Exclusive production of heavy meson pairs

A novel tool for study of the partonic content of the proton

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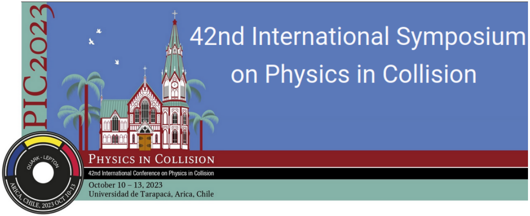
PRD 107, 034037 and [arXiv:2309.09748]

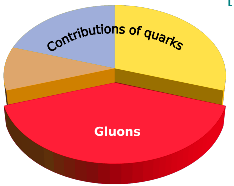


Hadrons in QCD

– Sophisticated strongly interacting dynamical systems
– Theoretical description challenging:
  * Many nontrivial nonperturbative phenomena (chiral symmetry breaking, dynamical masses and interaction vertices ...)
  * Can't evaluate everything from the first principles, have to rely on phenomenological estimates ...

- Quarks dominate, yet gluons contribute up to 40% of proton mass
  – Gluons don't interact directly with photons, leptons, couple only to quarks

[PRL 121, 212001]



– Gluonic contribution is deduced from their interactions with (confined) quarks, not easy to disentangle from experimental data

– Responsible for many nonlinear phenomena (saturation, ...), so understanding their dynamics is of primordial importance

(Generalized) parton distributions: theoretical aspects

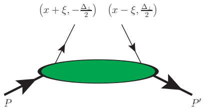
–Nonperturbative objects which encode information about **2-parton correlators**.

Might be reinterpreted in terms of hadron-parton amplitudes in helicity basis

*GPDs are different for each flavour, depend on 4 variables: x, ξ, t, μ^2. Dependence is not arbitrary:

**Dependence on μ^2 ⇒DGLAP

**Dependence on x, ξ ⇒positivity, polynomiality constraints



⇒Challenge for modelling ("dimensionality curse")

–Classification standardized since ∼2010 [PDG 2022, Sec 18.6]

– Leading twist-2 (dominant in many high-energy processes):

$$\int \frac{dz}{2\pi} e^{ix\bar{P}^+ z} \left\langle P' \left| \bar{\psi}\left(-\frac{z}{2}\right) \Gamma\, e^{i\int d\zeta n\cdot A} \psi\left(\frac{z}{2}\right) \psi \right| P \right\rangle = \bar{U}\left(P'\right) \mathcal{F}^{(\Gamma)} U(P)$$

Γ	$\mathcal{F}^{(\Gamma)}$
γ^+	$H\gamma^+ + E\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}$
$\gamma^+\gamma_5$	$\tilde{H}\gamma^+\gamma_5 + \tilde{E}\frac{\gamma_5\Delta^+}{2m}$

Γ	$\mathcal{F}^{(\Gamma)}$
$i\sigma^{+i}$	$H_T i\sigma^{+i} + \tilde{H}_T \frac{\bar{P}^+\Delta^i - \bar{P}^i\Delta^+}{m^2} +$ $+E_T\frac{\gamma^+\Delta^i - \gamma^i\Delta^+}{2m} + \tilde{E}_T\frac{\gamma^+\bar{P}^i - \gamma^i\bar{P}^+}{m}$

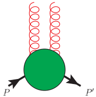
$$\bar{P} \equiv (P + P')/2 \qquad \Delta \equiv P' - P$$

*For gluons use operators $G^{+\alpha}G^+_\alpha$, $G^{+\alpha}\tilde{G}^+_\alpha$, $\mathbb{S}G^{+i}G^{+j}$ in left-hand side

Why do GPDs matter ?

