Neutrino mass models: Overview

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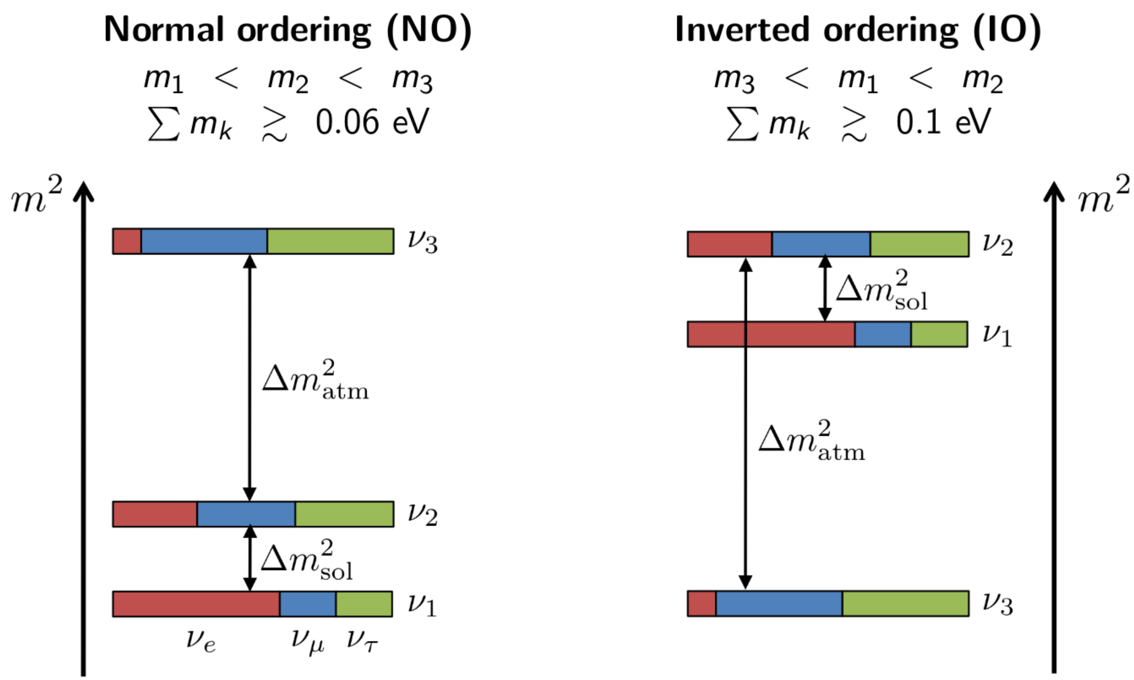
http://www.astroparticles.es/

Contents

$\mathcal{I}.$

Introduction

Neutrino oscillations



Nearly
25 years after
Super-K, 1998

2 Δm^2 and
all 3 θ_{ij}
measured with
high precision,
but ...

BUT, still unknown:

- Absolute mass scale? — Upper limit: ~ 1 eV (KATRIN), $\sim (0.1 - 0.2)$ eV ($0\nu\beta\beta$)
- Which hierarchy? — $\sim 2\ \sigma$ preference for NO
- CP phase? — Indication for $\delta \sim (3/2)\pi$? - But tension T2K/NOνA
- Majorana OR Dirac? — Unknown

Theoretical expectations

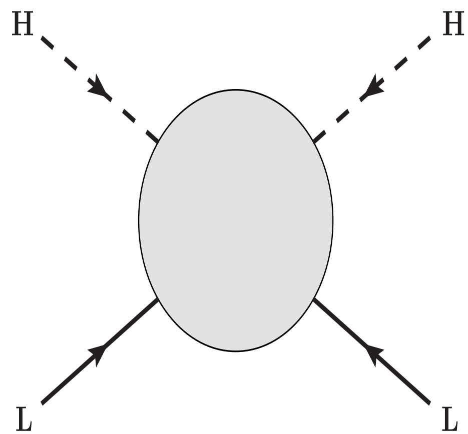
Majorana Neutrino mass

$$m_\nu \simeq \frac{(Yv)^2}{\Lambda}$$

Weinberg, 1979

Smallness of neutrino mass can be "explained" by:

$\Rightarrow$ High scale: Large Λ



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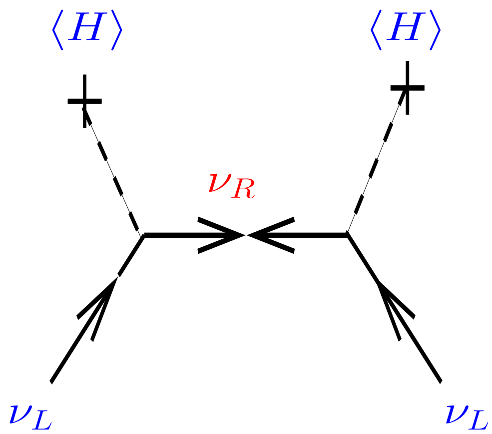
"classical" seesaw:
$\Lambda \sim 10^{(14-15)}$ GeV, $Y \sim 1$



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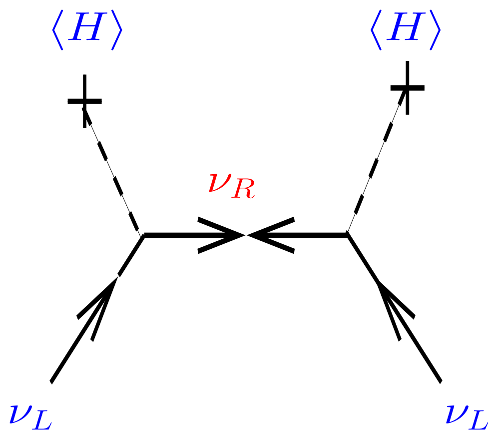

$\Rightarrow$ High scale: Large Λ
"classical" seesaw:
$\Lambda \sim 10^{(14-15)}$ GeV, $Y \sim 1$

OR:

$\Rightarrow$ $\Lambda \sim 100$ GeV and $Y \sim 10^{-6}$
"electro-weak scale" seesaw:

Theoretical expectations

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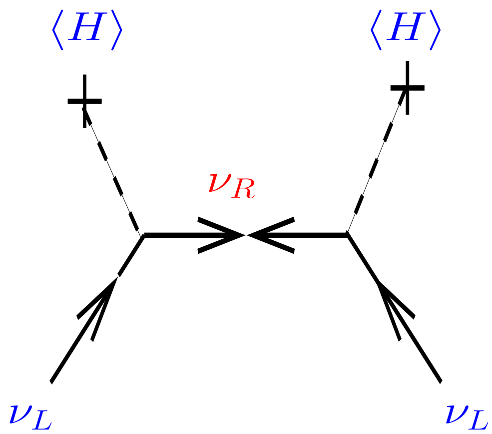

"A right-handed neutrino?"
"Heavy neutral lepton?"
"A nearly singlet fermion?"

Theoretical expectations

Majorana Neutrino mass

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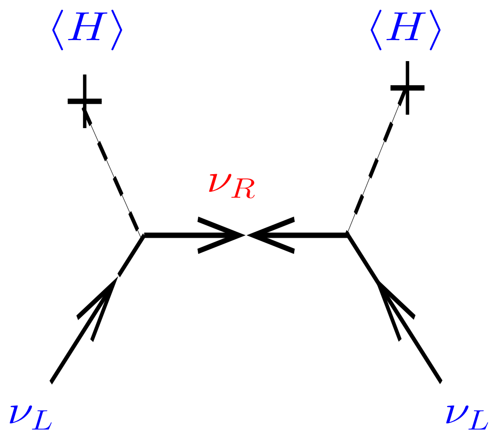


"A right-handed neutrino?"
"Heavy neutral lepton?"
"A nearly singlet fermion?"
A long-lived particle!

See talk by:
Giovanna Cottin

Theoretical expectations

Majorana Neutrino mass generated from an n-loop dimension d diagram:

$$m_\nu \simeq \frac{(Yv)^2}{\Lambda} \cdot \epsilon \cdot \left(\frac{Y^2}{16\pi^2}\right)^n \cdot \left(\frac{Yv}{\Lambda}\right)^{d-5}$$

Smallness of neutrino mass can be "explained" by:

$\Rightarrow$ High scale: Large Λ
"classical" seesaw

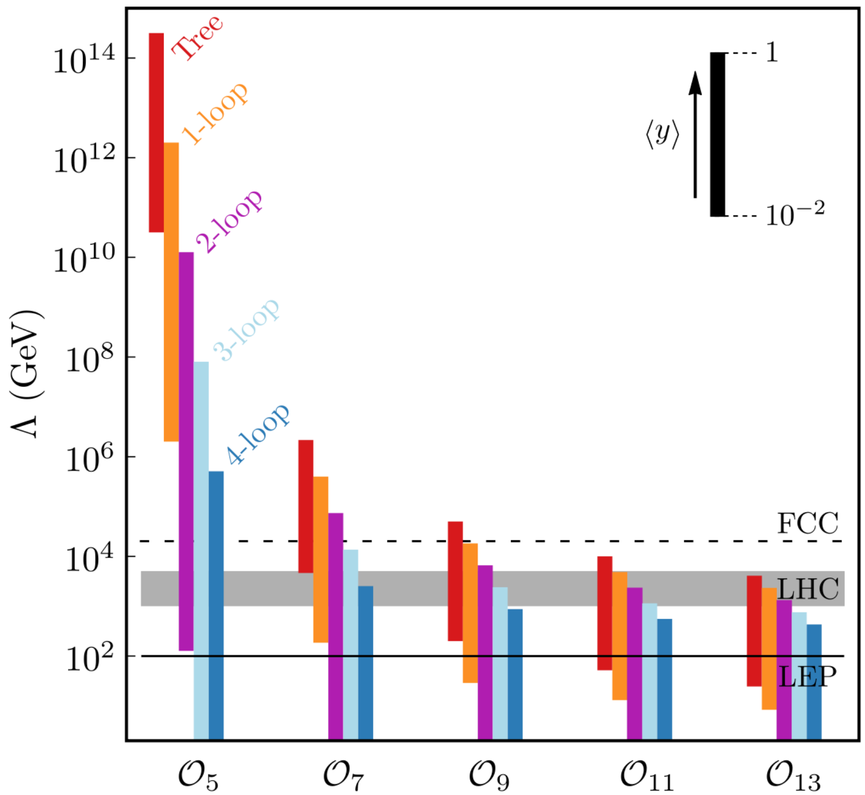
$\Rightarrow$ Loop factor: $n \geq 1$

$\Rightarrow$ Higher order: $d = 7, 9, 11$

$\Rightarrow$ Nearly conserved L,
i.e. small ϵ ("inverse seesaw")

