NNLL TMD evolution in the Parton Branching method

Ola Lelek on behalf of the PB team

12th October 2023

PIC 2023

Universiteit Antwerpen

Research Foundation Flanders
Opening new horizons

Introduction

- Hot topic: 3D hadron structure & Transverse Momentum Dependent (TMD) factorization
- the baseline TMD method: Collins–Soper–Sterman (CSS)
- recently new developments to include TMD physics in MCs:
  TMD Parton Branching (PB) method
  Hautmann, Jung, Lelek, Radescu, Zlebcik, Phys.Lett.B 772 (2017) 446 & JHEP 01 (2018) 070
- **Today: overview of the PB method and its development up to NNLL**

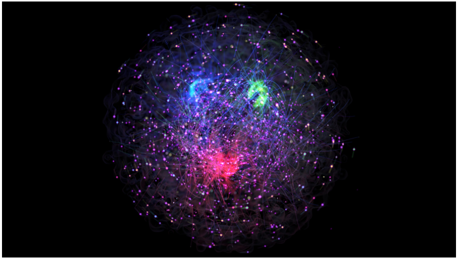
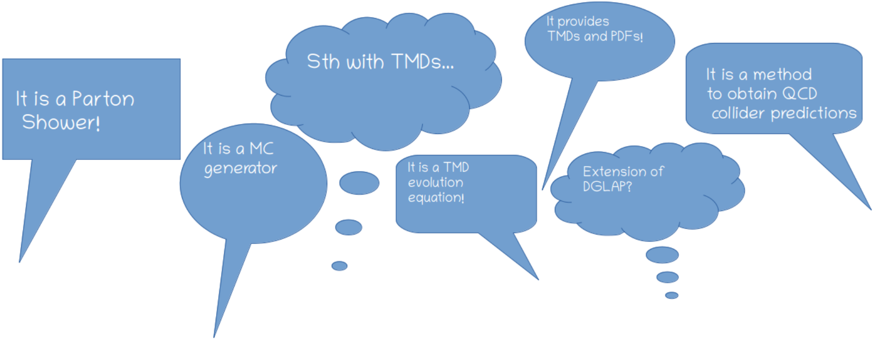


Image: James LaPlante/Sputnik Animation, MIT CAST & Jefferson Lab

What is the TMD Parton Branching method?



All this is true!

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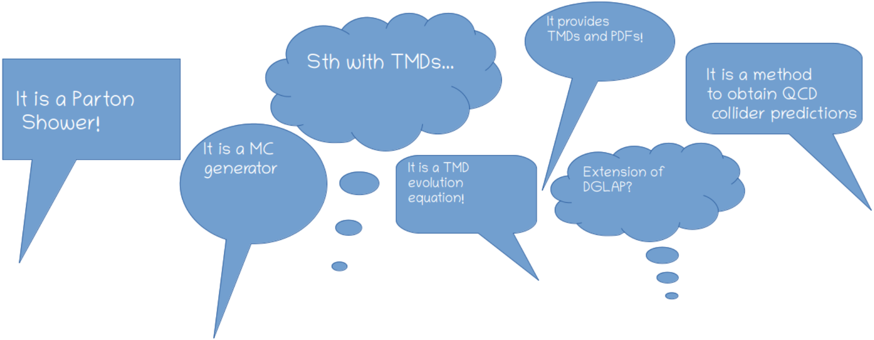
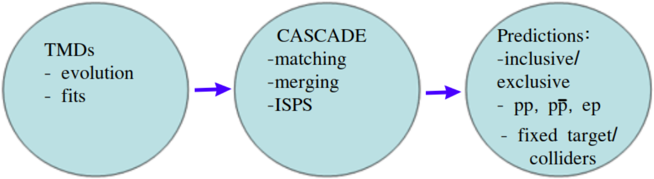
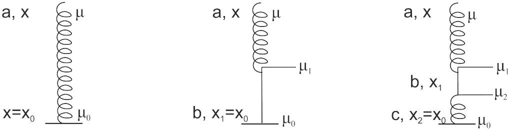


All this is true!

