Resolving tensions in cosmology via the modified measures approach to control vacuum energies

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I. Abstract and main references:

Abstract : Introducing modified measures redefines the scalar field potentials while providing spontaneous breaking of scale invariance. In this way one can obtain potentials with two flat regions , one suitable for inflation and the other suitable for the late universe. With two scalar fields the scalar potential can have three flat regions after spontaneous symmetry breaking, one for inflation and the other two for the late universe, showing the possibility of early dark energy, which has been invoked for the resolution of the H0 tension. Other phenomena present in the modified measures theory, like dark energy from fermions the avoidance of the 5^{th} force problem and the justification in terms of modified measure theory of the the phenomenological model of Afshordi et. al. for the resolution the H0 tension will be discussed.

References:

1. Unifying Inflation with early and late Dark Energy in Multi-Fields: Spontaneously broken scale invariant TMT , Eduardo Guendelman, Ramon Herrera, David Benisty, arXiv:2201.06470 [gr-qc], Phys. Rev. D 105, 124035 (2022) and references there, and show that these constructions justify the phenomenological model explained in

2. H0 tension as a hint for a transition in gravitational theory Nima Khosravi, Shant Baghram, Niayesh Afshordi, Natacha Altamirano, arXiv:1710.09366 [astro-ph.CO], Phys. Rev. D 99, 103526 (2019).

II. GRAVITY-MATTER FORMALISM WITH TWO INDEPENDENT NON-RIEMANNIAN VOLUME-FORMS

In this form, the action is given by

$$S = \int d^4x\, \Phi_1(A)\Big[R + L^{(1)}\Big] + \int d^4x\, \Phi_2(B)\Big[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}}\Big] , \tag{1}$$

where the following notations are used:

- The quantities $\Phi_1(A)$ and $\Phi_2(B)$ are two independent non-Riemannian volume-forms, *i.e.*, generally covariant integration measure densities on the underlying space-time manifold and are given by:

$$\Phi_1(A) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu A_{\nu\kappa\lambda}, \quad \Phi_2(B) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda} , \tag{2}$$

The functions $\Phi_{1,2}$ take over the role of the standard Riemannian integration measure density defined as $\sqrt{-g} \equiv \sqrt{-\det \|g_{\mu\nu}\|}$ and it is expressed in terms of the space-time metric $g_{\mu\nu}$.

The functions $R = g^{\mu\nu}R_{\mu\nu}(\Gamma)$ and $R_{\mu\nu}(\Gamma)$ correspond to the scalar curvature and the Ricci tensor

in the first-order (Palatini) formalism,

$$L^{(1)} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi_1\partial_\nu\varphi_1 - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi_2\partial_\nu\varphi_2 - V(\varphi_1,\varphi_2) \quad (3)$$

$$L^{(2)} = U(\varphi_1,\varphi_2) \quad (4)$$

where the scalar potential V is given by

$$V(\varphi_1,\varphi_2) = f_1\, e^{-\alpha_1\varphi_1} + g_1 e^{-\alpha_2\varphi_2}, \quad (5)$$

and the another scalar potential is defined as

$$U(\varphi_1,\varphi_2) = f_2\, e^{-2\alpha_1\varphi_1} + g_2\, e^{-2\alpha_2\varphi_2}, \quad (6)$$

where the quantities f_1, f_2, g_1, g_2,α_1 and α_2 are positive parameters.

- The function $\Phi(H)$ denotes the dual field strength of a third auxiliary 3-index antisymmetric tensor gauge field:

$$\Phi(H) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu H_{\nu\kappa\lambda}\,, \quad (7)$$

We mention the scalar potentials V and U have been chosen in such a way that the action given eq.(1) is invariant under global Weyl-scale transformations:

$$
\begin{aligned}
g_{\mu\nu} \to \lambda g_{\mu\nu} \quad , \quad \Gamma^{\mu}_{\nu\lambda} \to \Gamma^{\mu}_{\nu\lambda} \quad , \quad \varphi_1 \to \varphi_1 + \frac{1}{\alpha_1} \ln \lambda \, , \\
\varphi_2 \to \varphi_2 + \frac{1}{\alpha_2} \ln \lambda , A_{\mu\nu\kappa} \to \lambda A_{\mu\nu\kappa} \quad , \\
B_{\mu\nu\kappa} \to \lambda^2 B_{\mu\nu\kappa} \quad , \quad H_{\mu\nu\kappa} \to H_{\mu\nu\kappa} \, .
\end{aligned}
\tag{8}
$$

Note that this combination is invariant $\alpha_1\varphi_1 - \alpha_2\varphi_2 \to \alpha_1\varphi_1 - \alpha_2\varphi_2$, from eq.(8). Additionally, we observe that the requirement about the global Weyl-scale symmetry (8) uniquely fixes the structure of the non-Riemannian-measure gravity-matter action given by eq.(1).

