Modified gravity with 2 d.o.f. as a tool to address tensions in cosmology

1. Introduction
2. Type-I & -II minimally modified gravity
3. Massive gravity
4. Summary

Shinji Mukohyama (YITP, Kyoto U)

Introduction to Antonio’s talk
“How to address tensions in cosmology by modified gravity with 2 d.o.f.”

INTRODUCTION

Why modified gravity?

- Can we address **mysteries in the universe**?
  Dark energy, dark matter, inflation, big-bang singularity, cosmic magnetic field, and **tensions**
- Help constructing a **theory of quantum gravity**?
  Superstring, Horava-Lifshitz, etc.
- Do we really **understand GR**?
  One of the best ways to understand something may be to break (modify) it and then to reconstruct it.
- …

Proto-type of modified gravity: scalar-tensor theory

- Metric $g_{\mu\nu}$ + scalar field ϕ
- Jordan (1955), Brans & Dicke (1961), Bergmann (1968), Wagoner (1970), …
- Most general scalar-tensor theory of gravity with 2^{nd} order covariant EOM: Horndeski (1974)
- DHOST theories beyond Horndeski: Langlois & Noui (2016)
- U-DHOST theories beyond DHOST: DeFelice, Langlois, Mukohyama, Noui & Wang (2018)

Unwanted scalar d.o.f.

- Scalar-tensor theory propagates 3 (or more) physical d.o.f.: 2 tensor + 1 scalar
- Dynamics of scalar d.o.f. strongly constrained by solar-system experiments, binary pulsar measurements, etc.
- A possible way out: screening mechanism
- Another possibility: theory without scalar d.o.f., i.e. modified gravity with 2 d.o.f.

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Minimally modified gravity (MMG)

TYPE-I & -II MINIMALLY MODIFIED GRAVITY

not yet MMG

Type-I & type-II modified gravity

Katsuki Aoki, Antonio De Felice, Chunshan Lin, SM and Michele Oliosi, JCAP 01 (2019) 017

- Jordan (or matter) frame

$$I = \frac{1}{2}\int d^4x\sqrt{-g^{\mathrm{J}}}\,\left[\Omega^2(\phi)\,R[g^{\mathrm{J}}] + \cdots\right] + I_{\mathrm{matter}}[g^{\mathrm{J}}_{\mu\nu};\mathrm{matter}]$$

- Einstein-frame $g^{\mathrm{E}}_{\mu\nu} = \Omega^2(\phi)g^{\mathrm{J}}_{\mu\nu}$ K.Maeda (1989)

$$I = \frac{1}{2}\int d^4x\sqrt{-g^{\mathrm{E}}}\,\left[R[g^{\mathrm{E}}] + \cdots\right] + I_{\mathrm{matter}}[\Omega^{-2}(\phi)g^{\mathrm{E}}_{\mu\nu};\mathrm{matter}]$$

- Do we call this GR? No. This is a modified gravity because of **non-trivial matter coupling → type-I**
- There are more general scalar tensor theories where there is **no Einstein frame → type-II**

not yet MMG

Type-I & type-II modified gravity

Katsuki Aoki, Antonio De Felice, Chunshan Lin, SM and Michele Oliosi, JCAP 01 (2019) 017

- **Type-I:**
  There exists an Einstein frame
  Can be recast as GR + extra d.o.f. **+ matter, which couple(s) non-trivially**, by change of variables
- **Type-II:**
  No Einstein frame
  Cannot be recast as GR + extra d.o.f. + matter by change of variables

Type-I minimally modified gravity (MMG)

Katsuki Aoki, Chunshan Lin and SM, PRD98 (2018) 044022

- **# of local physical d.o.f. = 2**
- There exists an Einstein frame
- Can be recast as GR **+ matter, which couple(s) non-trivially**, by change of variables
- **The most general change of variables = canonical tr.**
- Matter coupling just after canonical tr. → breaks diffeo → 1st-class constraint downgraded to 2nd-class → leads to extra d.o.f. in phase space → inconsistent
- Gauge-fixing after canonical tr. → splits 1st-class constraint into pair of 2nd-class constraints
- **Matter coupling after canonical tr. + gauge-fixing → a pair of 2nd-class constraints remain → consistent**

Type-II minimally modified gravity (MMG)

- **# of local physical d.o.f. = 2**
- **No Einstein frame**
- Cannot be recast as GR + matter by change of variables
- **Are there such theories? Yes!**
- Example: Minimal theory of massive gravity (MTMG)
  Antonio De Felice and SM, PLB752 (2016) 302; JCAP1604 (2016) 028; PRL118 (2017) 091104
  See also arXiv:2206.03338 for a class of extended MTMG
- Other examples:
  Antonio De Felice, Andreas Doll and SM, *JCAP* 09 (2020) 034
  Katsuki Aoki, Mohammad Ali Gorji and SM, *PLB*810 (2020) 135843

Type-II minimally modified gravity (MMG)

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VCDM: a theory of type-II MMG

Antonio De Felice, Andreas Doll and Shinji Mukohyama [*JCAP* 09 (2020) 034]

- Simple construction with **a free function V(ϕ)**
  1. Hamiltonian of GR with 3+1 decomposition
  2. Canonical tr to a new frame
  3. Add a cosmological const in the new frame
  4. Gauge fix
  5. Inverse canonical tr back to the original frame
  6. Legendre tr to Lagrangian
  7. Add minimally-coupled matter fields (including CDM)

$$\mathcal{L} = N\sqrt{\gamma}\left[\frac{M_{\rm P}^2}{2}\left(R + K_{ij}K^{ij} - K^2 - 2V(\phi)\right) - \frac{\lambda^i_{\rm gf}}{N} M_{\rm P}^2\,\partial_i\phi - \frac{3M_{\rm P}^2\lambda^2}{4} - M_{\rm P}^2\lambda\,(K+\phi)\right]$$

- No Einstein frame, close relation to cuscuton

  [Katsuki Aoki, Francesco Di Filippo and SM, JCAP 05 (2021) 071] Afshordi-Chung-Geshnizjani 2007

