Study of thermodynamic variables in high-energy collisions using Tsallis Statistics

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Outline

★ Introduction

★ Tsallis Statistics

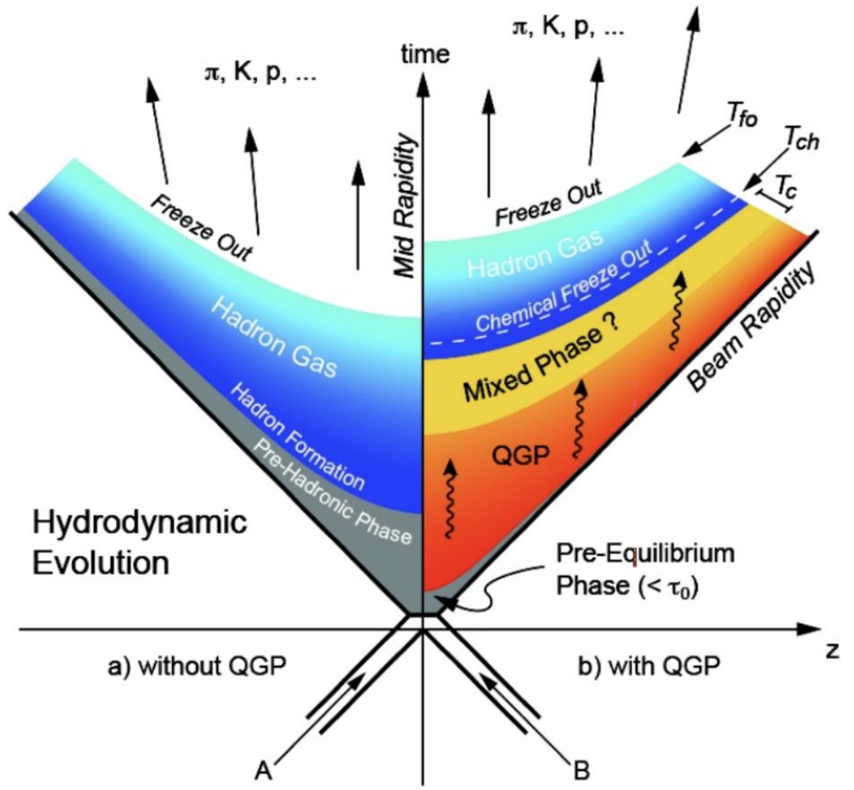
★ Fits to p_T-spectra

★ Thermodynamic Variables

★ Summary

Introduction

- ★ Hadronic fireball attains thermal equilibrium by *self-interacting constituents* in a process known as ***thermalization***
- ★ Interactions amongst hadrons can be of two types - ***elastic & inelastic***
- ★ ***Chemical Freeze-Out*** *(CFO)* → surface of last inelastic scattering
- ★ ***Kinetic Freeze-Out*** *(KFO)* → surface of last elastic scattering
- ★ Recent results suggest *formation of QGP matter* in small collision systems
- ★ Studying *thermodynamic properties* across different collision systems will aid in the understanding of QGP like effects in small systems



Courtesy:https://particlesandfriends.wordpress.com/2016/10/14/evolution-of-collisions-and-qgp/

★ How do various thermodynamic quantities at KFO vary as we move from small to large collision systems?

Tsallis Statistics

★ Tsallis statistics is a ***generalization of Boltzmann-Gibbs statistics*** to include ***non-equilibrium effects***

$$f(E, q, T, \mu) \equiv \left(1 + (q-1)\frac{E-\mu}{T}\right)^{-\frac{1}{q-1}}$$

★ ***q → 1*** will lead to Boltzmann-Gibbs statistics

★ Invariant yield in the Tsallis framework is given as:

$$\frac{d^2N}{dp_T d\eta} = 2\frac{V}{(2\pi)^3} p_T^2 \sum_{i=1}^{3} g_i \left[1 + (q-1)\frac{m_{T,i}}{T}\right]^{\frac{-q}{q-1}}$$

where *i* = pions, kaons, protons (‘*2*’ is to account for respective anti-particles)

★ Fit parameters: ***V, T,* and *q***

E = energy

g = degeneracy

μ = chemical potential (=0 for LHC energies)

T = temperature

q = non-extensivity parameter

V = volume

η = pseudorapidity

p = momentum

p_T = transverse momentum

m_T = transverse mass = $\sqrt{(p_T^2+m^2)}$

★ ***V, T,* and *q* are used to extract various thermodynamic properties at KFO in the hadronic fireball**

M D Azmi *et al.*, J. Phys. G: Nucl. Part. Phys. 47 (2020) 045001

Tsallis Statistics

★ Various thermodynamic quantities can be evaluated using the following relations:

$$\text{Entropy density: } s = -g\int \frac{d^3p}{(2\pi)^3}\left[f^q \ln_q f - f\right]$$

$$\text{Number density: } n = g\int \frac{d^3p}{(2\pi)^3} f^q$$

$$\text{Energy density: } \epsilon = g\int \frac{d^3p}{(2\pi)^3}\, E\, f^q$$

$$\text{Pressure: } P = g\int \frac{d^3p}{(2\pi)^3}\frac{p^2}{3E}\, f^q$$

E = energy

g = degeneracy

μ = chemical potential (=0 for LHC energies)

T = temperature

q = entropy index

V = volume

η = pseudorapidity

p = momentum

p_T = transverse momentum

m_T = transverse mass = $\sqrt{(p_T^2+m^2)}$

★ ***V*, *T*, and *q* are used to extract various thermodynamic properties at KFO in the hadronic fireball**