In the following we will use $\epsilon = 0$ and this case the equations of motion resulting from the variation of (1) w.r.t. affine connection $\Gamma^{\mu}_{\nu\lambda}$, are

$$
\int d^4 x \sqrt{-g} g^{\mu\nu} \left(\frac{\Phi_1}{\sqrt{-g}} \right) \left(\nabla_\kappa \delta\Gamma^{\kappa}_{\mu\nu} - \nabla_\mu \delta\Gamma^{\kappa}_{\kappa\nu} \right) = 0. \tag{9}
$$

Therefore, $\Gamma^{\mu}_{\nu\lambda}$ corresponds to a Levi-Civita connection

$$\Gamma^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\nu\lambda}(\bar{g}) = \frac{1}{2}\bar{g}^{\mu\kappa}\left(\partial_\nu \bar{g}_{\lambda\kappa} + \partial_\lambda \bar{g}_{\nu\kappa} - \partial_\kappa \bar{g}_{\nu\lambda}\right) \; , \quad (10)$$

w.r.t. to the Weyl-rescaled metric $\bar{g}_{\mu\nu}$:

$$\bar{g}_{\mu\nu} = \chi_1 \, g_{\mu\nu} \;\; , \;\; \text{and} \;\; \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}} \; . \quad (11)$$

Einstein frame

Also, from the variation of the action (1) w.r.t. auxiliary tensor gauge fields $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$ and $H_{\mu\nu\lambda}$ yields the equations, we have

$$\partial_\mu \Big[R + L^{(1)}\Big] = 0, \quad \partial_\mu \Big[L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}}\Big] = 0, \qquad \partial_\mu \Big(\frac{\Phi_2(B)}{\sqrt{-g}}\Big) = 0 \; , \quad (12)$$

whose solutions are given by

$$\frac{\Phi_2(B)}{\sqrt{-g}} \equiv \chi_2, \quad R + L^{(1)} = -M_1, \quad L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}} = -M_2. \quad (13)$$

Here the quantities M_1, M_2 and χ_2 are integration constants. However, the constants M_1 and M_2 are arbitrary and dimensional and χ_2 arbitrary and dimensionless.

We mention that the integration constant χ_2 in eq.(13) preserves global Weyl-scale invariance in eq.(8), whereas the appearance of the another integration constants M_1, M_2 signifies dynamical spontaneous breakdown of global Weyl-scale invariance under (8) due to the scale non-invariant solutions in eq.(13).

Also, varying the action (1) w.r.t. $g_{\mu\nu}$ and using relations (13) we have

$$\chi_1\left[R_{\mu\nu}+\frac{1}{2}\left(g_{\mu\nu}L^{(1)}-T^{(1)}_{\mu\nu}\right)\right]=\frac{\chi_2}{2}\left[T^{(2)}_{\mu\nu}+g_{\mu\nu}\ M_2-2R\,R_{\mu\nu}\right] \tag{14}$$

where χ_1 and χ_2 are defined in (11), and the quantities $T^{(1,2)}_{\mu\nu}$ correspond to the energy-momentum tensors of the scalar field Lagrangians with the standard definitions:

$$T^{(1,2)}_{\mu\nu}=g_{\mu\nu}L^{(1,2)}-2\frac{\partial}{\partial g^{\mu\nu}}L^{(1,2)}\ . \tag{15}$$

Now, taking the trace of eq.(14) and using again second relation of eq.(13), we find that the scale factor χ_1 becomes

$$\chi_1=2\chi_2\frac{T^{(2)}/4+M_2}{L^{(1)}-T^{(1)}/2-M_1}\ , \tag{16}$$

where $T^{(1,2)}=g^{\mu\nu}T^{(1,2)}_{\mu\nu}$.