  Wider range of applicability than cuscuton, e.g. GR solutions are allowed

  [Antonio De Felice, Kei-ichi Maeda, S.M. and Masroor C. Pookkillath, Phys.Rev.D 106 (2022) 2, 024028]
- V(ϕ) reconstructed from H(z) of FLRW background
- c_{GW} = 1, no extra dof
- May reduce H_0 tension → Antonio's talk

  [Antonio De Felice, Masroor C. Pookkillath and SM, PLB 816 (2021) 136201]

VCCDM: Weaker gravity for DM

Antonio De Felice and Shinji Mukohyama [JCAP 04 (2021) 018]

- Simple construction with **free functions $f_0(\phi)$ & $f_1(\phi)$**
  1. Hamiltonian of GR with 3+1 decomposition
  2. Canonical tr to a new frame
  3. Add a cosmological const **& dark matter** in the new frame
  4. Gauge fix
  5. Inverse canonical tr back to the original frame
  6. Legendre tr to Lagrangian
  7. Add minimally-coupled matter fields (no dark matter here)

$$\mathcal{L} = N\sqrt{\gamma}\left[\frac{M_{\rm P}^2}{2}\left(R + K_{ij}K^{ij} - K^2 - 2V(\phi)\right) - \frac{\lambda^i_{\rm gf}}{N}M_{\rm P}^2\,\partial_i\phi - \frac{3M_{\rm P}^2\lambda^2}{4} - M_{\rm P}^2\lambda\,(K+\phi)\right]$$

SM metric: $g_{\mu\nu}dx^\mu dx^\nu = -N^2dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$

DM metric: $g^{\rm eff}_{\mu\nu}dx^\mu dx^\nu = -\frac{N^2}{f_1^2}dt^2 + \frac{\gamma_{ij}}{f_0}(dx^i + N^i dt)(dx^j + N^j dt)$

- $f_0(\phi)$ & $f_1(\phi)$ reconstructed from $H(z)$ & $G_{DM}(z)/G_N$
- $c_{GW} = 1$, $G_{SM} = G_N$, no extra dof
- May reduce H_0 & S_8 tensions? → Antonio's talk

$$V(\phi) \equiv \frac{\bar{\Lambda}}{f_1 f_0^{3/2}}$$

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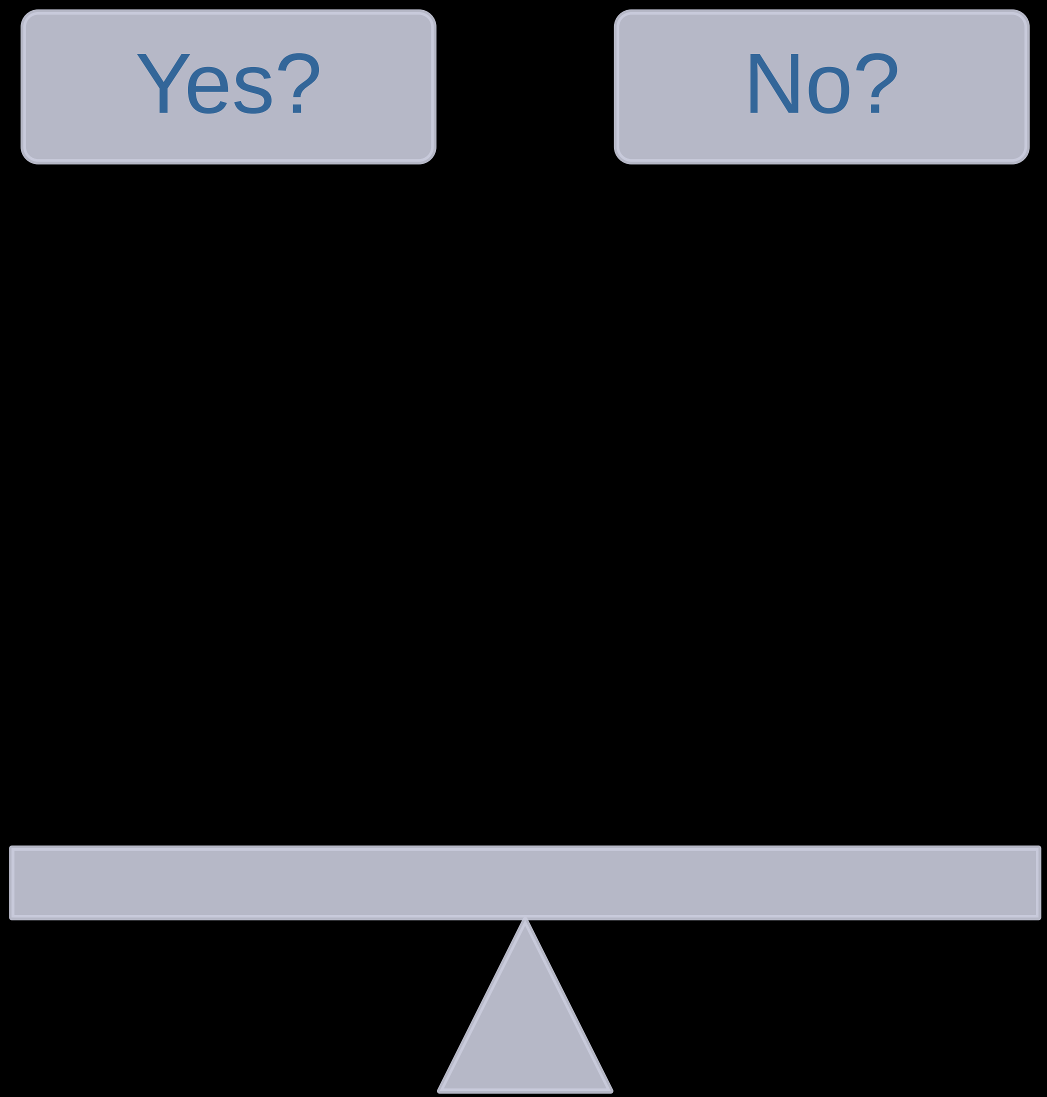
MASSIVE GRAVITY

Massive gravity in a nutshell

Simple question: Can graviton have mass?