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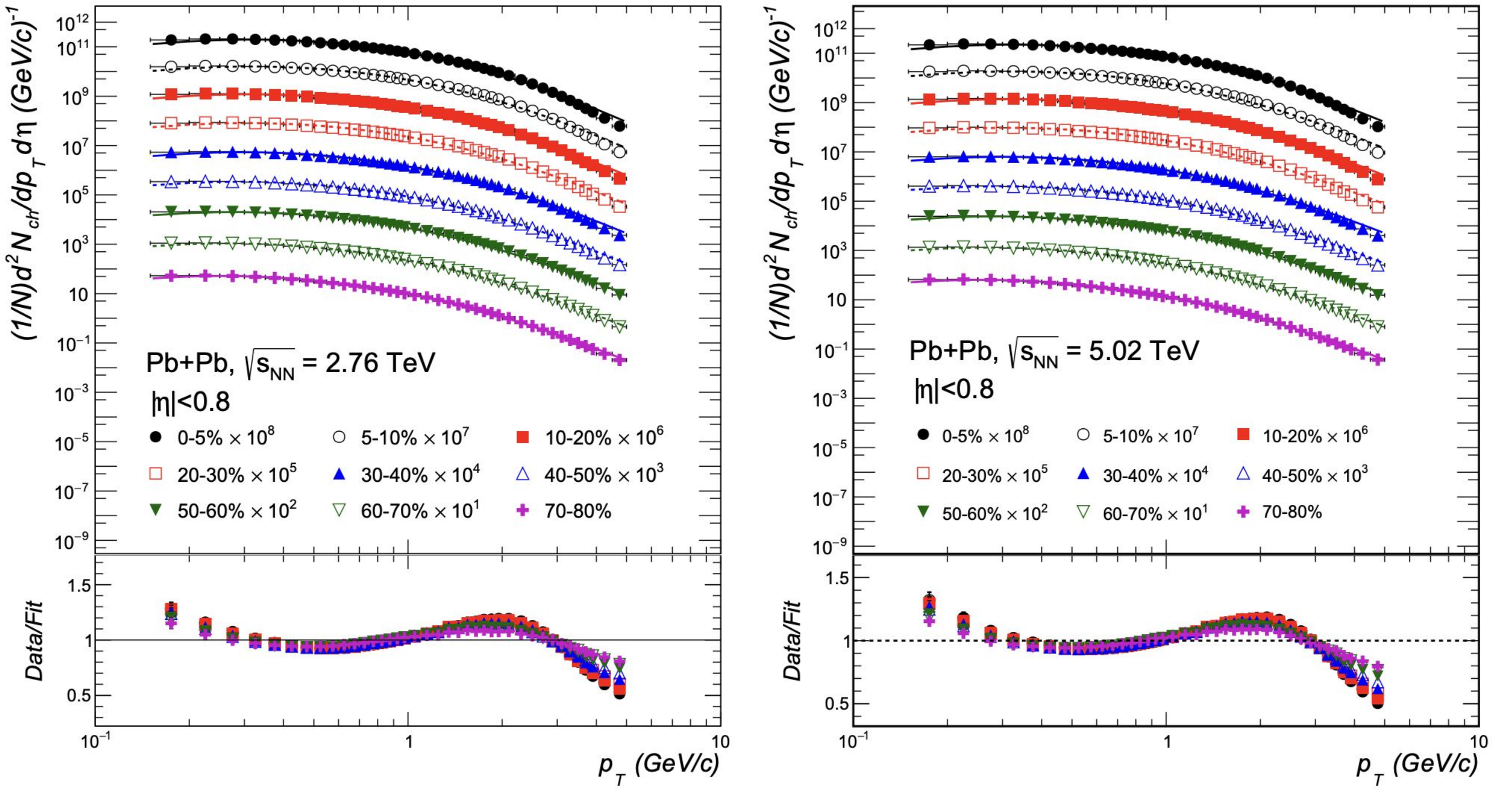
Tsallis Fits (Pb+Pb collisions)

Uniform treatment across various collision systems:

★ Studied the spectra in same pseudorapidity coverage $|\eta| < 0.8$

★ Limit the fit to $p_T < 5.0$ GeV/c (most high p_T particles come from hard processes)



★ **Charged hadron spectra in Pb+Pb collisions at $\sqrt{s_{NN}}$ = 2.76 and 5.02 TeV is well described by Tsallis statistics**

★ **Spectra in peripheral collisions is described more accurately compared to central collisions**

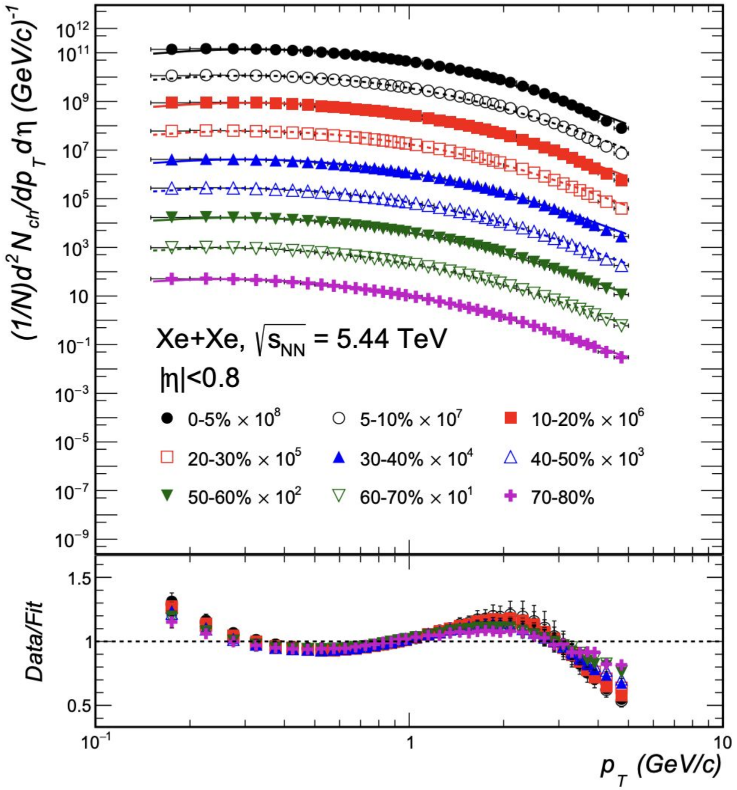
Data: ALICE - JHEP 2018, 13 (2018); PLB 788, 166 (2019); EPJ C 79 (2019)