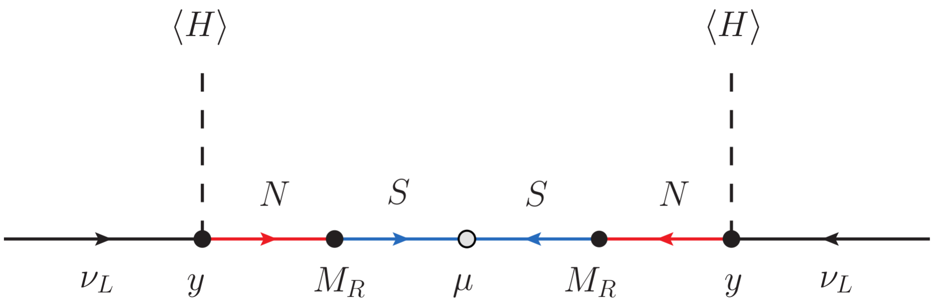
$\Rightarrow$ + "smallish" Y ($\sim \mathcal{O}(10^{-2} - 1)$?)

$\cdots$ or combination thereof



Seesaw variants

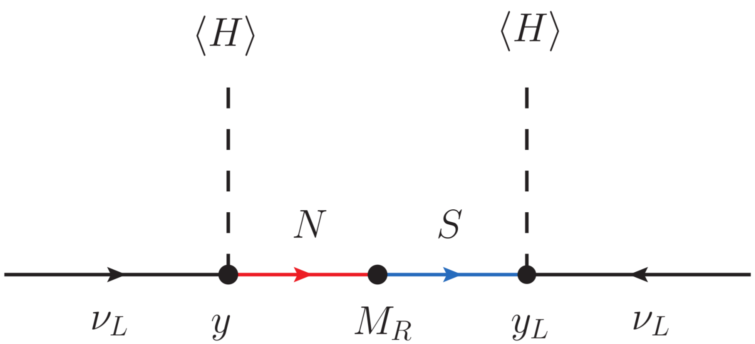
Inverse seesaw:



Mohapatra & Valle, 1986

$$m_\nu \propto \mu (yv)^2 / M_R^2$$

Linear seesaw:



Akhmedov et al. 1995

$$m_\nu \propto (yv)(y_L v)/M_R^2$$

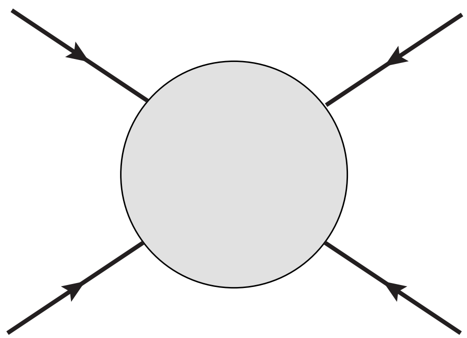
Diagramatically as type-I: Singlet fermionic decomposition

but suppression mechanism different !

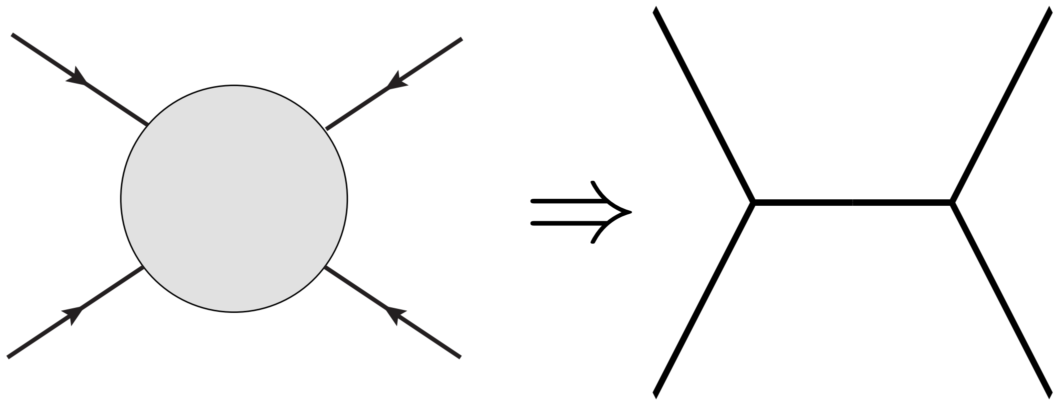
$\mathcal{II}.$

Higher dimensional operators

Seesaw reconsidered

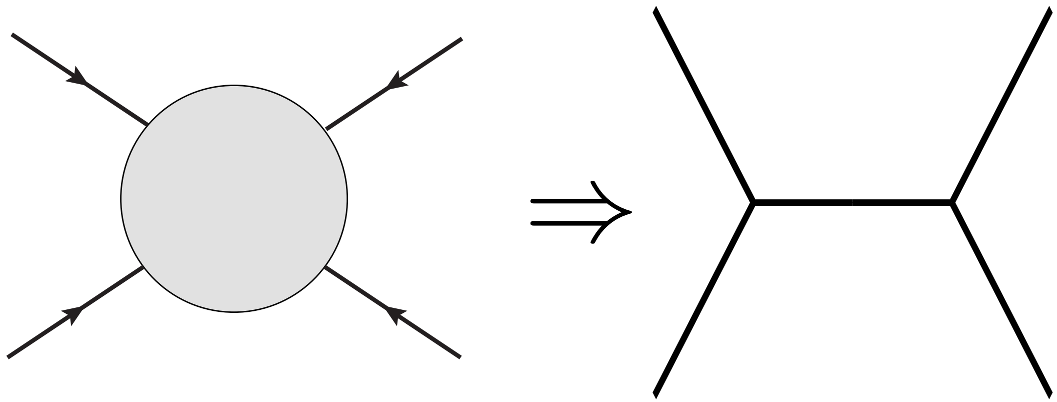


Seesaw reconsidered



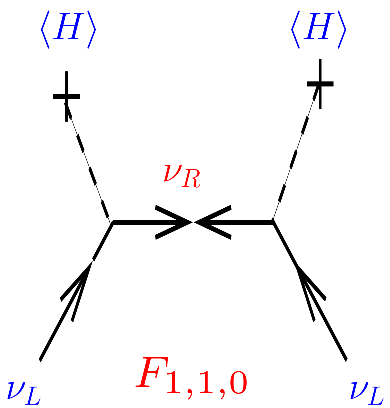
1 - topology with
0 - loops
4 - external legs

Seesaw reconsidered

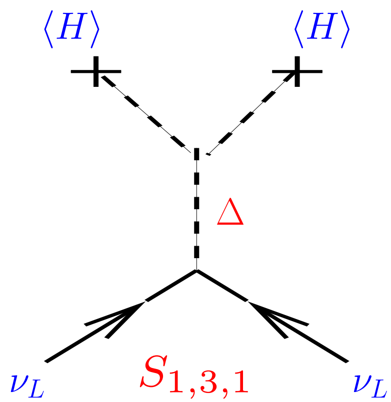


1 - topology with
0 - loops
4 - external legs

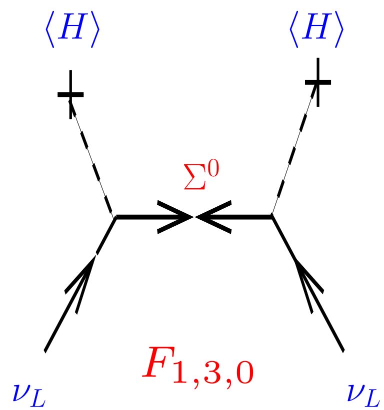
Fixing outside fields yields 3 diagrams:



seesaw type-I

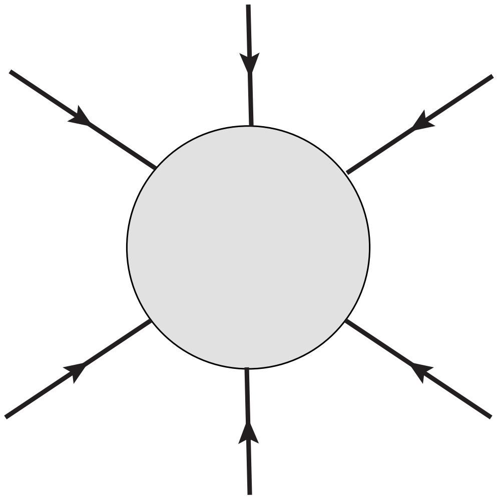


seesaw type-II



seesaw type-III

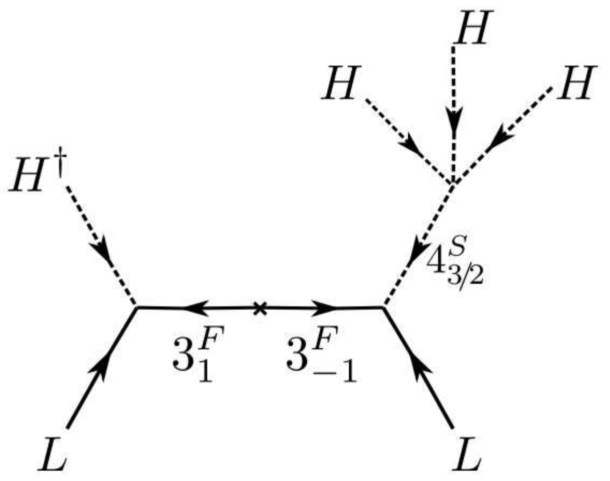
Dimension-7



$d = 7$ operator:

$$\mathcal{O}_7 = \frac{1}{\Lambda} LLHHHH^\dagger$$

Genuine $d = 7$ model



d=7:
Babu, Nandi
& Tavartkiladze, 2009 (BNT)

