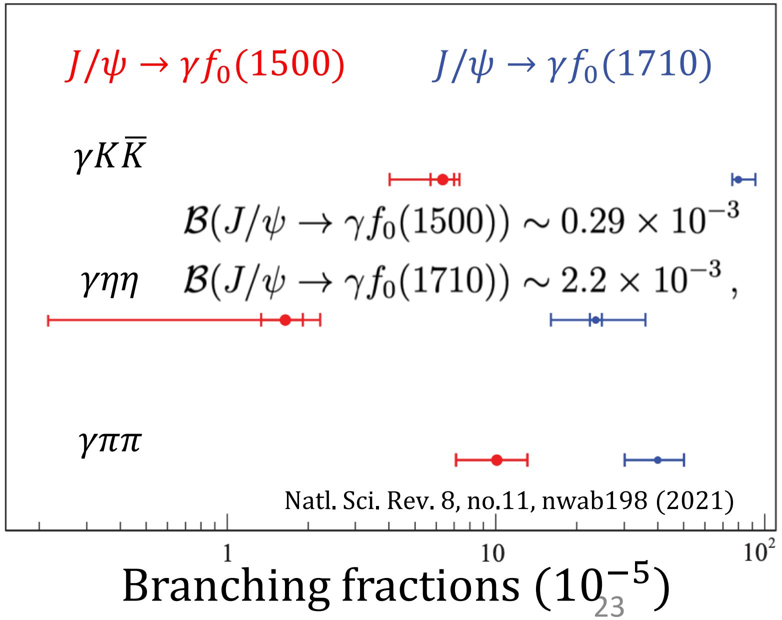
Phenomenology studies of coupled channel with BESIII data

PLB 816, 136227 (2021), EPJC 82, 80 (2022) [BESIII data]



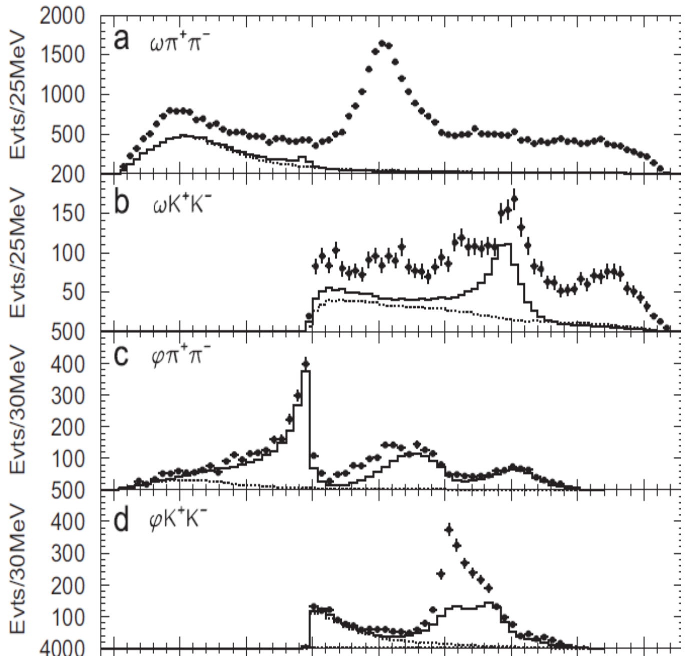
LQCD prediction of glueball spectrum





More scalars

$f_0(1710)/f_0(1790)$?

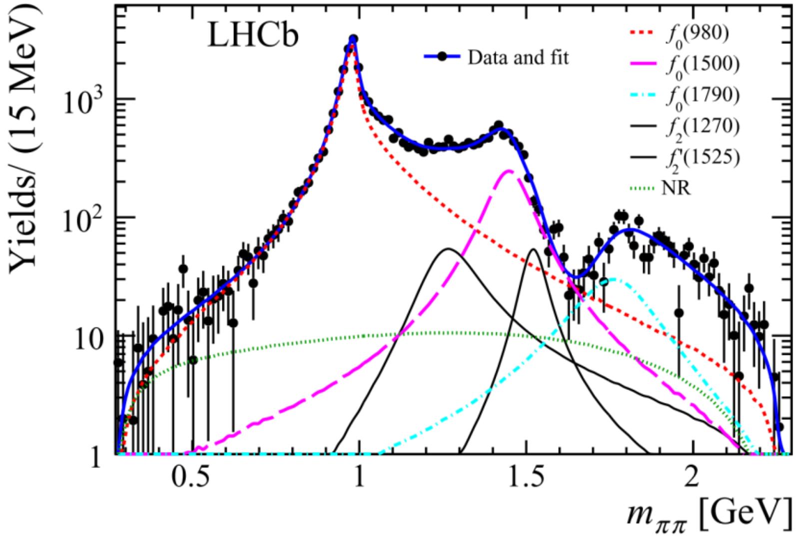


- ωK^+K^- ➜ Peak around 1700 MeV/c^2 (OZI rule: $n\bar{n}$ structure)
- $\phi\pi^+\pi^-$ ➜ Enhancement at 1790 MeV/c^2
- ϕK^+K^- ➜ No peak around 1700 MeV/c^2

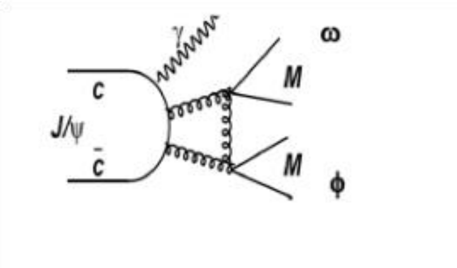
$f_0(1800)$

$B_s \to J/\psi f_0$ is selective for $s\bar{s}$

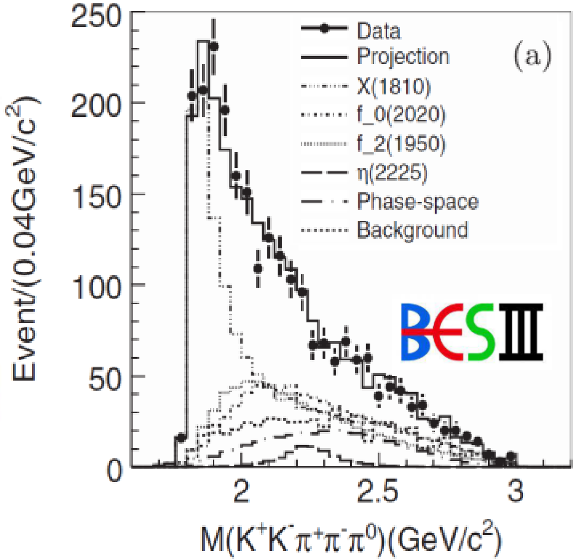
PLB 797 (2019) 134789



PRD 87, 032008(2013)