Many physical observables are constructed from bilinear partonic operators:

Example: Energy-momentum tensor



$$\bar{P} = \frac{P + P'}{2}$$

$$\Delta = P' - P$$

$$T^{\mu\nu} = -F^{\mu\alpha} F^{\nu}{}_{\alpha} + \frac{1}{4}\eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} + \frac{1}{2}\bar{\psi}\gamma^{\{\mu} iD^{\nu\}}\psi$$

$$\langle P' | T^{\mu\nu} | P \rangle =$$

$$\bar{U}(P')\left[A(t)\frac{\gamma^{\mu}\bar{P}^{\nu} + \gamma^{\nu}\bar{P}^{\mu}}{2} + B(t)\frac{\bar{P}^{\mu} i\sigma^{\nu\alpha}\Delta_{\alpha} + \bar{P}^{\nu} i\sigma^{\mu\alpha}\Delta_{\alpha}}{2M_N} + D(t)\frac{g^{\mu\nu}\Delta^2 - \Delta^{\mu}\Delta^{\nu}}{4M}\right] U(P)$$

General Relativity: $T^{\mu\nu}$ $\Rightarrow$couplings of graviton to proton, that's why A, B, D are called gravitational form factors

–"Classical" mechanics of continuum medium, rest frame of proton, config. space:

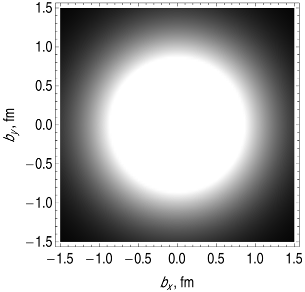
$$T^{ij}(\mathbf{r}) = \left(\frac{r^i r^j}{r^2} - \frac{\delta_{ij}}{3}\right)\underbrace{s(r)}_{\text{shear}} + \delta_{ij}\underbrace{p(r)}_{\text{pressure}} \Rightarrow D(t=0) \sim \int d^3r\, p(r) \sim \int dr\, r^2\, p(r)$$

Relation to GPDs:

$$\int_{-1}^{+1} dx\, x H(x,\xi,t) = A(t) + \xi^2 D(t), \quad \int_{-1}^{+1} dx\, x E(x,\xi,t) = B(t) - \xi^2 D(t),$$

$\Rightarrow$access to pressure, shear forces, energy and momentum density (via T^{00}, T^{0i}).
Can study separately contributions of gluons and quarks of each flavour

Why do GPDs matter ? (II)



Some other examples:

Baryonic/electromagnetic currents and relation to GPDs:

$$J^{\mu}_{\mathrm{baryonic}} = \bar{\psi}\gamma^{\mu}\psi, \qquad J^{\mu}_{\mathrm{em}} = \bar{\psi}\gamma^{\mu}\hat{Q}\psi$$

$$\Rightarrow \int_{-1}^{+1} dx \begin{pmatrix} H_q(x,\xi,t) \\ E_q(x,\xi,t) \\ \tilde{H}_q(x,\xi,t) \end{pmatrix} = \begin{pmatrix} F_1^q(t) \\ F_2^q(t) \\ G_A^q(t) \end{pmatrix}$$

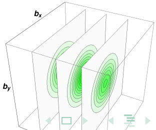
which are related by Fourier transform to densities of charge, magnetic momentum, axial charge ... can study independently each flavour

–Angular momentum density carried by quarks, gluons, and relation to GPDs:

$$M^{\mu\nu\rho} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}\gamma_{\sigma}\gamma_5\psi + \frac{1}{2}\bar{\psi}\gamma^{\mu}x^{[\nu}iD^{\rho]}\psi - 2\mathrm{Tr}\left[F^{\mu\alpha}x^{[\nu}F^{\rho]}_{\alpha}\right] - x^{[\nu}g^{\rho]\mu}\mathcal{L}_{\mathrm{QCD}}$$

$$J^a_q = \varepsilon^{abc}\int d^3r\, x^b\, T^{0c}_q \sim \varepsilon^{abc}\int d^3r\, M^{0bc}_q \sim \int_{-1}^{1} dx\, x\,(H_q(x,\xi,t) + E_q(x,\xi,t))$$

$\Rightarrow$ Study of GPDs = “3D tomography” of the hadron.



How can we study GPDs experimentally ?

