THE ETHERINGTON-HUBBLE RELATION

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Tension in Cosmology
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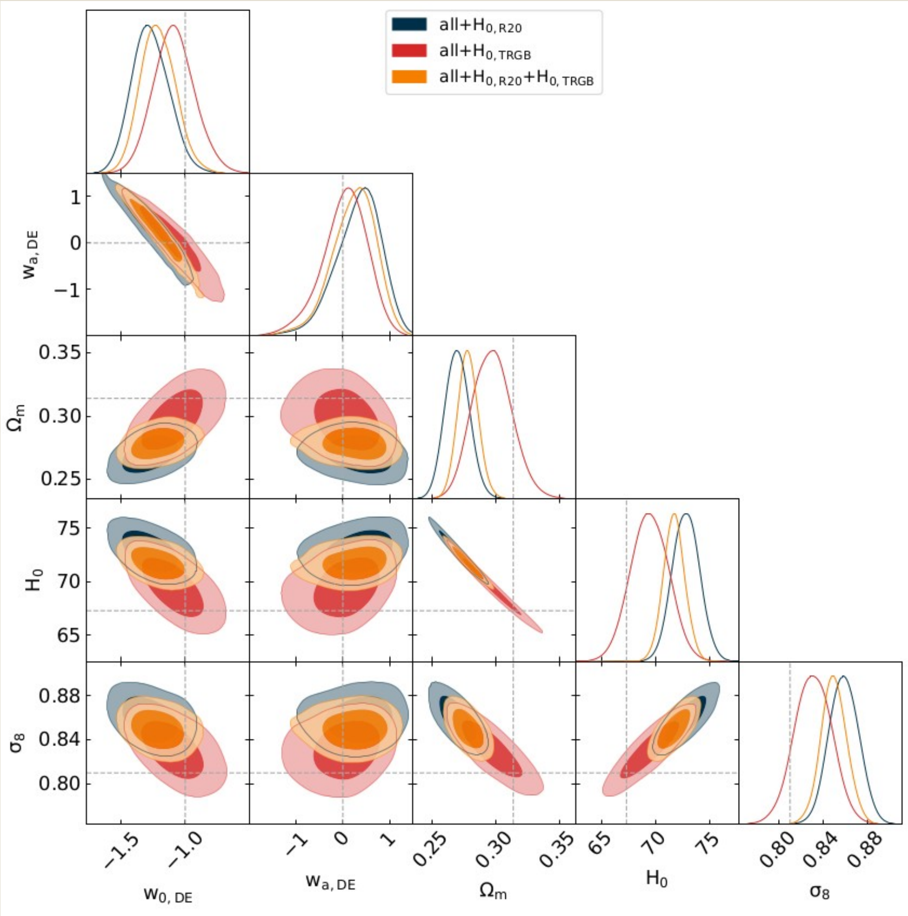
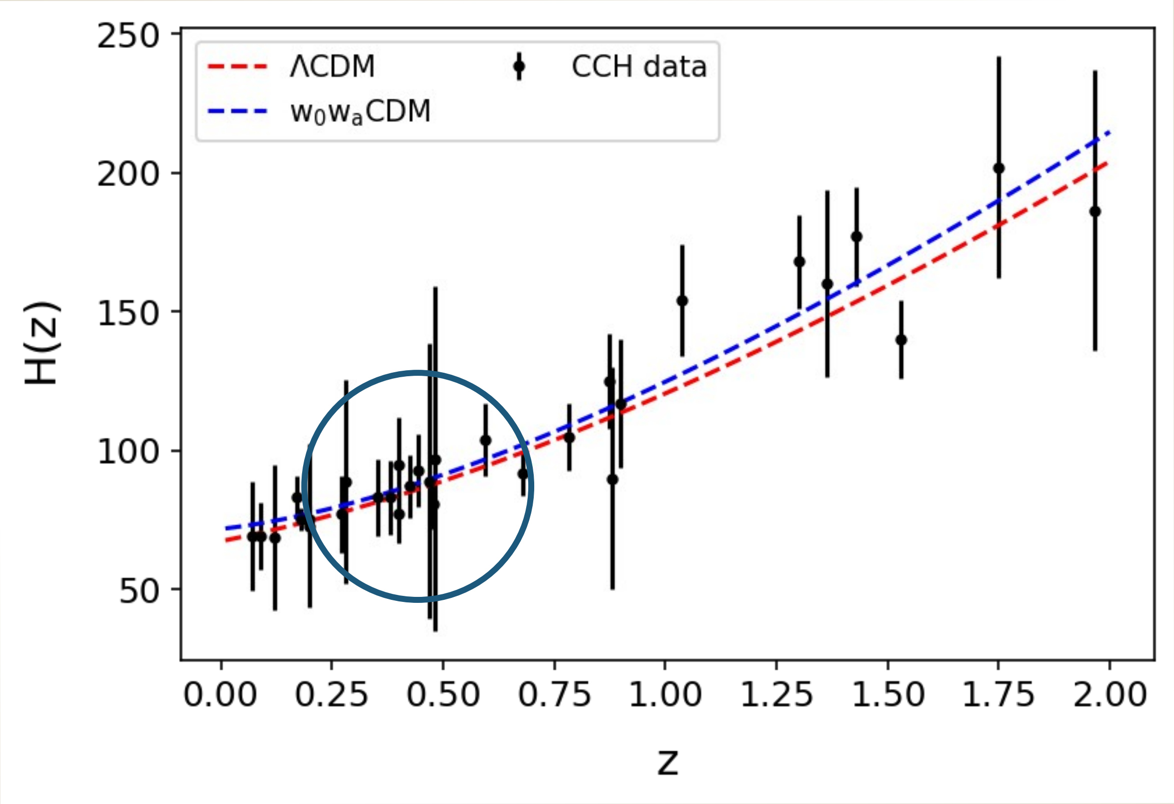
Hubble Trouble