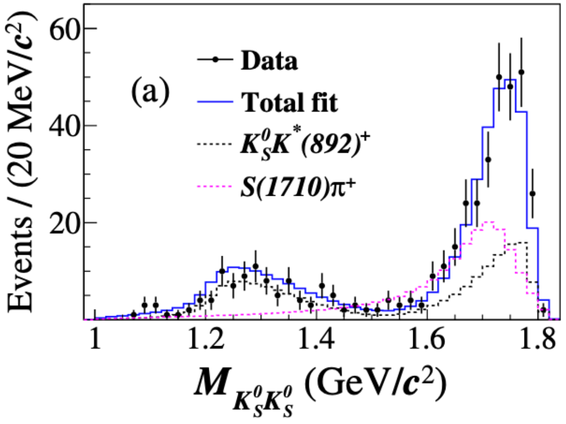
$J/\psi \to \gamma\omega\phi$ **(DOZI)**



$a_0(1817)$

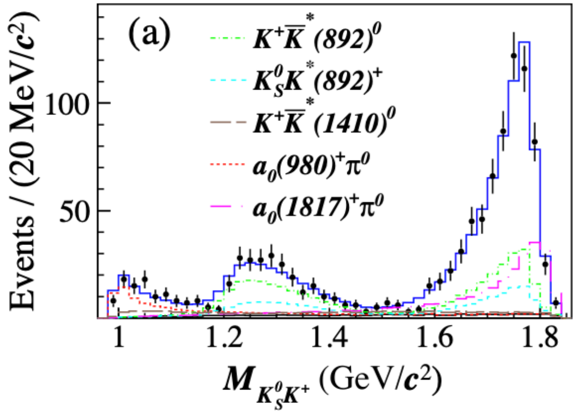
PRD105, L051103 (2022)

$D_s^+ \to K_S^0 K_S^0 \pi^+$

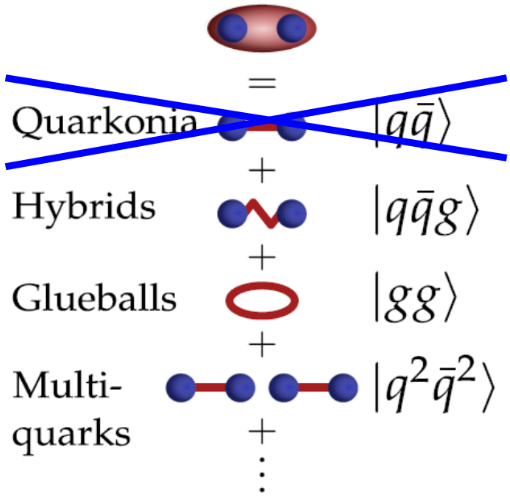


PRL129, 182001 (2022)

$D_s^+ \to K^0_S K^+ \pi^0$

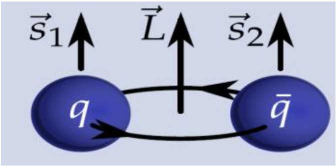


Light hadrons with exotic quantum numbers



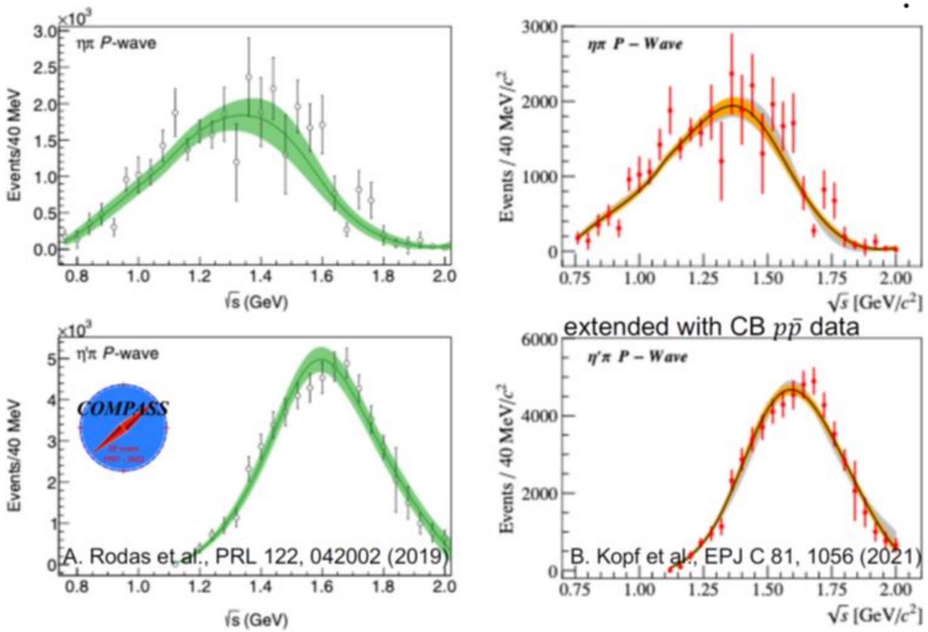
- **Unambiguous signature for exotics**
  - Light Flavor-exotic hard to establish
  - **Efforts concentrate on Spin-exotic**
    - **Forbidden for $q\bar{q}$:**

      $\mathbf{J^{PC} = 0^{--}, even^{+-}, odd^{-+}}$

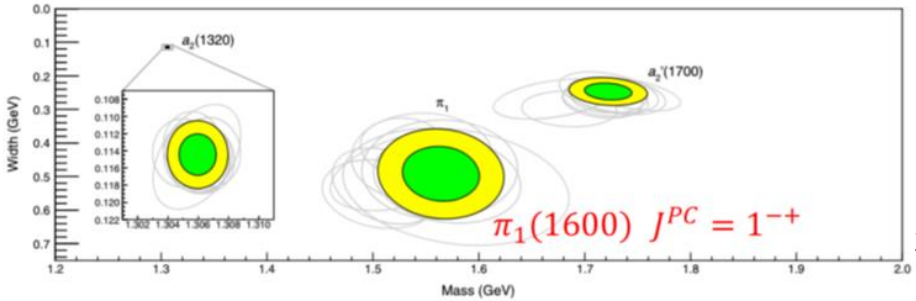


$\vec{J} = \vec{L} + \vec{S}$ $\mathrm{P} = (-1)^{L+1}$ $\mathrm{C} = (-1)^{L+S}$