TMD PB method in a nutshell



Evolution equation:



$$\widetilde{A}_a\left(x, k_\perp^2, \mu^2\right) = \Delta_a\left(\mu^2, \mu_0^2\right)\widetilde{A}_a\left(x, k_\perp^2, \mu_0^2\right) + \sum_b \int \frac{d^2\mu_{\perp 1}}{\pi \mu_{\perp 1}^2} \Theta\left(\mu_{\perp 1}^2 - \mu_0^2\right)\Theta\left(\mu^2 - \mu_{\perp 1}^2\right)$$

$$\Delta_a\left(\mu^2, \mu_{\perp 1}^2\right)\int_x^{z_M} dz P_{ab}^R\left(z, \mu_{\perp 1}^2\right)\widetilde{A}_b\left(\frac{x}{z}, |k_{\perp 1}|^2, \mu_{\perp 0}^2\right)\Delta_b\left(\mu_{\perp 1}^2, \mu_{\perp 0}^2\right) + \ldots$$

Probabilities $\Longleftrightarrow$ **solve by Monte Carlo** (MC) :

- $\Delta_a\left(\mu^2, \mu_0^2\right)$ – evolution from μ_0^2 to μ^2 without branchings
- $P_{ab}^R(z, \mu^2)$ – splitting of $b \to a$

z_M defines **resolvable and non-resolvable** branchings

transverse momentum **k calculated at each branching** $\rightarrow$ **TMD from parton branching**

Fits of PB distributions

Bermudez Martinez, Connor, Hautmann, Jung, Lelek, Radescu, Zlebcik, Phys.Rev.D 99 (2019) 7, 074008

The parameters of the initial distributions have to be obtained from the fits to data → **xFitter**

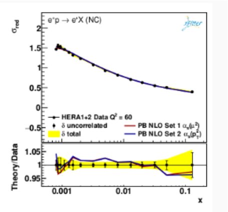
HERAPDF2.0 recipe: H1, ZEUS, Eur.Phys.J.C 75 (2015) 12, 580

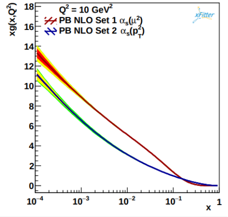
- data: **HERA H1 and ZEUS combined DIS** measurement
- range: $3.5 < Q^2 < 50000\ \mathrm{GeV}^2$, $4 \cdot 10^{-5} < x < 0.65$

Two scenarios, both very similar $\chi^2/\mathrm{d.o.f.} \approx 1.21$:

- Set1: $\alpha_s\left(\mu'^2\right)$, reproduces HERAPDF2.0 ✓
- Set2: $\alpha_s\left(q_\perp^2\right)$, different HERAPDF2.0 ✓

TMDs and iTMDs available in **TMDlib**
Eur.Phys.J.C 81 (2021) 8, 752





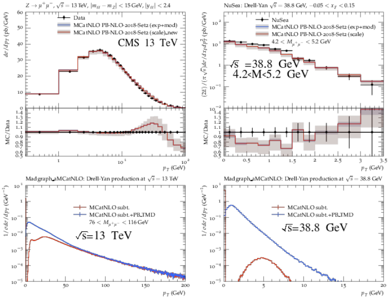
DY from fixed-target up to LHC

Eur.Phys.J.C 80 (2020) 7, 59

TMDs fitted to HERA data applied to DY

technical development:
matching TMDs to NLO ME

- standard MCatNLO: when ME matched with PS, **subtraction terms** (for soft and collinear contribution) used to avoid double counting
- PB TMDs similar to PS
  $\rightarrow$ subtraction needed for TMDs+NLO ME Phys.Rev.D 100 (2019) 7, 074027



- Low and middle $p_\perp$ spectrum well described.
At higher $p_\perp$ from Z+ jets important $\rightarrow$ see later

- **Good description of DY in different kinematic ranges**: NuSea, R209, Phenix, Tevatron, LHC. **No tuning/adjusting** of the method for different $\sqrt{s}$

- **"low $q_\perp$ crisis"** A. Bacchetta et al., Phys. Rev. D 100, 014018 (2019):
perturbative fixed order calculations in collinear factorization not able to describe DY p_T spectra at fixed target experiments for $p_T/m_{DY} \sim 1$ $\rightarrow$ we **confirm** this:

  - at larger masses and LHC energies the contribution from soft gluons in the region of $p_\perp/m_{DY} \sim 1$ is small and the spectrum driven by hard real emission.
  - at low DY mass and low $\sqrt{s}$ even in the region of $p_\perp/m_{DY} \sim 1$ the contribution of soft gluon emissions essential

DY at high pt

Recall: DY at high $p_\perp$: large corrections from higher orders

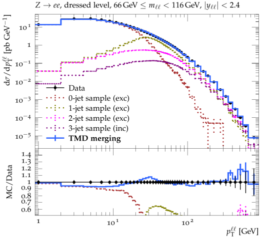
technical developments:

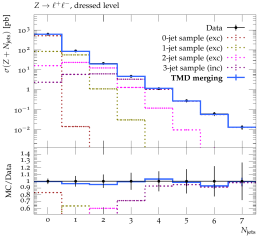
- **Cascade:** Initial State **TMD PS guided by the PB TMDs**
  Eur.Phys.J.C 81 (2021) 5, 425

  We start from a final parton a at a given x and μ and we it evolve back till μ_0

  $$\Pi_a\left(\mu^2,\mu_0^2\right) =$$
  $$\exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{\mathrm{d}\mu'^2}{\mu'^2} \int_0^1 \mathrm{d}z \, zP^R_{ab}\left(z,\mu^2\right) \frac{\widetilde{A}_b(x,k'_\perp,\mu')}{\widetilde{A}_a(x,k_\perp,\mu')}\right)$$

- **TMD merging** procedure developed: extension of MLM method to the TMD case





Bermudez Martinez et al., Phys.Lett.B 822 (2021) 136700

High energy kt-factorization & PB

Hautmann, Hentschinski, Keersmaekers, Kusina, Kutak, Lelek, Phys.Lett.B 833 (2022) 137276

Idea: **replace DGLAP P by TMD P**

-Concept from **high-energy factorization** (Catani & Hautmann $94'$), TMD Pqg calculated

-for finite $k_\perp'^2,\ k_\perp'^2 \sim \mathcal{O}(k_\perp^2)$:

expansion of TMD P in $(k_\perp'^2/\tilde{q}_\perp^2)^n$, with z-dependent coefficients

resummation of $\ln\frac{1}{z}$ at all orders in α_s via convolution with TMD gluon Green's functions

-Other channels by Gituliar, Hentschinski, Kusina, Kutak & Serino ($2015-2017$)

goal: incorporate **both small-x and Sudakov** contributions

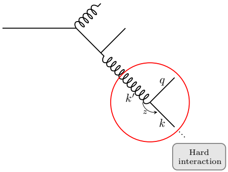
What to do with the Sudakov form factor?

newly constructed TMD Sudakov

$$\Delta_a\left(\mu^2,\mu_0^2\right) \ \rightarrow\ \Delta_a\left(\mu^2,\mu_{\perp 1}^2,k_\perp^2,\right)$$

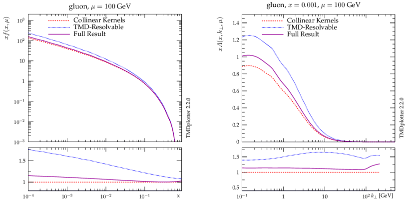
$$= \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{\mathrm{d}\mu'^2}{\mu'^2}\int_0^{z_M}\mathrm{d}z\ z\overline{P}_{ba}^R\left(z,k_\perp^2,\mu'^2\right)\right)$$

$\overline{P}$ - angular averaged P



momentum sum rule & unitarity crucial

Only with TMD Sudakov momentum sum rule satisfied



First parton branching algorithm to TMDs and PDFs which **includes TMD P and fulfils momentum sum rule**

first step towards a full TMD MC covering the small-x

Sudakov in PB

PB implements **AO**

- angles of emitted partons increase from the hadron side towards hard scattering

S. Catani, G. Marchesini, B. Webber (CMW):

scale associated with the emitted transverse momentum

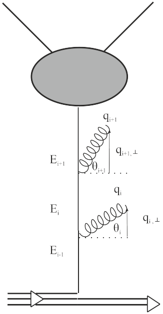
$q_\perp = (1-z)\mu'$ and $\alpha_s(q_\perp)$

AO assures PB TMDs do not have IR singularities

PB limits for iTMDs: $\widetilde{f}_a(x,\mu^2) = \int \mathrm{d}k_\perp^2 \widetilde{A}_a(x,k_\perp,\mu^2)$

Related issue: z_M

- $z_M = 1$ & $\alpha_s(\mu') \to$ **DGLAP**
- AO: q_0 – the minimal emitted transverse momentum for which a branching can be resolved
  $z_M(\mu') = z_{\mathrm{dyn}}(\mu') = 1 - q_0/\mu'$, LO P & $\alpha_s(q_\perp) \to$: **CMW**