- A 5 (!) standard deviation discrepancy in the measure of a fundamental constant
- Late and early time data disagreement but consistency within same redshift range
- Possible solutions ?? w0waCDM with w0 away from LCDM at 3 standard deviation would solve the problem
- Is this solution enough ? Not really as it doesn't give a physical explanation to the tension

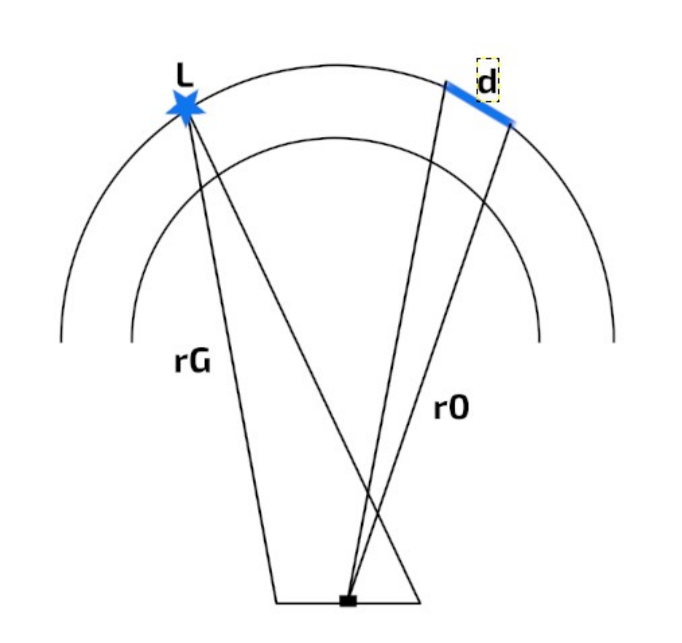


Distances and Parameters



- All parameters are fixed to Planck best fit for LCDM
- For w0waCDM :
  w0=-1.21
  wa=0.23
  H0~ 72
- Data do not distinguish between the two models

Etherington's reciprocity theorem



- Let $d\Omega_G$ be the solid angle subtended by a bundle of null geodesic diverging from a source and dS_G its cross-sectional area at some point, we then have:

$$dS = r_G^2 \, d\Omega_G \;\Rightarrow\; F = \frac{L}{4\pi r_G^2 (1+z)^2}$$

- Let $d\Omega_O$ be the solid angle subtended by a bundle of null geodesic diverging from the observer and dS_O its cross-sectional area at some point, we then have :

$$dS_O = r_O^2 \, d\Omega_O \;\Rightarrow\; r_O = d/\theta$$

- The reciprocity theorem relates the two distances

$$r_G = (1+z) r_0$$

Etherington's theorem : a thought experiment

- An observer at z =0 builds an experimental set-up to measure the luminosity of a source at redshift z. Therefore using the relation between flux and distances, the observer infers a distance

$$\log_{10} r_G (1+z) = \log_{10} d_L^{(1)}(z) = \frac{1}{5}\left(m_B(z) - M_B^{(1)} - 25\right)$$

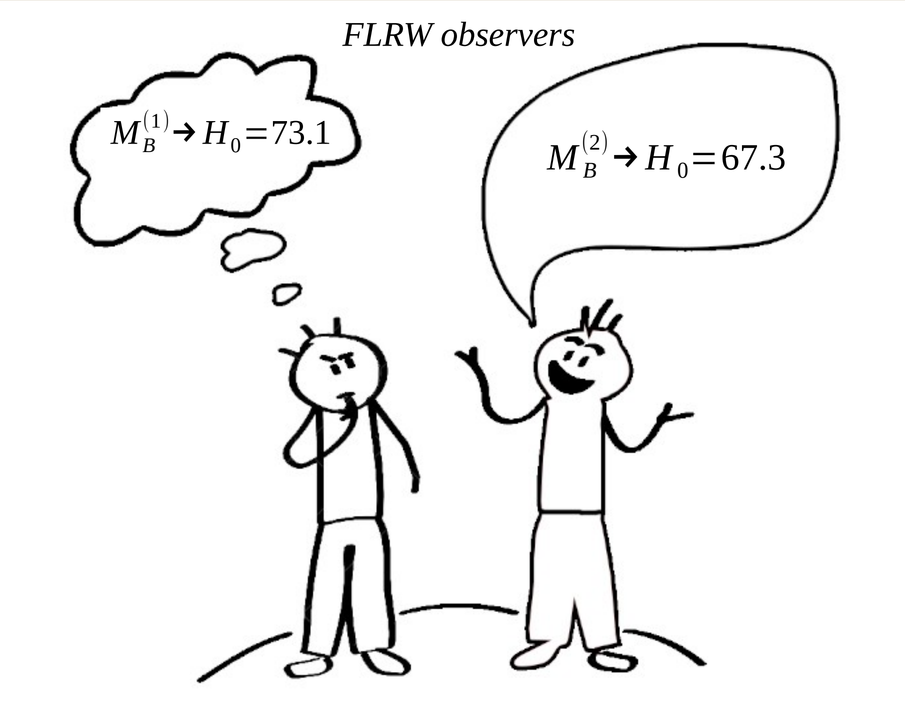
- An observer at z =0 builds an experimental set-up to instead measure angular distances from objects of known size. Then, observer 2 infers the source luminosity assuming the Etherington's theorem :