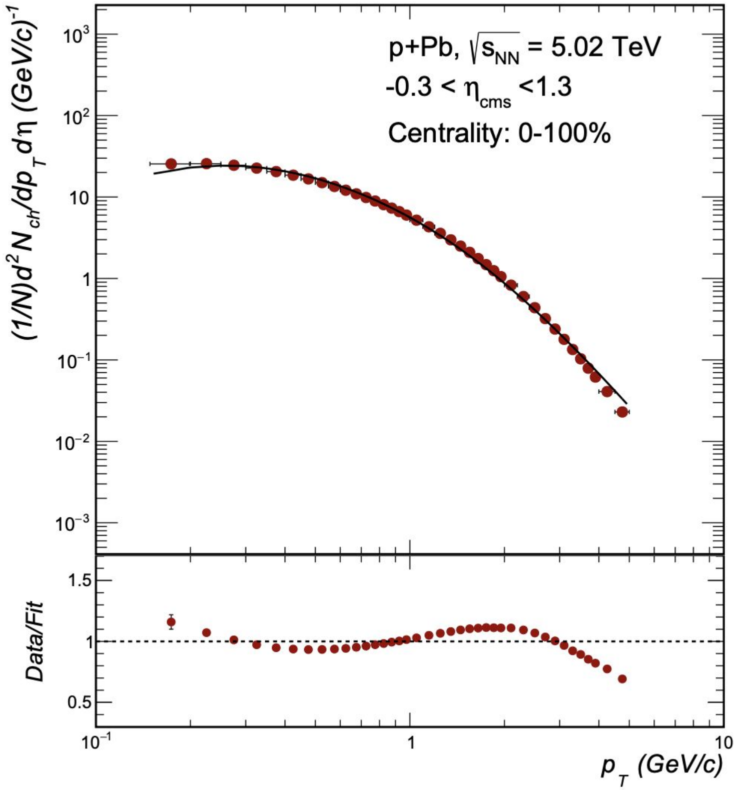
Tsallis Fits (Xe+Xe and p+Pb collisions)

Uniform treatment across various collision systems:

★ Studied the spectra in same pseudorapidity coverage $|\eta| < 0.8$

★ Limit the fit to $p_T < 5.0$ GeV/c (most high p_T particles come from hard processes)





★ **Charged hadron spectra in Xe+Xe at $\sqrt{s_{NN}}$ = 5.44 TeV and p+Pb collisions at $\sqrt{s_{NN}}$ = 5.02 TeV is well described by Tsallis statistics**

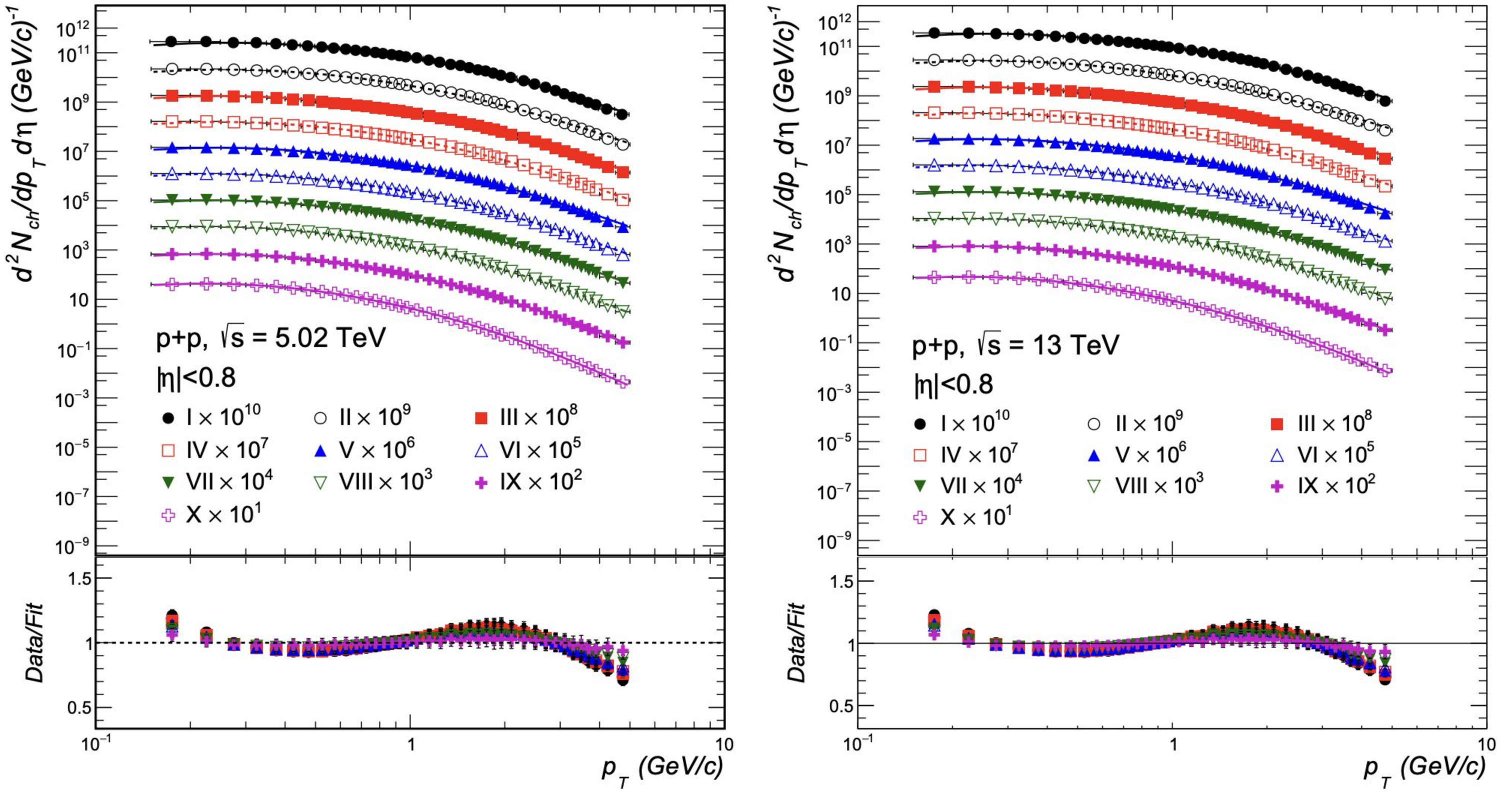
★ **Spectra in peripheral Xe+Xe collisions is described more accurately compared to central collisions**