Motivated by AO, **PB Sudakov factorized:**

$$\Delta_a(\mu^2,\mu_0^2) = \exp\left(-\int_{\mu_0^2}^{\mu^2}\frac{d\mu'^2}{\mu'^2}\left[\int_0^{z_{\mathrm{dyn}}(\mu')} dz\,\frac{k_q(\alpha_s)}{1-z} - d_q(\alpha_s)\right]\right) \times \exp\left(-\int_{\mu_0^2}^{\mu^2}\frac{d\mu'^2}{\mu'^2}\int_{z_{\mathrm{dyn}}(\mu')}^{z_M\approx 1} dz\,\frac{k_q(\alpha_s)}{1-z}\right).$$

both perturbative and non-perturbative regions are taken into account:

$$\Delta_a(\mu^2,\mu_0^2) = \Delta_a^{(\mathrm{P})}\left(\mu^2,\mu_0^2,q_0\right)\cdot\Delta_a^{(\mathrm{NP})}\left(\mu^2,\mu_0^2,\epsilon,q_0^2\right).$$

P: $z < z_{\mathrm{dyn}} \iff q_\perp > q_0$
NP: $z_{\mathrm{dyn}} < z < z_M$ ($z_M = 1-\epsilon$ with $\epsilon \ll 1$), $\iff q_\perp < q_0$
freeze α_s: $\alpha_s(q_\perp) \to \alpha_s(q_0)$

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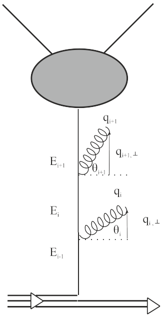

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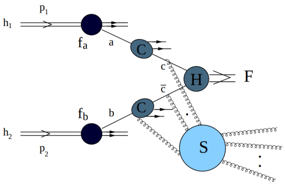
freeze α_s: $\alpha_s(q_\perp) \to \alpha_s(q_0)$

Perturbative low $p_\perp$ resummation in PB

PB perturbative Sudakov form factor

$$\Delta_a(Q^2, q_0^2)^{(P)} = \exp\left(- \int_{q_0^2}^{Q^2} \frac{dq_\perp^2}{q_\perp^2} \left(\int_0^{z_M = 1 - \frac{q_\perp}{Q}} dz \left(k_a(\alpha_s) \frac{1}{1-z} \right) - d_a(\alpha_s) \right) \right)$$



notice: $\int_0^{1-\frac{q_\perp}{Q}} dz \left(\frac{1}{1-z} \right) = \frac{1}{2} \ln \left(\frac{Q^2}{q_\perp^2} \right)$

Collins-Soper-Sterman (CSS1) Sudakov form factor:

$$\Delta_a^{\mathrm{CSS1}}(b, Q, Q_0, \mu_0) = \exp\left(- \int_{\mu_0^2}^{\mu_Q^2} \frac{d\mu'^2}{\mu'^2} \left(A_a(\alpha_s) \ln\left(\frac{Q^2}{\mu'^2} \right) + B_a(\alpha_s) \right) \right) \Delta_a^{\mathrm{NP}}$$

$$\frac{d\sigma}{dq_\perp} \sim \int d^2b \exp(ib \cdot q_\perp) \int dz_1 dz_2 H(Q^2) F_1(z_1, b, \text{scales}) F_2(z_2, b, \text{scales}) + Y$$

$$F = f \otimes C \otimes \sqrt{\Delta^{\mathrm{CSS}}}$$

We can compare: $k_a \Longleftrightarrow A_a$ and $d_a \Longleftrightarrow B_a$, order by order in α_s

- LL (A_1), NLL (A_2, B_1) coefficients in Sudakov the same in PB and CSS

NNLL:

B_2:

Renormalization group transformations mix the B, C, and H
$\overline{MS}$ resummation scheme: B corresponds to d
Difference coming from different schemes proportional to β_0

A_3:

double logarithmic part in PB: $P_{aa} = \frac{1}{1-z} k_a + \ldots$ (part of the DGLAP P)

collinear anomaly: **at NNLL k_a and A_a do not coincide** Becher & Neubert
$\rightarrow$ NNLL resummation in the PB Sudakov not achievable by implementing NNLO P
BUT can be done with **effective coupling!**