$$\log_{10} r_0 (1+z)^2 = \log_{10} d_A^{(2)}(z)(1+z)^2 = \frac{1}{5}\left(m_B(z) - M_B^{(2)} - 25\right)$$

Magnitude – Flux relation

$$m_B(z) - M_B \propto \log_{10} \frac{F(z=0)}{F(z)}$$

Etherington's theorem : a thought experiment



$$\Delta M_B = M_B^{(2)} - M_B^{(1)} = 5 \log_{10} \frac{d_L^{(1)}}{d_A^{(2)} (1+z)^2}$$

- Comparing their values of the source luminosity (in terms of the absolute magnitude), they will conclude that they agree on the measure of the luminosity if their measures of distances agree
- If their distances do not agree they will instead conclude the reciprocity theorem is broken (if any other source of discrepancy can be eliminated)
- If they both assume a FLRW metric they will, as well, conclude that their values of the Hubble constant differ

Distances and Parameters

- Observables constrained by data are related to cosmological distances in a complex way. Assuming an FLRW space-time those relations can be written as:
  - BAO $\frac{d_A(z)}{r_s}$ and $H(z) r_s \Rightarrow H(z) d_A(z)$
  - SNIa : $m_B(z) \propto 5 \log[H_0 d_L(z)]$
  - CCH: $H(z)$
- The Hubble parameter (and constant) is an observable as well as distances, parameters are model dependent:

$$\chi = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')} \qquad\qquad E(z) = \sum_{i=0}^{N} \Omega_i (1+z)^{\alpha_i}$$

- $H(z)$ cannot be eliminated as background cosmology is determined uniquely once both the expansion rate and the way photon propagates are known

The Etherington-Hubble relation

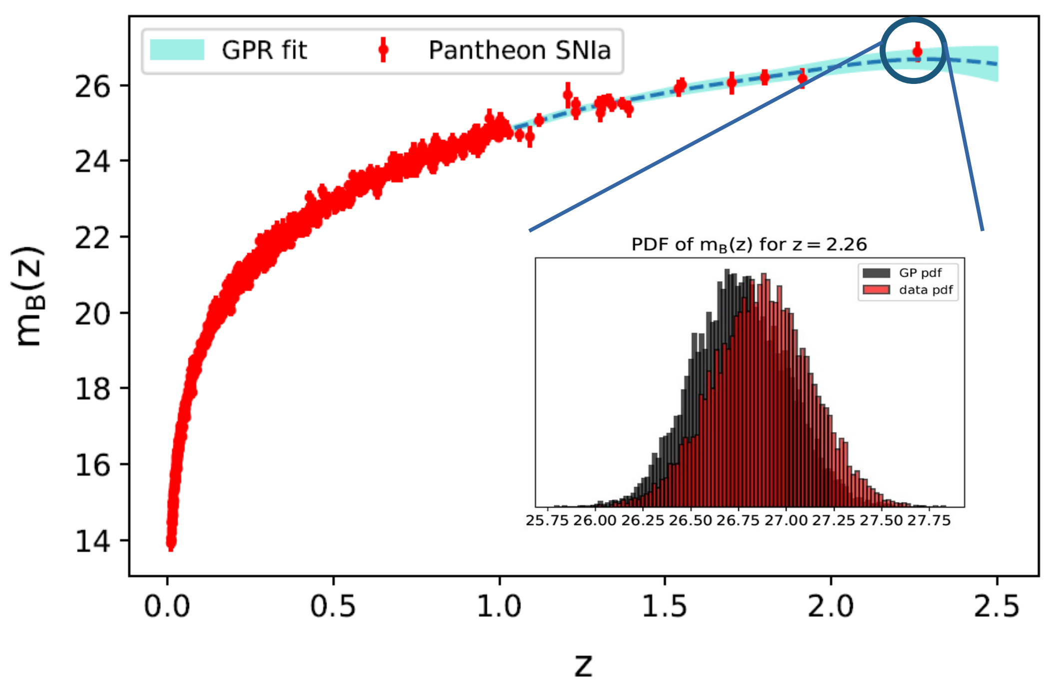
- Combining observables from SNIa, BAO and CCH, we obtain an estimator that can be used to test consistency among dataset (against a well-defined quantity) or to measure the Hubble constant (assuming the reciprocity theorem) :

$$\eta(z) H_0 = \frac{\left[H_0 d_L(z)\right]^{SNIa}}{\left[d_A(z)(1+z)^2\right]^{BAO+CCH}}$$