Data: ALICE - JHEP 2018, 13 (2018); PLB 788, 166 (2019); EPJ C 79 (2019)

Tsallis Fits (p+p collisions)

Uniform treatment across various collision systems:

- ★ Studied the spectra in same pseudorapidity coverage $|\eta| < 0.8$
- ★ Limit the fit to $p_T < 5.0$ GeV/c (most high p_T particles come from hard processes)

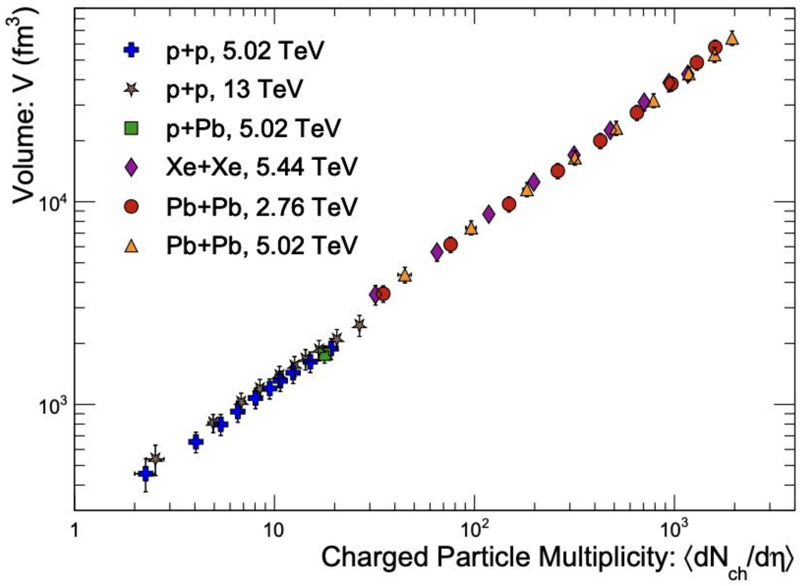


- ★ **Charged hadron spectra in p+p at √s = 5.02 and 13 TeV is well described by Tsallis statistics**
- ★ **Spectra in low multiplicity p+p collisions is described more accurately compared to high multiplicity collisions**

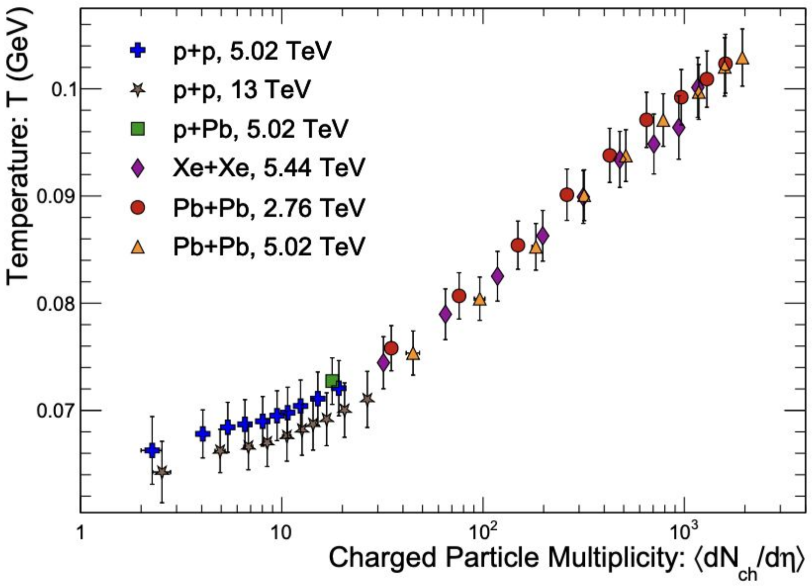
Data: ALICE - JHEP 2018, 13 (2018); PLB 788, 166 (2019); EPJ C 79 (2019)