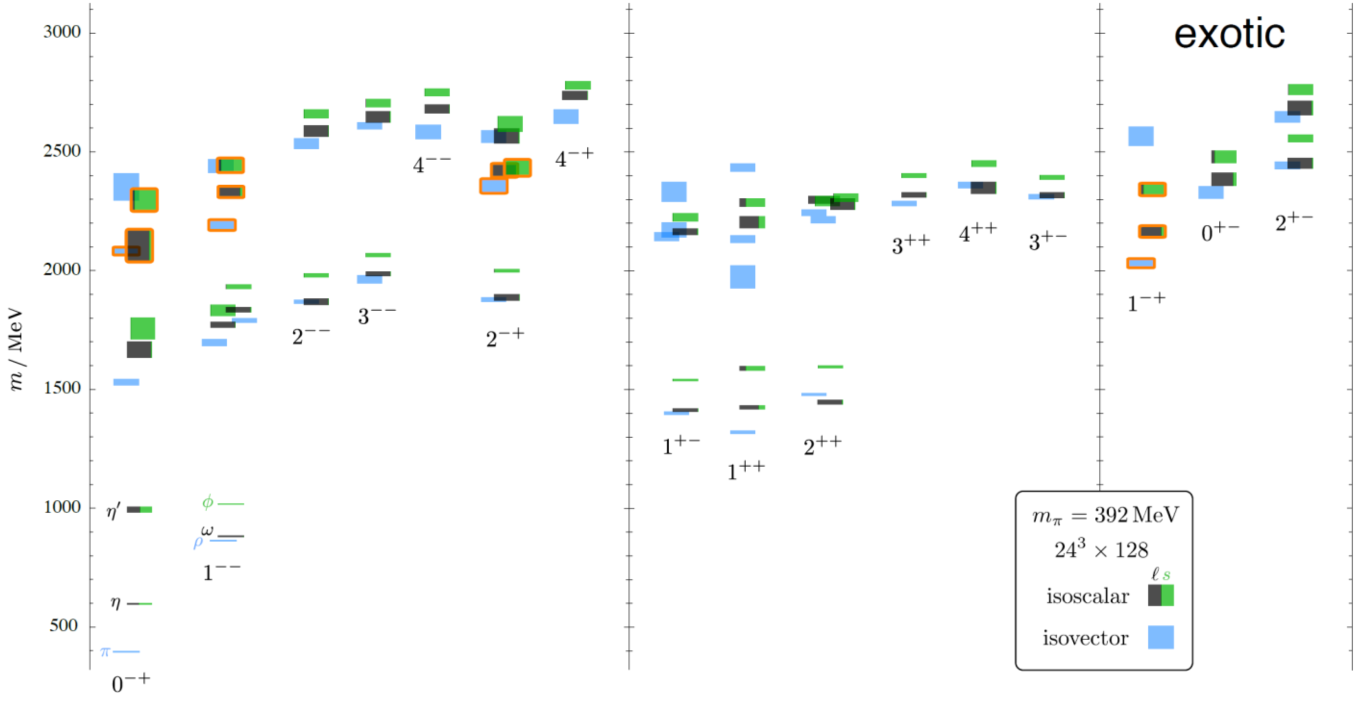
Allowed J^{PC}: $0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 2^{++}, \ldots$



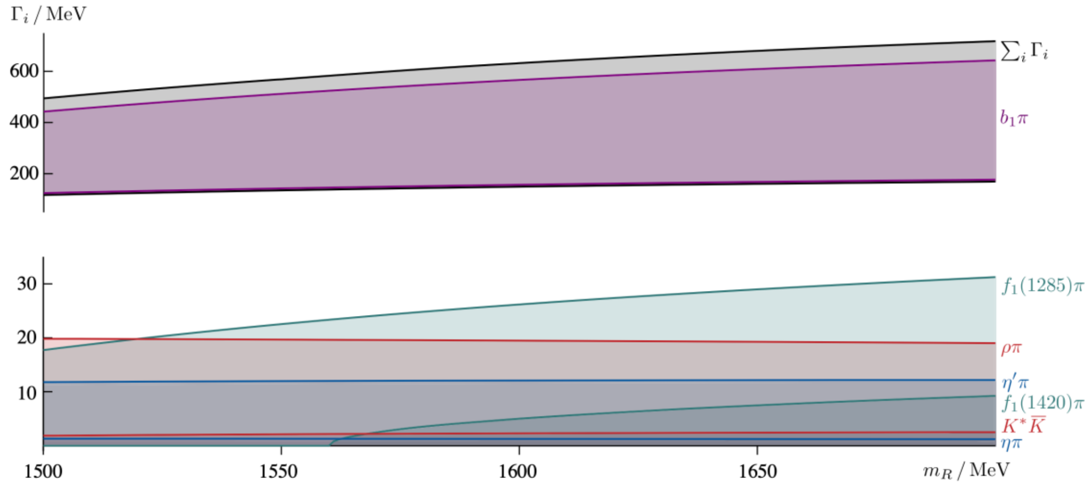
- **Only 3 candidates so far: All 1^{-+} isovectors**
  - $\pi_1(1400)$: seen in ηπ
  - $\pi_1(1600)$: seen in ρπ, η'π, $b_1\pi$, $f_1\pi$
  - $\pi_1(2015)$ (needs confirmation): seen in $b_1\pi$, and $f_1\pi$
- $\pi_1(1400)$ & $\pi_1(1600)$ can be explained as one pole, according to recent analyses



Hybrid from LQCD



PRD 88 094505(2013)



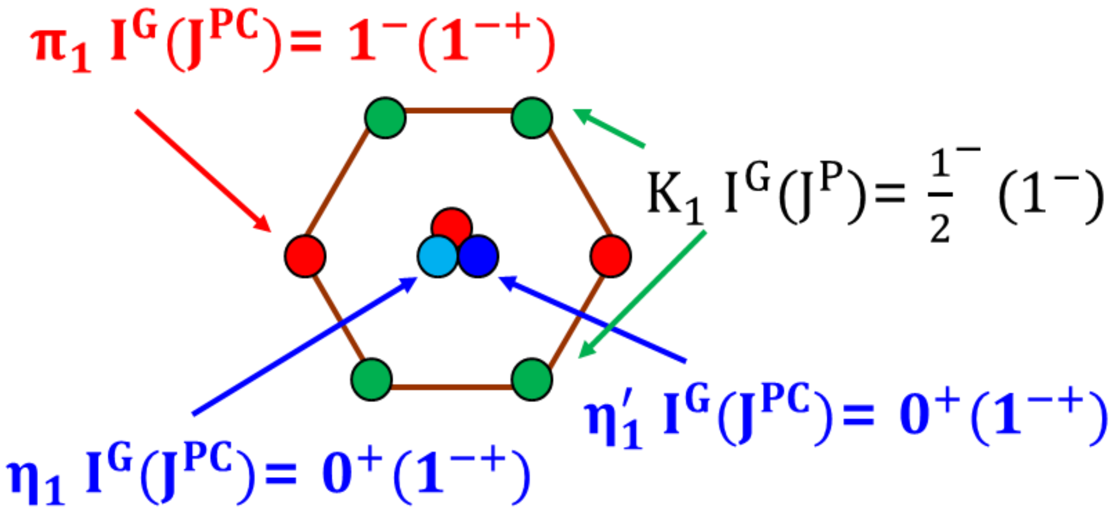
PRD 103, 054502(2021)