May lead to acceleration without dark energy



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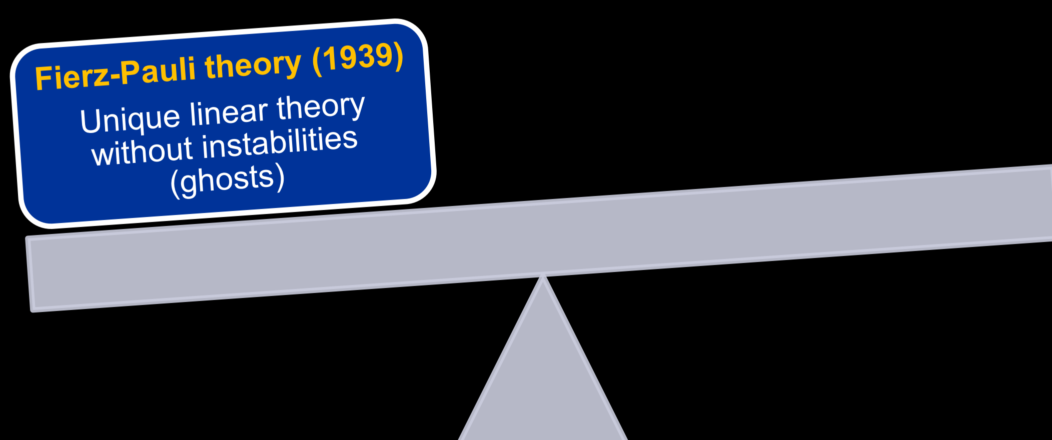

Massive gravity in a nutshell

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Yes?

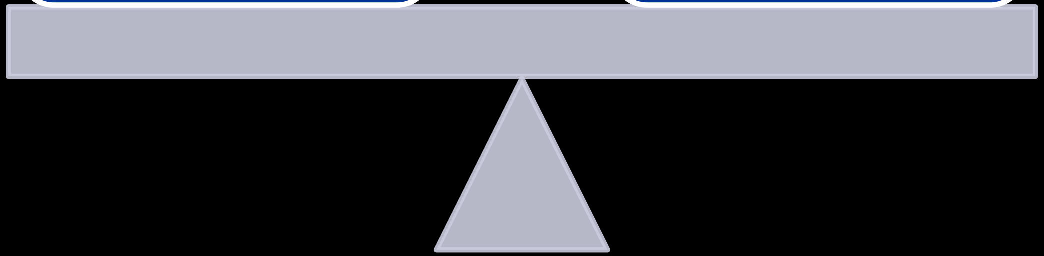
No?

Fierz-Pauli theory (1939)

Unique linear theory without instabilities (ghosts)

van Dam-Veltman-Zhakharov discontinuity (1970)

Massless limit ≠ General Relativity



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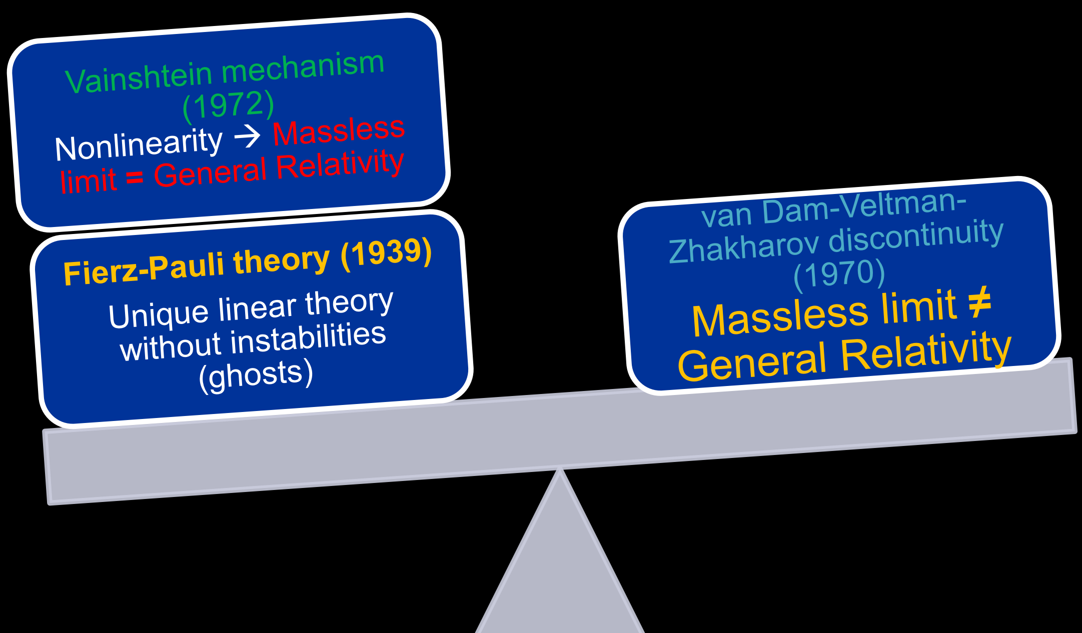


Massive gravity in a nutshell

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?