- Careful is needed in making use of the data and extrapolating parameters in order not to introduce any undesirable bias

[Renzi & Silvestri 2020; Renzi, Hogg, Giaré 2021]

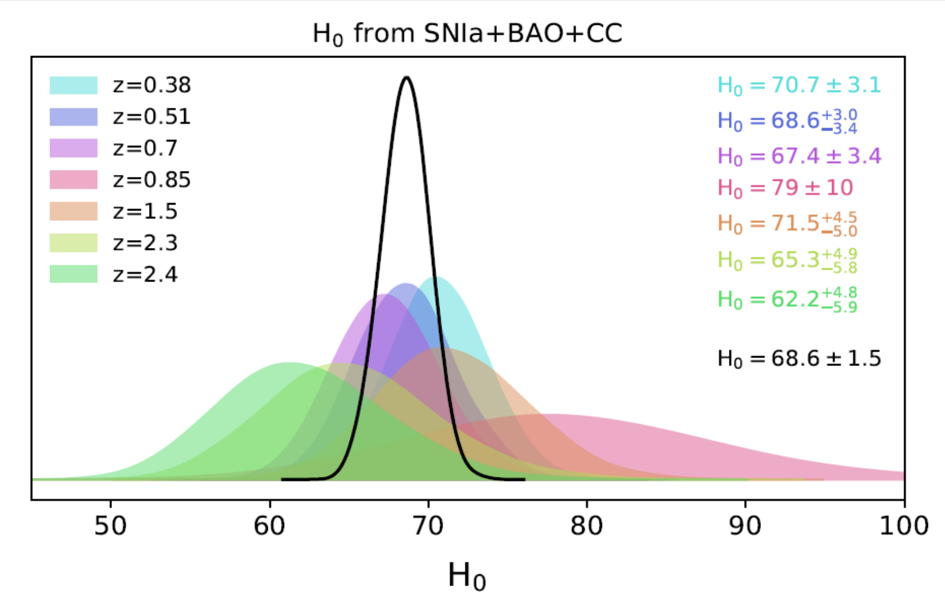
Gaussian Process Monte Carlo

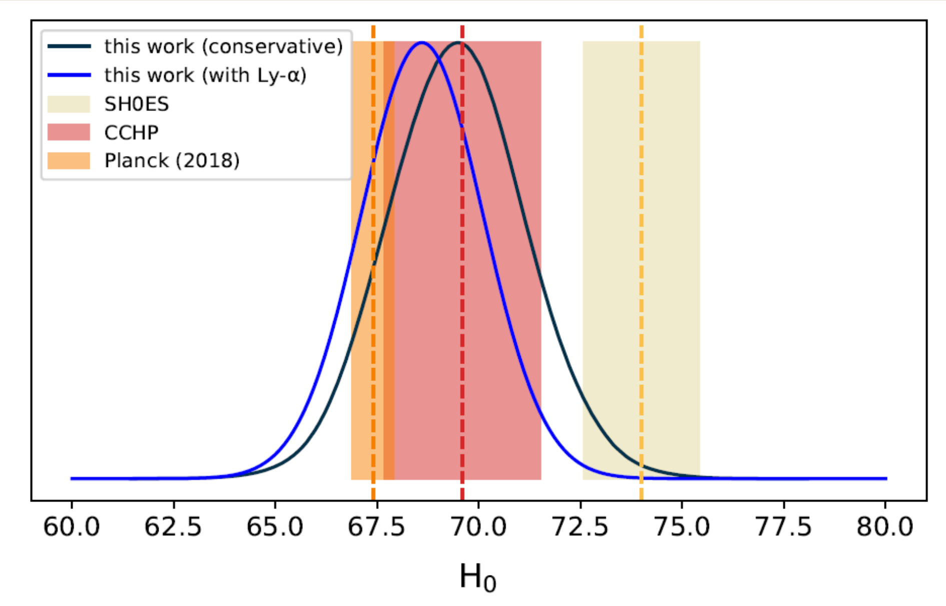


- GP to reconstruct a function given a set of data
- To preserve the information and avoid bias we propagate PDF samples
- PDFs and samples are related by :

