Boundary effects in the parity-breaking hadronic medium

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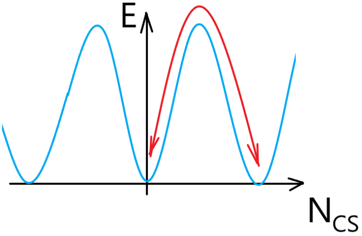
Local $\mathcal{CP}$ violation in QCD

When **finite volume** is considered a metastable axial topological charge may arise in hot QCD

$$T_5 = \frac{1}{8\pi^2} \int d^3x \epsilon_{jkl} \operatorname{Tr} \left[G^j \partial^k G^l - i\frac{2}{3} G^i G^k G^l \right]$$

Such fluctuations of the topological charge may appear in a fireball produced in the heavy-ion collisions. The lifetime for its fluctuation is comparable to the lifetime of the fireball and estimated to be $\Delta t \sim \tau_f \sim 5 - 10 fm/c$.

In the infinite volume limit this fluctuation is associated with a sphaleron (localized lump connecting vacua with different topological number) transitions



Chiral imbalance

The topological charge change is,

$$\Delta T_5 = \frac{1}{8\pi^2}\int_0^{\tau_f} dt \int\limits_{Vol} d^3x\,\mathrm{Tr}[G_{\mu\nu}\tilde{G}^{\mu\nu}] = \frac{1}{2\pi^2}\int_0^{\tau_f}\int\limits_{Vol} d^3x\partial_\mu K^\mu$$

Where $\tilde{G}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}G_{\alpha\beta}$. The effects of this local topological charge may be mimicked by introduction of the θ-term to the Lagrangian with $\theta(t,x)$, $\partial_t\theta = \mu_5$

For the massless quarks ($m_{u,d} \ll \tau_f^{-1}$) the quark axial current $J_5^\mu = i\bar{q}\gamma_5\gamma^\mu q$ satisfies the conservation law broken by the chiral anomaly

$$\partial_\mu J_5^\mu = \frac{N_f}{2\pi^2}\partial_\mu K^\mu \Rightarrow \frac{d}{dt}\Big(Q_5 - 2N_f T_5\Big) = 0$$

Thus, the local fluctuations of the topological charge should also produce the local fluctuations of $Q_5 = N_R - N_L$ - **chiral imbalance**

Chiral magnetic effect

In the peripheral heavy ion collisions these fluctuations result in a **chiral magnetic effect** [Kharzeev, McLerran, Warringa, 2008]

In the peripheral collision a large magnetic field is produced along the angular momentum.

Positively charged right-handed and negatively charged left-handed quarks tend to orient their spins and momenta along the magnetic field. Negatively charged lef-handed and positively charged right-handed orient in the opposite directions.

As result, in the presence of the chiral imbalance the electric current is produced along the direction of the magnetic field

What are effects in the **central collisions**?

Effective meson description on a chiral background

In the hadronic phase we may use the effective field theory description with the appropriate modifications

- Scalar and pseudoscalar mesons
  Chiral perturbation theory with the modified covariant derivative,

$$D_\alpha \mapsto D_\alpha - i\{\mu_5 \delta_{\alpha,0}, \cdot\}$$

- Vector mesons
  Vector dominance model with the extra term,

$$\epsilon^{\mu\nu\alpha\beta}\,\mathrm{Tr}\left[\zeta_\mu V_\nu V_{\mu\nu}\right]$$

  where $\zeta^\mu = \mu_5 \delta^{\mu 0}$

- Bottom-up holographic models - large charge difference for A_L^a and A_R^b 5d fields dual to the J_L^μ and J_R^μ currents, Chern-Simons interaction

Manifestations of the chiral imbalance in the hadronic medium

Using these effective field theories a number of phenomena inside the fireball was predicted by A.A.Andrianov and his coauthors

- For scalar mesons: parity violating decays $\eta, \eta' \to \pi\pi$
- Exotic parity violating decays of $\tilde{a}_0$, $\tilde{\pi}$ meson states inside the fireball
- Asymmetry of the photon polarizations in $\pi^{\pm}\gamma \to \pi^{\pm}\gamma$ scattering
- For vector mesons: the splitting of the longitudinal and transverse polarization masses
- An observable: anomalous yield of dileptons from the vector meson decays

