Charmonium decays at BESIII

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on behalf of BESIII Collaboration

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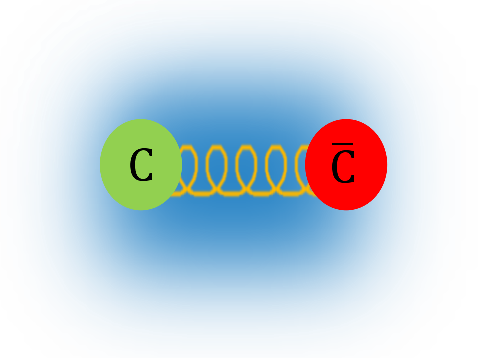
Why Study Charmonia?

- Charmonia
  - Simple configuration
    - ➢ Bound state of charm quark and its antiquark ($c\bar{c}$)
  - Difficult calculation → Perform non-perturbative calculation in
    - ➢ Hadronization of $c\bar{c}$ into charmonia and the subsequent decay
    - ➢ Because the momentum scale is much less than the charm mass
  - Experimental input becomes important



- Clean signature and high yield in experiment
  - Easy to be identified
  - Can be studied in huge range of final states (e.g. charm(less) hadron, charmonia + X)
- With numerous data, many to be done
  - Test or drive theoretical approaches
  - Help comprehending XYZ exotic particles
  - Find new charmonia
  - Probe the new physics
  - …..

This report only highlights this topic

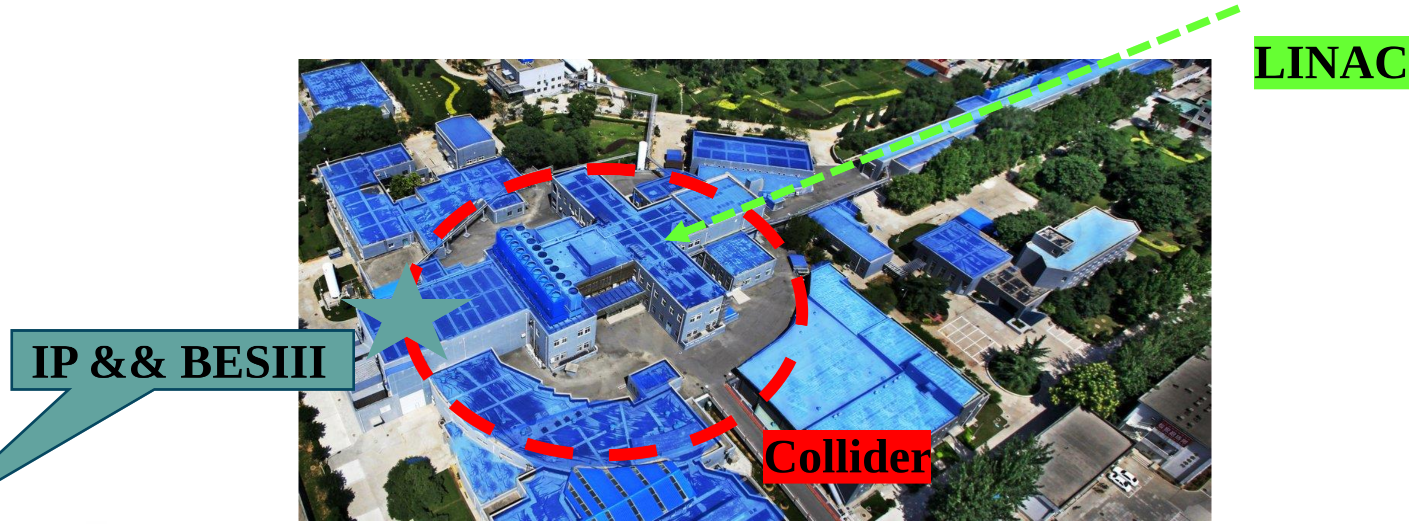
Outline

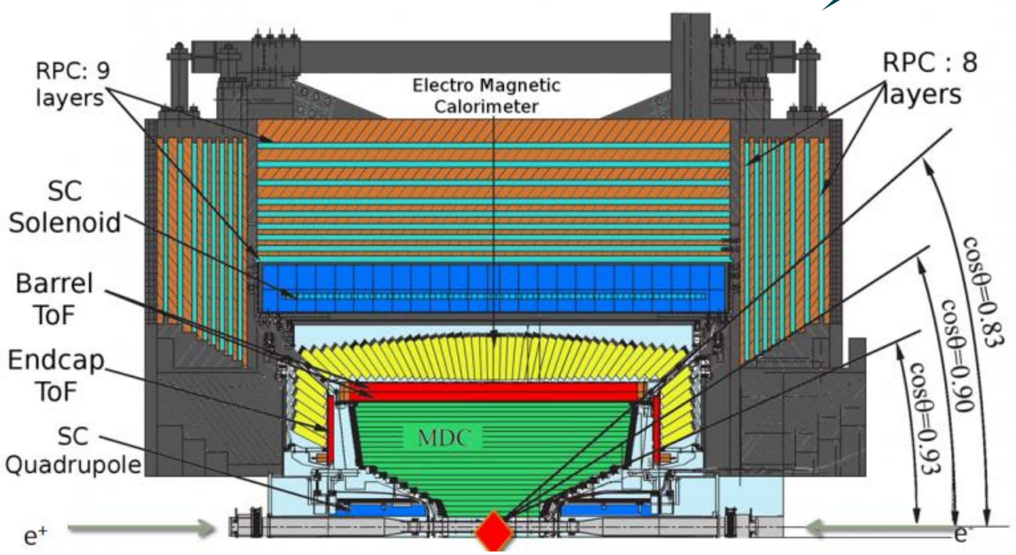
- BEPCII & BESIII & dataset
- Charmonia decaying to two baryons and one pseudoscalar particle ($B\bar{B}P$)
- $\psi(3686) \to \Sigma^-\bar{\Sigma}^+$ angular analysis
- Helicity amplitude analysis of $\chi_{cJ} \to \phi\phi$
- $\psi(3686) \to \gamma\eta_c(2S), \eta_c(2S) \to K\bar{K}\pi$ measurement
- Conclusion and outlook