Vainshtein mechanism (1972)
Nonlinearity → Massless limit = General Relativity

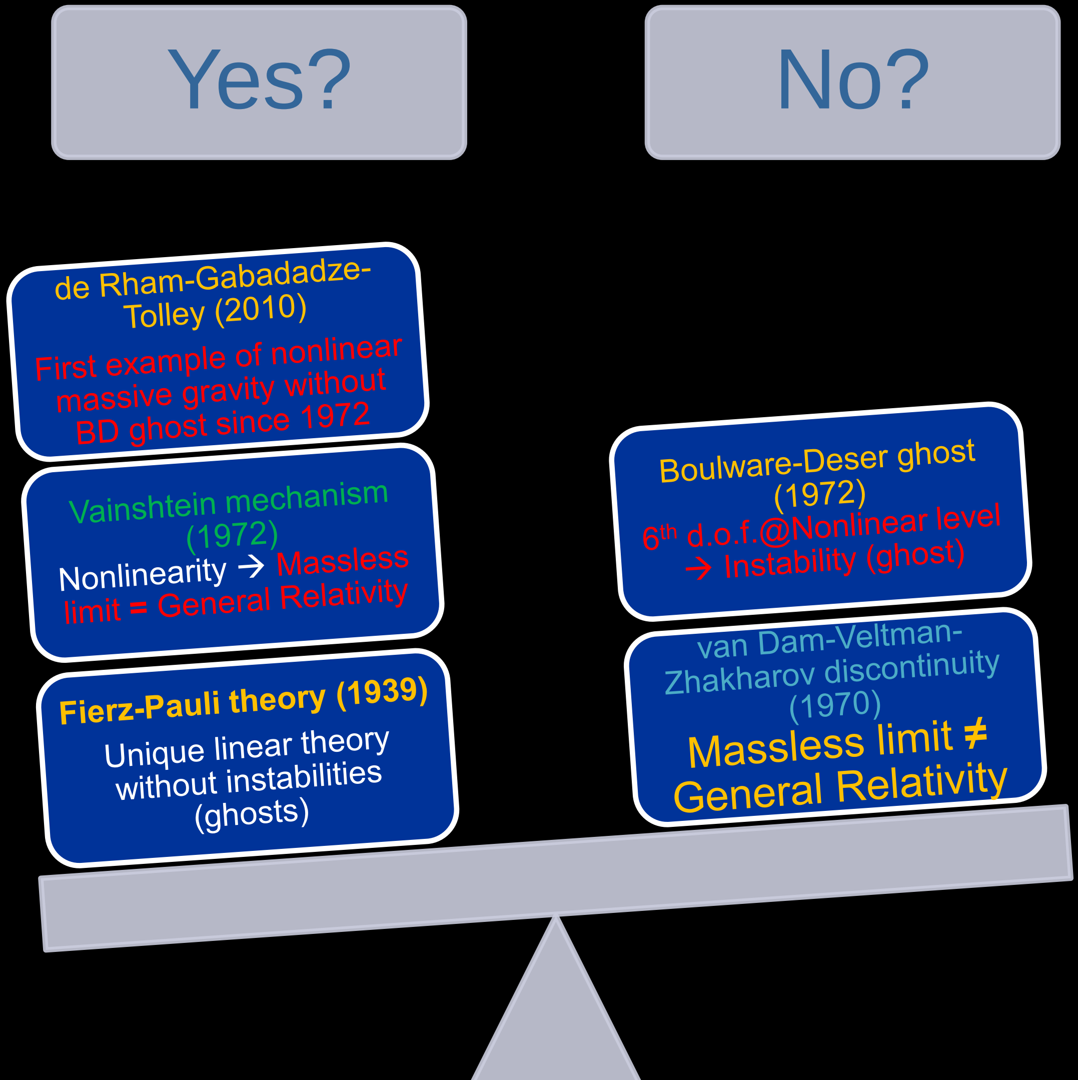
Fierz-Pauli theory (1939)
Unique linear theory without instabilities (ghosts)

Boulware-Deser ghost (1972)
6^{th} d.o.f.@Nonlinear level → Instability (ghost)

van Dam-Veltman-Zhakharov discontinuity (1970)
Massless limit ≠ General Relativity

Massive gravity in a nutshell

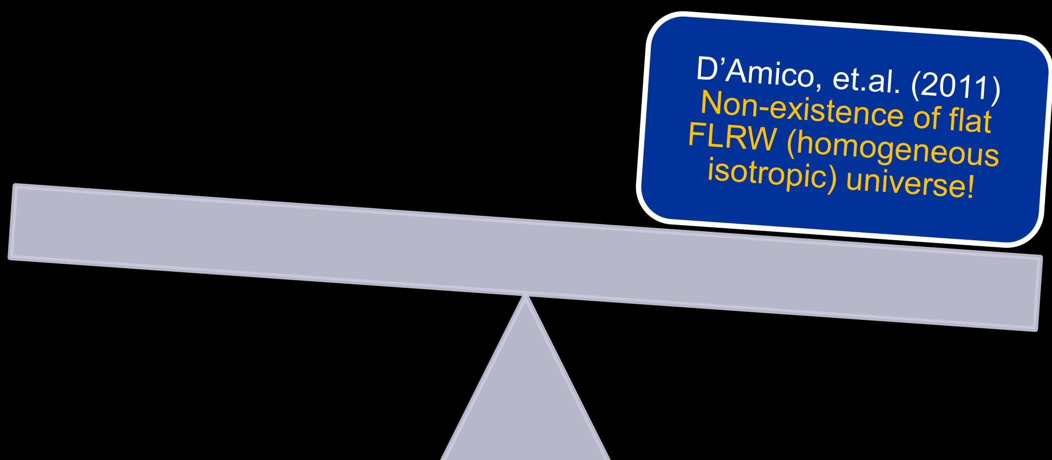
Simple question: Can graviton have mass?
May lead to acceleration without dark energy



Cosmological solutions in nonlinear massive gravity

Good?

Bad?