Lightest spin-exotic state: $\mathbf{1^{-+}}$

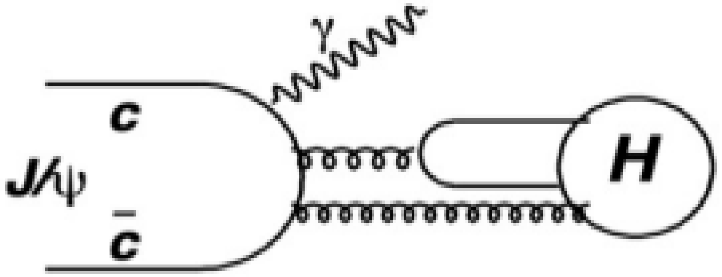
Decay width of $\mathbf{1^{-+}}$ hybrid π_1

1^{-+} Hybrids

- **Isoscalar 1^{-+} is critical to establish the hybrid nonet**
  - **Can be produced in the gluon-rich charmonium decays**
  - **Can decay to $\eta\eta'$ in P-wave**



PRD 83,014021 (2011), PRD 83,014006 (2011), EPJ.P 135, 945(2020)



$$\Gamma(J/\psi \to \gamma H) \sim O(\alpha\alpha_s^3)$$

→ Search for $\eta_1\ (1^{-+})$ in $J/\psi \to \gamma\eta\eta'$

Observation of An Exotic $\mathbf{1^{-+}}$ Isoscalar State $\boldsymbol{\eta_1(1855)}$

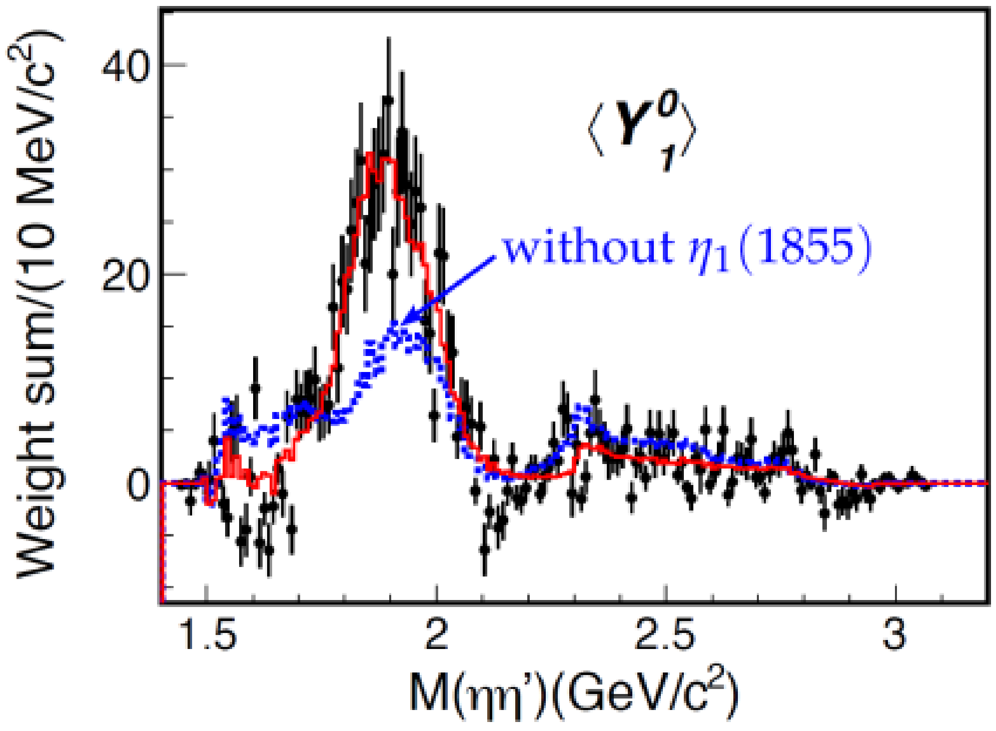
PRL 129 192002(2022) , PRD 106 072012(2022)

- **An isoscalar $\mathbf{1^{-+}}$, $\boldsymbol{\eta_1(1855)}$, has been observed in $\mathbf{J/\psi \to \gamma\eta\eta'}$ (>19σ)**

  $\mathrm{M} = (1855 \pm 9^{+6}_{-1})\ \mathrm{MeV/c^2}, \Gamma = (188 \pm 18^{+3}_{-8})\ \mathrm{MeV/c^2}$

  $\mathrm{B}(\mathrm{J}/\psi \to \gamma\eta_1(1855) \to \gamma\eta\eta') = \left(2.70 \pm 0.41^{+0.16}_{-0.35}\right) \times 10^{-6}$

  - **Mass consistent with hybrid on LQCD**



- **Inspired many interpretations:** **Hybrid/$\mathbf{K\bar{K}_1}$Molecule/Tetraquark?**
- Further more, suppression of $\mathrm{f_0}(1710) \to \eta\eta'$ supports $\mathrm{f_0}(1710)$ has a large overlap with glueball

Opens a new direction to completing the picture of spin-exotics

Synergies in new era of precision spectroscopy

- From serendipitous discoveries of new states to the systematic study of spectral properties and patterns
- High statistics → emergence of new properties/phenomena
- Test QCD with various probes

Heavy ⟷ Light

EM probe: e^+e^-: Belle II, BESIII; EIC; γp: GlueX, CLAS12

Hadronic probe: pp: LHCb, CMS, ATLAS; $\bar{p}p$: panda; πp: COMPASS, AMBER

Summary

- Understanding how hadron spectroscopy are emerged from QCD remains a key question in fundamental physics, which requires
  - Both heavy and light sectors
  - Complementary experimental information
- Lots of progress in the experimental study of hadron spectroscopy
- More results to come and lots of opportunities and challenges ahead
- Joint experimental-theoretical efforts needed to understand the hadron spectroscopy and the strong interaction