A lot of details and interesting topics can't be shown due to time limitation

BESIII Experiment at BEPC II

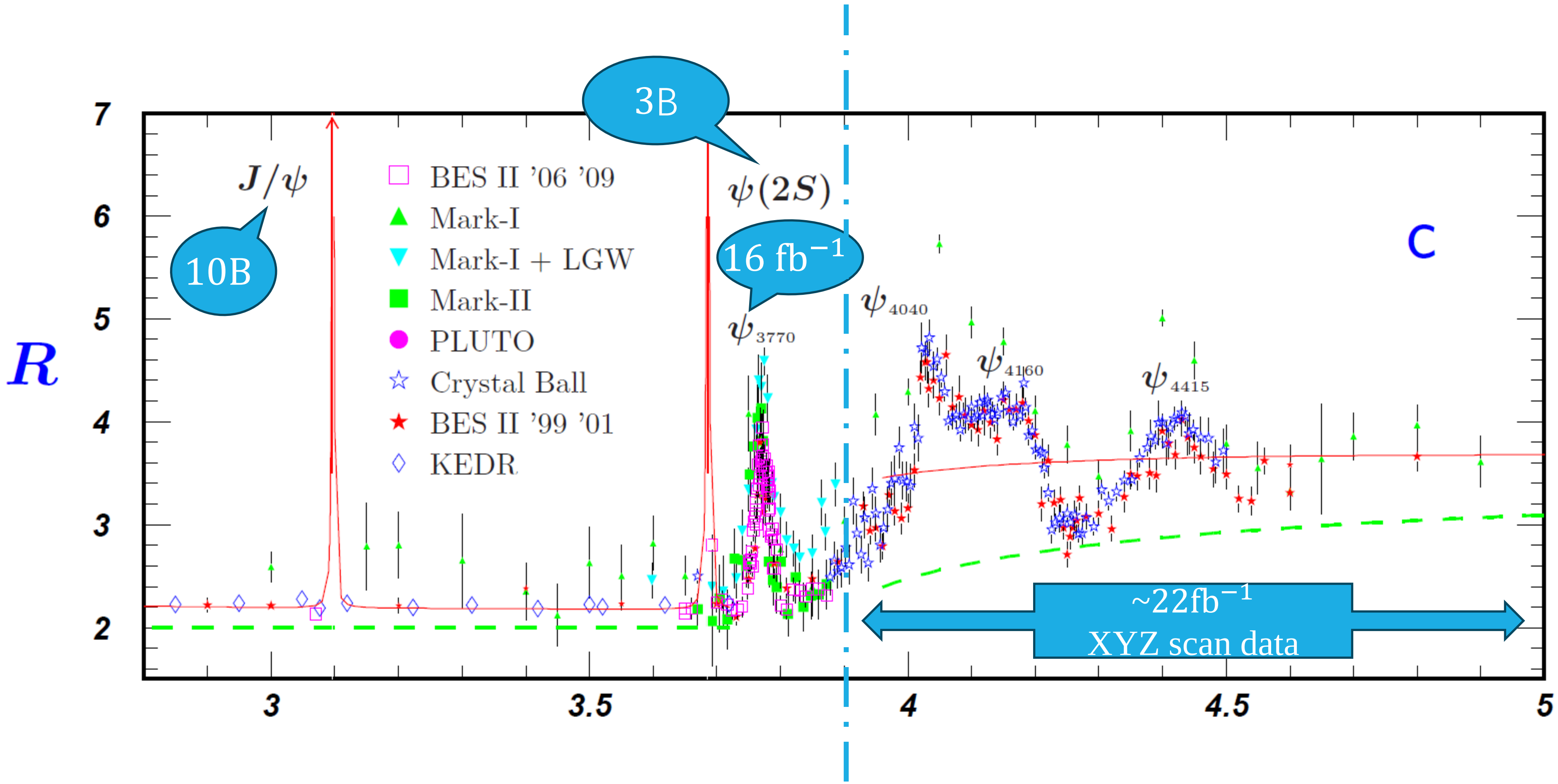
- BEPC II
  - CM energy: 2 ~ 5 GeV
  - Peak Luminosity:
    - ➢ $1.01 \times 10^{33}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$





- BESIII
  - Stable and reliable operation for 15 years!
  - Large acceptance, covering $93\% \times 4\pi$ solid angle
  - Good particle identification
  - High energy and momentum resolution

BESIII DataSet



- Rich dataset, can carry out sophisticated research
- Will accumulate more 4 fb^{-1} $\psi(3770)$ data in 2024 → total 20 fb^{-1} $\psi(3770)$ data

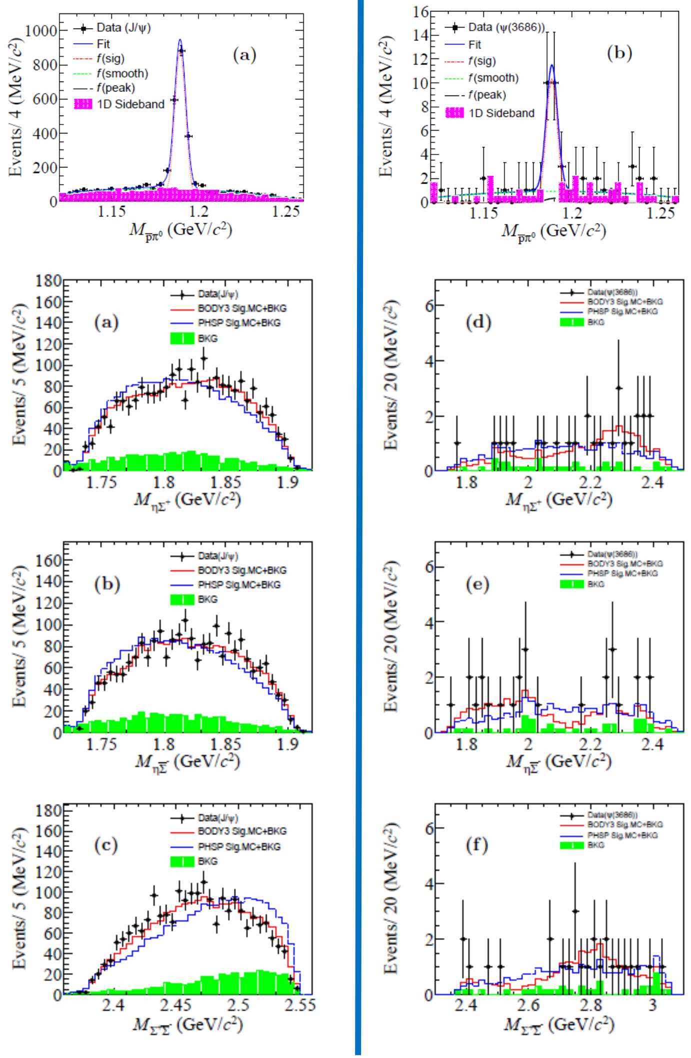
$J/\psi \rightarrow \eta\Sigma^+\bar{\Sigma}^-$ and $\psi(3686) \rightarrow \eta\Sigma^+\bar{\Sigma}^-$

Phys. Rev. D 106, 112007 (2022)

- $B\bar{B}P$ decays:
  - Experimentally driven with few theoretical predictions
  - Only 12% rule, i.e. $\frac{\mathcal{B}(\psi(3686)\rightarrow X)}{\mathcal{B}(J/\psi\rightarrow X)} \approx \frac{\mathcal{B}(\psi(3686)\rightarrow l^+l^-)}{\mathcal{B}(J/\psi\rightarrow l^+l^-)} \approx 12\ \%$, hints branching fraction (BF) ratio, where l stands for leptons
  - Provides a good opportunity to search for potential excited baryon states and unknown structures in $BB(P)$ mass spectra

- $J/\psi \rightarrow \eta\Sigma^+\bar{\Sigma}^-$ and $\psi(3686) \rightarrow \eta\Sigma^+\bar{\Sigma}^-(\Sigma^+ \rightarrow p\pi^0)$:
  - Dataset :
    - ➢ $(10087 \pm 44) \times 10^6$ J/ψ events
    - ➢ $(448.1 \pm 2.9) \times 10^6$ $\psi(3686)$ events (around one in ten full data)
  - Results
    - ➢ $\mathcal{B}(J/\psi \rightarrow \eta\Sigma^+\bar{\Sigma}^-) = (6.34 \pm 0.21 \pm 0.37) \times 10^{-5}$
    - ➢ $\mathcal{B}(\psi(3686) \rightarrow \eta\Sigma^+\bar{\Sigma}^-) = (9.59 \pm 2.37 \pm 0.61) \times 10^{-6}$
    - ➢ $\frac{\mathcal{B}(\psi(3686)\rightarrow\eta\Sigma^+\bar{\Sigma}^-)}{\mathcal{B}(J/\psi\rightarrow\eta\Sigma^+\bar{\Sigma}^-)} = (15.8 \pm 3.8)\%$, favour 12% rule
    - ➢ No enhancement near $\Sigma^+\bar{\Sigma}^-$ and $\eta\Sigma^+$ thershold is found