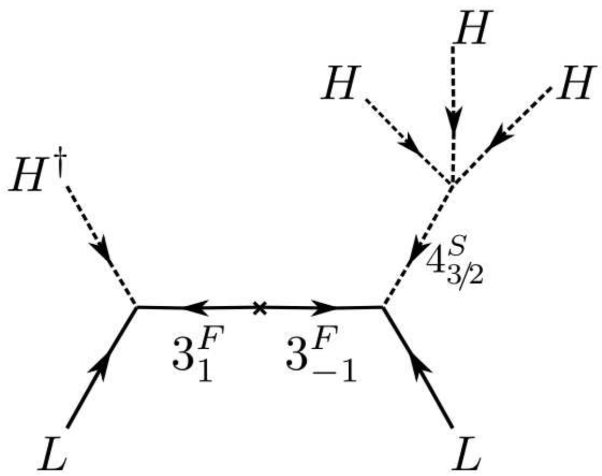
$\Rightarrow$ 3^F_1 = Fermionic triplet, hypercharge 1:

$$3^F_1 = (F_3^{++}, F_3^{+}, F_3^{0})$$

$\Rightarrow$ $4^S_{3/2}$ = Scalar quadruplet, hypercharge $3/2$

$$4^S_{3/2} = (S_4^{+++}, S_4^{++}, S_4^{+}, S_4^{0})$$

Genuine $d = 7$ model



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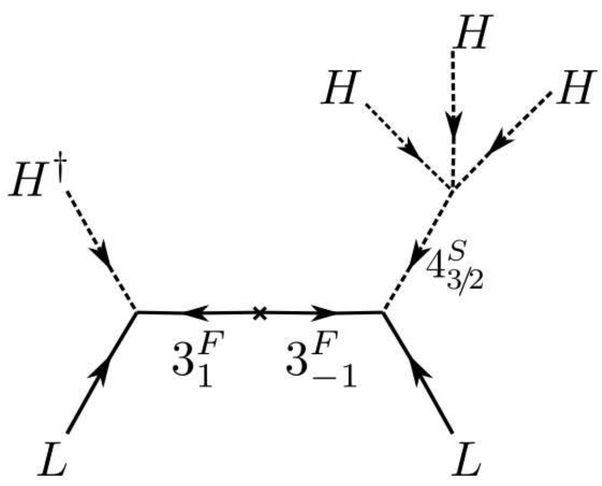
$$4^S_{3/2} = (S_4^{+++}, S_4^{++}, S_4^{+}, S_4^{0})$$

Neutral component of $4^S_{3/2}$ will acquire vev:

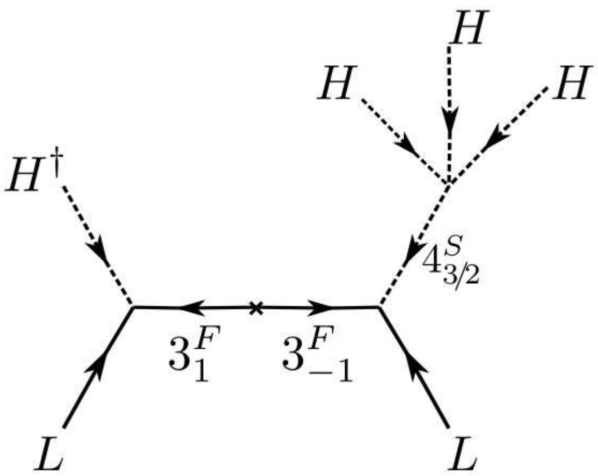
$$\langle S_4^0 \rangle \propto \lambda_5 \frac{v^3}{m_S^2}$$

Effectively
linear seesaw at $d = 7$

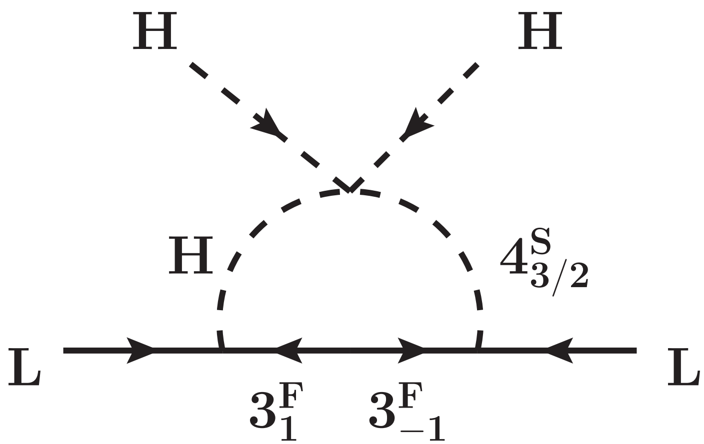
Tree versus loop



Tree versus loop



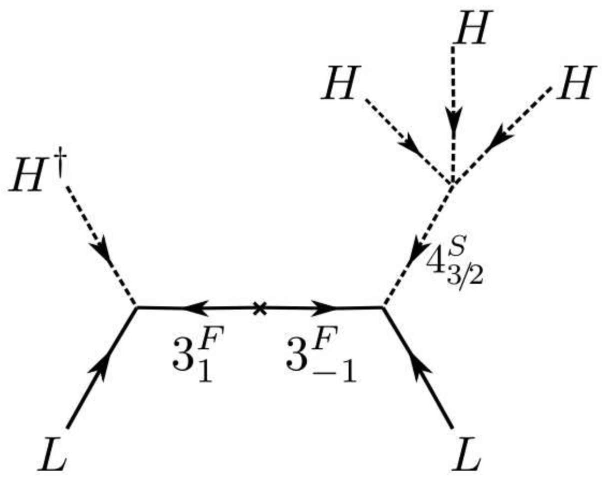
$\Rightarrow$



$$m_\nu \propto \frac{v^2}{\Lambda}\frac{v^2}{\Lambda^2}$$

$$m_\nu \propto \frac{1}{16\pi^2}\frac{v^2}{\Lambda}$$

Tree versus loop



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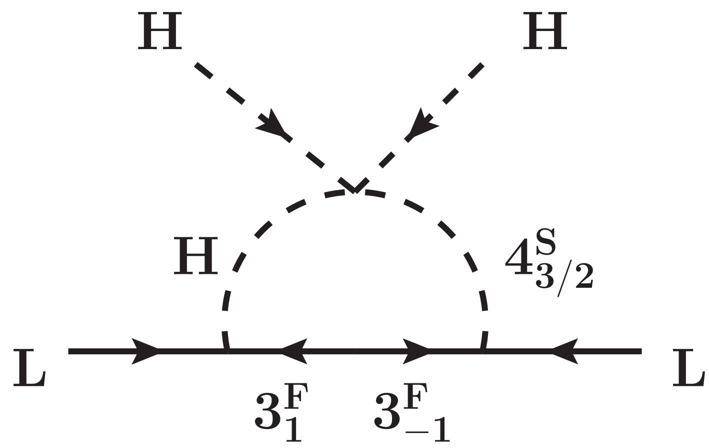


$$m_\nu \propto \frac{v^2}{\Lambda}\frac{v^2}{\Lambda^2}$$

$$m_\nu \propto \frac{1}{16\pi^2}\frac{v^2}{\Lambda}$$

Thus:

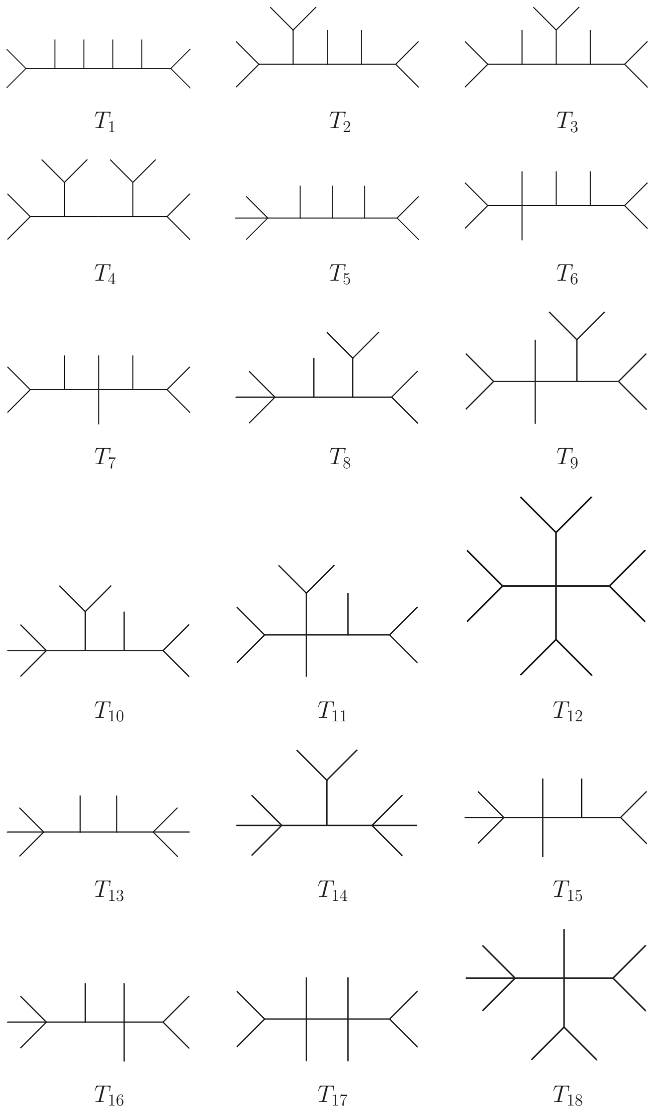
$\Lambda \le 2$ TeV

Otherwise loop larger than tree-level contribution!