Thus, considering the second relation of eq.(13) together with eq.(14), we obtain the Einstein-like form

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\frac{1}{2}g_{\mu\nu}\left(L^{(1)}+M_1\right)+\frac{1}{2}\left(T^{(1)}_{\mu\nu}-g_{\mu\nu}L^{(1)}\right)+\frac{\chi_2}{2\chi_1}\left[T^{(2)}_{\mu\nu}+g_{\mu\nu}\ M_2\right] \tag{17}$$

In this context, we can bring eqs.(17) into the standard form of Einstein equations for the metric $\bar{g}_{\mu\nu}$, i.e., the Einstein-frame gravity equations

$$R_{\mu\nu}(\bar{g})-\frac{1}{2}\bar{g}_{\mu\nu}R(\bar{g})=\frac{1}{2}T^{\rm eff}_{\mu\nu}, \tag{18}$$

in with the energy-momentum tensor (analogously to (15))

$$T^{\rm eff}_{\mu\nu}=g_{\mu\nu}L_{\rm eff}-2\frac{\partial}{\partial g^{\mu\nu}}L_{\rm eff}, \tag{19}$$

where the effective Einstein-frame scalar field Lagrangian:

$$L_{\rm eff}=\frac{1}{\chi_1}\left\{L^{(1)}+M_1+\frac{\chi_2}{\chi_1}\left[L^{(2)}+M_2\right]\right\}, \tag{20}$$

where $L^{(1,2)}$ represent Lagrangian densities defined as

$$L^{(1)}=\chi_1\left(X_1+X_2\right)-V\quad ,\quad L^{(2)}=U\ , \tag{21}$$

with the potentials V and U as in relations (3)-(4). Also, to write $L_{\rm eff}$ in terms of the Einstein-frame metric $\bar{g}_{\mu\nu}$ we consider the short-hand notation for the kinetic terms

$$X_1\equiv-\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\varphi_1\partial_\nu\varphi_1,\quad X_2\equiv-\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\varphi_2\partial_\nu\varphi_2. \tag{22}$$

By combining eqs.(16) and (19), and taking into account (21), we obtain

$$\chi_1=\frac{2\chi_2\left[U+M_2\right]}{(V-M_1)}\ . \tag{23}$$

From eqs.(23) and (20), we find at the explicit form for the Einstein-frame scalar Lagrangian $L_{\rm eff}$

$$L_{\rm eff} = X_1 + X_2 - U_{\rm eff}(\varphi_1, \varphi_2)\,, \tag{24}$$

in which the effective scalar potential $U_{\rm eff}(\varphi_1, \varphi_2)$ becomes

$$U_{\rm eff}(\varphi_1, \varphi_2) \equiv \frac{(V - M_1)^2}{4\chi_2\left[U + M_2\right]} = \frac{\left(f_1 e^{-\alpha_1\varphi_1} + g_1 e^{-\alpha_2\varphi_2} - M_1\right)^2}{4\chi_2\left[f_2 e^{-2\alpha_1\varphi_1} + g_2 e^{-2\alpha_2\varphi_2} + M_2\right]}\,. \tag{25}$$

We refer that choosing the "wrong" sign of the scalar potential U (Eq.(4)) in the initial non-Riemannian-measure gravity-matter action (1) is necessary to end up with the right sign in the effective potential (25) associated to scalar fields φ_1 and φ_2 in the physical Einstein-frame effective gravity-matter action given by eq.(24). On the other hand, the overall sign of the other initial scalar potential V (Eq.(4)) is in fact irrelevant since changing its sign does not alter the positivity of effective potential given by eq.(25).