Banfi, El-Menoufi & Monni; Catani, de Florian & Grazzini:

$$\alpha_s^{\rm eff} = \alpha_s \left(1 + \sum_n \left(\frac{\alpha_s}{2\pi}\right)^n \mathcal{K}^{(n)}\right)$$

$$\mathcal{K}^{(1)} = C_A \left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5}{9} N_f$$

$$\mathcal{K}^{(2)} = C_A^2 \left(\frac{245}{24} - \frac{67}{9}\zeta_2 + \frac{11}{6}\zeta_3 + \frac{11}{5}\zeta_2^2\right) + C_F N_f \left(-\frac{55}{24} + 2\zeta_3\right) + C_A N_f \left(-\frac{209}{108} + \frac{10}{9}\zeta_2 - \frac{7}{3}\zeta_3\right) - \frac{1}{27} N_f^2 + \frac{\pi\beta_0}{2}\left(C_A\left(\frac{808}{27} - 28\zeta_3\right) - \frac{224}{54} N_f\right)$$

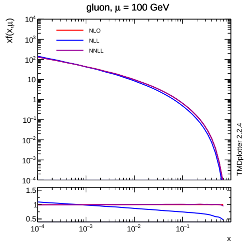
PB: recently implemented A_3 with $\alpha_s^{\rm eff}$

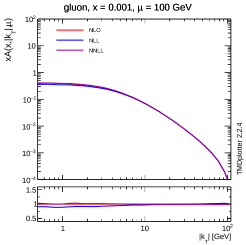
PB with A_3

Bermudez Martinez, Keersmaekers, Lelek, Mendizabal Morentin, Taheri Monfared, van Kampen

paper in preparation

NEW RESULTS

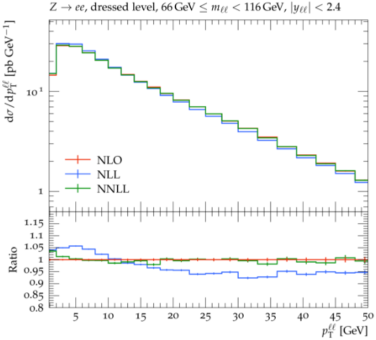




NLO: NLO P + 2 loop α_s

NLL: LO P + 2 loop α_s^{eff} with $\mathcal{K}^{(1)}$

NNLL: NLO P + 2loop α_s^{eff} with $\mathcal{K}^{(2)}$



Big effect between NLL and NLO

Effect between NLO and NNLL $\mathcal{O}(2\%)$

Non-perturbative Sudakov

Bermudez Martinez, Keersmaekers, Lelek, Mendizabal Morentin, Taheri Monfared, van Kampen

paper in preparation

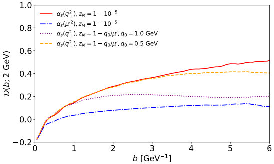
NEW RESULTS

if $z_M \approx 1-$ non – perturbative PB Sudakov included:

$$\Delta_a^{(\mathrm{NP})}(\mu^2, \mu_0^2, \epsilon, q_0) =$$
$$\exp\left(-\int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_{1-q_0/\mu'}^{1-\epsilon} dz \frac{k_a(\alpha_s)}{1-z}\right) =$$
$$\exp\left(-\frac{k_a(\alpha_s)}{2} \ln\left(\frac{\mu^2}{\mu_0^2}\right) \ln\left(\frac{q_0^2}{\epsilon^2 \mu_0 \mu}\right)\right)$$

Logarithmic structure as in CS kernel $\mathcal{D}$ of the modern CSS (CSS2)

$$\Delta_a^{\mathrm{CSS2}}(b, Q, Q_0, \mu_0) =$$
$$\exp\left(-\int_{\mu_0^2}^{\mu_Q^2} \frac{\mathrm{d}\mu'^2}{\mu'^2}\left(\gamma_k(\alpha_s)\ln\left(\frac{Q^2}{\mu'^2}\right) + \gamma_j(\alpha_s)\right)\right) \times$$
$$\exp\left(\mathcal{D}(b, \mu_0) \ln\frac{Q^2}{Q_0^2}\right)$$



- 4 models, differing in the amount of radiation, were studied
- radiation controlled via $\Delta_a^{(\mathrm{NP})}$ and α_s
- The amount of radiation has a huge impact on the extracted CS kernels

CS kernel from the PB approach extracted
using the method of Phys.Rev.D 106 (2022) 9

Summary & Conclusions

- **TMD Parton Branching**: a MC method to obtain QCD collider predictions based on TMDs
- PB: **TMD evolution** equation to obtain TMDs; **TMDs can be used in TMD MC generators** to obtain predictions

Discussed today: (very incomplete) overview of the PB applications

- TMD evolution equation
- fits to HERA DIS data
- application to inclusive DY at different $\sqrt{s}$, DY + jets
- Resummation
  - high energy kt- factorization with TMD splitting functions
  - low qt resummation
  - new result: NNLL TMD evolution

Thank you!

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