True for all high-d models

$d \geq 9$

Topologies at $d = 9$:



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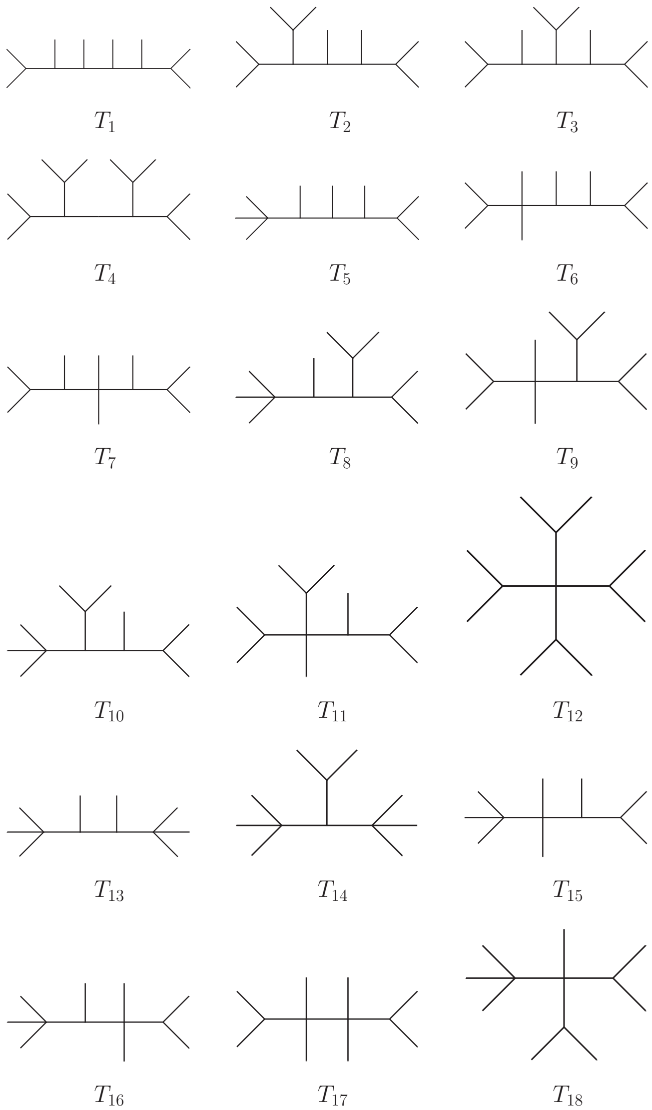
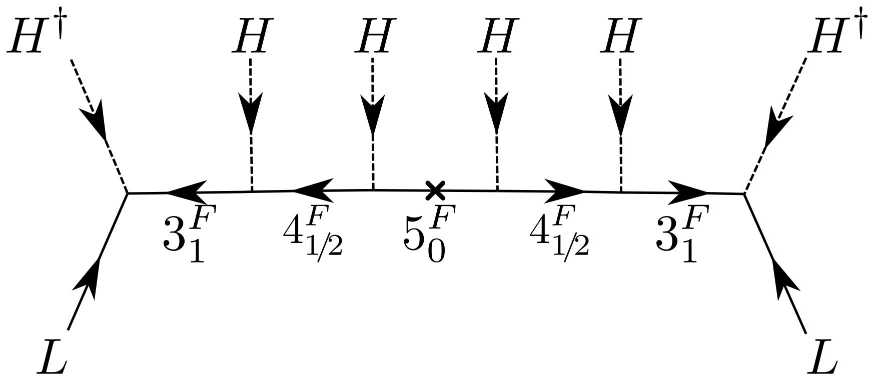


Anamiati et al.
JHEP 1812 (2018) 066

Numbers for tree-level models:

d:	Topologies:	Diagrams:	Genuine:
5	1	3	3
7	5	9	1
9	18	66	2
11	92	504	2
13	576	4199	4

Genuine ‘high’-d models

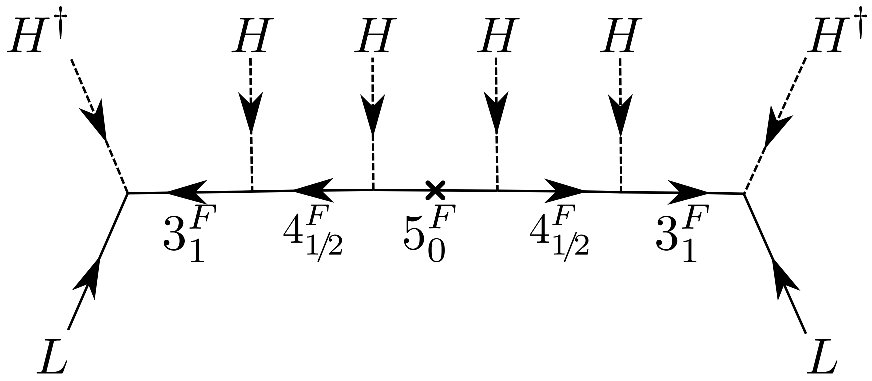


No exotic scalars!

Only 3 vector-like fermions:

3_1^F , $4_{1/2}^F$ and 5_0^F

Genuine 'high'-d models



No exotic scalars!

Only 3 vector-like fermions:

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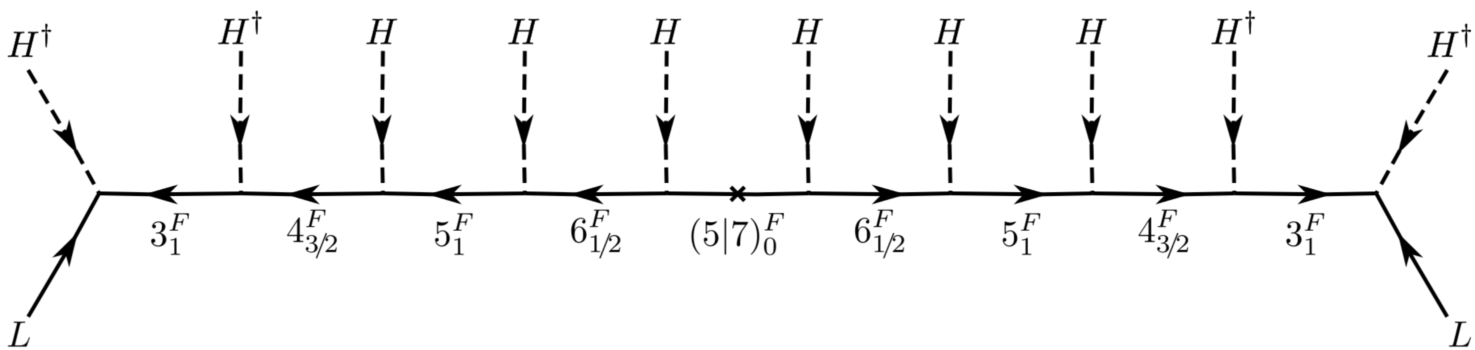
$d = 13$ model!

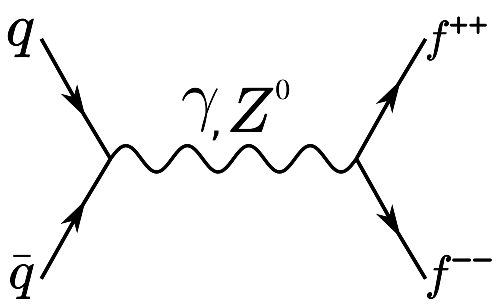


Neutrino mass suppressed by $m_\nu \propto (\frac{v}{\Lambda})^{d-4} v$

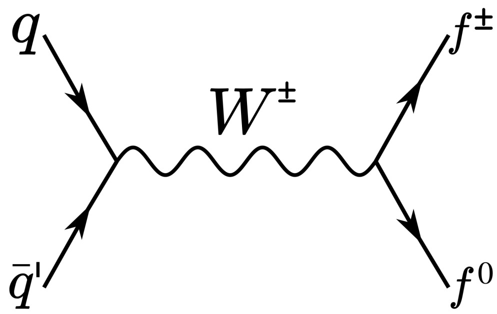
All high-d models need large representations: Large x-section at LHC!