$J/\psi \rightarrow \eta\Sigma^+\bar{\Sigma}^-$ | $\psi(3686) \rightarrow \eta\Sigma^+\bar{\Sigma}^-$

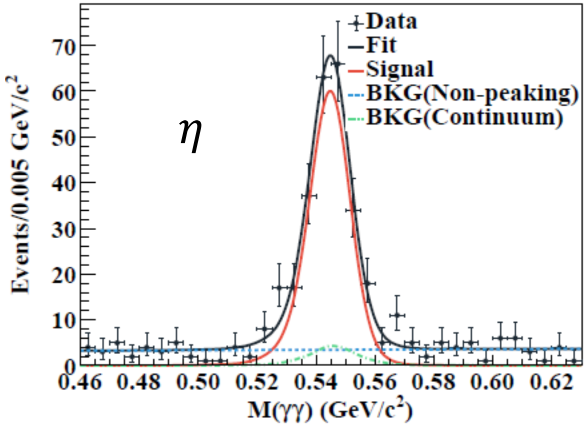


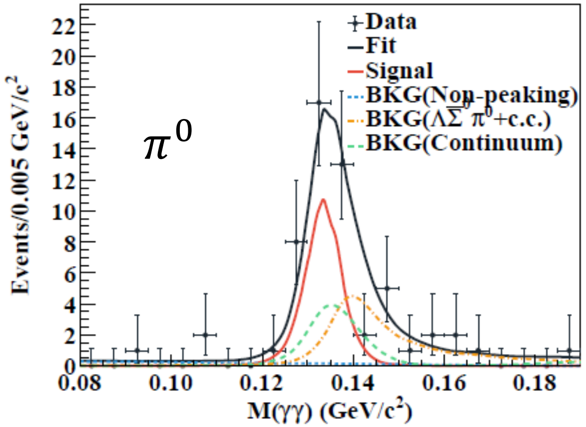
$\psi(3686) \to \eta\Lambda\bar{\Lambda}, \pi^0\Lambda\bar{\Lambda}$ and $\omega\Lambda\bar{\Lambda}$

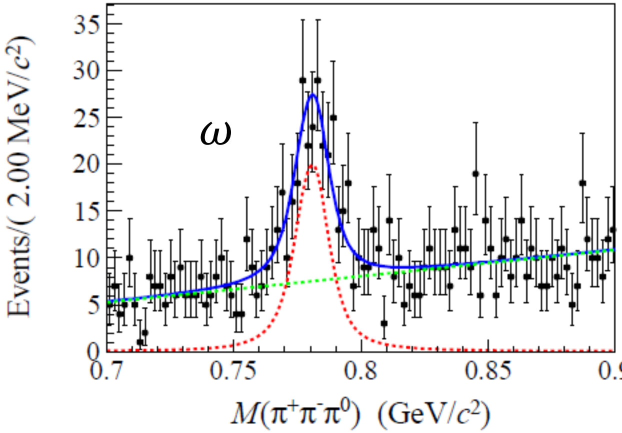
Only use $(448.1 \pm 2.9) \times 10^6$ $\psi(3686)$ events

Phys. Rev. D 106, 072006 (2022)   Phys. Rev. D 106, 112011 (2022)

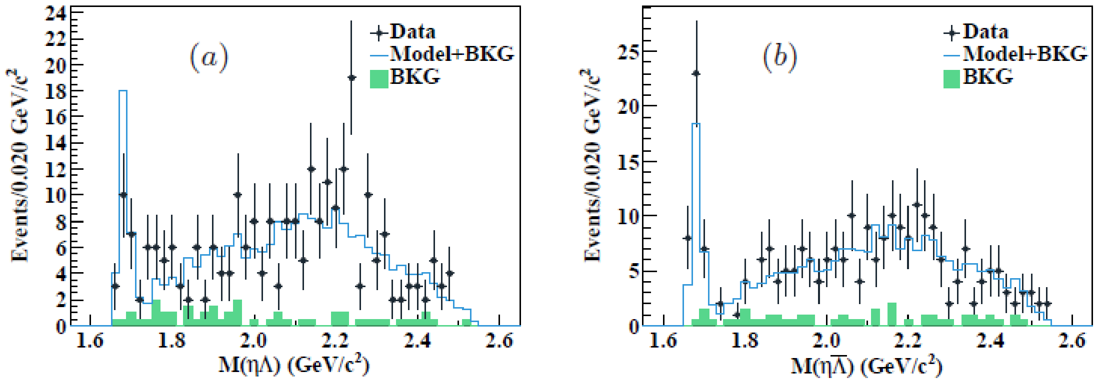
Isospin violating decay







$\Lambda(1670)$ is found in $\eta\Lambda(\bar{\Lambda})$ mass spectra



- $M = 1672 \pm 5 \pm 6$ MeV/c^2
- $\Gamma = 38 \pm 10 \pm 19$ MeV

} Be consistent with PDG

- $\mathcal{B}(\psi(3686) \to \eta\Lambda\bar{\Lambda}) = (2.34 \pm 0.18 \pm 0.52) \times 10^{-5}$
  - $\mathcal{B}(\psi(3686) \to \Lambda(1670)\bar{\Lambda}) \times \mathcal{B}(\Lambda(1670) \to \eta\Lambda) = (1.29 \pm 0.31 \pm 0.62) \times 10^{-5}$
  - BF ratio = $(1.4 \pm 0.7)\%$, disfavour 12% rule
- $\mathcal{B}(\psi(3686) \to \pi^0\Lambda\bar{\Lambda}) = (1.42 \pm 0.39 \pm 0.59) \times 10^{-6}$
  - Stat. sig = 3.7 σ, upper limit (UL) at 90% C.L. on BF = 2.47×10^{-5}
  - BF ratio = $(2.3 \pm 0.6)\%$, disfavour 12% rule
- $\mathcal{B}(\psi(3686) \to \omega\Lambda\bar{\Lambda}) = (3.30 \pm 0.34 \pm 0.29) \times 10^{-5}$
  - An evidence of Λ^* in $\omega\Lambda$ is found (shown in backup slides)
  - No phase space for $J/\psi \to \omega\Lambda\bar{\Lambda}$

More experimental and theoretical involvement is necessary for $B\bar{B}P$

$\psi(3686) \to \Sigma^-\bar{\Sigma}^+$ angular analysis

Only use $(448.1 \pm 2.9) \times 10^6$ $\psi(3686)$ events

JHEP12 (2022) 016

- Angular distribution of $e^+e^- \to \psi \to BB$
  - $1 + \alpha_\psi \cos^2\theta_B$, derived from the general helicity formalism
  - α_ψ is expected to be 1 due to the helicity conservation rule
  - Masses of the baryon and quark must be taken into considered into α_ψ calculation → α_ψ measurement becomes crucial
- Differential cross section is given as →
  - α_ψ (denoted as α_{Σ^-} later) is a free parameter in fit
  - $\alpha(\bar{\alpha})$ is set as 0.068(-0.068) according to PDG
  - $\Delta\Phi$ is set as 0 by assuming no polarization

for $\psi(3686) \to \Sigma^-\bar{\Sigma}^+, \Sigma^- \to n\pi^-, \bar{\Sigma}^+ \to \bar{n}\pi^+$