Cosmological solutions in nonlinear massive gravity

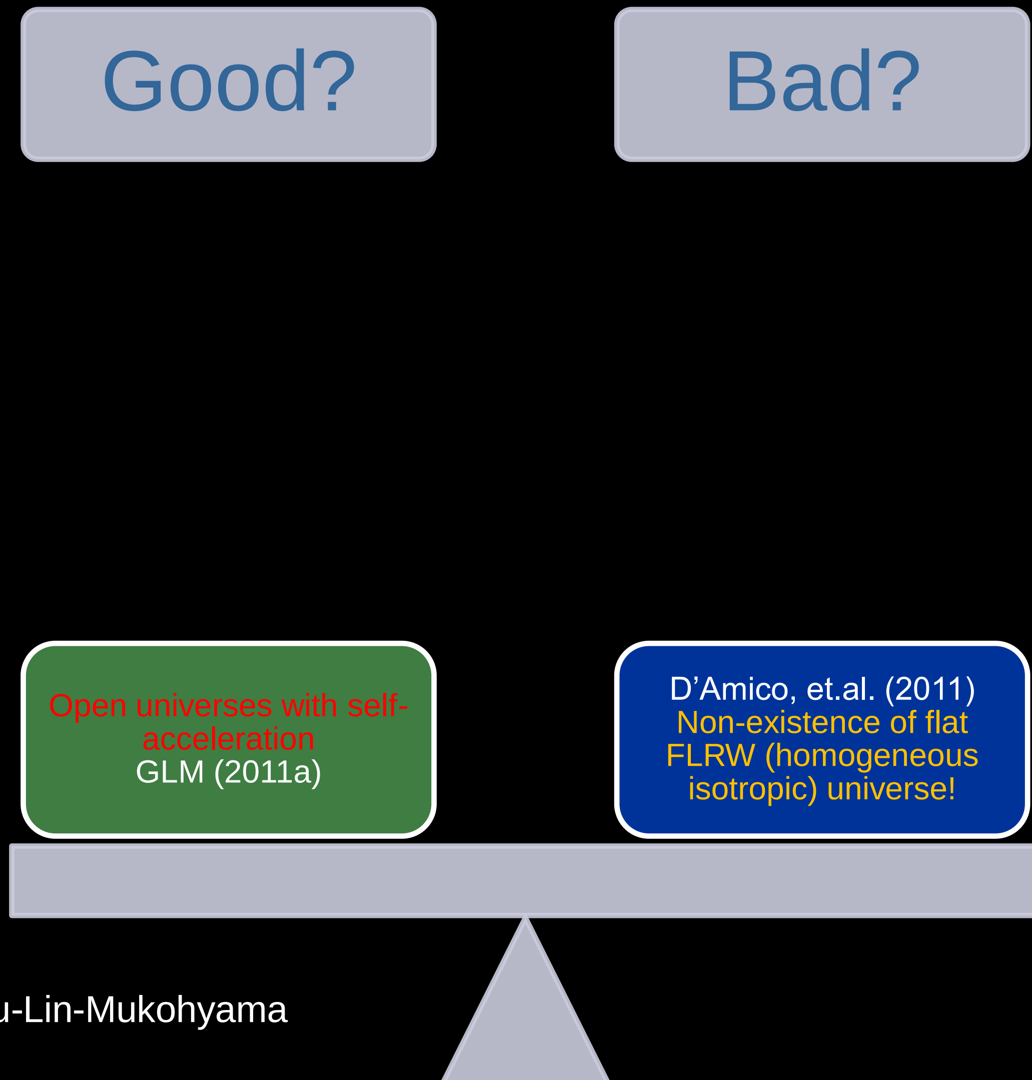
Good?

Bad?

Consistent Theory found in 2010 but

No Viable Cosmology?

Cosmological solutions in nonlinear massive gravity

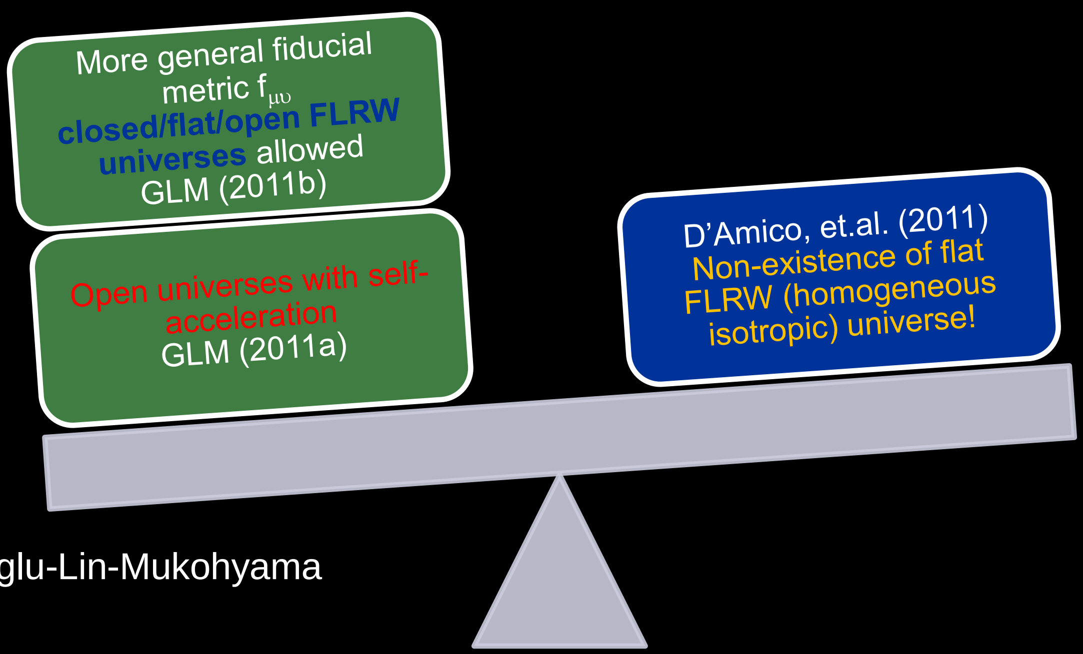


GLM = Gumrukcuoglu-Lin-Mukohyama

Cosmological solutions in nonlinear massive gravity

Good?

Bad?