Fit Parameters

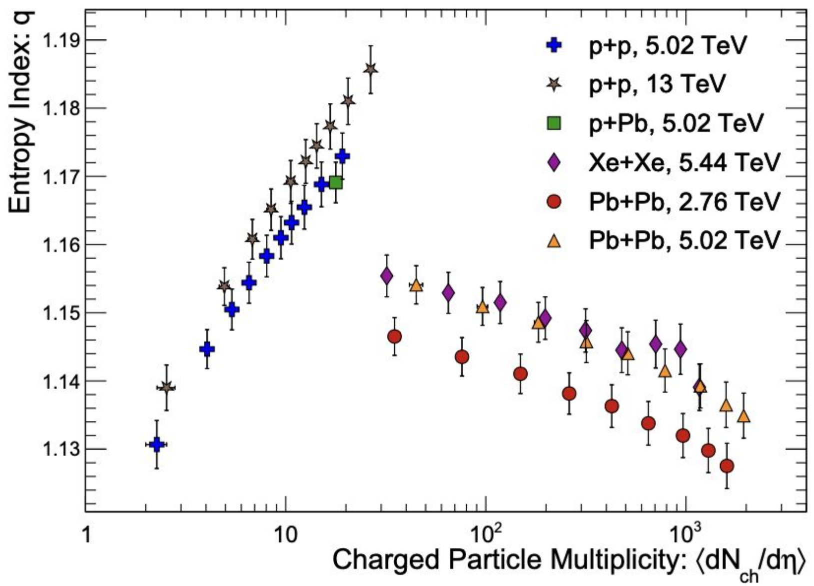
Volume (V)



Temperature (T)



Entropy index (q)

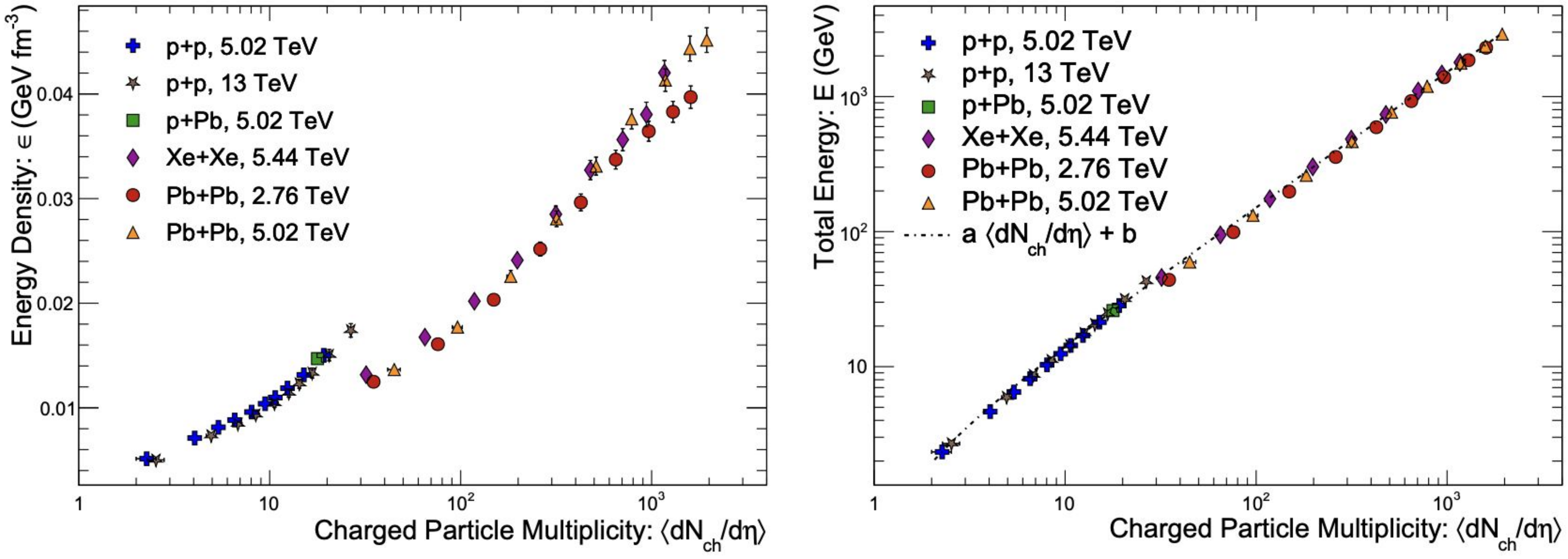


$<dN_{ch}/d\eta>$ acts an indicator of the system size

- ★ V increase with increasing $<dN_{ch}/d\eta>$
- ★ *Rate of rise of V is slower in small collision systems*
- ★ T increase with increasing $<dN_{ch}/d\eta>$
- ★ Behaviour of q with $<dN_{ch}/d\eta>$ discriminates between small & large collision systems
- ★ q is higher for higher collision energies in similar systems