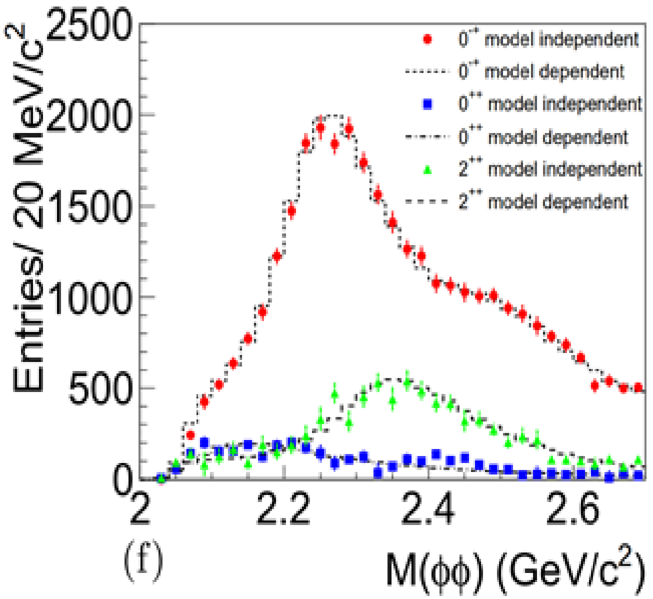
Thank you for your attention

Tensor glueball candidate

$$\Gamma(J/\psi \to \gamma G_{2^+}) = 1.01(22)keV$$
$$\Gamma(J/\psi \to \gamma G_{2^+})/\Gamma_{tot} = 1.1 \times 10^{-2}$$

CLQCD, Phys. Rev. Lett. 111, 091601 (2013)

BESIII $J/\psi \to \gamma\phi\phi$ with 1.3B J/ψ



Experimental results

$$\mathbf{Br(J/\psi \to \gamma f_2(2340) \to \gamma\eta\eta) = \left(3.8^{+0.62+2.37}_{-0.65-2.07}\right) \times 10^{-5}}$$

BESIII PRD 87,092009 (2013)

$$\mathbf{Br(J/\psi \to \gamma f_2(2340) \to \gamma\phi\phi) = \left(1.91 \pm 0.14^{+0.72}_{-0.73}\right) \times 10^{-4}}$$

BESIII PRD 93, 112011 (2016)

$$\mathbf{Br(J/\psi \to \gamma f_2(2340) \to \gamma K_s K_s) = \left(5.54^{+0.34+3.82}_{-0.40-1.49}\right) \times 10^{-5}}$$

BESIII PRD 98,072003 (2018)

$$\mathbf{Br(J/\psi \to \gamma f_2(2340) \to \gamma\eta'\eta') = \left(8.67 \pm 0.70^{+0.16}_{-1.67}\right) \times 10^{-6}}$$

BESIII PRD 105,072002 (2022)

$f_2(2010)$, $f_2(2300)$ and $f_2(2340)$ stated in $\pi^- p$ reactions are observed with a strong production of $f_2(2340)$

Consist with WA102@CERN

It is desirable to search for more decay modes

Observation of An Isoscalar 1^{-+} State $\eta_1(1855)$ in $J/\psi \to \gamma\eta\eta'$

- Angular distribution as a function of $M(\eta\eta')$ expressed **model-independently**

$$\langle Y_l^0 \rangle \equiv \sum_{i=1}^{N_k} W_i Y_l^0(cos\theta_\eta^i)$$

- Related to the spin-0(S), spin-1(P), spin-2(D) amplitudes in $\eta\eta'$ by:

$$\sqrt{4\pi}\langle Y_0^0\rangle = S_0^2 + P_0^2 + P_1^2 + D_0^2 + D_1^2 + D_2^2,$$

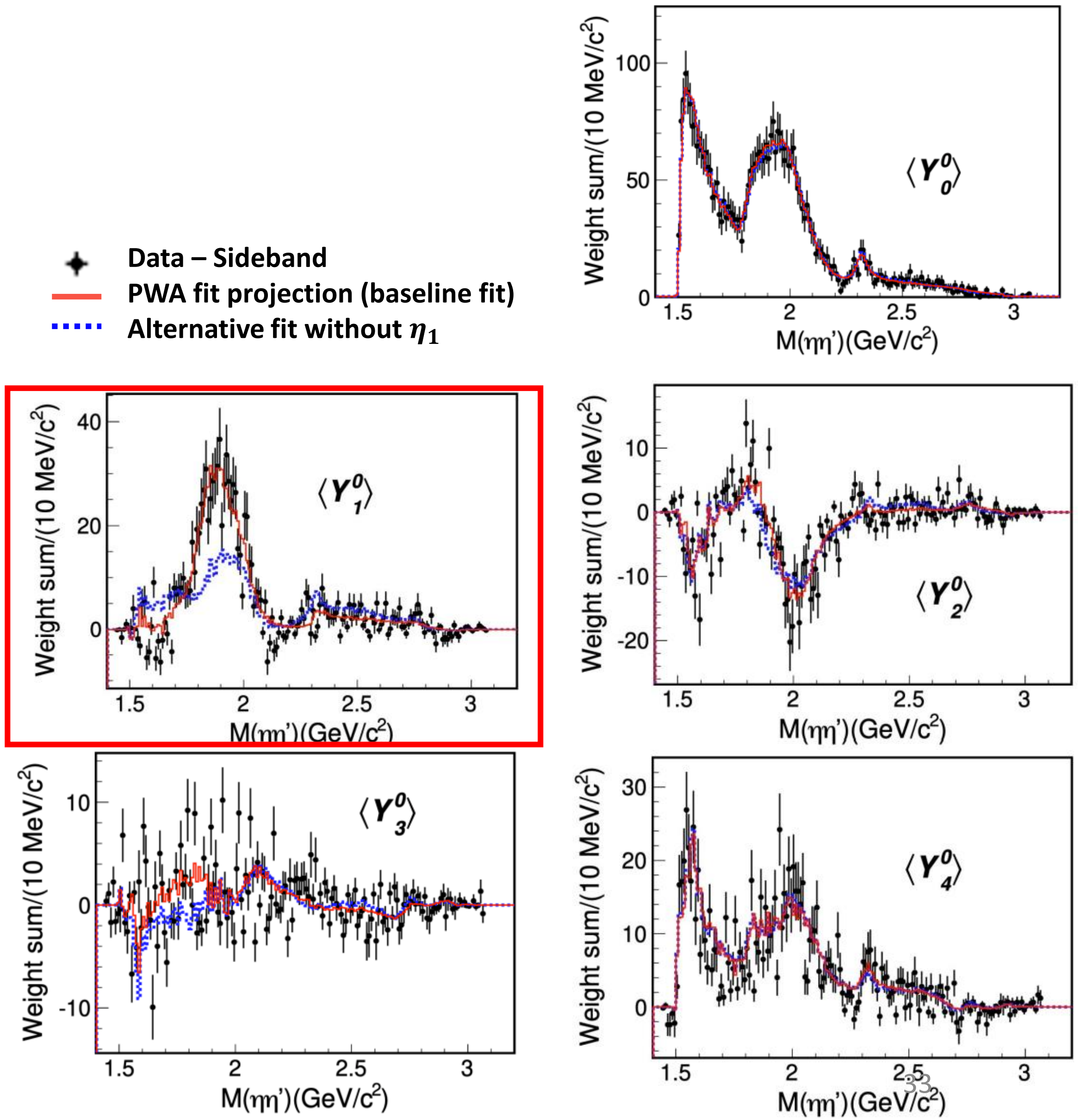
$$\sqrt{4\pi}\langle Y_1^0\rangle = 2S_0P_0\cos\phi_{P_0} + \frac{2}{\sqrt{5}}(2P_0D_0\cos(\phi_{P_0}-\phi_{D_0}) + \sqrt{3}P_1D_1\cos(\phi_{P_1}-\phi_{D_1})),$$