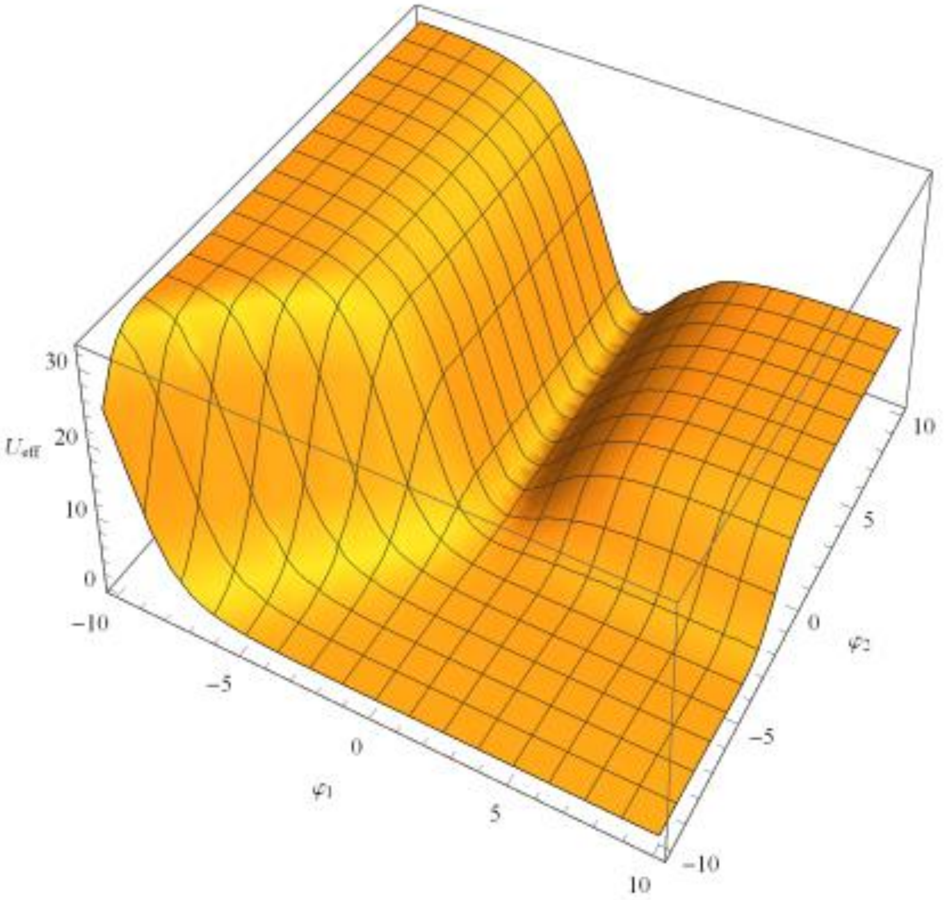

FIG. 1: *The effective potential with three flat regions. One flat region refers to the inflationary phase and the other region refers to dark energy. The third could be another early dark energy phase.Here we have used a positive value for M_1*

III. FLAT REGIONS OF THE EFFECTIVE SCALAR POTENTIAL

A. Flat Regions values

We mention that the important feature of the effective potential U_{eff} (see eq.(25)) is the presence of three infinitely large flat regions – for large positive values of the fields φ_1 and φ_2. For large positive values of φ_1 and φ_2, we have for the effective potential reduces to

$$U_{\text{eff}}(\varphi_1, \varphi_2) \simeq U_{(++)} \equiv \frac{M_1^2}{4\chi_2 M_2} . \tag{26}$$

For the case in which we only have large negative φ_1:

$$U_{\text{eff}}(\varphi_1, \varphi_2) \simeq U_{(\varphi_1 \to -\infty)} \equiv \frac{f_1^2}{4\chi_2 f_2} . \tag{27}$$

In the other flat region in which we only have large negative φ_2:

$$U_{\text{eff}}(\varphi_1, \varphi_2) \simeq U_{(\varphi_2 \to -\infty)} \equiv \frac{g_1^2}{4\chi_2 g_2} . \tag{28}$$

Fig 1 shows a qualitative example for the three fat regions. The flat regions (26), (27) and (28) correspond to the evolution of the early and the late universe, respectively, provided we choose the ratio of the coupling constants in the original scalar potentials versus the ratio of the scale-symmetry breaking integration constants to obey:

$$\frac{M_1^2}{M_2} \gg \frac{f_1^2}{f_2}, \text{ and } \frac{M_1^2}{M_2} \gg \frac{g_1^2}{g_2}, \tag{29}$$

which makes the vacuum energy density of the early universe $U_{(++)}$ much bigger than that of the late universe.

On the other hand, from the cosmological perturbations together with the Planck data [82–87], we have that the first flat region of the effective potential is approximately

$$U_{(++)} \sim M_1^2/\chi_2 M_2 \sim 6\pi^2 \, r \, \mathcal{P}_S \sim 10^{-8} \ , \tag{30}$$

(in units of M_{Pl}^4), where the r denotes the tensor to scalar ratio and $\mathcal{P}_S$ corresponds to the scalar power perturbation. Let us recall that, since we are using units where $G_{\text{Newton}} = 1/16\pi$, in the present case the Planck mass $M_{Pl} = \sqrt{1/8\pi G_{\text{Newton}}} = \sqrt{2}$.

In order to study the dynamics of the universe, we consider that the metric corresponds to the standard flat Friedmann-Lemaitre-Robertson-Walker space-time metric given by:

$$ds^2 = -dt^2 + a^2(t)\Big[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\Big], \tag{31}$$

where $a(t)$ denotes the scale factor. Thus, the associated Friedmann equations (recall the presently used units $G_{\text{Newton}} = 1/16\pi$) result

$$\frac{\ddot{a}}{a} = -\frac{1}{12}(\rho + 3p) \quad , \quad H^2 = \frac{1}{6}\,\rho \quad , \quad H \equiv \frac{\dot{a}}{a} \, , \tag{32}$$

where H is the Hubble parameter. Also, the quantities ρ and p are defined as

$$\rho = \frac{1}{2}\,\dot{\varphi}_1^2 + \frac{1}{2}\,\dot{\varphi}_2^2 + U_{\text{eff}}(\varphi_1, \varphi_2) \, , \tag{33}$$

$$p = \frac{1}{2}\,\dot{\varphi}_1^2 + \frac{1}{2}\,\dot{\varphi}_2^2 - U_{\text{eff}}(\varphi_1, \varphi_2), \tag{34}$$

and denote the total energy density and pressure of the scalar fields $\varphi_1 = \varphi_1(t)$ and $\varphi_2 = \varphi_2(t)$, respectively.