$$U[0,1] - \int_{-\infty}^{x} PDF(x')\,dx' = 0$$

Results : Measuring H0

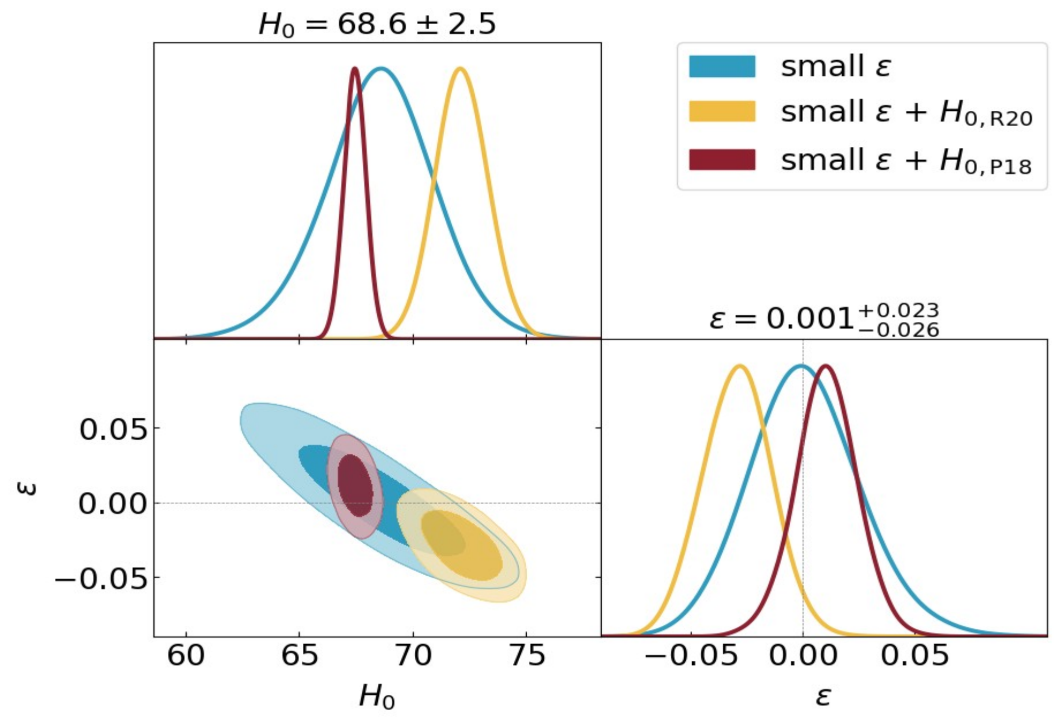




$$\eta(z)=1 \;\Rightarrow\; H_0=[H_0 d_L(z)]^{SNIa}/[d_A(z)(1+z)^2]^{BAO+CCH}$$

[Renzi & Silvestri 2020]

Results : Testing FLRW paradigm



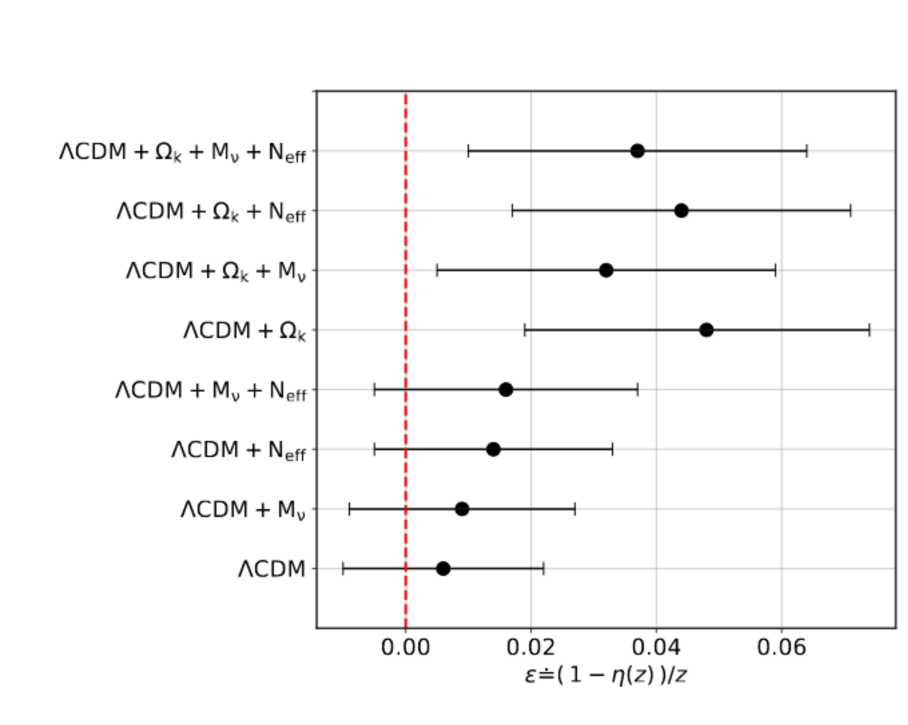
- We assume small linear deviations from the standard values of the Etherington constant

$$\eta(z) = 1 + \epsilon z$$

- Even small deviations can impact the value of H0

[Renzi, Hogg, Giaré 2021]