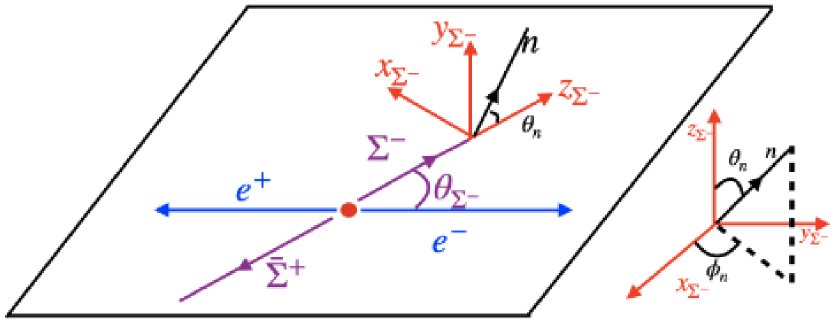
$$W(\xi) = t_0(\xi) + \alpha_\psi t_5(\xi)$$ → Σ^- scattering angle

$$+ \alpha\bar{\alpha}(t_1(\xi) + \sqrt{1-\alpha_\psi^2}\cos(\Delta\Phi)t_2(\xi) + \alpha_\psi t_6(\xi))$$ → three spin correlation

$$\sqrt{1-\alpha_\psi^2}\sin(\Delta\Phi)(\alpha t_3(\xi) - \bar{\alpha} t_4(\xi)).$$ → polarzation term

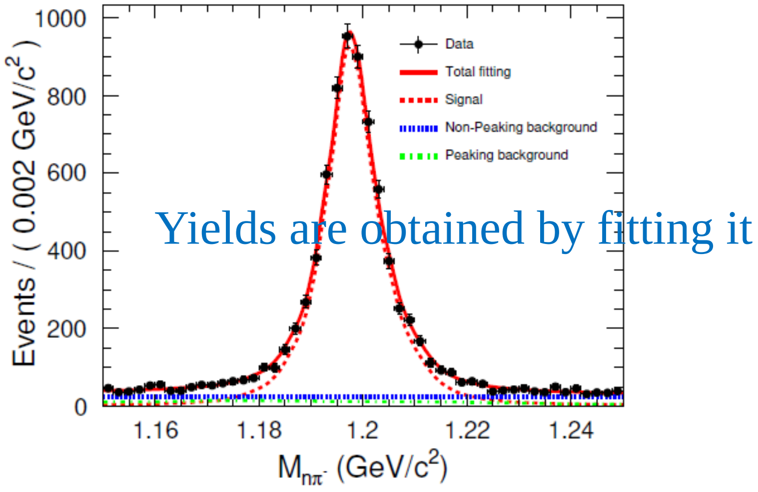
$$t_0(\xi) = 1$$
$$t_1(\xi) = \sin^2\theta \sin\theta_n \sin\theta_{\bar{n}} \cos\phi_n \cos\phi_{\bar{n}} + \cos^2\theta \cos\theta_n \cos\theta_{\bar{n}}$$
$$t_2(\xi) = \sin\theta\cos\theta(\sin\theta_n \cos\theta_{\bar{n}} \cos\phi_n + \cos\theta_n \sin\theta_{\bar{n}} \cos\phi_{\bar{n}})$$
$$t_3(\xi) = \sin\theta\cos\theta \sin\theta_n \sin\phi_n$$
$$t_4(\xi) = \sin\theta\cos\theta \sin\theta_{\bar{n}} \sin\phi_{\bar{n}}$$
$$t_5(\xi) = \cos^2\theta$$
$$t_6(\xi) = \cos\theta_n \cos\theta_{\bar{n}} - \sin^2\theta \sin\theta_n \sin\theta_{\bar{n}} \sin\phi_n \sin\phi_{\bar{n}}$$

angle definition ←



$\psi(3686) \rightarrow \Sigma^-\bar{\Sigma}^+$ angular analysis

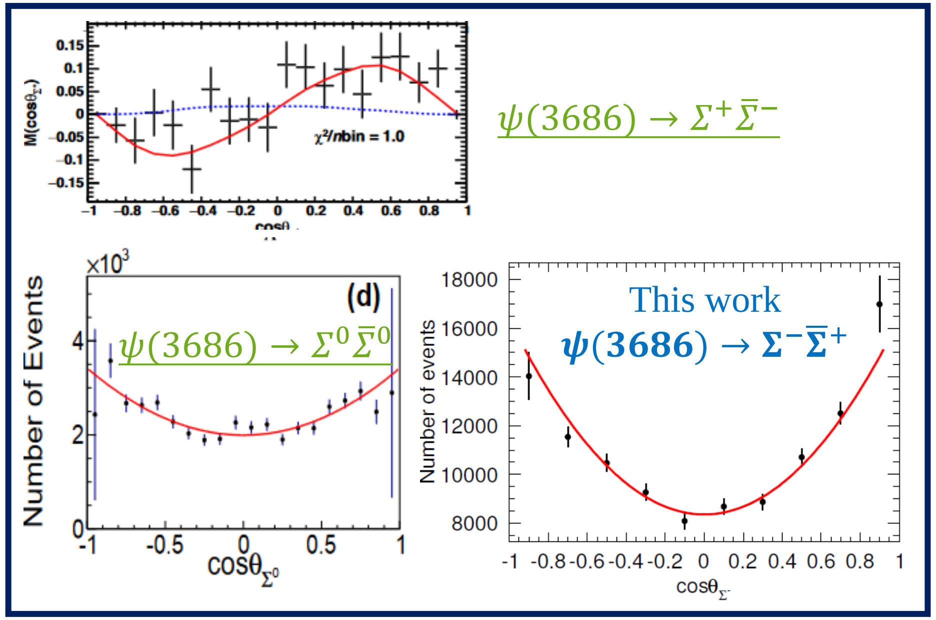
- $n(\bar{n})$ reconstruction
  - Total four-momentum conservation + known Σ^- mass + n mass → n ($\bar{n}$) four-momentum can be obtained by kinematic fit
  - For n, three-momentum are free in the fit
  - For $\bar{n}$, polar and azimuthal angles (momentum direction) are used as input



- Results
  - Measured BF is consistent with prediction and has same order with its isospin partners ones
  - α_{Σ^-} is consistent with prediction, but have significant different from α_{Σ^+} and α_{Σ^0}

Decay mode	Br($\times 10^{-4}$)	Angular parameter α_B	Br prediction($\times 10^{-4}$) [5]
$\psi(3686) \rightarrow \Sigma^+\bar{\Sigma}^-$	$2.52 \pm 0.04 \pm 0.09$ [40]	$0.682 \pm 0.030 \pm 0.011$ [21]	2.29 ± 0.15
$\psi(3686) \rightarrow \Sigma^0\bar{\Sigma}^0$	$2.44 \pm 0.03 \pm 0.11$ [22]	$0.71 \pm 0.11 \pm 0.04$ [22]	2.37 ± 0.09
$\psi(3686) \rightarrow \Sigma^-\bar{\Sigma}^+$	$2.82 \pm 0.04 \pm 0.08$	$0.96 \pm 0.09 \pm 0.03$	2.46 ± 0.13