Fermion production at LHC

Example diagrams:



pair production



associated production

Fermion production at LHC

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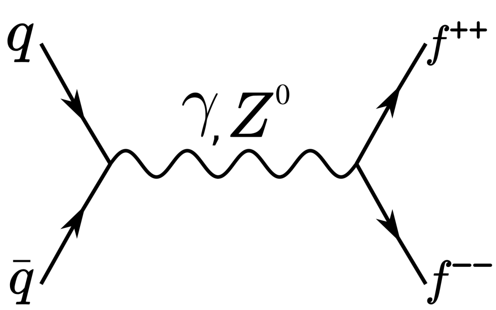
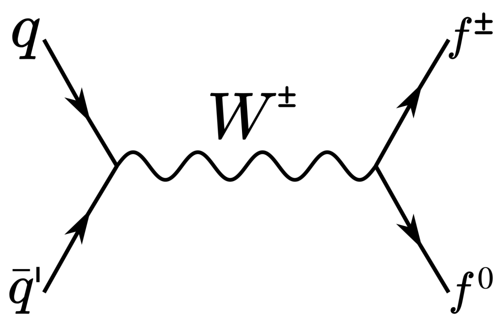
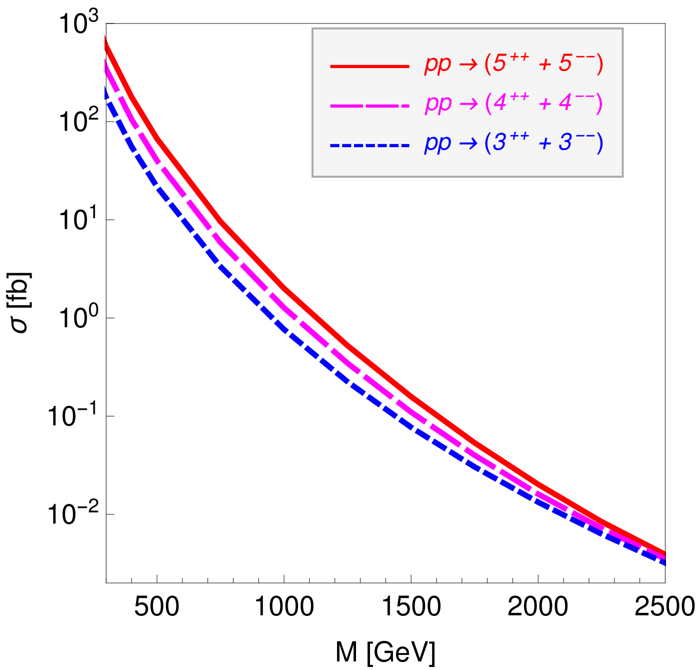


pair production



associated production

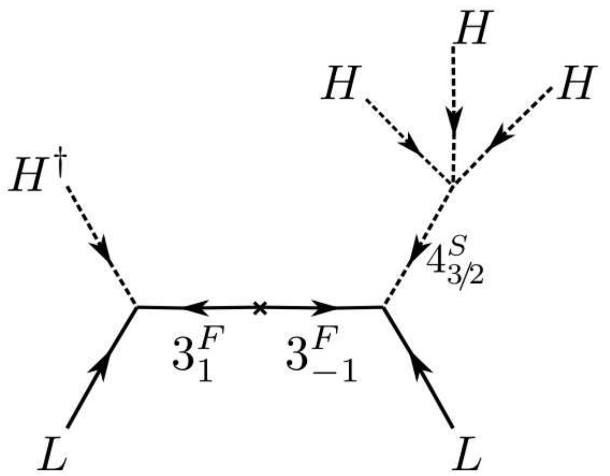


LHC @ 13 TeV and $3/$ab:

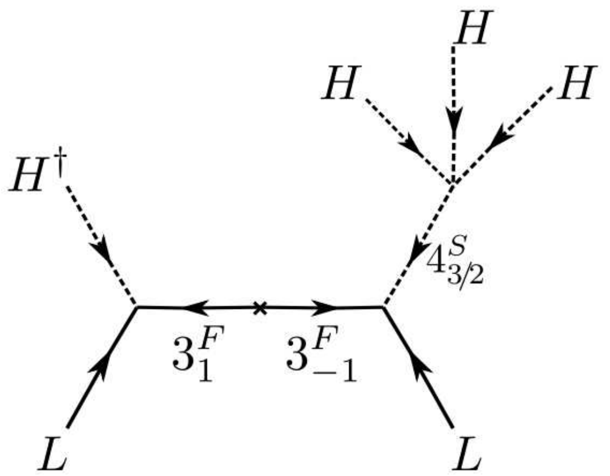
$F_5^{++}F_5^{--}$ - 60 events
$F_4^{++}F_4^{--}$ - 48 events
$F_3^{++}F_3^{--}$ - 40 events

for $M_F = 2$ TeV
(before cuts)

BNT at LHC



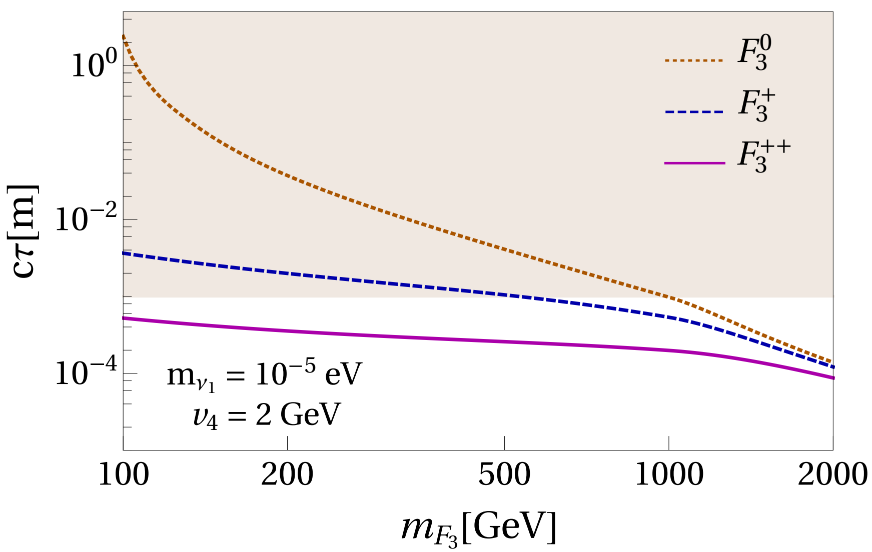
BNT at LHC

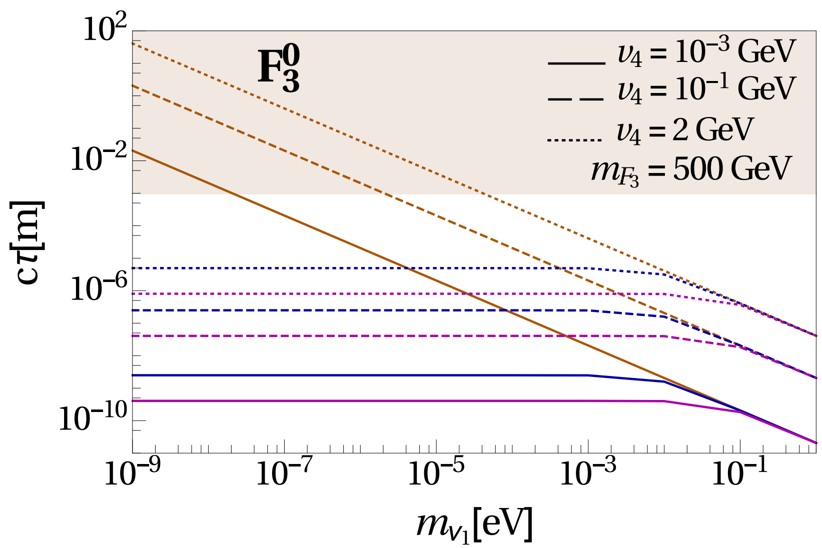


Note:
$\Rightarrow$ Decay lengths of F_3^{++}, F_3^{+} and F_3^{0} differ

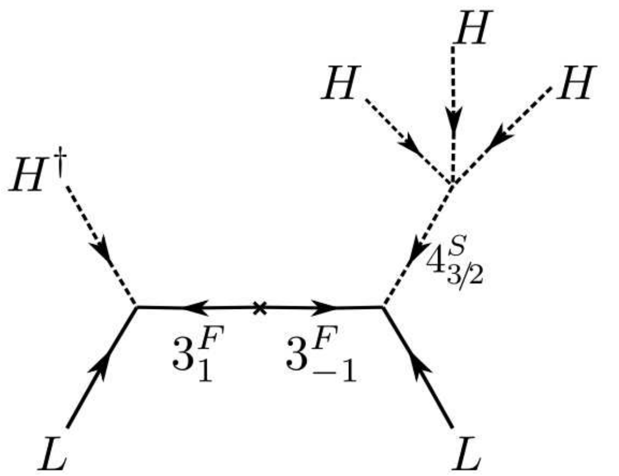
Decay lengths in BNT model, fermions:

No upper limit on $c\tau(F_3^0)$





BNT at LHC



Note:

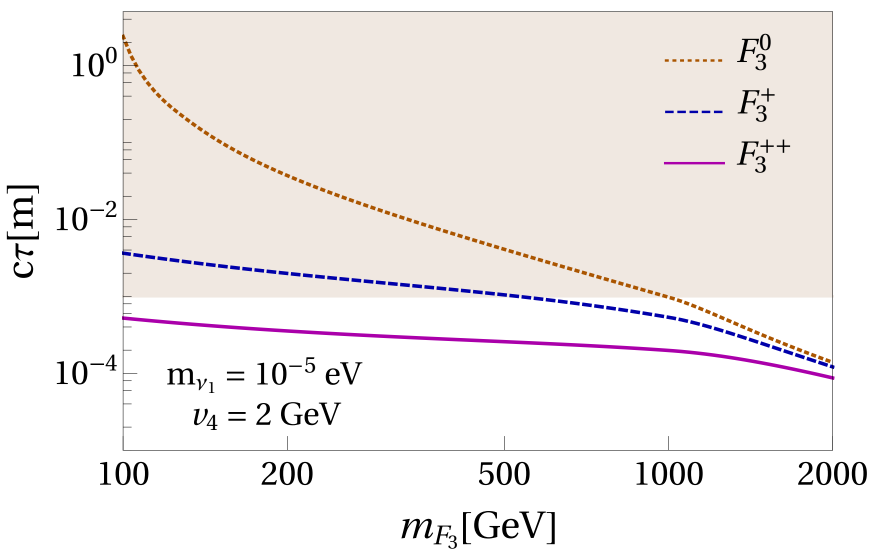
$\Rightarrow$ Decay lengths of F_3^{++}, F_3^+ and F_3^0 differ

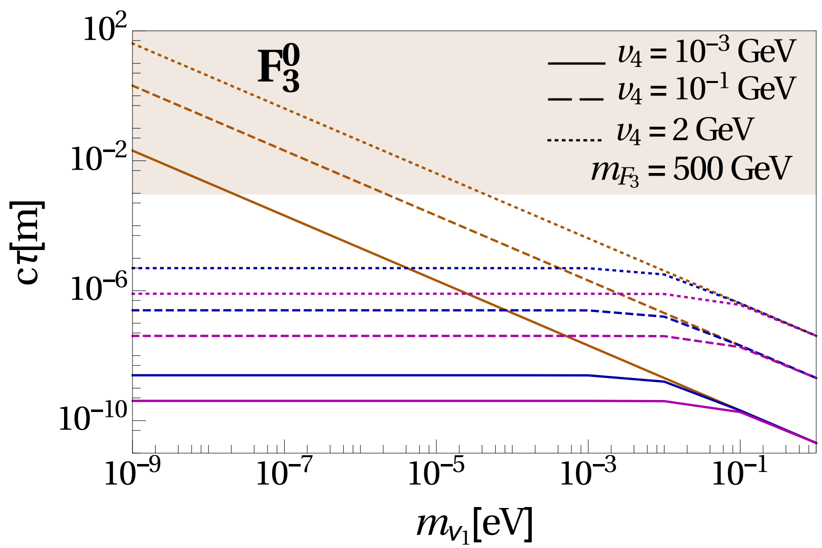
Similar for all high-d models, lightest neutral fermion:

$c\tau \propto 1/m_{\nu_1}$!

Decay lengths in BNT model, fermions:

No upper limit on $c\tau(F_3^0)$

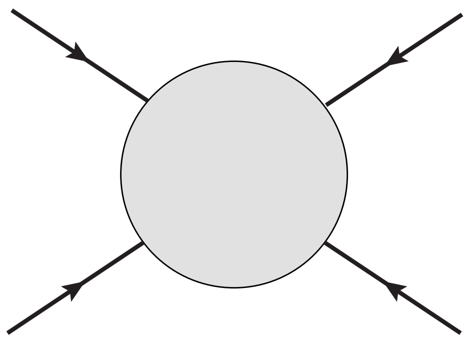




$\mathcal{III}.$

Loop models

Seesaw reconsidered



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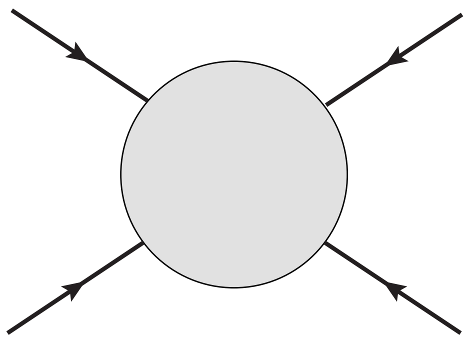
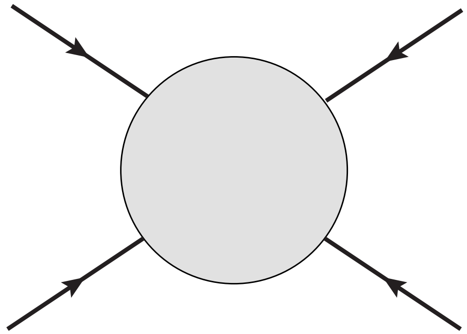


$\Rightarrow$

6 - topology with
1 - loops
4 - external legs

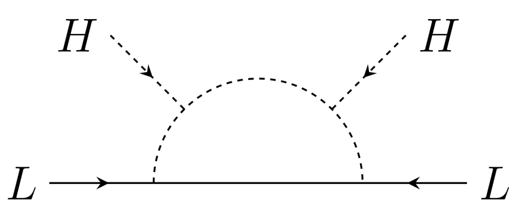
Seesaw reconsidered



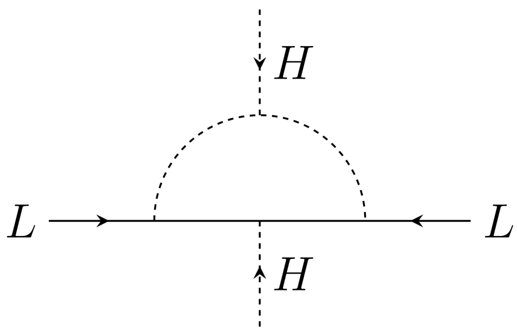
$\Rightarrow$

6 - topology with
1 - loops
4 - external legs

But only 4 genuine diagrams:

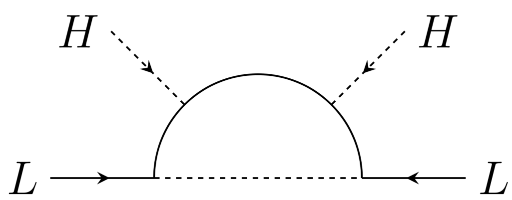


T-I-1

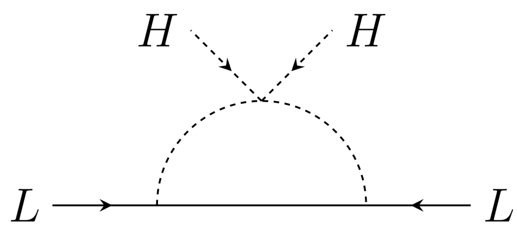


T-I-2

Bonnet et al.
JHEP 07 (2012) 153



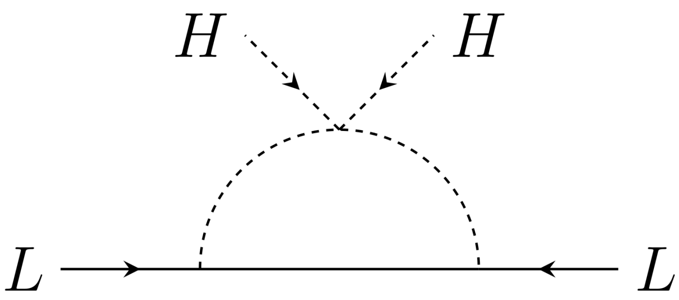
T-I-3



T-III

Filling diagrams ...

Consider one (famous) example:



Filling diagrams ...

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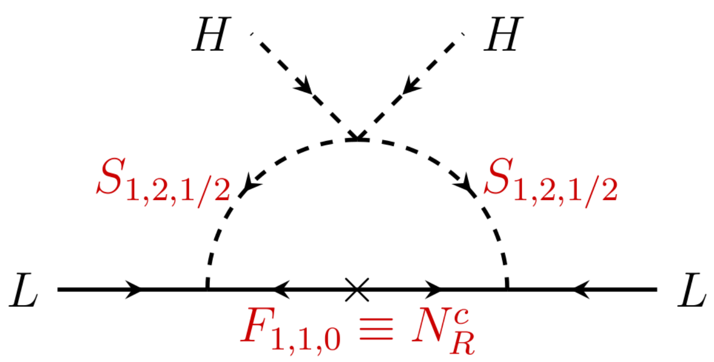

E. Ma, 2006
"scotogenic model"

Add Z_2 symmetry
loop particles odd

S=Scalar, F=Fermion

Filling diagrams ...