Thermodynamic Parameters

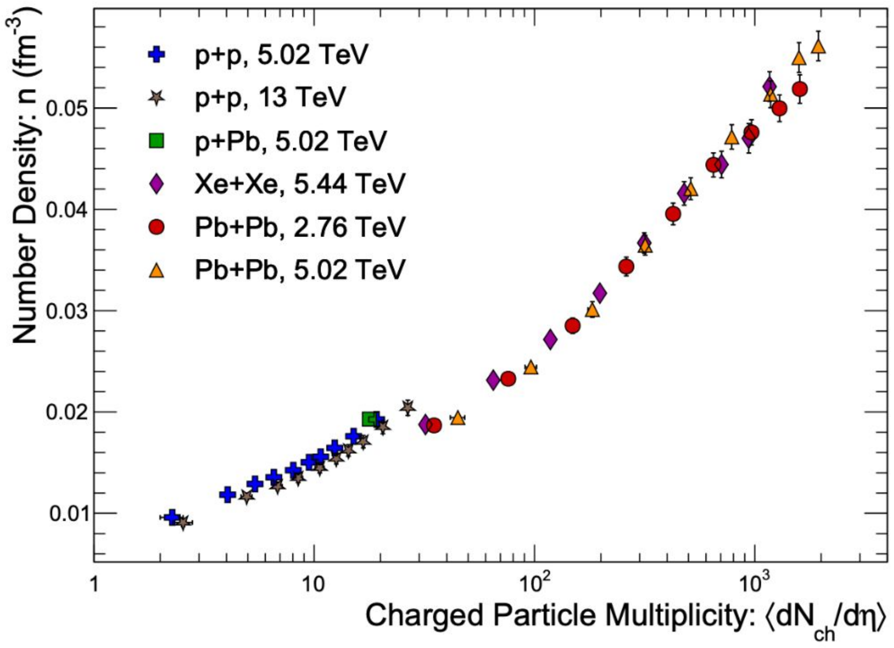
Energy

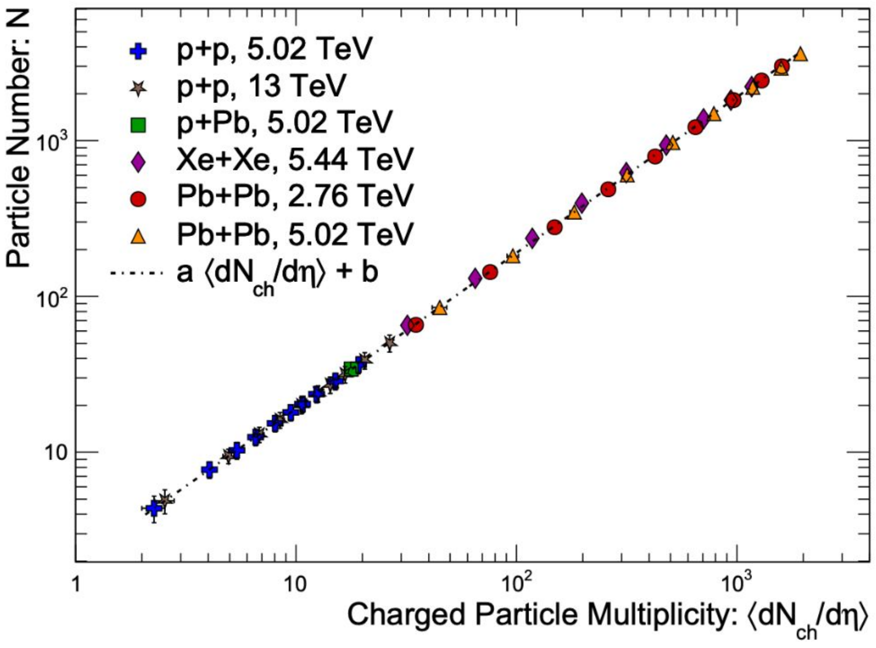


- ★ **Energy density increases with increasing $<dN_{ch}/d\eta>$**
- ★ **Rate of rise of energy density is different in small and large collision systems**
  - ○ **Discontinuity around the common multiplicity region**
- ★ **Linear scaling is observed when total energy $E(=\varepsilon V)$ is studied as a function of $<dN_{ch}/d\eta>$**