Experimental constraints on GPDs:

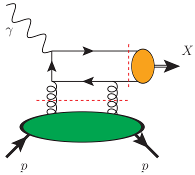
1) Special limits (PDF, form factors)

$$q(x) = \lim_{\xi, t \to 0} H(x, \xi, t)$$

$$F(t) = \int dx\, H(x, \xi, t)$$

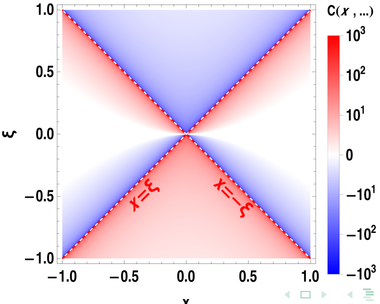
2) $2 \to 2$ **processes** (DVCS, DVMP, TCS, WACS, ...)

– Rely on factorization (separation) of amplitude onto:
  * soft hadron-dependent correlators (blobs), and
  * perturbative process-dependent parts



– Amplitude is a convolution of GPD with process-dependent coef. function:
$\mathcal{A} = \int dx\, C\,(x, \xi)\, H\,(x, \xi, ...)$

– Predominantly sensitive to GPDs at $x = \pm\xi$ boundary
  * Similar behaviour for all $2 \to 2$ processes, and after we take into account NLO corrections, ...



Can we fix GPD experimentally ?

Constraints on GPDs from DVCS (spin-0 target for simplicity):

$$\underbrace{\mathcal{A}}_{\text{experiment}} = \int dx \; \underbrace{C(x,\xi)}_{\text{pert. theory}} \; \underbrace{H(x,\xi,...)}_{?} \qquad (1)$$

– DVCS: can extract $\mathcal{A}$ (both real and imaginary parts) due to interference with Bethe-Heitler ($\Rightarrow$asymmetries). In other channels usually know only $|\mathcal{A}|^2$

– A typical "inverse problem", need to solve inhomogeneous integral equation (1).

* Major concern: existence of zero-modes, "homogeneous" solutions of (1).

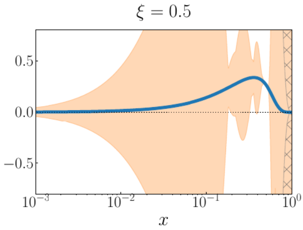
– Recent discoveries [PRD 103, 114019 (2021), PRD 108 (2023) 3, 036027]:

There are nontrivial **homogeneous** solutions of (1) which satisfy all the theoretical constraints on GPDs

$\Rightarrow$Deconvolution is impossible

$\Rightarrow$Compton FFs don't fix uniquely the GPDs

$\Rightarrow$Significant uncertainty in reconstructed GPDs, even if experimental DVCS data are measured *exactly* and we *assume* that kernel $C(x,\xi)$ does not get loop corrections (except those which we took into account)



$\Rightarrow$Need multichannel analysis: each process has its own kernel $C_a(x,\xi)$, null-spaces of different kernels don't overlap in general

New opportunity: $2 \to 3$ processes

Process:

$$\gamma^{(*)} + p \to h_1 + h_2 + p$$

States h_1, h_2 are light hadrons or photons, many possibilities studied in the literature:

$\gamma\pi$, $\gamma\rho$	[2212.00655, 2212.01034, JHEP 11 (2018) 179; 02 (2017) 054]
$\gamma\gamma$	[JHEP 08 (2022) 103; PRD **101**, 114027; **96**, 074008]
$\gamma\gamma^* \to \gamma\bar{\ell}\ell$	[Phys. Rev. D 103 (2021) 114002]
$\pi\rho$	[Phys.Lett.B 688 (2010) 154-167]

Main strength:

−Can vary independently kinematics of h_1, h_2 to probe GPDs at $x \neq \xi$

Weakness:

−Cross-section significantly smaller than for $2 \to 2$ processes, especially for states with additional γ in final state ($\sim$picobarns/(...)).