Worth further investigation



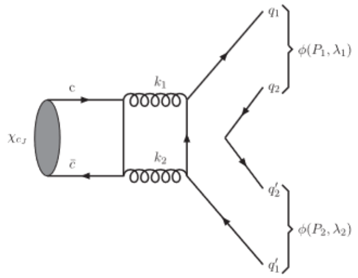
Helicity amplitude analysis of $\chi_{cJ} \to \phi\phi$

JHEP05 (2023) 069

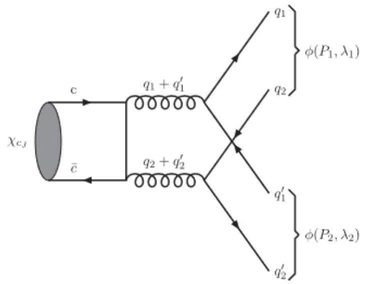
- pQCD prediction for a charmonia decaying into two hadron → $\mathcal{B}[\psi(\lambda) \to h_1(\lambda_1)h_2(\lambda_2)] \sim \left(\frac{\Lambda^2_{QCD}}{m_c^2}\right)^{|\lambda_1+\lambda_2|+2}$
- If the light-quark mass is neglected, the vector-gluon coupling conserves quark helicity leading to the helicity selection rule (HSR), i.e.
  - $\lambda_1 + \lambda_2 = 0$
  - if the helicity configuration does not satisfy it, the BF will be suppressed, as in the case of $\chi_{c1} \to \phi\phi$
- BESIII observed similar BFs of three decays, contrary to HSR
  - Many models have been proposed to interpret this result (and have used data as input)
  - Ratios of the helicity amplitudes F_{λ_1,λ_2} are found to be effective in the discrimination between them

Decay channel	$\chi_{c0} \to \phi\phi$	$\chi_{c2} \to \phi\phi$		
Parameter	x	ω_1	ω_2	ω_4
pQCD	0.293 ± 0.030	0.812 ± 0.018	1.647 ± 0.067	0.344 ± 0.020
${}^3P_0$	0.515 ± 0.029	1.399 ± 0.580	0.971 ± 0.275	0.406 ± 0.017
$D\bar{D}$ loop	0.359 ± 0.019	1.285 ± 0.017	5.110 ± 0.057	0.465 ± 0.002
	$\|F^0_{1,1}\|/\|F^0_{0,0}\|$	$\|F^2_{0,1}\|/\|F^2_{0,0}\|$	$\|F^2_{1,-1}\|/\|F^2_{0,0}\|$	$\|F^2_{1,1}\|/\|F^2_{0,0}\|$

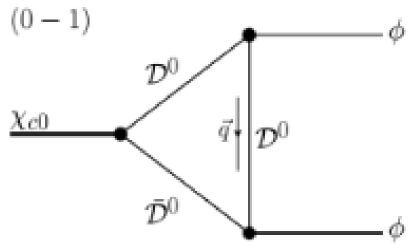
pQCD



${}^3P_0$



$D\bar{D}$ loop



Helicity amplitude analysis of $\chi_{cJ} \to \phi\phi$

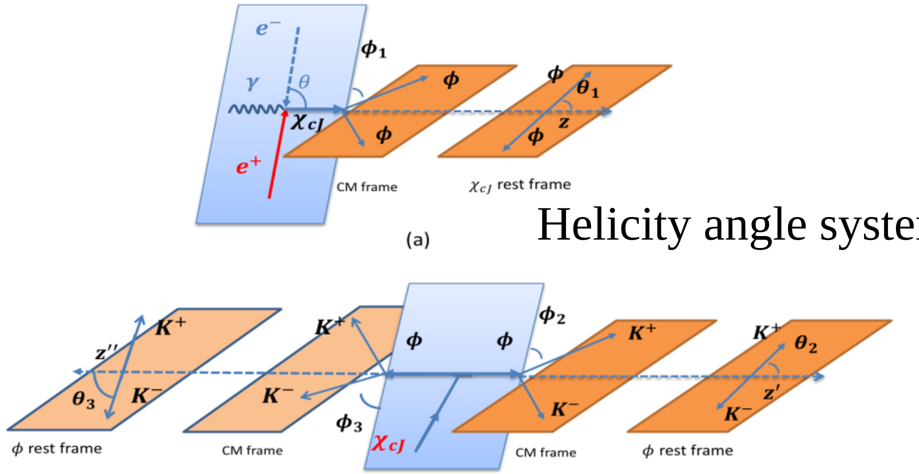
- Amplitude analysis
  - Formula: Helicity amplitude expanded with covariant tensor →
    - Relativistic Breit-Wigner function with damping factors
  - Resonance:
    - Relativistic Breit-Wigner function with damping factors
    - Mass resolution is considered
  - Non resonance (NR)
    - Only consider 0^+ and 0^-
    - χ_{c1} and χ_{c2} widths are quite narrow → their interference with a non-resonant contribution is neglected

$$A = \langle \vec{p}\lambda_1; -\vec{p}\lambda_2 |\mathcal{M}| JM\rangle$$
$$= 4\pi\left(\frac{\mu}{p}\right)^{\frac{1}{2}} \langle \phi\theta\lambda_1\lambda_2|JM\lambda_1\lambda_2\rangle \langle JM\lambda_1\lambda_2|\mathcal{M}|JM\rangle$$
$$= N_J F^J_{\lambda_1,\lambda_2} D^{J*}_{M,\lambda}(\phi,\theta,0), \lambda = \lambda_1 - \lambda_2,$$

$$F_{\lambda_1,\lambda_2} = \sum_{LS} g_{LS}\sqrt{\frac{2L+1}{2J+1}}\langle L0S\lambda|J\lambda\rangle\langle \eta_1\lambda_1\eta_2-\lambda_2|S\lambda\rangle r^L B_L(r)/B_L(r_0)$$

Partial waves

Decay	Partial waves (LS)
$\psi(3686) \to \gamma\chi_{c0}$	(01) , (21)
$\psi(3686) \to \gamma\chi_{c1}$	(01) , (21) , (22)
$\psi(3686) \to \gamma\chi_{c2}$	(01) , (21) , (22) , (23) , (43)
χ_{c0} or NR$(0^+) \to \phi\phi$	(00) , (22)
χ_{c1} or NR$(1^+) \to \phi\phi$	(01) , (21) , (22)
χ_{c2} or NR$(2^+) \to \phi\phi$	(02) , (20) , (21) , (22) , (42)
NR$(0^-) \to \phi\phi$	(11)
$\phi \to K^+K^-$	(10)