$$\sqrt{4\pi}\langle Y_2^0\rangle = \frac{1}{7\sqrt{5}}(14P_0^2 - 7P_1^2 + 10D_0^2 + 5D_1^2 - 10D_2^2) + 2S_0D_0\cos\phi_{D_0},$$

$$\sqrt{4\pi}\langle Y_3^0\rangle = \frac{6}{\sqrt{35}}(\sqrt{3}P_0D_0\cos(\phi_{P_0}-\phi_{D_0}) - P_1D_1\cos(\phi_{P_1}-\phi_{D_1})),$$

$$\sqrt{4\pi}\langle Y_4^0\rangle = \frac{1}{7}(6D_0^2 - 4D_1^2 + D_2^2).$$

- **Narrow structure** in $\langle Y_1^0\rangle$
  - ➢**Cannot be described by resonances in $\gamma\eta(\eta')$**
  - **$\eta_1(1855) \to \eta\eta'$ needed**

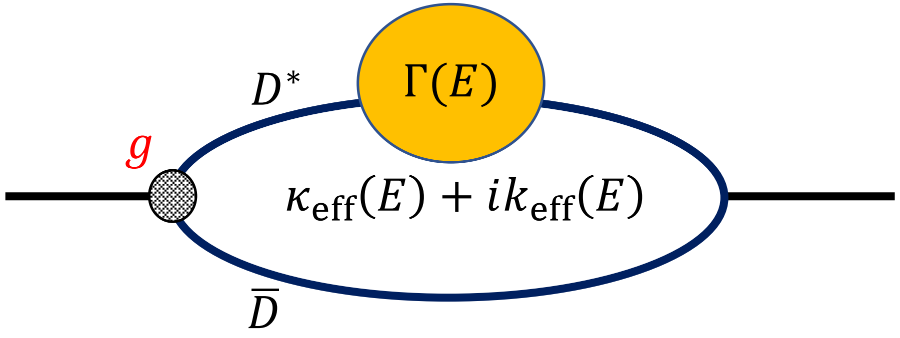


Lineshape parameterization

$$\frac{d\mathrm{Br}(D^0\bar{D}^0\pi^0)}{dE} = \boldsymbol{B}\frac{1}{2\pi}\times\frac{g*k_{\mathrm{eff}}(E)}{|D(E)|^2}\times\mathrm{Br}(D^{*0}\to D^0\pi^0)$$

$$\frac{d\mathrm{Br}(\pi^+\pi^-J/\psi)}{dE} = \boldsymbol{B}\frac{1}{2\pi}\times\frac{\Gamma_{\pi^+\pi^-J/\psi}}{|D(E)|^2}$$

[C. Hanhart, PRD 81, 094028 (2010)]



$$D(E) = E - E_X + \frac{1}{2}g*\left(\kappa_{\mathrm{eff}}(E) + ik_{\mathrm{eff}}(E) + \kappa^c_{\mathrm{eff}}(E) + ik^c_{\mathrm{eff}}(E)\right) + \frac{i}{2}\Gamma_0$$

$$k_{\mathrm{eff}}(E) = \sqrt{\mu_p}\sqrt{\sqrt{(E-E_R)^2+\Gamma^2/4}+E-E_R}$$

$$\kappa_{\mathrm{eff}}(E) = -\sqrt{\mu_p}\sqrt{\sqrt{(E-E_R)^2+\Gamma^2/4}-E+E_R} + \sqrt{\mu_p}\sqrt{\sqrt{(E_X-E_R)^2+\Gamma_X^2/4}-E_X+E_R}$$

$$\Gamma_0 = \Gamma_{\pi^+\pi^-J/\psi} + \Gamma_{known} + \Gamma_{unknown}$$

$$E_X = M_X - (m_{D^0} + m_{\bar{D}^0} + m_{\pi^0})$$

$\boldsymbol{B}$: the global normalization

* superscript c: charged $D^{*+}D^-$
* Due to the limited statistics, $\boldsymbol{\Gamma_{unknown}/\Gamma_{\pi^+\pi^-J/\psi}}$ is fixed [Chunhua Li, Chang-Zheng Yuan, PRD 100(2019) 094003]

Key features:

- Model independent
- Including the $D^*\bar{D}$ self energy terms
- Including the width of D^*
- Including the coupled channel effect
- Fit parameters: g, $\Gamma_{\pi^+\pi^-J/\psi}$, M_X

The effective range expansion

[S. Weinberg, Phys. Rev. 137, B672 (1965)]

$$a = -\frac{2(1-Z)}{(2-Z)}\frac{1}{\gamma} + \mathcal{O}(\beta^{-1})$$

$$r_e = -\frac{Z}{1-Z}\frac{1}{\gamma} + \mathcal{O}(\beta^{-1})$$

Z: field renormalization constant
- $Z = 0$: pure bound (composite) state
- $Z = 1$: pure elementary state

$\beta^{-1} \approx \frac{1}{m_\pi} \approx$ 1.4 fm, for both deuteron and the $X(3872)$