Thermodynamic Parameters

Particle Number

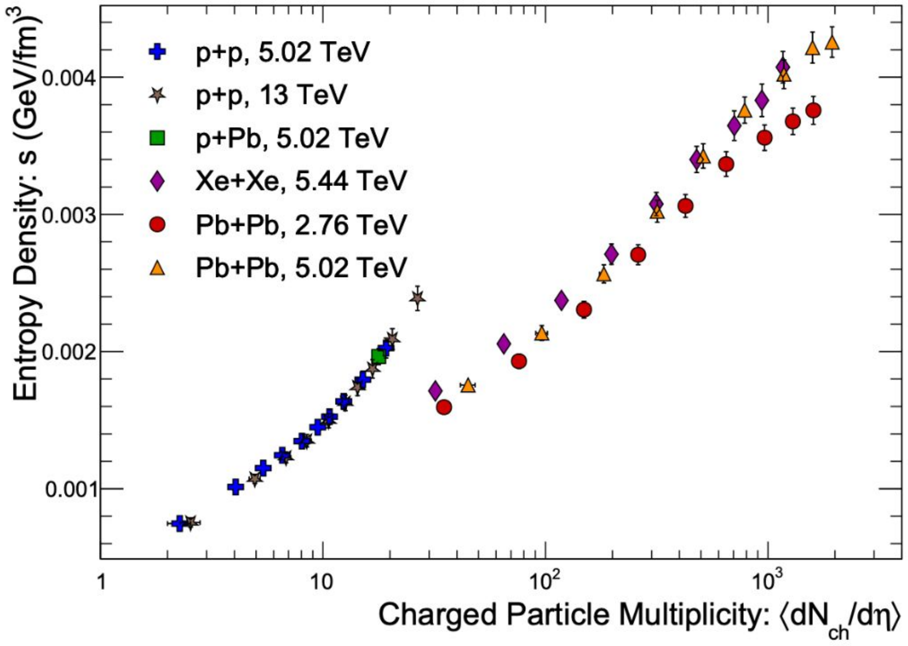


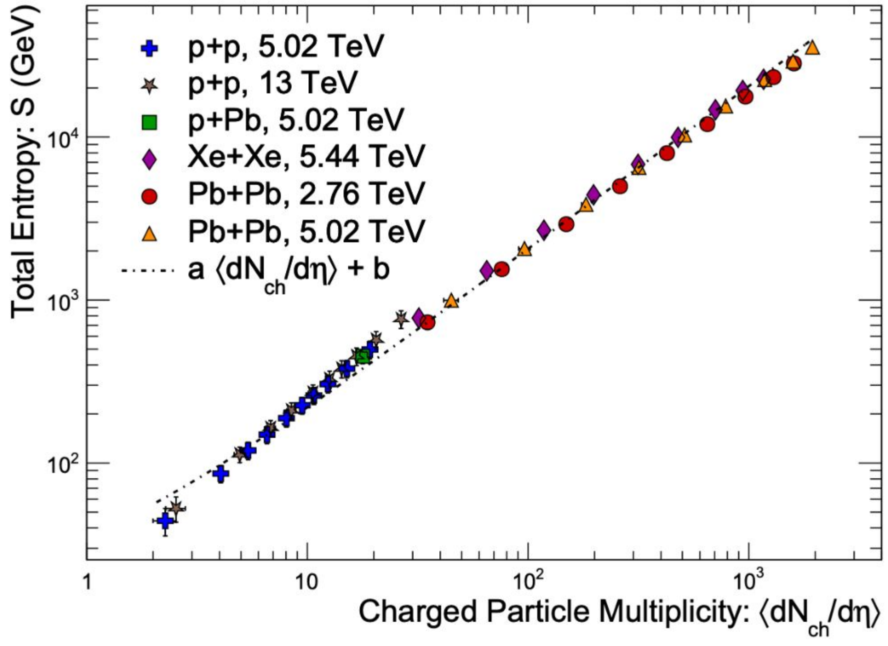


- ★ **Particle number density increases with increasing $<dN_{ch}/d\eta>$**
- ★ **Rate of rise of particle number density is different in small and large collision systems**
  - ○ **Discontinuity around the common multiplicity region**
- ★ **Linear scaling is observed when total particle number $N(=nV)$ is studied as a function of $<dN_{ch}/d\eta>$**

Thermodynamic Parameters

Entropy

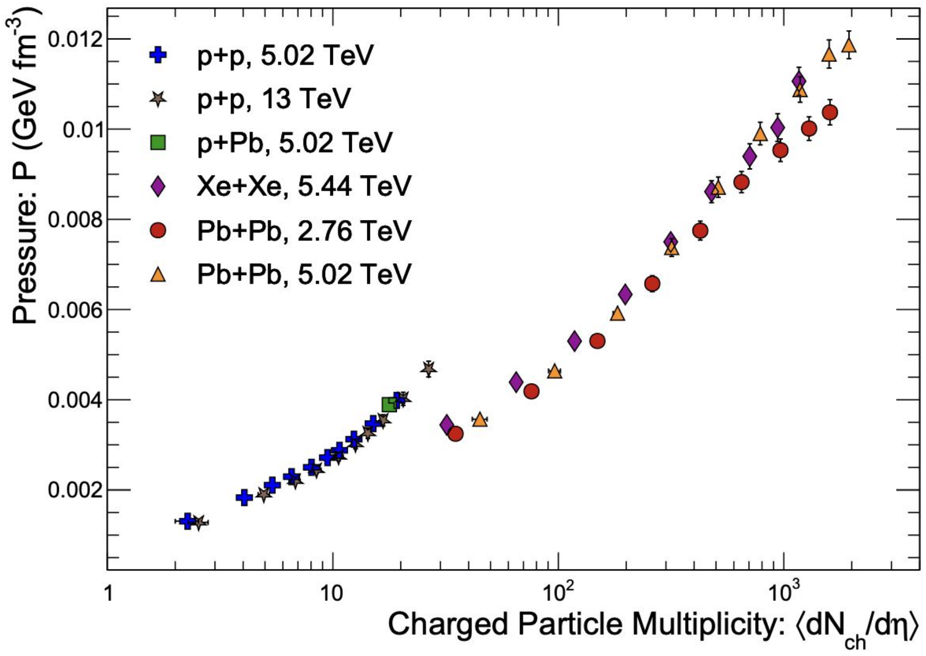




- ★ **Entropy density increases with increasing** $<dN_{ch}/d\eta>$
- ★ **Rate of rise of particle number density is different in small and large collision systems**
  - ○ **Discontinuity around the common multiplicity region**
- ★ **Linear scaling is observed when total entropy** $S(=sV)$ **is studied as a function of** $<dN_{ch}/d\eta>$

Thermodynamic Parameters

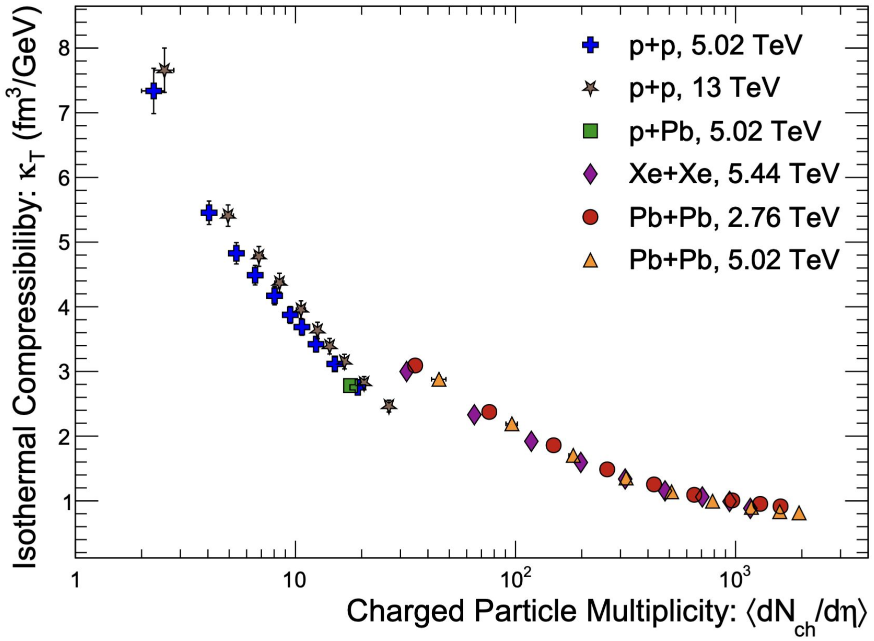
Pressure



- ★ **Pressure increase with increasing $<dN_{ch}/d\eta>$**
- ★ **Pressure rises more rapidly in small collisions than in large collisions → higher initial densities in small collisions → larger pressure exerted against the surrounding environment**