B. Slow Roll approximation

In the context of the slow roll inflation, we can introduce the standard "slow-roll" parameters [27, 28]:

$$\varepsilon \equiv -\frac{\dot{H}}{H^2}, \quad \eta_1 \equiv -\frac{\ddot{\varphi}_1}{H\,\dot{\varphi}_1}\,, \text{ and } \quad \eta_2 \equiv -\frac{\ddot{\varphi}_2}{H\,\dot{\varphi}_2}, \tag{38}$$

and under the slow-roll approximation ε, η_1 and $\eta_2 \ll 1$, thus one ignores the terms with $\ddot{\varphi}_{1,2}$, so that the φ_1, φ_2-equations of motion together with the second Friedmann eq.(32) simplify to:

$$3H\,\dot{\varphi}_1 + \partial U_{\rm eff}/\partial\varphi_1 \simeq 0, \quad 3H\,\dot{\varphi}_2 + \partial U_{\rm eff}/\partial\varphi_2 \simeq 0, \qquad H^2 \simeq \frac{1}{6}U_{\rm eff}\,. \tag{39}$$

Since now the fields φ_1 and φ_2 evolve on the first flat region of $U_{\rm eff}$ for large positive values (26), we can consider that the effective potential during inflationary scenario can be approximated to,

$$U_{eff}(\varphi_1,\varphi_2) \simeq \frac{M_1^2 - 2M_1(f_1 e^{-\alpha_1\varphi_1} + g_1 e^{-\alpha_2\varphi_2})}{4\chi_2 M_2}. \tag{40}$$

Here we have used the expansion of the effective potential given eq.(25) .

In the following we will introduce the number of $e-$folds N defined as $N = \ln(a/a_f)$ where a_f corresponds to the scale factor at the end of the inflation, that is, at the end of inflation $N = 0$. Thus, from eqs.(39) and (40) can be rewritten as,

$$\frac{d\varphi_1}{dN} = \frac{6M_1\alpha_1 f_1\, e^{-\alpha_1\varphi_1}}{[M_2^2 - 2M_1(f_1 e^{-\alpha_1\varphi_1} + g_1 e^{-\alpha_2\varphi_2})]}, \tag{41}$$

and

$$\frac{d\varphi_2}{dN} = \frac{6M_1\alpha_2 g_1\, e^{-\alpha_2\varphi_2}}{[M_2^2 - 2M_1(f_1 e^{-\alpha_1\varphi_1} + g_1 e^{-\alpha_2\varphi_2})]}. \tag{42}$$

Dividing these two equations we get a relation between the scalar fields φ_1 and φ_2 given by,

$$e^{\alpha_1\varphi_1} d\varphi_1 = \frac{f_1\alpha_1}{g_2\alpha_2}\, e^{\alpha_2\varphi_2} d\varphi_2. \tag{43}$$

Notice that the symmetry breaking constants M_1 and M_2 dropped from this equation. The integration of this equation introduces a new constant of integration C

$$e^{\alpha_1\varphi_1} = \frac{f_1\alpha_1^2}{g_1\alpha_2^2}\, e^{\alpha_2\varphi_2} + C. \tag{44}$$

In the following we will consider that the integration constant $C = 0$.

Now, we can redefine two new scalar fields ϕ_1 and ϕ_2, in terms of the scalar fields φ_1 and φ_2, such that

$$\phi_1 = \frac{\alpha_1\varphi_1 - \alpha_2\varphi_2}{\sqrt{\alpha_1^2+\alpha_2^2}}, \quad \text{and} \quad \phi_2 = \frac{\alpha_2\varphi_1 + \alpha_1\varphi_2}{\sqrt{\alpha_1^2+\alpha_2^2}}. \tag{45}$$

Thus, this transformation is orthogonal, $\dot{\phi}_1^2 + \dot{\phi}_2^2 = \dot{\varphi}_1^2 + \dot{\varphi}_2^2$, where ϕ_1 is invariant and ϕ_2 transforms under a scale transformation.