$\gamma = \sqrt{2\mu E_b}$

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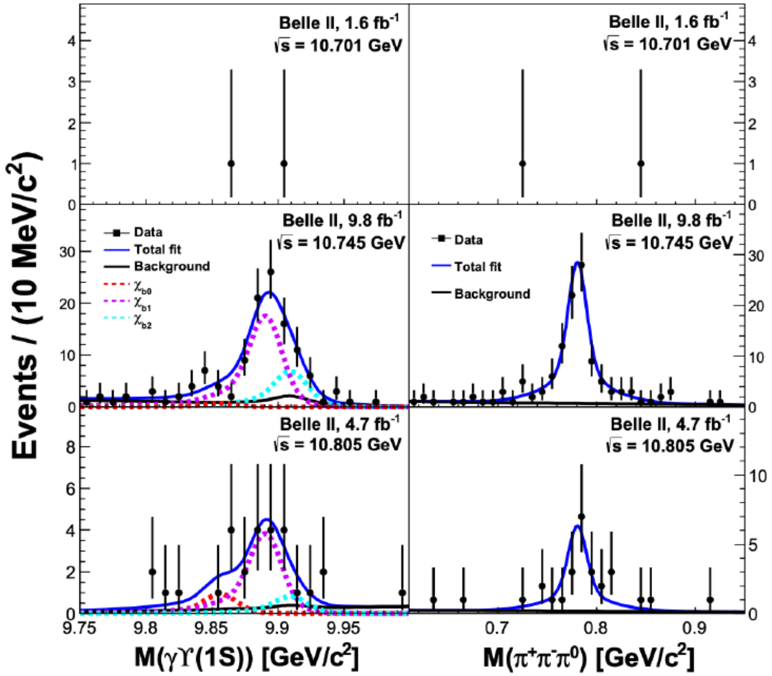
Parameters	$X(3872)$	deuteron
Nearby threshold	$D^{*0}\bar{D}^0$	pn
a	$-16.5^{+7.0}_{-27.6}{}^{+5.6}_{-27.7}$ fm	-5.41 fm
r_e	$-4.1^{+0.9}_{-3.3}{}^{+2.8}_{-4.4}$ fm	1.75 fm
Range correction	negligible	important for r_e
Z	≈ 0.18	-

➡ Different sign, may suggest an elementary $c\bar{c}$ core [A. Esposito PRD 105, L031503]

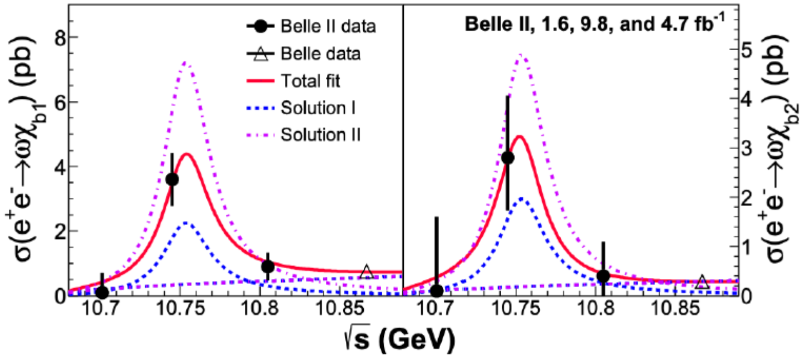
➡ Close to 0 but can not be solved model-independently due to the range correction

Effective Range Expansion → scattering length a and effective range r_e

Observation of $\Upsilon(10753) \to \omega\chi_{bJ}$ [PRL 130, 091902 (2023)]



Channel	$\sqrt{s}$ (GeV)	N^{sig}
$e^+e^- \to \omega\chi_{b0}$	10.701	$0.0^{+1.1}_{-0.0}$
$e^+e^- \to \omega\chi_{b1}$		$0.0^{+2.1}_{-0.0}$
$e^+e^- \to \omega\chi_{b2}$		$0.1^{+2.2}_{-0.1}$
$e^+e^- \to \omega\chi_{b0}$	10.745	$3.0^{+5.5}_{-4.7}$
$e^+e^- \to \omega\chi_{b1}$		$68.9^{+13.7}_{-13.5}$
$e^+e^- \to \omega\chi_{b2}$		$27.6^{+11.6}_{-10.0}$
$e^+e^- \to \omega\chi_{b0}$	10.805	$3.6^{+3.8}_{-3.1}$
$e^+e^- \to \omega\chi_{b1}$		$15.0^{+6.8}_{-6.2}$
$e^+e^- \to \omega\chi_{b2}$		$3.3^{+5.3}_{-3.8}$

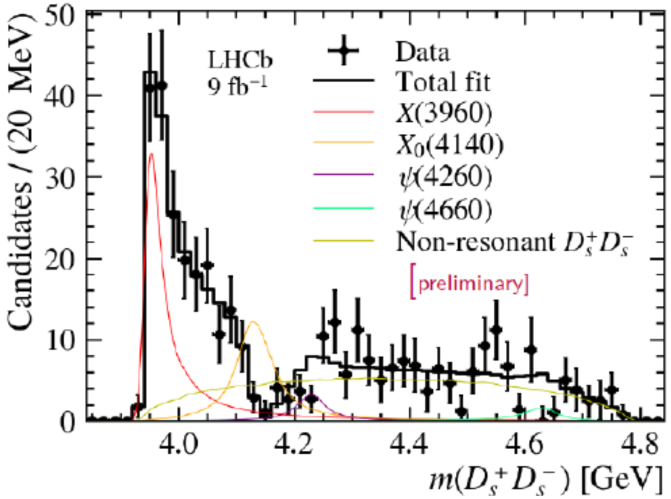


- ➢ $\sigma_B(e^+e^- \to \omega\chi_{b1})/\sigma_B(e^+e^- \to \omega\chi_{b2}) = 1.3 \pm 0.6$ at $\sqrt{s} = 10.745$ GeV
- ✓ Contradicts expectation of pure D-wave of 15
- ✓ 1.8 σ difference to S-D mixture of 0.2
- ➢ $\Upsilon(10753) \to \omega\chi_{bJ}$ & $\Upsilon(10860) \to \Upsilon(nS)\pi\pi$ are different states

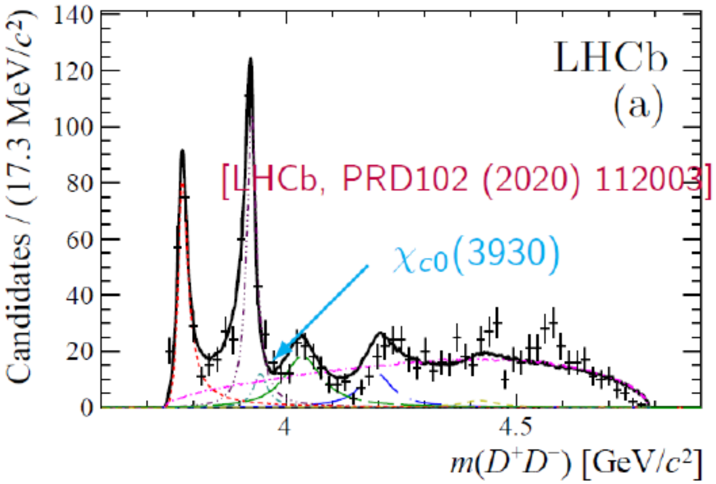
X(3960)=X(3930)?   X(3960)=X(3915)?