Vector dominance model

Lowest radiatively induced one-loop lagrangian for QED and ρ and ω mesons in presence of the chiral imbalance [V.Kovalenko,A.Andrianov,V.Andrianov]

$$\mathcal{L}_{\rm eff} = -\frac{F_{\mu\nu}F^{\mu\nu}}{4} - \frac{\omega_{\mu\nu}\omega^{\mu\nu}}{4} - \frac{\rho_{\mu\nu}\rho^{\mu\nu}}{4} + \frac{5m_V^2 e^2 A_\mu A^\mu}{9g^2} - \frac{m_V^2 e A^\mu \omega_\mu}{3g} - \frac{m_V^2 e A^\mu \rho_\mu}{g} + \frac{\zeta^2 N_c V^\nu V_\nu}{24\pi^2 m^2} - \frac{N_c \epsilon_{\delta\gamma\mu\nu}\zeta^\mu V^\nu V^{\delta\gamma}}{8\pi^2} + \frac{V_{\gamma\lambda} N_c V_\mu^\lambda \zeta^\gamma \zeta^\mu}{12\pi^2 m^2} + \frac{1}{2} m_V^2 \omega_\mu \omega^\mu + \frac{1}{2} m_V^2 \rho_\mu \rho^\mu$$

$$V_\mu \equiv -eA_\mu Q + \frac{1}{2} g_\omega \omega_\mu \mathrm{I}_q + \frac{1}{2} g_\rho \rho_\mu \lambda_3$$

If $\zeta_\mu = \partial_\mu \theta$ the model may be rewritten in a gauge-invariant way with help of the Stuckelberger mechanism

Diagonalization

Let us introduce the rotated field

$$V^\mu \mapsto \begin{pmatrix} \frac{3g/e}{\sqrt{\frac{9g^2}{e^2}+10}} & -\frac{10e/g}{\sqrt{\frac{100e^2}{g^2}+90}} & 0 \\ \frac{1}{\sqrt{\frac{9g^2}{e^2}+10}} & \frac{3}{\sqrt{\frac{100e^2}{g^2}+90}} & -\frac{3}{\sqrt{10}} \\ \frac{1}{\sqrt{\frac{9g^2}{e^2}+10}} & \frac{9}{\sqrt{\frac{100e^2}{g^2}+90}} & \frac{1}{\sqrt{10}} \end{pmatrix} \begin{pmatrix} A_\mu \\ \omega_\mu \\ \rho_\mu \end{pmatrix}$$

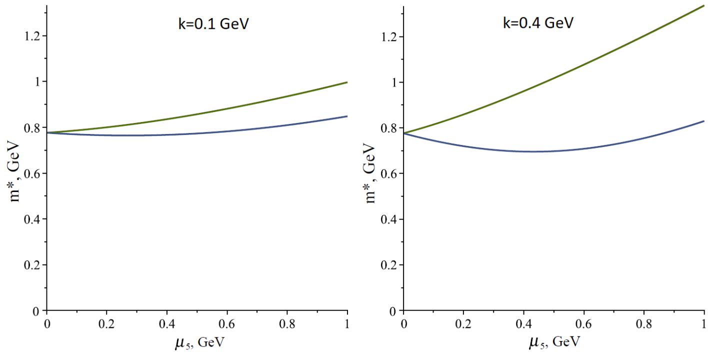
Note that this matrix does not depend on ζ_μ. Then vector fields decouple,

$$-\frac{1}{4}\left(1+\xi\frac{\zeta^2}{m^2}\right)V^{\mu\nu}V_{\mu\nu}+\xi\frac{\zeta_\nu\zeta^\rho}{2m^2}V^{\nu\lambda}V_{\rho\lambda}-\frac{3}{2}\xi\zeta_\nu V_\lambda\tilde{V}^{\nu\lambda}+\frac{1}{2}(m^2+\xi\zeta^2)V_\nu V^\nu$$

with specific ξ and m^2 for the different components of V^μ

The one-loop effects

ξ controls the one-loop radiative contribution to the effective Lagrangian. At low momenta transverse to $\vec{\zeta}$ the main contribution comes from $(m^*)^2 = m^2 + m\xi\zeta^2$ combination



Simplified model

To get qualitative understanding of the behavior of the vector mesons in the hadronic medium, let us neglect one-loop terms

A massive electrodynamics with θ-term

$$S = \int d^4x \Big(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\theta(x)F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{1}{2}m^2A^\mu A_\mu + A^\mu\partial_\mu B + \frac{1}{2}B^2\Big)$$

The auxiliary field B is introduced to make a massive model self-consistent by the Stückelberg mechanism. It gives,

$$(\Box + m^2)(\partial_\mu A^\mu) = 0$$

Such model was used in [A.A.Andrianov, V.A.Andrianov, D. Espriu, S.S.Kolevatov] and [A.A.Andrianov, S.S. Kolevatov, R.Soldati] with,