Notice that in this case, the scale invariant combination $\alpha_1\varphi_1 - \alpha_2\varphi_2$ gets determined, which corresponds to fixing the scalar field ϕ_1 defined in (45), this scalar field is scale invariant and is given by

$$\phi_1 = \frac{1}{\sqrt{\alpha_1^2+\alpha_2^2}} \ln\left[\frac{f_1\alpha_1^2}{g_1\alpha_2^2}\right] = \text{constant}. \tag{46}$$

We will not review all the inflationary aspects of this model because this conference emphasizes the late universe, however the inflation is unified with the late universe and the slow roll inflation trajectory determines the vacuum of the late universe!!

Notice that the slow roll trajectory defined by (46) which for a given constant defines a straight line in the (φ_1, φ_2) plane in the top vacuum and for another constant defines another parallel line in the top vacuum. We can then choose the line we desire so as to fall in one of the two lower vacuua from the top vacuum.

V. EVOLUTION TO DARK ENERGY AND DARK MATTER

In this section we will analyze the evolution of the dark energy and dark matter as a remnant of the early universe. After the inflation period has ended there must be a period of particle creation that will produce dark matter as well as ordinary matter, this can be achieved in many different even in the case of one scalar field coupled to different measures [92]. In this section we add now a dark matter particles contribution, defined in a scale invariant form by the matter action defined as

$$S_m = \int (\Phi_1 + b_m e^{\kappa_1 \phi_2} \sqrt{-g}) L_m d^4x, \qquad (66)$$

where b_m is a constant that defines the strength to the coupling of ϕ_2 to $\sqrt{-g}$, coupling to Φ_2 does not give a physically different situation, since still Φ_2 and $\sqrt{-g}$ are proportional. Also, the matter Lagrangian density L_m is given by

$$L_m = -\sum_i m_i \int e^{\kappa_2 \phi_2} \sqrt{g_{\alpha\beta} \frac{dx_i^\alpha}{d\lambda} \frac{dx_i^\beta}{d\lambda}} \frac{\delta^4(x - x_i(\lambda))}{\sqrt{-g}} d\lambda, \qquad (67)$$

here the constants κ_1 and κ_2 satisfy the condition of scale invariance and the quantity m_i denotes the mass parameter of the "*i-th*" particle. This invariace determines the coupling constants to be equal to $\kappa_1 = -\frac{\alpha_1 \alpha_2}{\sqrt{\alpha_1^2 + \alpha_2^2}}$ and $\kappa_2 = -\frac{1}{2}\kappa_1$.

Under these conditions the presence of matter induces a potential for the scalar field ϕ_2 since there is a scalar field dependence ϕ_2 which multiplies a "density of matter"contribution which is ϕ_2 independent. The scalar field ϕ_2 dependence is of the form,

$$(e^{-\frac{1}{2}\kappa_1 \phi_2} \Phi_1 + b_m e^{\frac{1}{2}\kappa_1 \phi_2} \sqrt{-g}). \qquad (68)$$

Such potential is extremized by the condition

$$\Phi_1 - b_m e^{\kappa_1 \phi_2} \sqrt{-g} = 0, \qquad (69)$$

interestingly enough the same condition eliminates all kind of non canonical anomalous effects, like the appearance of pressure in the contribution to the energy momentum from the particles, see section (VII). Also the constraint equation that was used to determine the ratio of the measures Φ_1 and $\sqrt{-g}$ becomes unaffected by the presence of the dust when the condition above (69) is satisfied, see section (VII), so we can use equation (23) and in the late universe, neglecting M_1 and M_2, we obtain an equation that determines ϕ_1. Analogous effects were recognized in a scale invariant two measure model of gravity, matter and one scalar field in [93] to obtain the avoidance of the Fifth Force Problem, which the ϕ_2, the "dilaton", could possibly cause, since it is a massless field. Here the the avoidance of the Fifth Force Problem is also achieved field ϕ_1 adjusts itself so as to satisfy the above equation. In this context, we find that the equation for ϕ_1 is given by

$$2\chi_2 f_2 e^{-\frac{\alpha_1^2}{\sqrt{\alpha_1^2+\alpha_2^2}}\phi_1} + 2\chi_2 g_2 e^{\frac{\alpha_1^2}{\sqrt{\alpha_1^2+\alpha_2^2}}\phi_1} = b_m f_1 + b_m g_1 e^{\sqrt{\alpha_1^2+\alpha_2^2}\phi_1}. \qquad (70)$$