Results: Planck-BAO tension

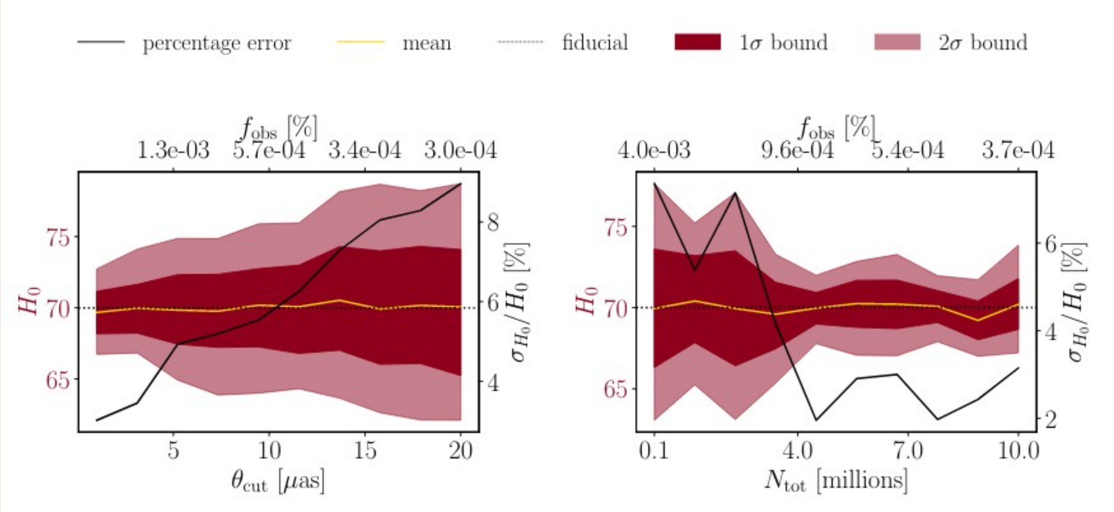


- We can build a consistency test for cosmological models and data, summarizing them in terms of H0

- Curvature pulls the tension with late-time data higher.

[Renzi, Hogg, Giaré 2021]

BH Shadows for H0



[Renzi & Martinelli 2022]

- Relative errors on the shadow size and the mass of the BH fixed to 7%

$$d_L^{BH} = (1+z)^2 \frac{2\sqrt{3} M_{BH}}{c^2 \theta_{BH}}$$

- Final constraints on H0 depends on minimal angular resolution and BH distribution

$$P(x,z) = \frac{4\pi A \chi(z)^2}{N_{tot} H(z)} \Phi(x,z)$$

Summary

- The Etherington-Hubble relation can signal inconsistencies between datasets casting tensions into a well-defined quantity.
- It also hints to a more complicated resolution of the Hubble tension than simply abandoning the Lambda-Cold Dark Matter paradigm
- The framework we have presented works for an FLRW spacetime, the extension to more complicated spacetime can be done.
- The extension of this framework to higher redshift is feasible but requires to take into account that CMB data assumes a standard temperature-redshift scaling
  - Not an easy task as any variation of the temperature scaling will generate spectral distortions on the CMB spectrum

The Flux-Distance relation

- Stems from surface brightness conservation in geometric optics and recognizing that redshift is a property of an observer that move along its worldline as the Universe expands

$$1+z=\frac{(u^a k_a)_{emitter}}{(u^a k_a)_{observer}}$$

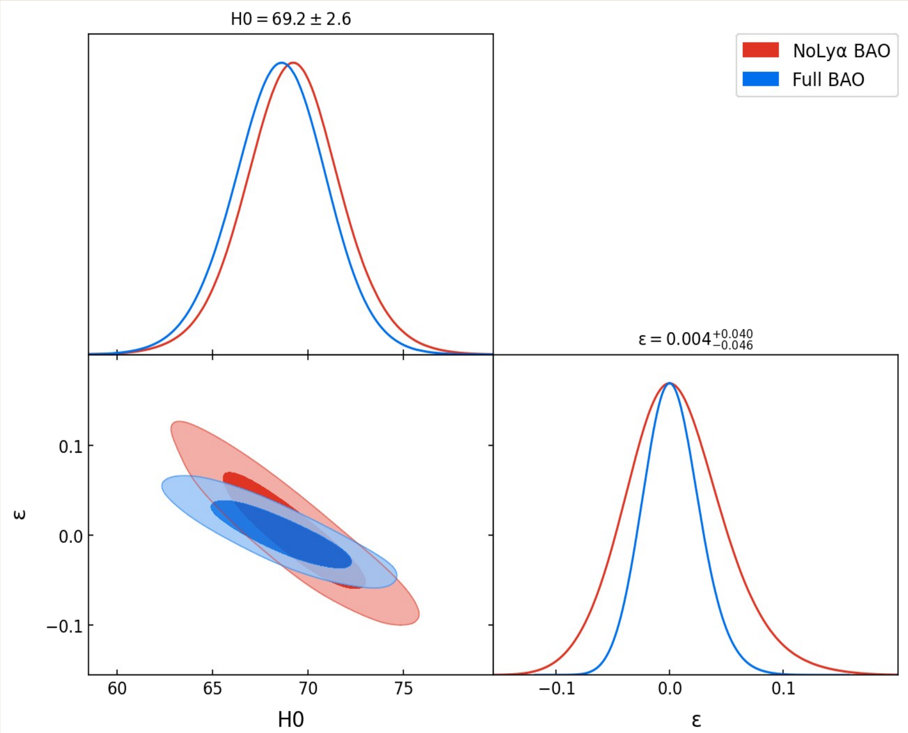
$$F \propto \left[\frac{(u^a k_a)_{observer}}{(u^a k_a)_{emitter}}\right]^2 \frac{1}{dS}$$

$$F=\frac{const}{(1+z)^2 dS}$$

$u^a \rightarrow observer\ 4-velocity$
$k_a \rightarrow photon\ wave-vector$
$dS \rightarrow observer\ cross-sectional\ area$