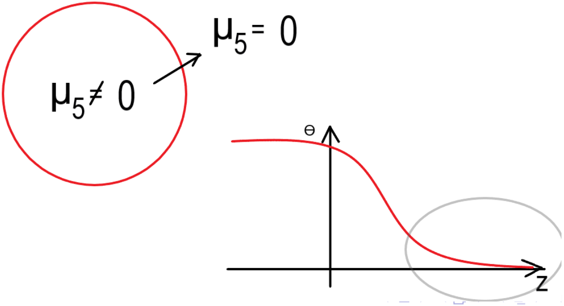
$$\partial_\mu\theta \sim \zeta_\mu x^\mu \theta\Big(-\zeta_\alpha x^\alpha\Big)$$

Bubble boundary

Such model was used to study two processes:

- The transition of the effective meson states to the ordinary mesons after the fireball decay
- The propagation of the meson states through the spatial boundary of the fireball

However the ansatz considered may be associated only with the very edge of the fireball boundary. Also μ_5 corresponds to $\partial_t\theta$.



Near the boundary

Assume that the boundary thickness is much smaller than the fireball

$$A^\mu(t,x,y,z) = \int d^2k A^\mu(t,z)\exp(ik_x x + ik_y y),$$

$$A_\pm = A^x \pm iA^y, \quad k_\pm = k_x \pm ik_y$$

Then the equations take the form,

$$(\Box + m^2)A^t = -\frac{1}{2}(\partial_z\theta)\Big[k_- A_+ - k_+ A_-\Big],$$

$$(\Box + m^2)A^z = \frac{1}{2}(\partial_t\theta)\Big[k_- A_+ - k_+ A_-\Big],$$

$$(\Box + m^2)A_\pm = \pm k_\pm\Big[(\partial_t\theta)A^z + (\partial_z\theta)A^t\Big] \pm i\Big[(\partial_t\theta)\partial_z A_\pm - (\partial_z\theta)\partial_t A_\pm\Big]$$

For simplicity: momentum directed perpendicularly to the boundary i.e. $k_\pm = 0$. Then $A_\pm$ decouple both from the equations on A^t and A^z.

Circular polarizations

$$(\partial_t^2 - \partial_z^2 + m^2)A_\pm = \pm i\Big[(\partial_t\theta)\partial_z A_\pm - (\partial_z\theta)\partial_t A_\pm\Big]$$

A_- satisfies the same equation but in the reverse time.

Assume that $\partial_t\theta$ is small compared to $\partial_t A_+$ (adiabatic regime)

This would correspond to μ_5 being small compared to $\omega \sim m$

$$A_+ = a(t,z)\exp\Big(-i\int dt\,\omega(t)\Big)$$

a and $\omega(t)$ are slowly varying at the same rate as θ.

Adiabatic regime

In the leading approximation $i(\partial_t\theta)$ neglected,

$$a \simeq a_0, \quad \omega \simeq \omega_0$$

$$-\partial_z^2 a_0 + \left[m^2 - \omega_0^2 + \omega_0(\partial_z\theta)\right] a_0 = 0$$

Schrödinger equation though with the potential proportional to the spectral parameter ω_0

Sample shape for θ - the transition between two homogeneous regions

$$\theta(t,z) = \bar{\theta}(t) + \Delta\theta(t) \cdot \tanh\left[\mu(t)\Big(z - z_0(t)\Big)\right],$$

$$\theta \underset{z\to-\infty}{\longrightarrow} \bar{\theta} - \Delta\theta, \quad \theta \underset{z\to+\infty}{\longrightarrow} \bar{\theta} + \Delta\theta,$$

For this shape of θ the potential becomes the Pöschl-Teller one,

$$\left[-\partial_z^2 + m^2 - \omega_0^2 + \frac{\omega_0\mu\cdot\Delta\theta}{\cosh^2\mu(z-z_0)}\right]a_0 = 0, \tag{1}$$

The potential has the continuous spectrum of the wavelike solutions for $\varepsilon > 0$ i.e. $|\omega_0| > m$.

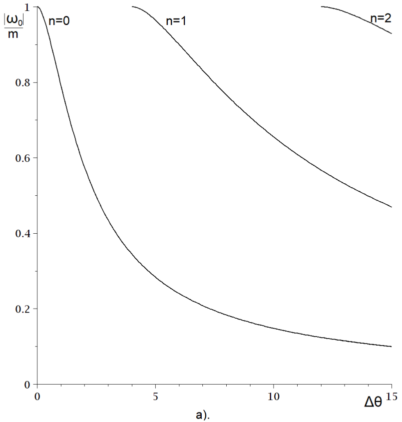
In the band $|\omega_0| < m$ the bound states may exist. This is true for $\omega\mu\cdot\Delta\theta < 0$,

$$\varepsilon_n = -\mu^2(\lambda - n - 1)^2, \quad \lambda(\lambda-1) = \left|\frac{\omega_0\Delta\theta}{\mu}\right|$$

where n is integer from 0 to the largest integer $n_{max} \leq \lambda - 1$,

$$\lambda^4 - 2\lambda^3 + \left(1+\Delta\theta^2\right)\lambda^2 - 2(n+1)\Delta\theta^2\lambda + \Delta\theta^2\left((n+1)^2 - \frac{m^2}{\mu^2}\right) = 0$$

Bound states spectrum



(a) $m/\mu = 0.5$ (b) $m/\mu = 1.0$

Disappearance of the levels - nonadiabaticity!