Thus, eq.(70) determines the value of ϕ_1 to be a given constant solving this equation and then the velocity of the scalar field ϕ_1 is zero i.e. $\dot{\phi}_1 = 0$. In order to determine the value of the scalar field ϕ_1 we consider $x = e^{\frac{\alpha_1^2\phi_1}{\sqrt{\alpha_1^2+\alpha_2^2}}}$ then Eq.(70) can be rewritten as

$$2\chi_2 g_2 x^2 - b_m g_1 x^{\frac{2\alpha_1^2+\alpha_2^2}{\alpha_1^2}} - b_m f_1 x + 2\chi_2 f_2 = 0, \qquad (71)$$

interestingly enough, the field ϕ_2 drops from this equation. This is quite reasonable since the field ϕ_2 undergoes a shift under the scale transformation, so if we were to determine the field ϕ_2 , that would correspond to a breaking of scale invariance, but now we are working in a phase with exact scale invariance, since we are neglecting the scale symmetry breaking constants M_1 and M_2. The field ϕ_2 is decoupled from matter, which is a consequence of the elimination of the 5th force .

In order to obtain a solution for the scalar field ϕ_1 from eq.(70) or (71) we consider that for very large value of ϕ_1 or equivalently $x \to \infty$ the dominate terms of eq.(71) are

$$2\chi_2 g_2 x^2 - b_m g_1 x^{\frac{2\alpha_1^2+\alpha_2^2}{\alpha_1^2}} \sim 0, \quad \text{then} \quad x \sim \left(\frac{2\chi_2 g_2}{g_1 b_m}\right)^{(\alpha_1/\alpha_2)^2}, \qquad (72)$$

where for consistency, we must choose the quantity $(\chi_2 g_2/g_1 b_m) \to \infty$. Here the value of the scalar field ϕ_1 at this point is

$$\phi_{1(+)} \sim \frac{\sqrt{\alpha_1^2+\alpha_2^2}}{\alpha_2^2} \ln\left[\frac{2\chi_2 g_2}{f_1 b_m}\right]. \qquad (73)$$

Now in the region in which the scalar field $\phi_1 \to -\infty$ or $x \to 0$ we have that the dominant terms are

$$-b_m f_1 x + 2\chi_2 f_2 \sim 0, \quad \text{and} \quad x \sim \left(\frac{2\chi_2 f_2}{f_1 b_m}\right) \to 0, \qquad (74)$$

and the value of the scalar field at this point is

$$\phi_{1(-)} \sim \frac{\sqrt{\alpha_1^2+\alpha_2^2}}{\alpha_1^2} \ln\left[\frac{2\chi_2 f_2}{f_1 b_m}\right]. \qquad (75)$$

In what follows of this section we study the dynamics of the dark energy and as defined before, with the equations for the ratio of the two measures obtained in the absence of dark matter (23) still being valid, so we

VII. CONDITIONS FOR CANONICAL DUST BEHAVIOR BEYOND THE BACKGROUND CASE

In our previous considerations we have only considered cases where the scalar fields and the dust are distributed homogeneously in the Universe and we have also chosen the scalar field ϕ_1 by the observation that the presence of matter induces a potential for the scalar field ϕ_2 since there is a scalar field dependence ϕ_2 which multiplies a "density of matter"contribution which is ϕ_2 independent and the result of such minimization lead us to a value of ϕ_1 defined by eq. (69), which in turn lead us to a dust behavior for our model of point particles coupled in a scale invariant fashion. Here we will go a bit deeper, following the method studied in [93] for a single scalar field

$$n(\vec{x}) = \sum_i \frac{1}{\sqrt{-g_{(3)}}} \delta^{(3)}(\vec{x} - \vec{x}_i(\lambda)), \qquad (104)$$

where $g_{(3)} = \det(g_{kl})$. We transform to the Einstein frame where this transformation causes the transformation of the particle density

$$\bar{n}(\vec{x}) = (\chi_1)^{-3/2} n(\vec{x}). \qquad (105)$$

The gravitational equations take the standard GR form

$$G_{\mu\nu}(\bar{g}_{\alpha\beta}) = \frac{\kappa}{2} T^{eff}_{\mu\nu}, \qquad (106)$$

where $G_{\mu\nu}(\bar{g}_{\alpha\beta})$ is the Einstein tensor in the Riemannian space-time with the metric $\bar{g}_{\mu\nu}$. The components of the effective energy-momentum tensor are as follows

$$T^{eff}_{00} = \left(\dot{\phi}_1^{\,2} - \bar{g}_{00} X_1\right) + \left(\dot{\phi}_2^{\,2} - \bar{g}_{00} X_2\right) + \bar{g}_{00}\left[U_{eff}(\phi_1, \phi_2; \chi_1, M_1, M_2) + \frac{3\chi_1 e^{-\frac{\kappa_1 \phi_2}{2}} + b_m e^{\frac{\kappa_1 \phi_2}{2}}}{2\sqrt{\chi_1}} m\,\bar{n}\right], \qquad (107)$$