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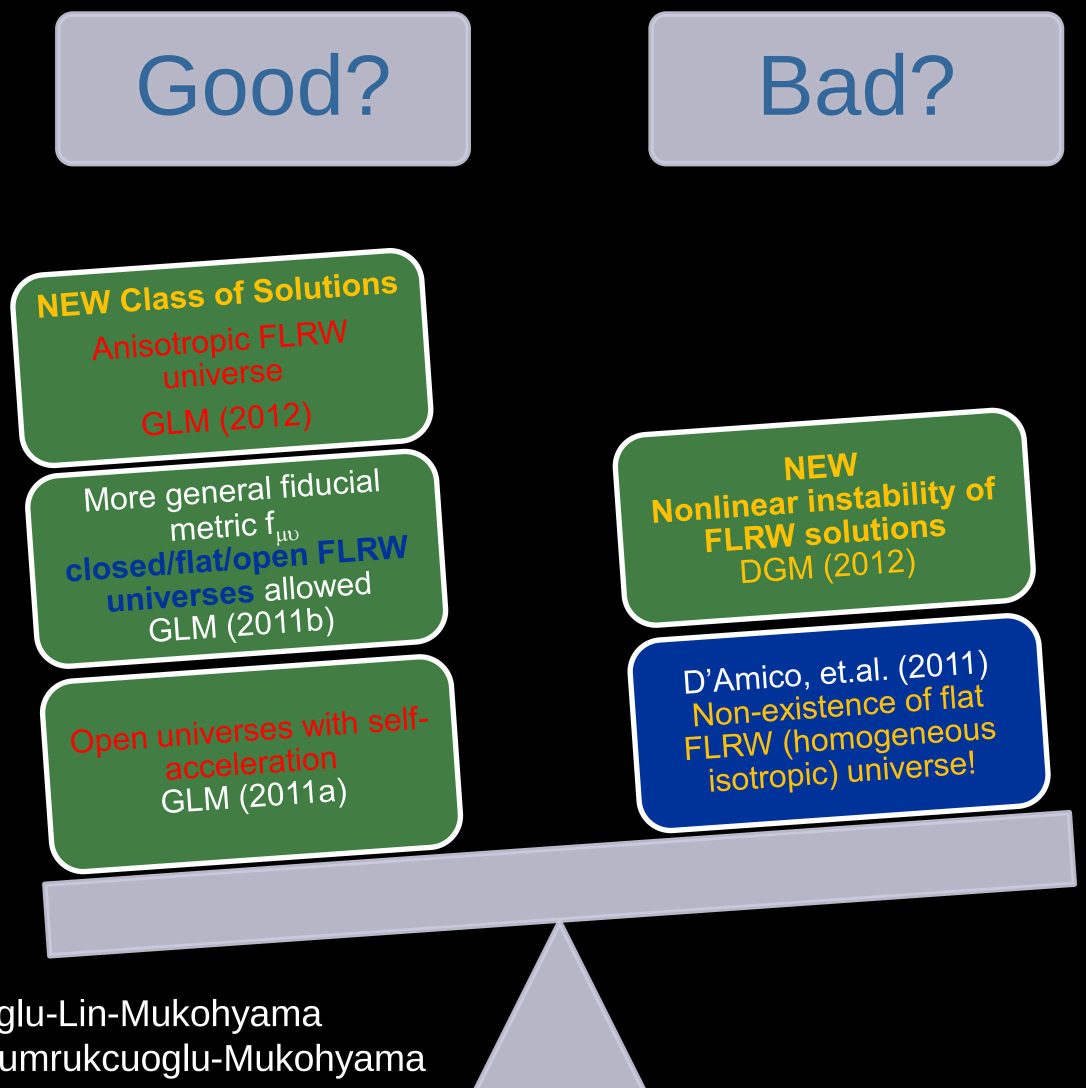
Cosmological solutions in nonlinear massive gravity

Good?	Bad?
More general fiducial metric $f_{\mu\nu}$ **closed/flat/open FLRW universes** allowed GLM (2011b)	**NEW Nonlinear instability of FLRW solutions** DGM (2012)
Open universes with self-acceleration GLM (2011a)	D'Amico, et.al. (2011) Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

Cosmological solutions in nonlinear massive gravity



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Minimal theory of massive gravity (MTMG)

De Felice & Mukohyama, PLB752 (2016) 302; JCAP1604 (2016) 028

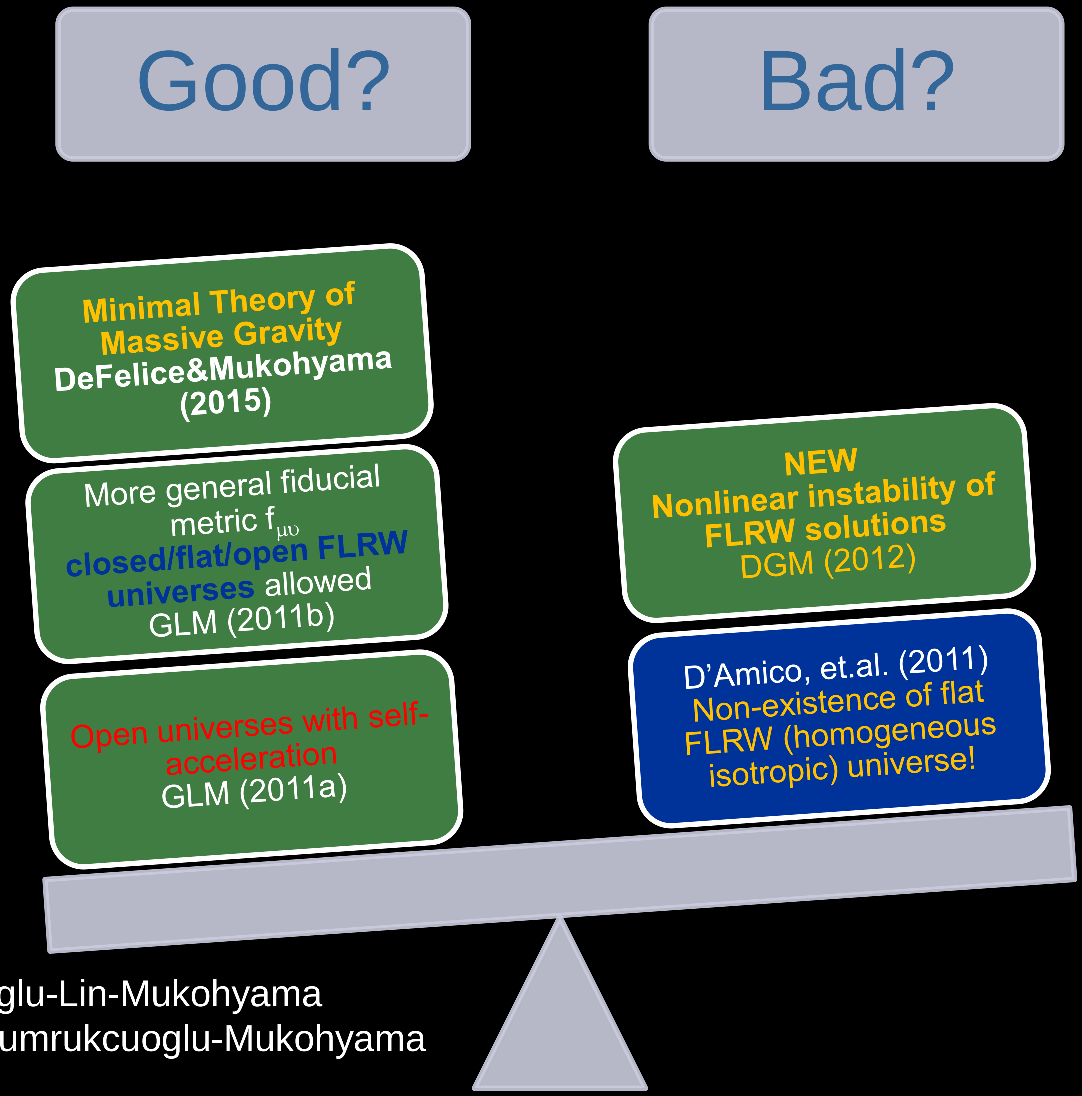
- 2 physical dof only = massive gravitational waves
- exactly same FLRW background as in dRGT
- no BD ghost, no Higuchi ghost, no nonlinear ghost
- positivity bound does not apply