Time dependence influence

At small times we may approximate the influence of the $\partial_t \theta(t, z) \equiv v(t, z)$ term with the adiabatically changing perturbation that changes the frequencies and profile functions,

$$\omega = \omega_0 + \omega_1, \quad a = a_0 + a_1, \quad \omega_1, a_1 \sim |v|$$

The perturbation is non-Hermitian and results in,

$$\omega_1 = -\frac{i}{2} \frac{\int_{-\infty}^{+\infty} dz \, (\partial_z v) a_0^2}{\int_{-\infty}^{+\infty} dz \, (2\omega_0 + \partial_z \theta) a_0^2}$$

For state $n = 0$ in our sample potential,

$$\omega_{1,n=0} = -\frac{i}{2} \frac{\frac{\lambda - 1}{2\lambda - 1} \mu (\partial_t \Delta\theta)}{\omega_0 + \frac{\lambda - 1}{2\lambda - 1} \mu \Delta\theta}$$

This means that the θ time-dependence results in the damping or amplification of the bound states

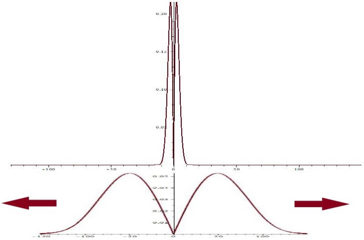
Non-adiabatic effect - explosion of the bound state

Assume that $\Delta\theta$ decreases with time

Adiabatic: the level goes up and its profile becomes wider until it becomes delocalized when the level touches the continuous spectrum

The breakdown of the adiabatic approximation: when the bound state level becomes too close to the continuous spectrum it explodes

The resulting wave packets have spectrum close to the spectrum of the bound state at the moment of its explosion



Schrodinger form of the equation

One may rewrite the evolution in the Schrodinger form

$$i\partial_t \Psi = \mathcal{H}\Psi, \quad i\partial_t \Psi = \mathcal{H}\Psi$$

where $\mathcal{H}$ is not Hermitian but satisfies the time-dependent pseudo-Hermiticity relation

$$\rho\mathcal{H} = \mathcal{H}^\dagger \rho - i\partial_t \rho, \quad \rho = \begin{pmatrix} -(\partial_z \theta) & 1 \\ 1 & 0 \end{pmatrix}$$

and thus admits the conserved norm $(\Psi_1, \Psi_2) = \int_{-\infty}^{+\infty} dz\, \Psi_1^\dagger \rho \Psi_2$ under which the solutions with different ω are approximately orthogonal (with error $\sim \mu_5$.

Explosion moment analytical estimate

Now we can use the usual method to study the breakdown of the adiabatic approximation. Decompose Ψ into the superposition of $\mathcal{H}$ eigenfunctions

$$\Psi = \sum_n c_n(t)\Psi_n e^{-i\int dt\,\omega_n}$$

Putting it into our equation we get the mixing between levels $n \neq m$,

$$\frac{i\dot{\omega}}{(\omega_m^* - \omega_n)^2}\int_{-\infty}^{+\infty} dz\,\Psi_m^\dagger(\partial_t\rho)\Psi_n - \frac{1}{\omega_m^* - \omega_n}\int_{-\infty}^{+\infty} dz\,\Psi_m^\dagger\rho(\partial_t\mathcal{H})\Psi_n$$

The adiabatic regime breaks when one of this terms becomes large which happens at,

$$\Delta\theta \sim n(n+1)\frac{\mu}{m} + \min\left[\left(\frac{4\mu_5}{m}(2n+1)\right)^{2/3}, (2n+1)\sqrt{\frac{2\mu_5}{m}}\right]$$

Conclusions

- We argue that the bound currents near the transition region of the fireball exist
- The present analysis was using the adiabatic approximation. Estimate for its validity implies that it should work for sufficiently high $\Delta\theta$ (which should be present in a sufficiently long-living fireball)
- When the adiabatic regime breaks the bound state explodes which may serve as a signature for their existence
- Impact on the signatures for the chiral imbalance?
  Resonances in the transition probabilities? Signatures of the explosions of boundary currents? Work in progress (interactions, pumping by background)

Thank you for your attention