and

$$T^{eff}_{ij} = (\phi_{1,k}\phi_{1,l} - \bar{g}_{kl} X_1) + (\phi_{2,k}\phi_{2,l} - \bar{g}_{kl} X_2) + \bar{g}_{kl}\left[U_{eff}(\phi_1, \phi_2, \chi_1, M_1, M_2) + \frac{\chi_1 e^{-\frac{\kappa_1 \phi_2}{2}} - b_m e^{\frac{\kappa_1 \phi_2}{2}}}{2\sqrt{\chi_1}} m\,\bar{n}\right]. \qquad (108)$$

$$\frac{1}{\sqrt{-\bar{g}}}\partial_\mu \left[\sqrt{-\bar{g}}\bar{g}^{\mu\nu}\partial_\nu \phi_2\right] + \frac{\partial U_{eff}}{\partial \phi_2} = \kappa_1 \frac{\chi_1 e^{-\frac{\kappa_1 \phi_2}{2}} - b_m e^{\frac{\kappa_1 \phi_2}{2}}}{2\sqrt{\chi_1}} m\,\bar{n}.$$

In the above equations, the scalar field χ_1 is determined as a function $\chi_1(\phi_1, \phi_2, \bar{n})$ by means of the following constraint:

$$\frac{\chi_1 (M_1 + V) - 2\chi_2 (U + M_2)}{(\chi_1)^2} = \frac{\chi_1 e^{-\frac{\kappa_1 \phi_2}{2}} - b_m e^{\frac{\kappa_1 \phi_2}{2}}}{2\sqrt{\chi_1}} m\,\bar{n}. \qquad (112)$$

Therefore General Relativity is restored for HIGH MATTER DENSITIES.

In summary a "miracle " takes place here, the same combination $\chi_1 e^{-\frac{\kappa_1 \phi_2}{2}} - b_m e^{\frac{\kappa_1 \phi_2}{2}}$ appears in the right hand side of equations (112), (111) and in the anomalous pressure contribution produces by the dust displayed in (108). The vanishing of $\chi_1 e^{-\frac{\kappa_1 \phi_2}{2}} - b_m e^{\frac{\kappa_1 \phi_2}{2}}$ was also obtained in our simplified considerations in eq. (69) from the condition of minimization of the matter induced potential for ϕ_2, which (111) expresses in its full generality.

The justification in terms of modified measure theory of the the phenomenological model of Afshordi et. al. for the resolution the H0 tension

Nima Khosravi, Shant Baghram, Niayesh Afshordi, Natacha Altamirano, arXiv:C[astro-ph.CO], Phys. Rev. D 99, 103526 (2019).

III. ÜΛCDM COSMOLOGY

In this section, we propose a cosmological model which is a natural consequence of über-gravity model. According to Fig. 2, we see that the über-gravity leads to a very simple model for the gravity as

$$\text{Gravity} \simeq \begin{cases} R = R_0 & \rho < \rho_{\text{über}} \\ \Lambda\text{CDM} & \rho > \rho_{\text{über}} \end{cases} \quad (5)$$

which we call üΛCDM. In this scenario, if matter density $\rho > \rho_{\text{über}}$ then it sees pure GR with a cosmological constant, while if $\rho < \rho_{\text{über}}$ then the metric is constrained to have constant Ricci scalar i.e. R_0, which is a free parameter in our model presented in Eq.(4). We should mention that the above argument does not depend on the radiation content of the universe since the radiation is trace-free and has no contribution to our conclusion based on Fig. 2.

The sharp transition in our model is representative of a fam-

General Relativity is restored for HIGH MATTER DENSITIES

THEIR PROPOSED ACTION FOR LOW DENSITY

scalar-tensor action representing the cosmological era after the transition in über-gravity:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[\xi\,(R - R_0) - \lambda\right] + \mathcal{L}_m\,, \quad (10)$$

(EOM) for this action are:

$$-\frac{g_{ab}}{2}\xi(R-R_0)+\frac{\lambda}{2}g_{ab}+\xi R_{ab}-[\nabla_a\nabla_b-g_{ab}\Box]\xi=8\pi G T_{ab} \quad (11)$$

$$R - R_0 = 0\,. \quad (12)$$

Their formulation for low density is exactly a two measure theory where the modfied measure couples to R and the equation R= M= constant

is obtained obtained naturally.

Also there is no need to suddenly change theory for high density, since the modified measure theory automatically reverts to General Relativity for high density.

Thank you for your attention !!!