Three steps to the Minimal Theory

1. Fix local Lorentz to realize ADM vielbein in dRGT
2. Switch to Hamiltonian
3. Add 2 additional constraints

(It is easy to go back to Lagrangian after 3.)

Cosmological solutions in nonlinear massive gravity



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Extended Minimal theories of massive gravity

De Felice & Mukohyama & Pookkillath,
arxiv:2206.03338

- 2 physical dof only = massive gravitational waves
- FLRW background more general than dRGT
- no BD ghost, no Higuchi ghost, no nonlinear ghost
- positivity bound does not apply
- better $G_{eff}(z)$ than original MTMG → Antonio's talk

Three steps to the Extended Minimal Theory

1. Start with GR Hamiltonian
2. Add general graviton mass term
3. Add 2 additional constraints

(It is easy to go to Lagrangian after 3.)

SUMMARY

Summary of MMG part

- Minimal # of d.o.f. in modified gravity = 2
  can be saturated → **minimally modified gravity (MMG)**
- **MMG evades many observational/experimental constraints**
- Type-I MMG: $^{\exists}$Einstein frame
  Type-II MMG: no Einstein frame
- All type-I MMG are obtained as
  GR + canonical tr. + gauge-fixing (+ adding matter)
  Rich phenomenology: w_{DE}, G_{eff}, etc.
- Examples of type-II MMG: VCDM and its extension (VCCDM)
  GR + canonical tr. + cc & DM + gauge-fixing + inverse canonical tr.
  $V(f)$ & $f_1(f)$ reconstructed from $H(z)$ & $G_{eff}(z)$
  May reduce H_0 & S_8 tension?
- Type-Ia&-IIa MMG: $c_{gw} = c_\gamma$
  Type-Ib&-IIb MMG: $c_{gw} \neq c_\gamma$

Phenomenologically important &
Theoretically useful refinement

Summary of massive gravity part

- dRGT theory is free from BD ghost but its cosmology suffers from strong coupling and ghost instability.
  Stable cosmology requires either (i) new class of cosmological solutions or (ii) extended theories.
- MTMG and extended MTMG (also MTBG) provide stable nonlinear completion of massive gravity cosmology.
  → Testing grounds for gravitational phenomena that can be probed by GWs.
  > Blue-tilted & amplified primordial GW
  > Graviton Oscillation
  > Massive graviton DM and GW
  and more …
  May address tensions? → Antonio's talk

Modified gravity with 2 d.o.f. may be useful as a tool to address tensions in cosmology

4 concrete examples
VCDM, VCCDM, MTMG, extended MTMG

Enjoy Antonio's talk
"How to address tensions in cosmology by modified gravity with 2 d.o.f."

Thank you!

Backup slides

Refined classification

[arXiv: 2103.15044 w/ Katsuki Aoki & Francesco Di Filippo]

	Having Einstein frame	No Einstein frame
$c_T^2(k^2) = 1$	**Type-Ia** $g_{\mu\nu} \propto \tilde{g}_{\mu\nu}$	**Type-IIa**
$c_T^2(k^2) \neq 1$	**Type-Ib** $g_{\mu\nu} \not\propto \tilde{g}_{\mu\nu}$	**Type-IIb** MTMG: $\omega^2 = k^2 + m^2$ 4DEGB: $\omega^2 = k^2 + k^4/\Lambda^2$

Implication of GW170817 on gravity theories @ late time

$X = -\partial^\mu\phi\partial_\mu\phi$

- $|(c_{gw} - c_\gamma)/ c_\gamma| < 10^{-15}$
- Horndeski theoy (scalar-tensor theory with 2nd-order eom):
  Among 4 free functions, $G_4(\phi,X)$ & $G_5(\phi,X)$ are strongly constrained. Still $G_2(\phi,X)$ & $G_3(\phi,X)$ are free.
  $G_3(\phi,X)$ may be constrained due to GW-DE interactions [Creminelli, Tambalo, Vernizzi, Yingcharoenrat 2019]
- Generalized Proca theory (vector-tensor theory):
  Among 6 (or more) free functions, $G_4(X)$ & $G_5(X)$ are strongly constrained. Still $G_2(X,F,Y,U)$, $G_3(X)$, $G_6(X)$, $g_5(X)$ are free.

$X = -A^\mu A_\mu$

- Horava-Lifshitz theory (renormalizable quantum gravity):
  The coefficient of $R^{(3)}$ is strongly constrained
  → IR fixed point with $c_{gw} = c_\gamma$? How to speed up the RG flow?
- Ghost condensation (EFT of scalar-tensor theory in Minkowski/de Sitter):
  No additional constraint
- Massive gravity (simplest modification of GR):
  Upper bound on graviton mass ≈ 10^{-22}eV
  Much weaker than the requirement from acceleration

c.f. "All" gravity theories (including general relativity):
The cosmological constant is strongly constrained ≈ 10^{-120}.