Is $X(3960)$ the same as $\chi_{c0}(3930)$ from D^+D^-?

$B^+ \to (D_s^+ D_s^-) K^+$ by LHCb:

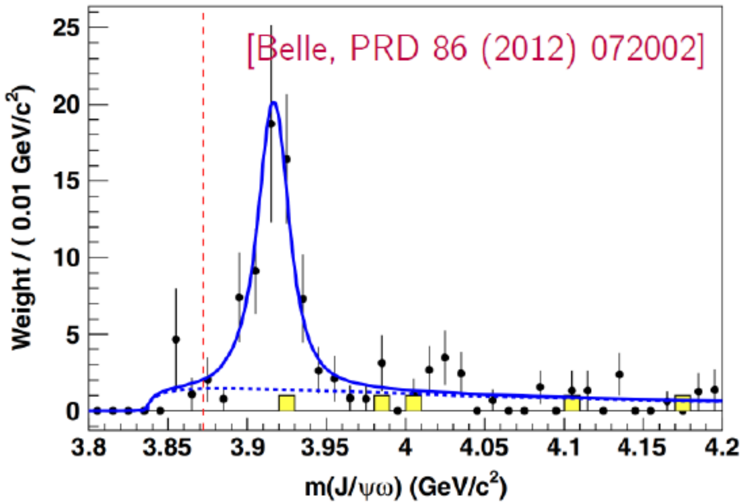


$B^+ \to (D^+ D^-) K^+$ by LHCb:



Is $X(3960)$ the same as $\chi_{c0}(3915)$?

$\gamma\gamma \to J/\psi\omega$ by Belle:



	M [MeV]	Γ [MeV]	J^{PC}
$X(3960)$	$3955 \pm 6 \pm 12$	$48 \pm 17 \pm 10$	0^{++}
$\chi_{c0}(3930)$	3924 ± 2	17 ± 5	

➢Same particle?

$\mathcal{FF}$: Fit fraction

$$\frac{\Gamma(X \to D^+D^-)}{\Gamma(X \to D_s^+D_s^-)} = \frac{\mathcal{B}(B^+ \to D^+D^-K^+)\times\mathcal{FF}^X_{B^+\to D^+D^-K^+}}{\mathcal{B}(B^+ \to D_s^+D_s^-K^+)\times\mathcal{FF}^X_{B^+\to D_s^+D_s^-K^+}} = 0.29 \pm 0.09 \pm 0.10 \pm 0.08$$

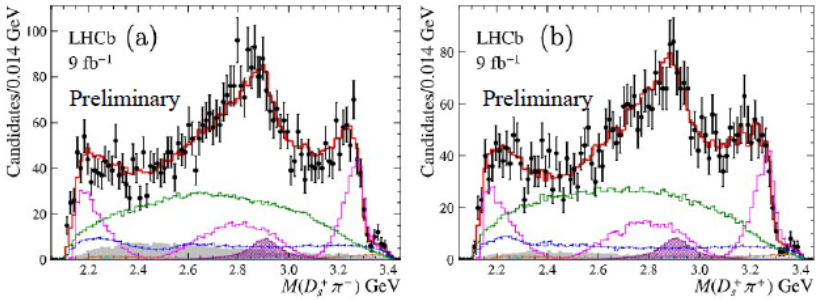
✓Creation of $s\bar{s}$ from vacuum is suppressed wrt $u\bar{u}$ or $d\bar{d}$

✓$X \to D_s^+D_s^-$ has smaller phase-space factor than $X \to D^+D^-$

$\Rightarrow X$ has an exotic nature! Candidate for $c\bar{c}s\bar{s}$

➢Different particles?

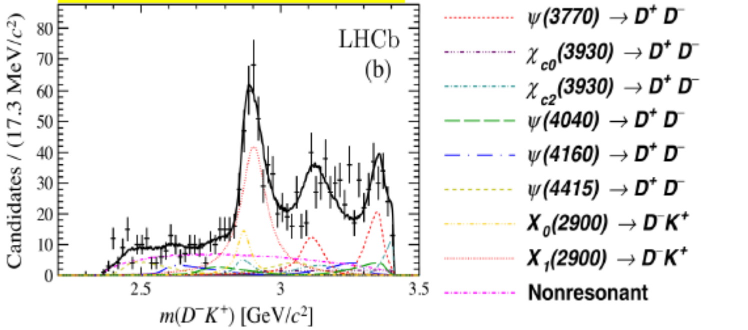
LHCb-PAPER-2022-026 ; LHCb-PAPER-2022-027



$T^a_{c\bar{s}0}(2900)^0[c\bar{s}\bar{u}d]$ $\quad$ $T^a_{c\bar{s}0}(2900)^{++}[c\bar{s}u\bar{d}]$

PRD 102, 112003 (2020)

$B^+ \to D^+ D^- K^+$



$X_0(2900), X_1(2900)[cs\bar{u}\bar{d}]$

	Mass (GeV)	Width (GeV)	J^P
$T^a_{c\bar{s}0}(2900)^0$ & $T^a_{c\bar{s}0}(2900)^{++}$	$2.908 \pm 0.011 \pm 0.020$	$0.136 \pm 0.023 \pm 0.020$	0^+
$X_0(2900)/T_{cs0}(2900)$	$2.866 \pm 0.007 \pm 0.002$	$0.057 \pm 0.012 \pm 0.004$	0^+
$X_1(2900)/T_{cs1}(2900)$	$2.904 \pm 0.005 \pm 0.001$	$0.110 \pm 0.011 \pm 0.004$	1^-

$T^a_{c\bar{s}0}(2900)$ v.s. $X_0(2900)$

- ✓ Similar mass, but width and flavor contents are different.
- ✓ $T^a_{c\bar{s}1}(2900)$?
- ✓ $T^a_{c\bar{s}0}(2900)^{++} \to D^+K^+$?
- ✓ $T^a_{c\bar{s}0}(2900)^+ \to D_s^+\pi^0, D_s^+\pi^+\pi^-$?

- **no isospin relation: $[c\bar{s}u\bar{d}]$ v.s. $[cs\bar{u}\bar{d}]$**
- **U-spin relation: $[c\bar{s}\bar{u}d]$ v.s. $[c\bar{d}\bar{u}s]$**
- **$T^a_{c\bar{s}0}(2900)$** mass and width larger than **$T_{cs0}(2900)$**