Thermodynamic Parameters

Isothermal Compressibility

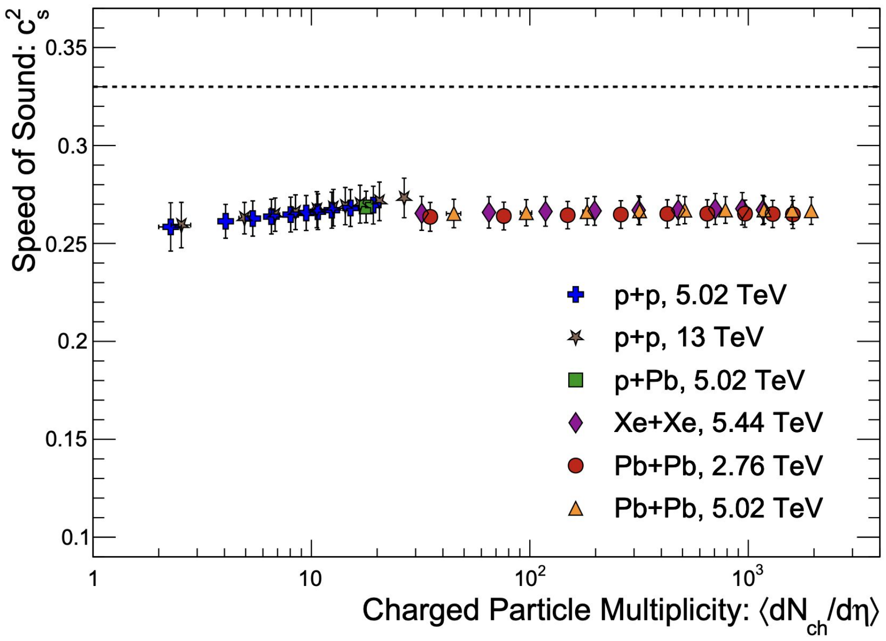


$$\kappa_T = -\frac{1}{V}\frac{\partial V}{\partial P}\Big|_T$$

$\kappa_{T,Ideal\ Fluid} = 0$; $\kappa_{T,H2O} = 6.62 \times 10^{42}\ fm^3/GeV$; $\kappa_{T,Hg} = 5.33\times10^{41}\ fm^3/GeV$

Rana A. Fine *et al.*, JCP. 59, 5529 (1973) ; D. Sahu *et al.*, EPJA 58, 78 (2022)

Speed of Sound



$$c_s^2 = \frac{\partial P}{\partial \epsilon}\Big|_s$$

For ideal gas: $c_s^2 = ⅓\times c^2$

E. Ferrer *et al.*, NPA 1031, 122608 (2023)

- ★ **Isothermal compressibility is a measure of the extent to which the volume of a system changes in response to external pressure**
- ★ **κ_T decreases as $\langle dN_{ch}/d\eta\rangle$ increases → higher $\langle dN_{ch}/d\eta\rangle$ requires higher pressure to achieve a small change in volume**
- ★ **c_s^2 increases as we move towards higher higher $\langle dN_{ch}/d\eta\rangle$ → suggesting near ideal behaviour at higher $\langle dN_{ch}/d\eta\rangle$**

Summary

- ★ We have studied the charged particle spectra in the framework of Tsallis Statistics in the following systems:
  - Pb+Pb at $\sqrt{s_{NN}}$ = 2.76 and 5.02 TeV ; Xe+Xe at $\sqrt{s_{NN}}$ = 5.44 TeV ; p+Pb at $\sqrt{s_{NN}}$ = 5.02 TeV ; p+p at $\sqrt{s}$ = 5.02 and 13 TeV
- ★ Tsallis fits are found to be in good agreement with the data
- ★ We have studied various thermodynamic variables as a function of charged particle multiplicity ($\langle dN_{ch}/d\eta \rangle$):
  - Volume (V) increases with increasing $\langle dN_{ch}/d\eta \rangle$, however, the rate of rise of volume is slower for small collision systems
  - Temperature (T) increases with increasing $\langle dN_{ch}/d\eta \rangle$
  - Entropy index (q) increases in small collision systems while it decreases in large collision systems with increasing $\langle dN_{ch}/d\eta \rangle$
  - Energy density (ε), number density (n), entropy density (s), and pressure (P) increase with increasing $\langle dN_{ch}/d\eta \rangle$ with a discontinuity between small and large collision systems around the common multiplicity region
  - A linear dependence of total energy ($E=\varepsilon V$), total particle number ($N=nV$), and total entropy ($S=sV$) is observed with $\langle dN_{ch}/d\eta \rangle$
  - Isothermal compressibility (κ_T) decreases while squared speed of sound (c_s^2) increases with increasing $\langle dN_{ch}/d\eta \rangle$ → suggesting near ideal behaviour at higher $\langle dN_{ch}/d\eta \rangle$

★ **Thermodynamic variables in small collision systems have a different behaviour compared to large collision systems around the common multiplicity region**

Thank you