Opportunity for EIC:

−Within reach of EIC, not studied by HERA (DESY) due to insufficient statistics

*For typical (integrated) luminosity $\mathcal{L} \sim 100\,\mathrm{fb}$ yields $N = \mathcal{L} \times \mathrm{pb} \gtrsim 10^5$ events

Our suggestion: $2 \to 3$ processes with heavy mesons

Exclusive photoproduction of heavy meson pairs

$$\gamma^{(*)} + p \to h_1 + h_2 + p$$

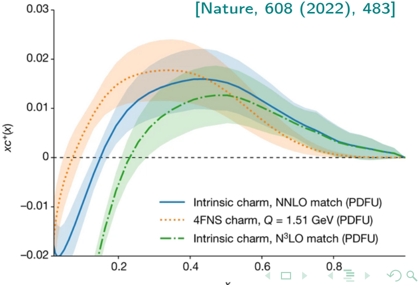
– Focus on D -mesons and quarkonia with opposite sJ^P (e.g. $D^+ D^{*-}$, $J/\psi\,\eta_c$), largest cross-section
  - *Dominant contribution from unpolarized chiral even GPDs H_q, H_g for D-mesons, only gluons for quarkonia
  - *In $m_Q \to \infty$ limit, can use heavy spin-favor symmetry for D-mesons, NRQCD for quarkona, theoretically understood
  - *Heavy quark mass plays the role of natural hard scale, $\alpha_s(m_Q) \ll 1$. No need to impose constraints on virtuality Q^2, ... (Bjorken regime)

There is essentially no heavy quarks inside protons. The "intrinsic" charm does not exceed a few per cent.

$\Downarrow$

All final state heavy quarks stem from photon/gluon fragmentation

[Nature, 608 (2022), 483]



Our focus: EIC kinematics

► Typical values of variables ξ, x_B

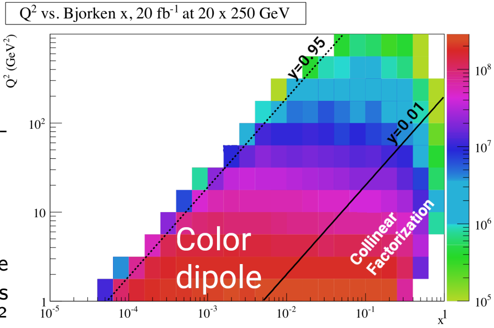
$$x_B, \xi \in \left(10^{-4},\, 1\right)$$

$$Q^2 \in \left(0,\, 10^3\right)\ \mathrm{GeV}^2$$

– Focus on dominant photoproduction region:

$$Q^2 \approx 0 \quad \left(Q^2 \ll M_Q^2\right)$$

For $Q^2 \lesssim (M_1 + M_2)^2$ the Q^2-dependence of $\sigma_{\gamma p \to M_1 M_2 p}$ is weak, and for $Q^2 \gg (M_1 + M_2)^2$ (Bjorken regime) the cross-section is suppressed as $\lesssim 1/Q^6$, too small



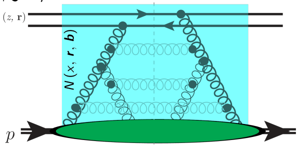
Kinematics vs. choice of framework:

– Partonic language is adequate for $x_B, \xi \gtrsim 10^{-2}$

– For smaller x_B, ξ onset of saturation, should rely on CGC/dipole picture, with built-in saturation

*Probe dipole cross-section instead of GPD

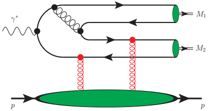
$$N\left(x,\, \boldsymbol{r},\, \boldsymbol{b}\right)$$

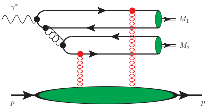


Schematic illustration of multigluon diagrams which for $x \ll 1$ lead to saturation

Exclusive photoproduction of mesons pairs

Quarkonia pairs ($J/\psi\,\eta_c$, $J/\psi\,\chi_c$, $B_c^+ B_c^-$, $J/\psi\,\eta_b$, $\Upsilon(1S)\,\eta_c$)
—Only gluons contribute
—Two main production mechanisms, depending on heavy flavour content:

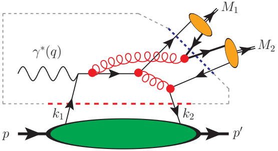


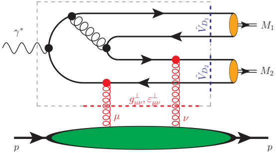


Summation over all possible gluon attachments is implied

Can vary quantum numbers and kinematics of produced quarkonia in order to disentangle effects due to wave function and get detailed info about the target

***D*-meson pairs** ($D^+ D^{*-}$, $D^0 \bar{D}^{*0}$, $D_s^+ D_s^{*-}$ and homonymous B-meson pairs)
—Both light quarks and gluons contribute on equal footing at this order





Summation over all possible permutations of photon, gluon vertices is implied
In the right diagram, should sum contributions with photon attached to heavy or light quark lines

(!) For quark sector, only one light flavour contributes (d, u, s for meson pairs $D^+ D^{*-}$, $D^0 \bar{D}^{*0}$, $D_s^+ D_s^{*-}$ respectively). Important for flavour separation

Hadronization into heavy mesons

Quarkonia: use NRQCD (justified in heavy quark mass limit)

- Use dominant color singlet projectors $\hat{V}_{J/\psi}$, $\hat{V}_{\eta_c}$ to project out contributions of $\bar{Q}Q$ pairs with proper quantum numbers:

$$\left(\hat{V}^{[1]}_{\eta_c}\right)_{ij} \approx -\sqrt{\frac{\left\langle \mathcal{O}_{\eta_c}\left({}^1S_0^{[1]}\right)\right\rangle}{m_Q}}\,\frac{\delta_{ij}}{4N_c}\left(\frac{\hat{p}}{2}-m_Q\right)\gamma_5$$

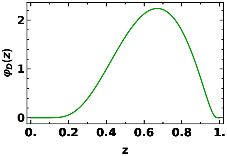
$$\left(\hat{V}^{[1]}_{J/\psi}\right)_{ij} \approx \sqrt{\frac{\left\langle \mathcal{O}_{J/\psi}\left({}^3S_1^{[1]}\right)\right\rangle}{m_Q}}\,\frac{\delta_{ij}}{4N_c}\hat{\varepsilon}^{*}_{J/\psi}(p)\left(\frac{\hat{p}}{2}+m_Q\right)$$

$$\left\langle \mathcal{O}_{\eta_c}\left({}^1S_0^{[1]}\right)\right\rangle = \frac{1}{2}\left\langle \mathcal{O}_{J/\psi}\left({}^3S_1^{[1]}\right)\right\rangle\left(1+\mathcal{O}\left(\Lambda/m_Q\right)\right).$$

D-**mesons**: use heavy spin-flavour symmetry.

– The DAs for all mesons are close to each other, though spin structure differs:

$$f_D\varphi_D\left(z,\mu^2\right)\left(\frac{1+\hat{v}}{2}\gamma_5\right),\quad f_D\varphi_D\left(z,\mu^2\right)\left(\frac{1+\hat{v}}{2}\right)\hat{\varepsilon}\left(p\right)$$



– Phenomenology: The DAs are quite broad, not δ-functions

Amplitudes in collinear factorization picture

Evaluation is straightforward, amplitude (squared):

$$\sum_{\text{spins}} \left| \mathcal{A}^{(\mathfrak{a})}_{\gamma p \to M_1 M_2 p} \right|^2 = \frac{1}{(2-x_B)^2} \left[4\,(1-x_B) \left(\mathcal{H}_{\mathfrak{a}} \mathcal{H}^*_{\mathfrak{a}} + \tilde{\mathcal{H}}_{\mathfrak{a}} \tilde{\mathcal{H}}^*_{\mathfrak{a}} \right) - x_B^2 \left(\mathcal{H}_{\mathfrak{a}} \mathcal{E}^*_{\mathfrak{a}} + \mathcal{E}_{\mathfrak{a}} \mathcal{H}^*_{\mathfrak{a}} + \right.\right.$$