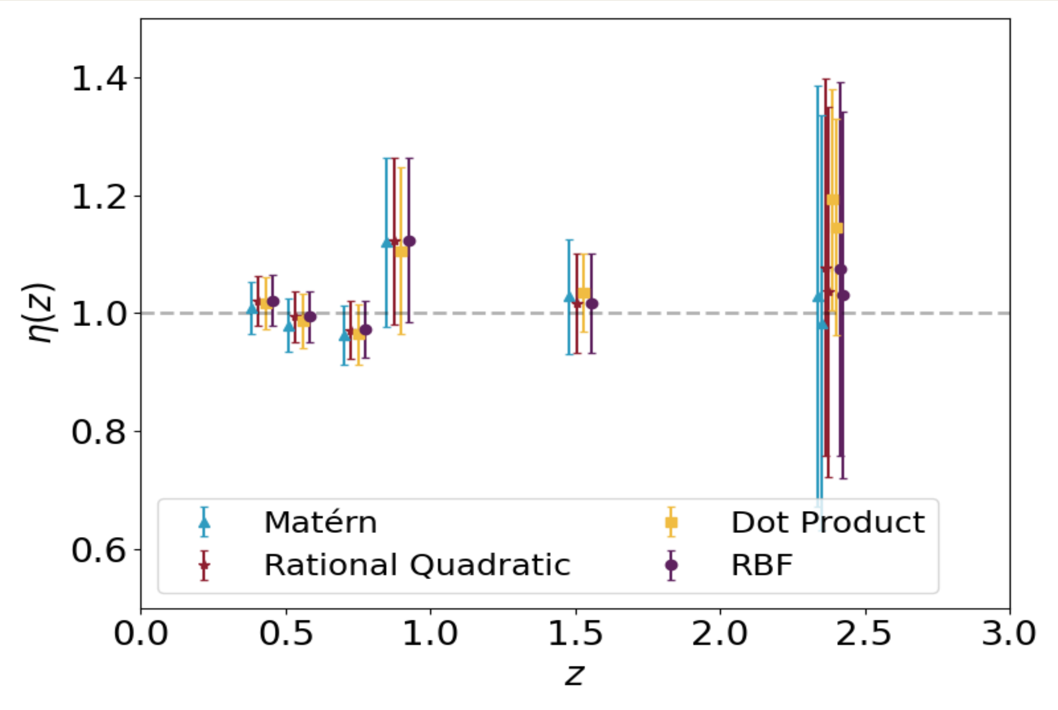
- The flux of a source is define up to a constant, the luminosity, which is an intrinsic property of the source
- As solid angle are conserved along the geodesic, one first calculate the luminosity on a unit 2-sphere around the source ie. $L=4\pi F_G$
- Then we relate the observer cross-sectional area to the solid angle at the source by define the so-called *galaxy area distance* : $dS=r_G^2 d\Omega_G$
- Finally, using the definition of flux leads to $F=\frac{L}{4\pi(1+z)^2 r_G^2}$

Results: Planck-BAO tension



- Curvature pulls the tension with late-time data higher.
- Well-known tension between Planck and BAO data in measuring curvature density
- LyA-BAO does not add any significant pull to this tension

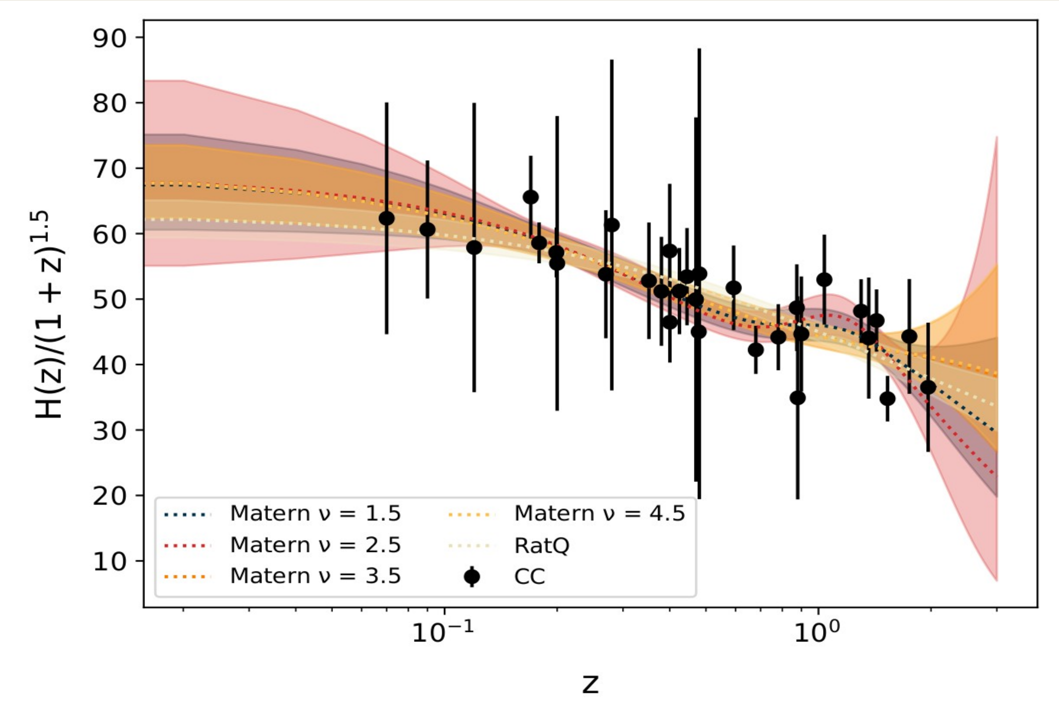
Kernel Comparison



- It is important to asses that the results is stable against different kernel representations