Helicity angle system

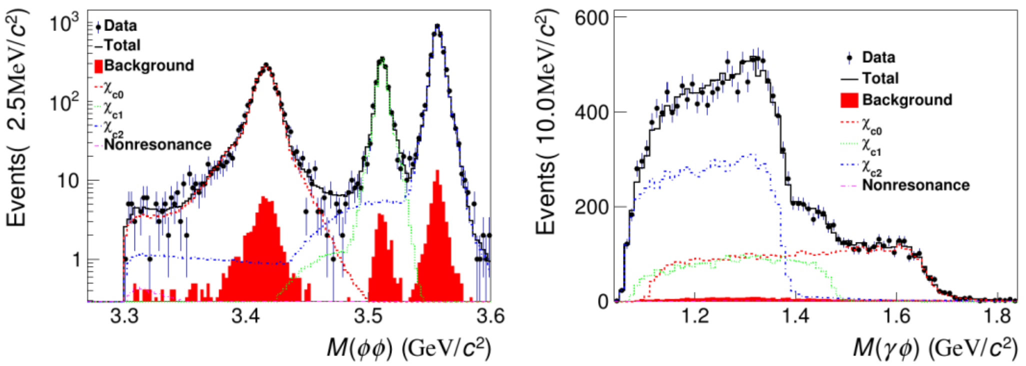
Angular and partial wave analysis

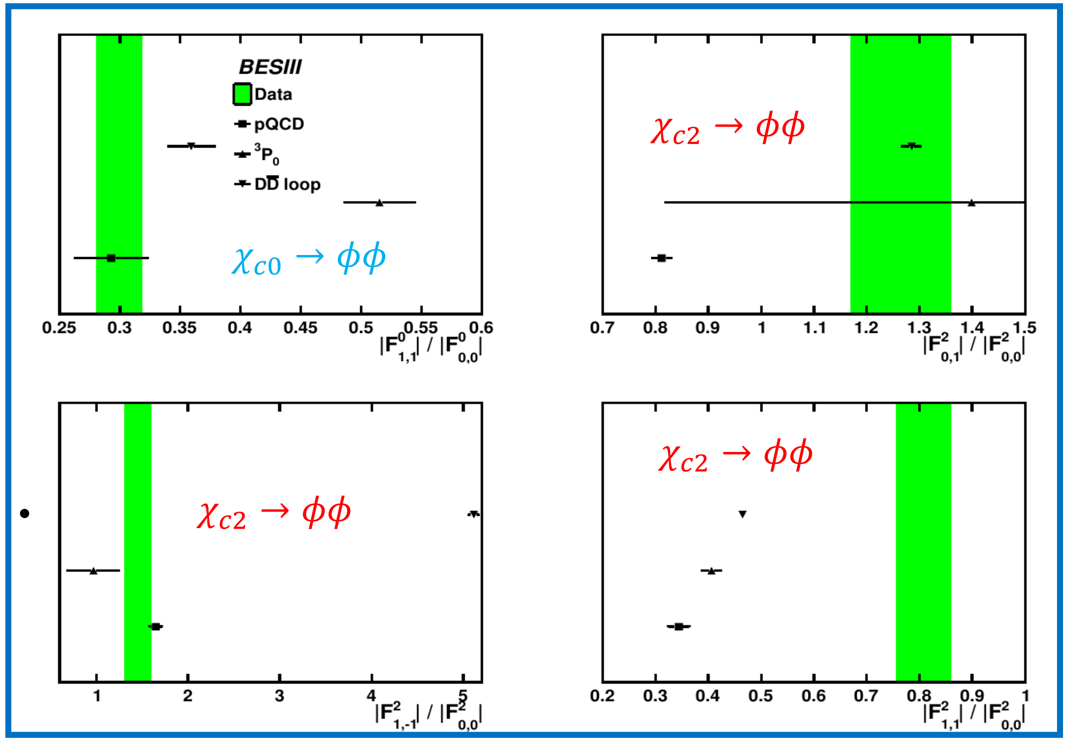
- χ_{c0} mass and width float in the fit, χ_{c1} and χ_{c2} ones are fixed to PDG
- BF results are consistent with BESIII previous results and PDG values

Decay Mode	2011 BESIII [8]	this work	PDG value [22]
$\mathcal{B}[\chi_{c0} \to \phi\phi](\times 10^{-4})$	7.8±0.4±0.8	$8.59 \pm 0.27 \pm 0.20$	8.0±0.7
$\mathcal{B}[\chi_{c1} \to \phi\phi](\times 10^{-4})$	4.1±0.3±0.5	$4.26 \pm 0.13 \pm 0.15$	4.2±0.5
$\mathcal{B}[\chi_{c1} \to \phi\phi](\times 10^{-4})$	10.7±0.4±1.2	$12.67 \pm 0.28 \pm 0.33$	10.6±0.9

- Helicity amplitude ratios
  - $|F^0_{1,1}|/|F^0_{0,0}| = 0.299 \pm 0.003 \pm 0.019$
  - $|F^2_{0,1}|/|F^2_{0,0}| = 1.265 \pm 0.054 \pm 0.079$
  - $|F^2_{1,-1}|/|F^2_{0,0}| = 1.450 \pm 0.097 \pm 0.104$
  - $|F^2_{1,1}|/|F^2_{0,0}| = 0.808 \pm 0.051 \pm 0.009$
  - The $D\bar{D}$ loop model can be ruled out
  - Other predictions also have sizeable difference

With $(27.08 \pm 0.14) \times 10^8$ $\psi(3686)$ events, a fuller understanding of this inconsistency can be provided!





$\psi(3686) \to \gamma\eta_c(2S), \eta_c(2S) \to K\bar{K}\pi$

use full $(27.08 \pm 0.14) \times 10^8$ $\psi(3686)$ events

ArXiv: 2309.14689, released in Sep 28th 2023

- $\eta_c(2S)$
  - Limited knowledge about its decay → the sum of its BFs measured experimentally is only about 3%
  - Uncertainties of its mass and width are also not small
    - PDG: $m_{\eta_c(2S)} = (3637.5 \pm 1.1)$ MeV/c^2, $\Gamma_{\eta_c(2S)} = (11.3^{+3.2}_{-2.9})$ MeV
  - $J^{PC} = 0^{-+}$, cannot be produced directly via e^+e^- annihilation, but through $\psi(3686)$ radiative transition
    - Radiative transition requires charmed-quark spin-flip, thus it proceeds via a magnetic (M1) transition
  - The M1 transition BF and partial width calculated by different theoretical models vary

		Mass (MeV/c^2)	$\mathcal{B}(\psi(3686) \to \gamma\eta_c(2S))$ ($\times 10^{-4}$)	$\Gamma(\psi(3686) \to \gamma\eta_c(2S))$ (keV)
Non-relativistic potential (NR)	NR model [1]	3630	7.14 ± 0.19	0.21
Godfrey-Isgur relativized Potential (GI)	GI model [1]	3623	5.80 ± 0.16	0.17
	Meson loop correction [2]	N/A	2.72 ± 1.00	0.08 ± 0.03
	Light-front quark model [3]	3637	3.9	0.11
	Effective field theory [12]	N/A	0.6 – 36.0	N/A