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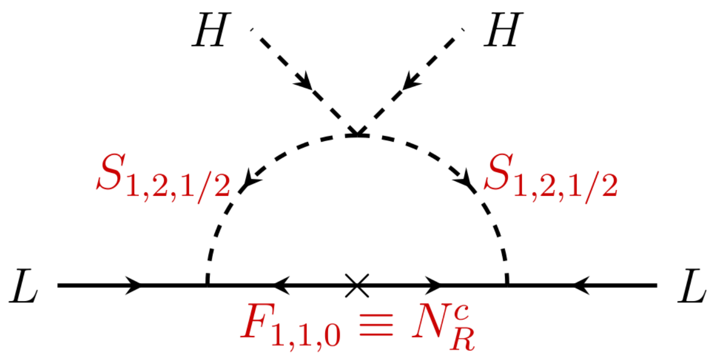
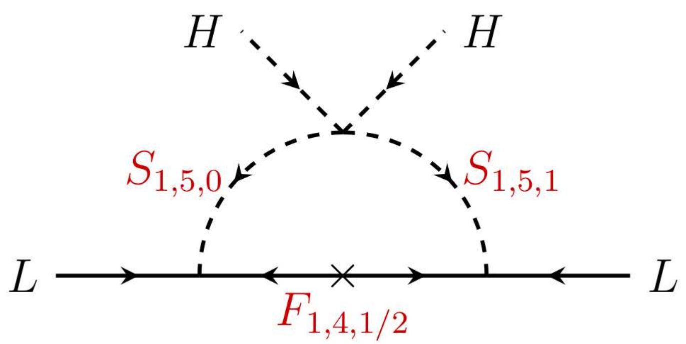


E. Ma, 2006
"scotogenic model"

Add Z_2 symmetry
loop particles odd

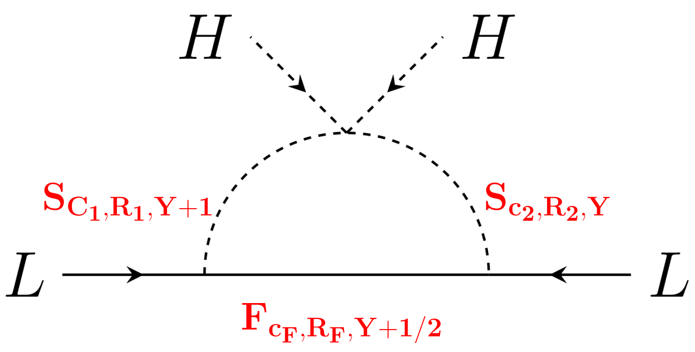
S=Scalar, F=Fermion

Clearly, not the only possibility! One more example:



Filling diagrams ...

In general:



Conditions:

For $SU(3)$: (i) $\mathbf{C_1} \otimes \mathbf{c_F} = \mathbf{1} \oplus \cdots$ + (ii) $\cdots$

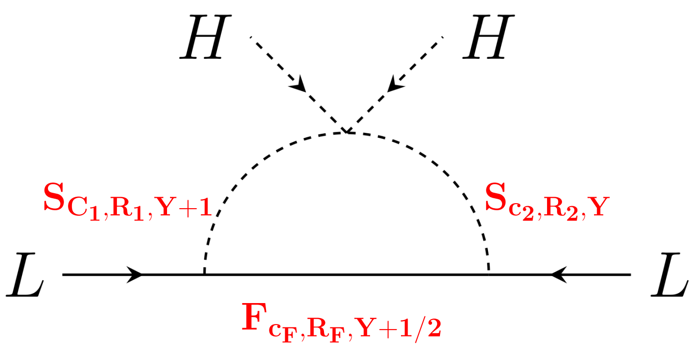
For $SU(2)$: (i) $\mathbf{R_1} \otimes \mathbf{R_F} = \mathbf{2} \oplus \cdots$ + (ii) $\cdots$

For $U(1)_Y$: Y - free parameter

$\cdots$

Filling diagrams ...

In general:



Infinite series of models?

Conditions:

For $SU(3)$: (i) $\mathbf{C_1} \otimes \mathbf{c_F} = \mathbf{1} \oplus \cdots$ + (ii) $\cdots$

For $SU(2)$: (i) $\mathbf{R_1} \otimes \mathbf{R_F} = \mathbf{2} \oplus \cdots$ + (ii) $\cdots$

For $U(1)_Y$: Y - free parameter

$\cdots$

Filling diagrams ...

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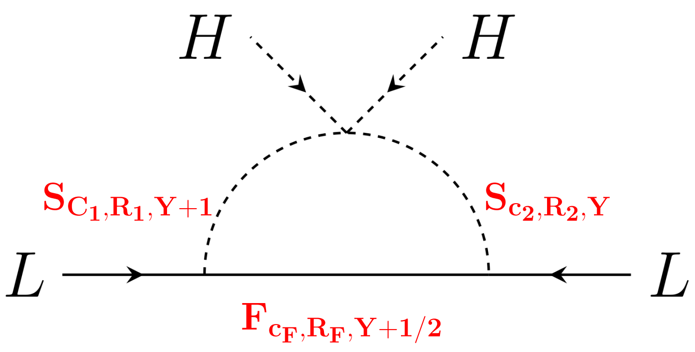

Infinite series
of models?

Cutoffs!

(i) Phenomenological
constraints

(ii) Theoretical
arguments

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Selection criteria

(i) Phenomenological constraint:

(ii) Theoretical arguments:

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No stable charged particles

PDG: No stable, charged relics observed in mass range $M \sim (1 - 10^5)$ GeV

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(a) "Exit" particles
Any particle with linear coupling to two or more SM fields

J. de Blas et al.
1711.10391
"Granada dictionary"

(b) Dark matter candidate
Any multiplet with neutral state (must be lightest member)

S. Bottaro et al.
2107.09688 & 2205.04486

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(ii) Theoretical arguments:

No Landau poles

Adding large multiplets to SM field content one (or more) α_i goes to infinity below M_G

$\cdots$ others $\cdots$

Exits

Definition: Exit: Particle the couples linearly to 2 (or 3) SM fields

Scalar exits

Name	$\mathcal{S}^{(a)}$	$\mathcal{S}_1$	$\mathcal{S}_2$	φ	Ξ	$\Xi_1/\Delta^{(a,b)}$	$\Theta_1^{(c)}$	$\Theta_3^{(c)}$
Irrep	$(1,1,0)$	$(1,1,1)$	$(1,1,2)$	$(1,2,\frac{1}{2})$	$(1,3,0)$	$(1,3,1)$	$(1,4,\frac{1}{2})$	$(1,4,\frac{3}{2})$

Name	ω_1	ω_2	ω_4	Π_1	Π_7	ζ
Irrep	$(3,1,-\frac{1}{3})$	$(3,1,\frac{2}{3})$	$(3,1,-\frac{4}{3})$	$(3,2,\frac{1}{6})$	$(3,2,\frac{7}{6})$	$(3,3,-\frac{1}{3})$

Name	Ω_1	Ω_2	Ω_4	Υ	Φ
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"Exits" appear in tree-level decompositions of $d=6$ SMEFT, see: de Blas et al., arXiv:1711.10391

Fermion exits

Name	$N^{(a)}$	E	Δ_1	Δ_3	$\Sigma^{(a)}$	Σ_1
Irrep	$(1,1,0)$	$(1,1,-1)$	$(1,2,-\frac{1}{2})$	$(1,2,-\frac{3}{2})$	$(1,3,0)$	$(1,3,-1)$

Name	U	D	Q_1	Q_5	Q_7	T_1	T_2
Irrep	$(3,1,\frac{2}{3})$	$(3,1,-\frac{1}{3})$	$(3,2,\frac{1}{6})$	$(3,2,-\frac{5}{6})$	$(3,2,\frac{7}{6})$	$(3,3,-\frac{1}{3})$	$(3,3,\frac{2}{3})$

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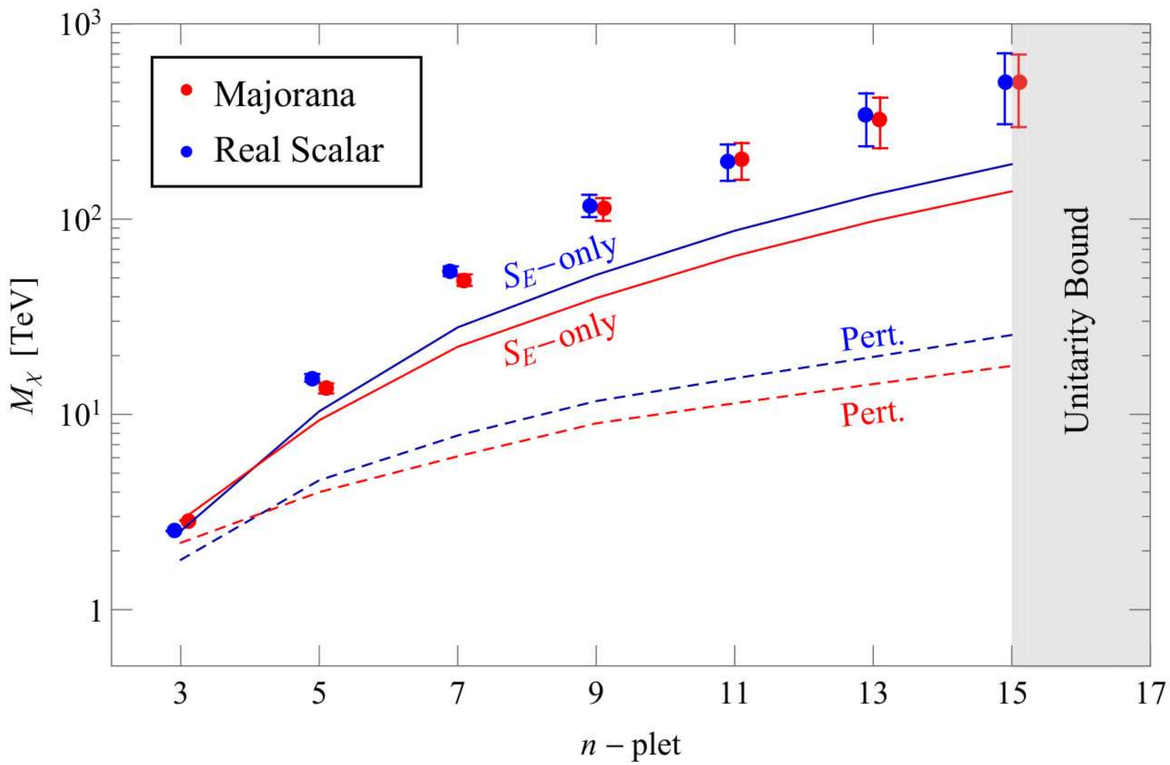
Arbeláez et al.; arXiv:2205.13063
406 models with "exit" particles