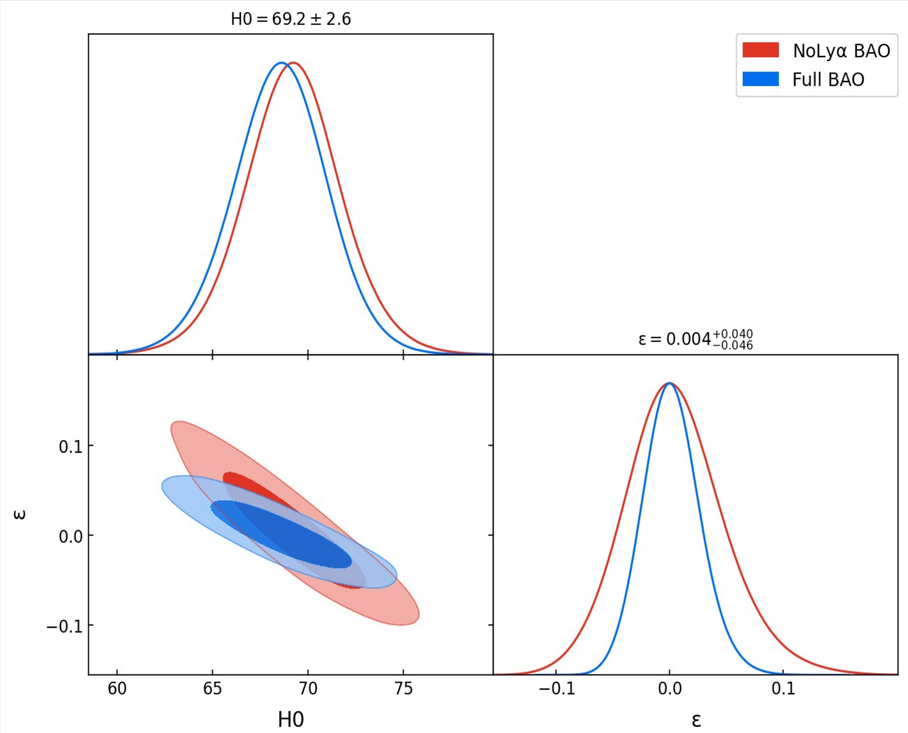
- In this work the same kernel is used for BAO and SNIa

Kernel Comparison



- It is important to asses that the results is stable against different kernel representations
- In this work the same kernel is used for CCH and SNIa

Results: Planck-BAO tension



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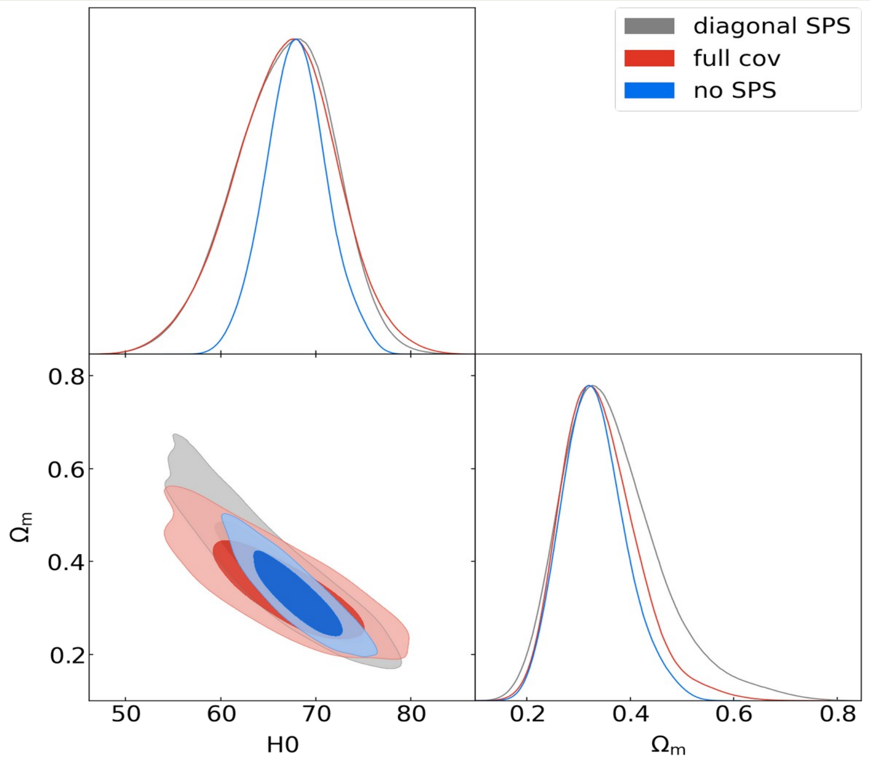
The impact of CCH covariance



- The Hubble constant is not affected by the inclusion of the CCH covariance
- The value of the matter density shift of the 5% from grey to red contours
- The chi-square also shift from 12 to 16, however this is still a factor of two lower than expected