$$\left.\left. + \tilde{\mathcal{H}}_{\mathfrak{a}} \tilde{\mathcal{E}}^*_{\mathfrak{a}} + \tilde{\mathcal{E}}_{\mathfrak{a}} \tilde{\mathcal{H}}^*_{\mathfrak{a}} \right) - \left(x_B^2 + (2-x_B)^2 \frac{t}{4m_N^2} \right) \mathcal{E}_{\mathfrak{a}} \mathcal{E}^*_{\mathfrak{a}} - x_B^2 \frac{t}{4m_N^2} \tilde{\mathcal{E}}_{\mathfrak{a}} \tilde{\mathcal{E}}^*_{\mathfrak{a}} \right],$$

$$\{\mathcal{H}_{\mathfrak{a}},\, \mathcal{E}_{\mathfrak{a}}\} = \int dx\, dz_1\, dz_2 \sum_{\kappa=q,g} C^{(\kappa)}_{\mathfrak{a}}(x,\, z_1,\, z_2,\, y_1,\, y_2) \{H_\kappa,\, E_\kappa\} \Phi_{M_1}(z_1) \Phi_{M_2}(z_2),$$

$$\left\{\tilde{\mathcal{H}}_{\mathfrak{a}},\, \tilde{\mathcal{E}}_{\mathfrak{a}}\right\} = \int dx\, dz_1\, dz_2 \sum_{\kappa=q,g} \tilde{C}^{(\kappa)}_{\mathfrak{a}}(x,\, z_1,\, z_2,\, y_1,\, y_2) \left\{\tilde{H}_\kappa,\, \tilde{E}_\kappa\right\} \Phi_{M_1}(z_1) \Phi_{M_2}(z_2),$$

– For quarkonia effectively replace $\Phi_M(z) \to \mathrm{const}\,\delta(z-1/2)$ (or derivatives of δ for some higher order terms).

– Summation over quarks and gluons implied

– Disregard chiral-odd transversity GPDs (not known, should be negligible in small-t kinematics)

Results for coefficient function

$$\{\mathcal{H}_{\mathfrak{a}},\,\mathcal{E}_{\mathfrak{a}}\} \sim \int dx \underbrace{\int dz_1 dz_2\, C_{\mathfrak{a}}\left(x,\,\xi,\Delta y,\,z_1,z_2\right)\varphi_D\left(z_1\right)\varphi_D\left(z_2\right)}_{C_{\mathfrak{a}}^{(\mathrm{int})}(x,\xi,\Delta y)} \{H_g,\,E_g\},$$

- Structure function $C_{\mathfrak{a}}(x,\,...)$:

$$C_{\mathfrak{a}} \sim \sum_{\ell} \frac{\mathcal{P}_{\ell}\left(x,...\right)}{\mathcal{Q}_{\ell}(x,...)}$$

where $\mathcal{P}_\ell$, $\mathcal{Q}_\ell$ are polynomials of order $n_\ell \lesssim 3$ as a function of x.

– Each term might have up to 3 poles in the integration region $|x| < 1$

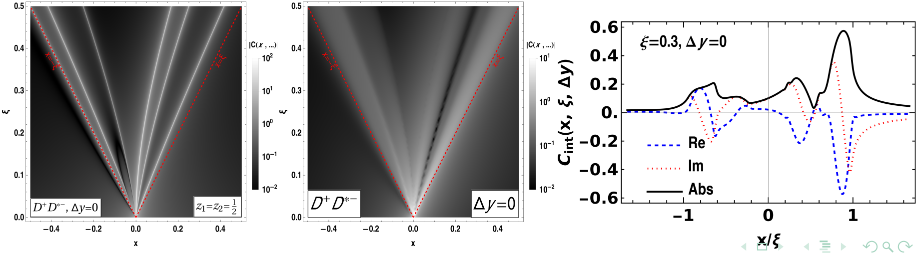
– Position of poles depends on kinematics $(\xi,\,\Delta y,\,z_1,\,z_2)$

– Poles do NOT overlap for $m_Q \neq 0$, so integrals exist in Principal Value sense

Density plot of coefficient function:

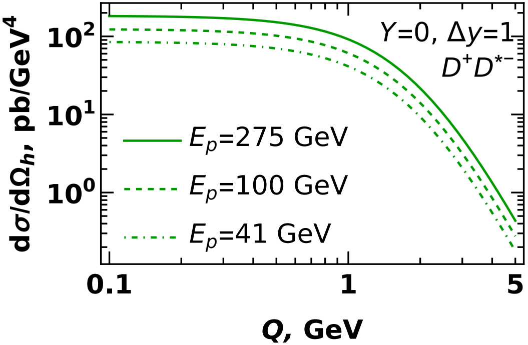
– Poles are seen as bright white lines in the left plot, all in ERBL region ($|x| \lesssim \xi$)

– *D*-**mesons**: convolution with DAs$\Rightarrow$poles are smoothed out (central and right plots)



Results for Q^2-dependence

- Focus on $D^+ D^{*-}$ mesons for brevity (similar dependence for other D- and B-mesons, and for heavy quarkonia pairs)
  - Many good GPD parametrizations are known from the literature, use Kroll-Goloskokov for definiteness

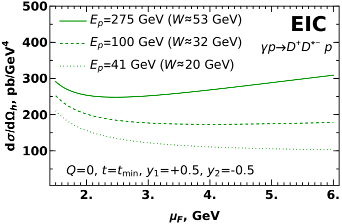


- The Q^2-dependence is controlled by $\mathcal{M}_{12} = \sqrt{(p_1 + p_2)^2} \gtrsim 2M_D \approx 4\,\mathrm{GeV}$
  - very mild dependence for $Q^2 \lesssim \mathcal{M}_{12}^2$, yet $d\sigma \sim 1/Q^6$ for $Q^2 \gg \mathcal{M}_{12}^2$
  - Transition scale largely independent on W

Dependence on factorization scale $\mu_F = \mu_r = \mu$

– Physical observables should not depend on μ, yet when we cut pert. series, such dependence appears due to omitted higher order terms

* At LO dependence on μ due to $\alpha_s(\mu)$, DGLAP evolution of GPDs



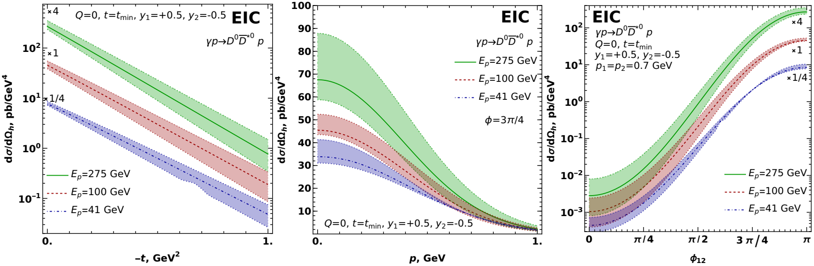
– At small W (large x_B) dependence is mild

– At large W (small x_B) dependence is more and more pronounced, since the omitted higher order loop corrections become more relevant, and the μ-dependence gets stronger

– We'll assume for definiteness that $\mu_F \approx m_D \approx 2\,\mathrm{GeV}$, yet consider uncertainty choice varying μ_F in the range $m_D/2 \lesssim \mu_F \lesssim 2m_D$

Results for t-dependence

– The t-dependence of $d\sigma/d\Omega_h$ largely reflects dependence implemented in GPDs

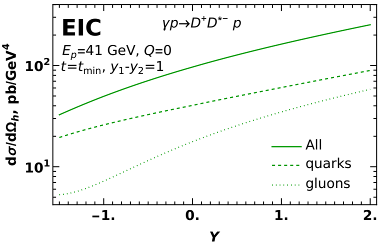


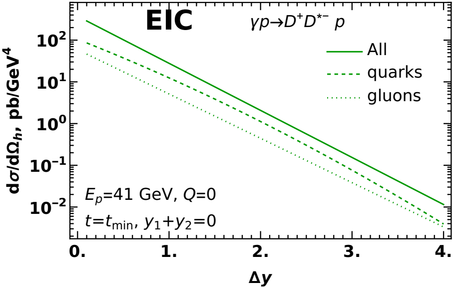
* Predominantly D-meson pairs are produced in back-to-back kinematics, with small p_T, as could be understood from

$$t = \Delta^2 = -\frac{4\xi^2 m_N^2 + \left(\boldsymbol{p}_1^\perp + \boldsymbol{p}_2^\perp\right)^2}{1-\xi^2}$$