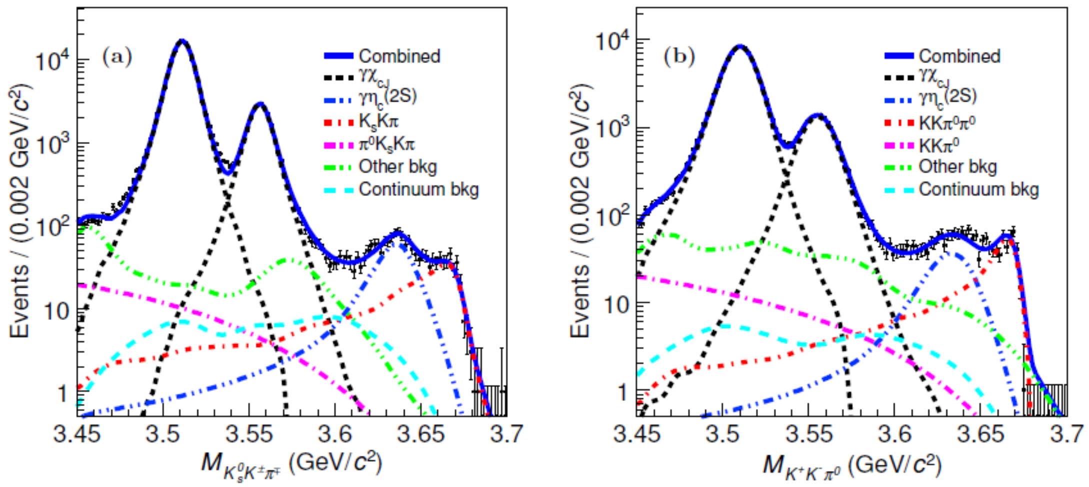
  - Experimental measurements on M1 transition BF have large uncertainties
    - By a global fit: $(7^{+3.4}_{-2.5}) \times 10^{-4}$; By PDG: $(7 \pm 5) \times 10^{-4}$ → Difficult to validate the theoretical calculations. Precise measurement is required!

$\psi(3686) \to \gamma\eta_c(2S), \eta_c(2S) \to K\bar{K}\pi$

- $\eta_c(2S) \to K\bar{K}\pi$ ($K^+K^-\pi^0$ and $K_sK^\pm\pi^\mp$)
  - Yields are obtained from simultaneous fit to $M(K\bar{K}\pi)$ in the range of (3.45-3.70) GeV/c^2
  - Signal model: $\eta_c(2S)$ M1 transition lineshape (details are in backup slides)
    - ➢ $(E_\gamma^3 \times BW\left(M(K\bar{K}\pi)\right) \times f_d(E_\gamma) \times \varepsilon(M(K\bar{K}\pi))) \otimes$ Double Gaussian (utilized to describe the resolution)
    - ➢ Mass and width of $\eta_c(2S)$ are floating and sharing parameters in the fit
  - $\mathcal{B}(\psi(3686) \to \gamma\eta_c(2S)) \times \mathcal{B}(\eta_c(2S) \to K\bar{K}\pi)$, stat. uncertainty only
    - ➢ $\eta_c(2S) \to K_sK^+\pi^- + c.c$: $(3.23 \pm 0.20) \times 10^{-6}$
    - ➢ $\eta_c(2S) \to K^+K^-\pi^0 + c.c$: $(1.61 \pm 0.10) \times 10^{-6}$

  BFs ratio ≈2, which agrees with isospin symmetry expectation
  It will be taken into consideration in all subsequent calculations

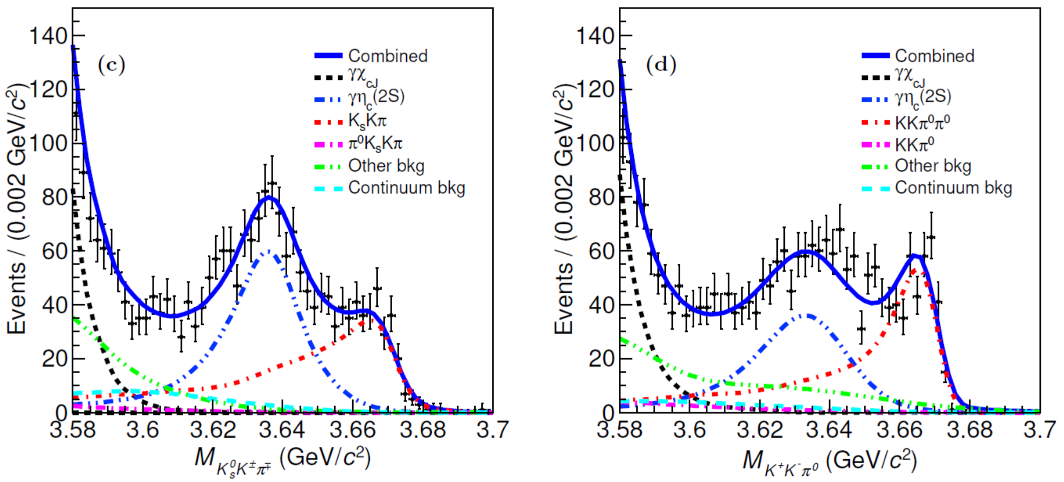
- Sum over the total seven $K\bar{K}\pi$ channels BFs (with set BFs ratios between them), i.e.
  - $K^+K^-\pi^0(2)$, $K_sK_s\pi^0(1)$, $K_LK_L\pi^0(1)$
  - $K_sK^+\pi^-(2)$, $K_sK^-\pi^+(2)$, $K_LK^+\pi^-(2)$, $K_LK^-\pi^+(2)$,
  - Total $\mathcal{B}\left(\psi(3686) \to \gamma\eta_c(2S)\right) \times \mathcal{B}\left(\eta_c(2S) \to K\bar{K}\pi\right)$
    - ➢ $(0.97 \pm 0.06 \pm 0.09) \times 10^{-6}$ (stat. + sys. uncertainty)



$\psi(3686) \to \gamma\eta_c(2S), \eta_c(2S) \to K\bar{K}\pi$

- Results
  - $\eta_c(2S)$ mass and width are measured with better precision to PDG
  - $\Gamma(\psi(3686) \to \gamma\eta_c(2S))$ is determined by using total width of $\psi(3686)$
    - ➢ consistent with all the theoretical predictions within two standard deviations → cannot distinguish them
  - By combining $\mathcal{B}(\eta_c(2S) \to K\bar{K}\pi) = (1.86^{+0.68}_{-0.49})\%$,
    - ➢ $\mathcal{B}(\psi(3686) \to \gamma\eta_c)) = (5.2 \pm 0.3 \pm 0.5^{+1.9}_{-1.4})$
    - ➢ Uncertainty is dominated by the last term, which is from the quoted value of $\mathcal{B}(\eta_c(2S) \to K\bar{K}\pi)$
  - The precision of the $\mathcal{B}(\eta_c(2S) \to K\bar{K}\pi)$ measurement needs further improvement!