DM candidates

The list of possible DM multiplets is finite:

Cirelli et al., 2006
Bottaro et al., 2021
Bottaro et al., 2022



⇒ Cross-section for reproducing $(\Omega h^2)_{DM}$ violates unitarity for $n > 13$

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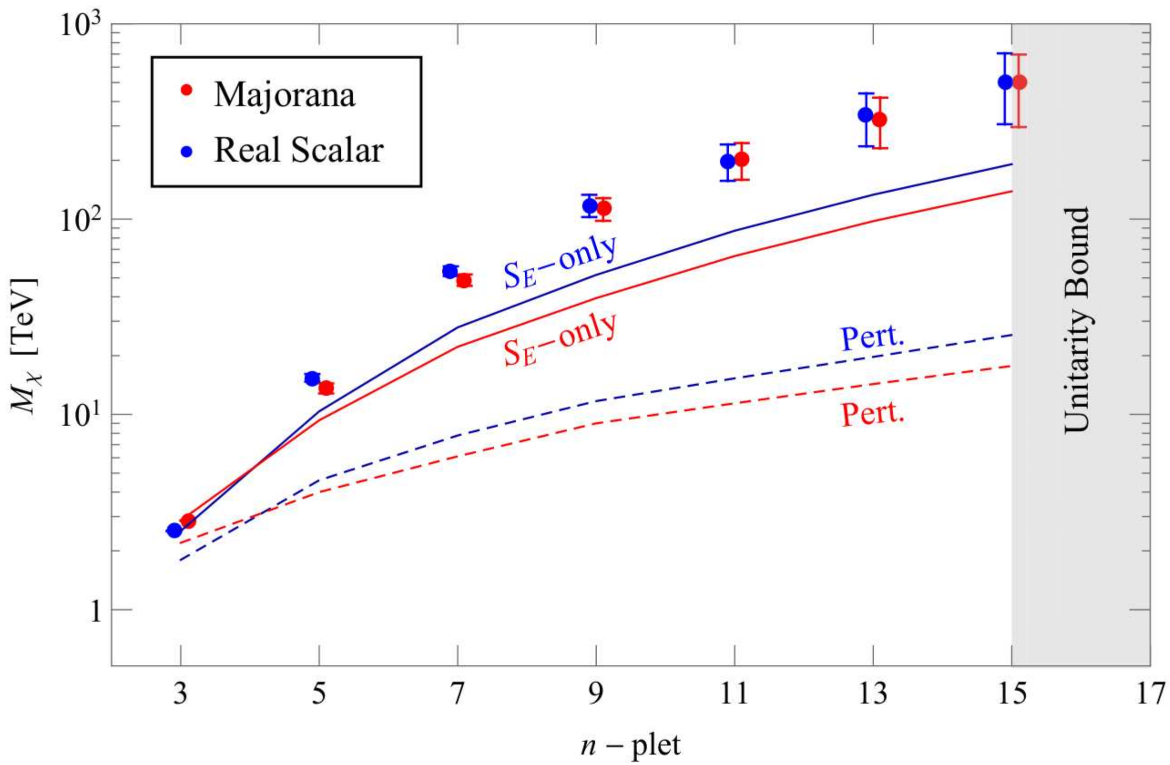

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arXiv:2205.13063
318 models
with dark matter

$\Rightarrow$ Cross-section for reproducing $(\Omega h^2)_{DM}$ violates unitarity for $n > 13$

Perturbativity

Still 724 = 406 + 318 models! - Can we reduce that number?

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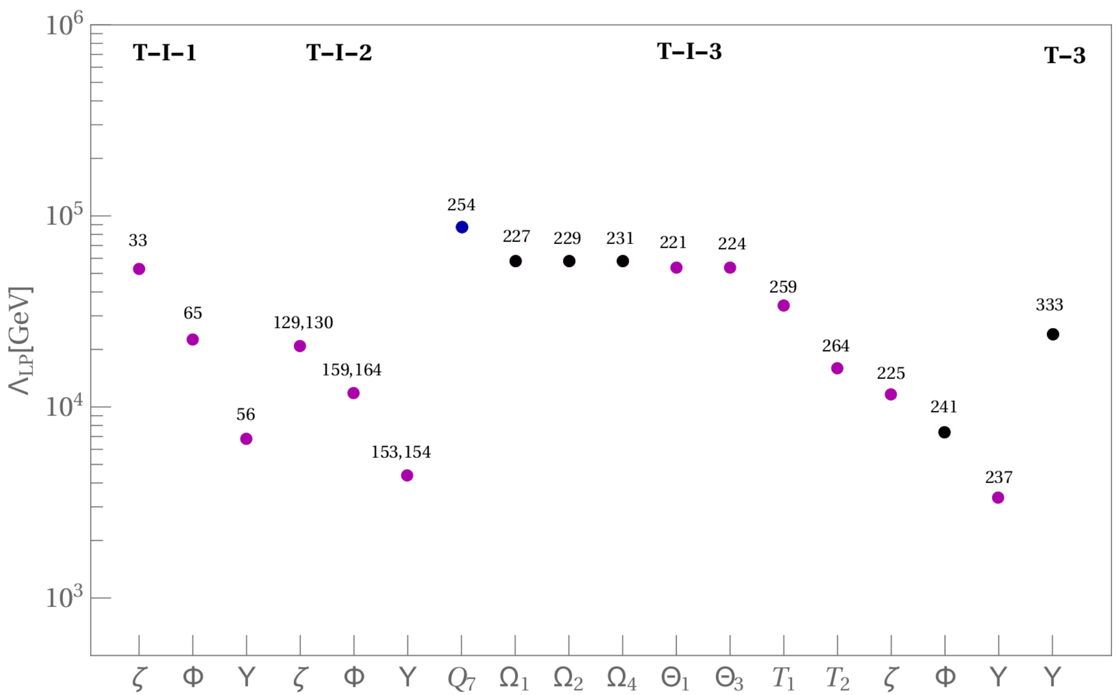
Add condition that models should be perturbative up to GUT scale

Putting scale of new physics to $\Lambda = 1$ TeV for this plot:



Λ_{LP}: energy scale, where one $\alpha_i > (4\pi)$

Numbers correspond to model numbers in tables

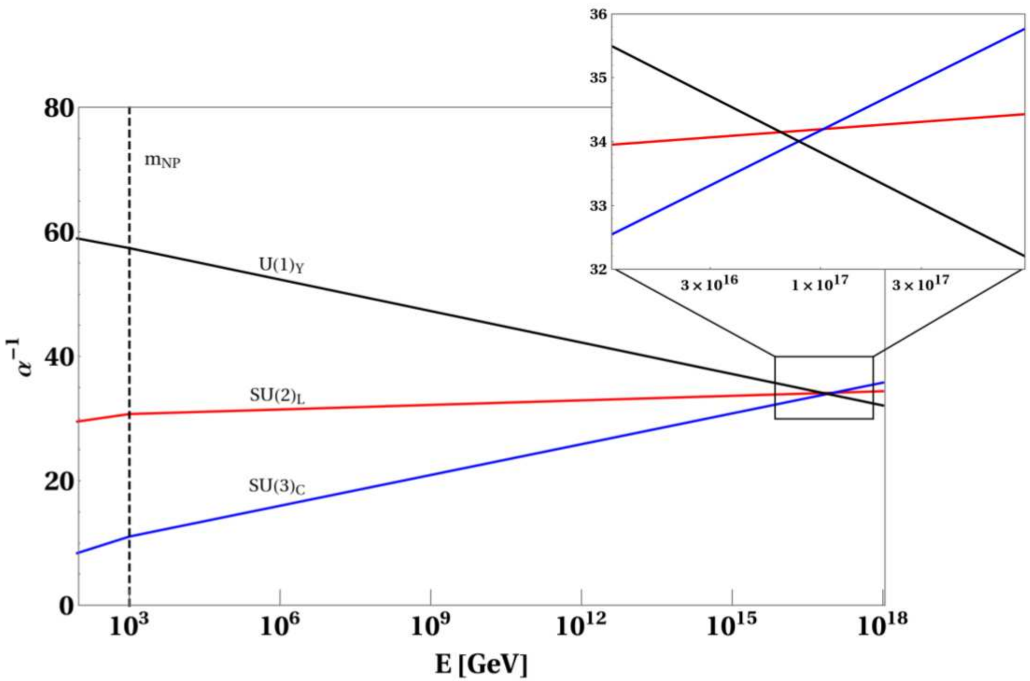
Accepting this condition:

$\Rightarrow$ only 57 models out of 406 survive - exit class

$\Rightarrow$ only 59 out of the 318 survive - DM class

Unification

One more curious comment:



model contains:
$(D, Q_1, \Pi_1, \omega_1)$

$\Rightarrow$ Only one of all models unifies gauge couplings above $E = 10^{15}$ GeV

$\Rightarrow$ 12 more models "unify" but at low energy (proton decay!)

$\mathcal{IV}$.

2-loops, 3-loops, 4-loops

$\mathcal{IV}.$

2-loops, 3-loops, 4-loops

Unfortunately ... No time!

Ask me offline, if you are interested.

Conclusions

⇒ Tree-level $d = 5$ seesaw simplest possibility for Majorana m_ν

⇒ Higher-dimensional tree-level models require low scale Λ
Advantage: testable!

⇒ Loop models: Many phenomenologically consistent models exist.
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⇒ Long way to go to identify origin of neutrino mass!

Conclusions

"When you have eliminated all which is impossible, then whatever remains, however improbable, must be the truth"

Arthur Conan Doyle