* The colored band reflects uncertainty due to choice of the scale $m_D/2 \lesssim \mu_F \lesssim 2m_D$

Results for rapidity dependence





The increase of average rapidity

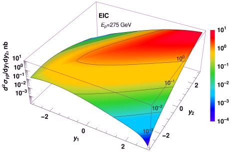
$$Y = \frac{y_1 + y_2}{2}$$

implies larger invariant energy W, smaller x_B, ξ

– The quark contributions dominate at larger x_B (negative $Y \lesssim -1$), the gluons become more important at smaller x_B (positive Y)

– The cross-section drops rapidly as a function of Δy, since increase of Δy implies larger recoil to proton Δ_L and $|t_{\rm min}|$, $|t| = |\Delta^2|$

Cumulative y_1, y_2 dependence:

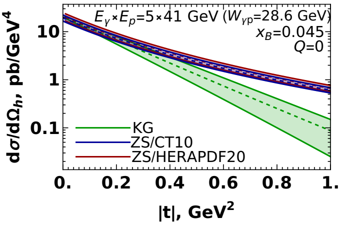


Dependence on choice of GPD

– Compare Kroll-Goloskokov (KG) and Zero Skewness (ZS) parametrizations

$$H_g(x,\xi,t) = g(x)\,F_N(t)$$

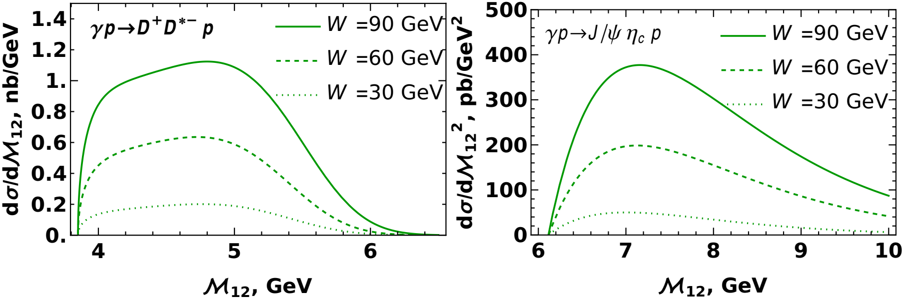
* Makes sense since $\xi \ll 1$ for *photo*production @EIC
* Gluon PDF $g\left(x,\,\mu^2\right)$ is taken from CT10 and HERAPDF20 fits



– At small t results agree within theoretical uncertainty ($\pm 20\%$)
– At larger t results differ quite significantly due to shapes of KG and ZS parametrizations.
  * The shape of GPD in KG is affected by $\sim x^{\alpha' t}$ factors even for $\xi \approx 0$
– Similar behaviour is observed for other energies

$\Rightarrow$ The process might be used to distinguish the GPD models

Results for invariant mass dependence



– Pronounced peak at $M_{12} \approx 7\,\mathrm{GeV}$ for quarkonia pairs, and $M_{12} \approx 4.5$ GeV for D mesons

**Small relative momentum of mesons, $p_{\rm rel} \lesssim 2-3\,\mathrm{GeV}$

Summary

Exclusive production of heavy meson pairs might be used as a new probe of the GPD models:

- **Quarkonia pairs**: probe gluon GPDs
- D-**meson pairs**: probe gluon and quark GPDs of just 1 light quark flavor (u, d or s)
  - Sensitive to behaviour in the ERBL region $|x| \lesssim \xi$. Almost no contribution from outside

- The cross-section is large enough for experimental studies
  - On par with $\gamma^{(*)}p \to \gamma\pi^0\, p$, $\gamma^{(*)}p \to \gamma\rho^0\, p$ suggested by other authors

– *Our studies can be extended to B-mesons and bottomonia production. The cross-sections have similar dependence on kinematic variables, but the numerically are too small (sub-picobarn level).*

Thank You for your attention!

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