Fit result in the range only containing $\eta_c(2S)$ signal



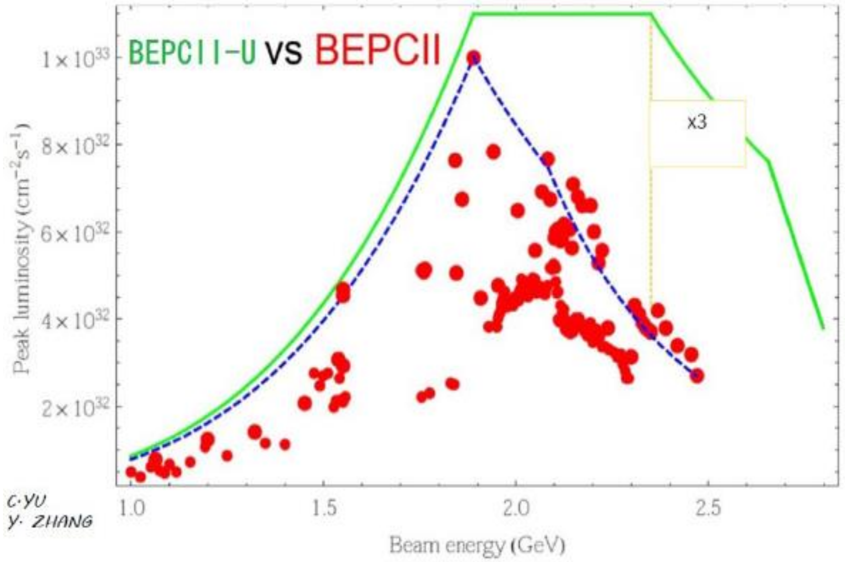
	Mass (MeV/c^2)	Width (MeV)	$\mathcal{B}(\psi(3686) \to \gamma\eta_c(2S))$ ($\times 10^{-4}$)	$\Gamma(\psi(3686) \to \gamma\eta_c(2S))$ (keV)
This work	$3637.8 \pm 0.8 \pm 0.2$	$10.5 \pm 1.7 \pm 3.5$	$5.2 \pm 0.3 \pm 0.5\ ^{+1.9}_{-1.4}$	$0.15\ ^{+0.06}_{-0.04}$
BESIII (2012)	$3637.6 \pm 2.9 \pm 1.6$	$16.9 \pm 6.4 \pm 4.8$	$6.8 \pm 1.1 \pm 4.5$	0.20 ± 0.14
World average	3637.6 ± 1.2	$11.3\ ^{+3.2}_{-2.9}$	7 ± 5	0.21 ± 0.15

Conclusion and outlook

- BESIII has carried out many experiments with rich physical results
  - But many analyses do not use all of the $\psi(3686)$ data → New ones are in progress
  - Research on more $J/\psi, \psi(3686), \chi_{cJ} \to BB(P)$ decays are conducting

- Some results are not currently explained in theoretical terms, or are not explained by theory
  - E.g. 12% rule and helicity selection rule
  - Need more experimental and theoretical collaboration

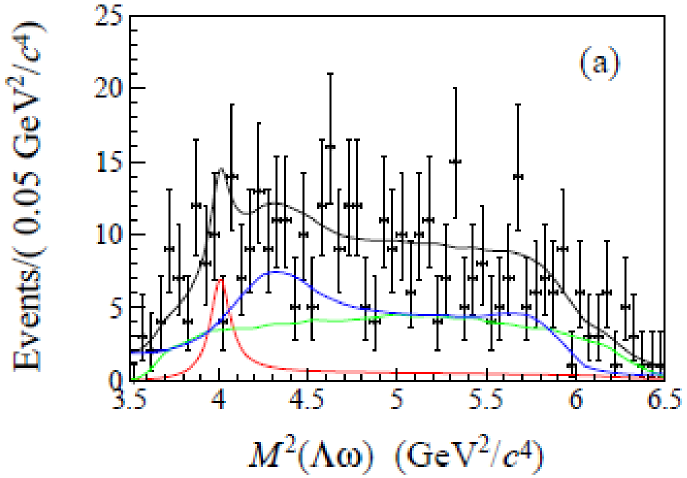
- Upgrade to the BEPCII (BEPCII-U)
  - Luminosity: triple as BEPCII
  - CM energy from 5 to 5.6 GeV (for charmed baryons)
  - Also provides an opportunity to search for new Charmonia and their decays

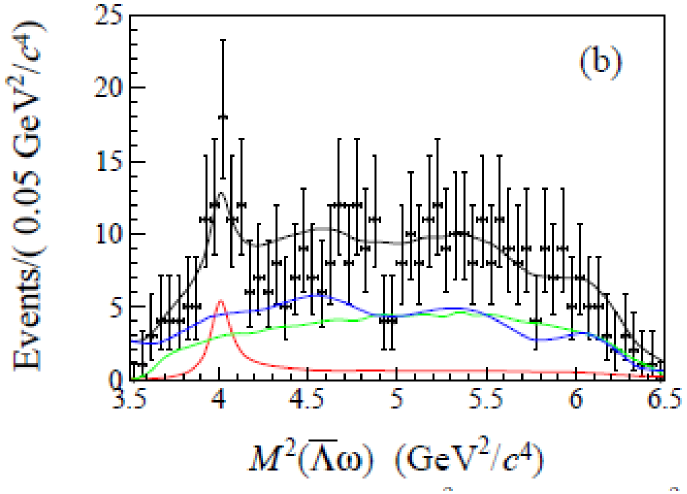


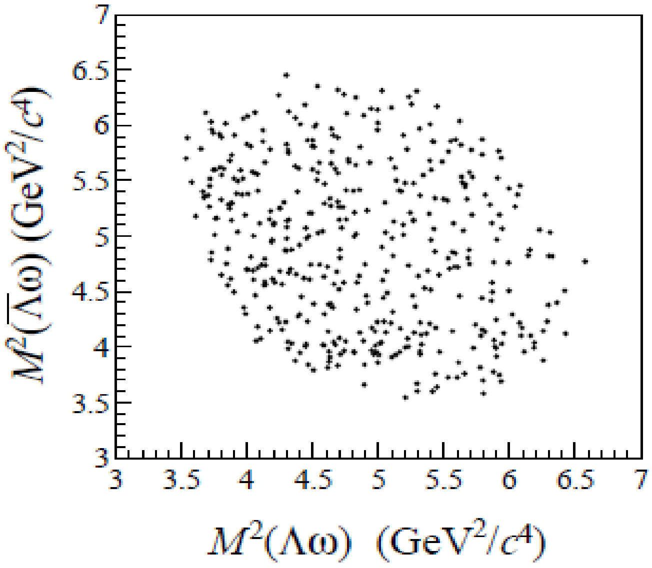
Thanks for your attentions!

Backup

$\psi(3686) \to \omega\Lambda\bar{\Lambda}$







- Λ^*
  - Stats. sig = 2.3 σ
  - $M = 2.001 \pm 0.007$ GeV/c^2
  - $\Gamma = 0.036 \pm 0.014$ GeV
  - UL $\mathcal{B}(\psi(3686) \to \omega\Lambda^* \to \omega\Lambda\bar{\Lambda}) = 1.40 \times 10^{-5}$

$\psi(3686) \to \gamma\eta_c(2S), \eta_c(2S) \to K\bar{K}\pi$

- $\eta_c(2S)$ M1 transition lineshape
  - $E_\gamma^3 \times BW\left(M(K\bar{K}\pi)\right) \times f_d(E_\gamma) \times \varepsilon(M(K\bar{K}\pi)))$
  - $E_\gamma = (m^2_{\psi(3686)} - M^2_{K\bar{K}\pi})/(2m_{\psi(3686)})$, the energy of the transition photon in the rest frame of the $\psi(3686)$
  - BW, Breit-Wigner function with floating width and mean
  - $f_d(E_\gamma) = E_0^2/(E_\gamma E_0 + (E_\gamma - E_0)^2\ )$, damping function proposed by KEDR experiment
    - ➢ In order to suppress the diverging tail raised by the term of E_γ^3
  - $E_0 = = (m^2_{\psi(3686)} - m^2_{\eta_c(2S)})/(2m_{\psi(3686)})$, mean energy of the transition photon
  - $\varepsilon(M(K\bar{K}\pi))$, efficiency function, obtained from MC simulation