Refined classification

[arXiv: 2103.15044 w/ Katsuki Aoki & Francesco Di Filippo]

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Proof of the absence of Einstein frame in cuscuton/VCDM

1. GWs → cuscuton/VCDM is of type-Ia or type-IIa
2. GR + conformal-type canonical tr. → most general type-Ia MMG
3. Vacuum Bianchi-I universes → cuscuton/VCDM is not of type-Ia
4. 1 & 3 → cuscuton/VCDM is of type-IIa, thus no Einstein frame

Refined classification

[arXiv: 2103.15044 w/ Katsuki Aoki & Francesco Di Filippo]

	Having Einstein frame	No Einstein frame
$c_T^2(k^2) = 1$	**Type-Ia** $g_{\mu\nu} \propto \tilde{g}_{\mu\nu}$	**Type-IIa** Cuscuton/VCDM: $\omega^2 = k^2$
$c_T^2(k^2) \neq 1$	**Type-Ib** $g_{\mu\nu} \not\propto \tilde{g}_{\mu\nu}$	**Type-IIb** MTMG: $\omega^2 = k^2 + m^2$ 4DEGB: $\omega^2 = k^2 + k^4/\Lambda^2$

Proof of the absence of Einstein frame in cuscuton/VCDM

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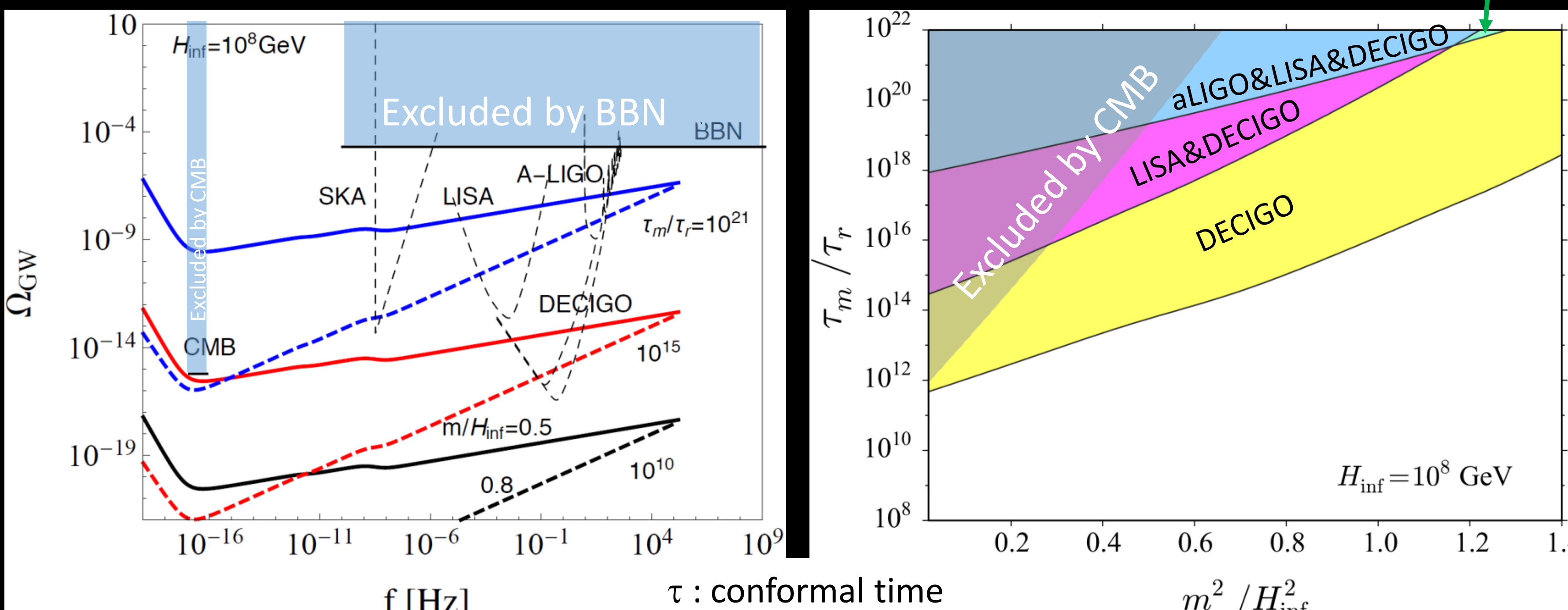
Blue-tilted & amplified primordial GW from MTMG

Fujita, Kuroyanagi, Mizuno, Mukohyama, PLB789 (2019) 215

Fujita, Mizuno, Mukohyama, JCAP 01 (2020) 023

- Simple extension: $c_i \rightarrow c_i(\phi)$ with $\phi = \phi(t)$
- m large until t_m ($t_{reh} < t_m < t_{BBN}$) but small after t_m
  cf. no Higuchi bound in MTMG
- Suppression of GW in IR due to large m → **blue spectrum**
- $\rho_{GW} \propto a^{-3}$ for $t_{reh} < t < t_m$ → **amplification relative to GR**

aLIGO& DECIGO



Minimal theory of bigravity (MTBG)

De Felice, Larrouturou, Mukohyama, Oliosi, arXiv:2012.01073.

- 4 physical dof only = massless & massive GWs
- exactly same FLRW backgrounds as in HRBG
- no BD ghost, no Higuchi ghost, no strong coupling

Three steps to the Minimal Theory

1. Fix local Lorentz to realize ADM vielbeins in HRBG
2. Switch to Hamiltonian
3. Add 4 (= 5-1) additional constraints carefully

(It is easy to go back to Lagrangian after 3.)

The very first example of completely stable & cosmologically viable theory of nonlinear bigravity. A testing ground for gravitational phenomena, e.g. graviton oscillation, that can be probed by GWs.

PHYSICAL REVIEW D **94,** 024001 (2016)

Massive gravitons as dark matter and gravitational waves

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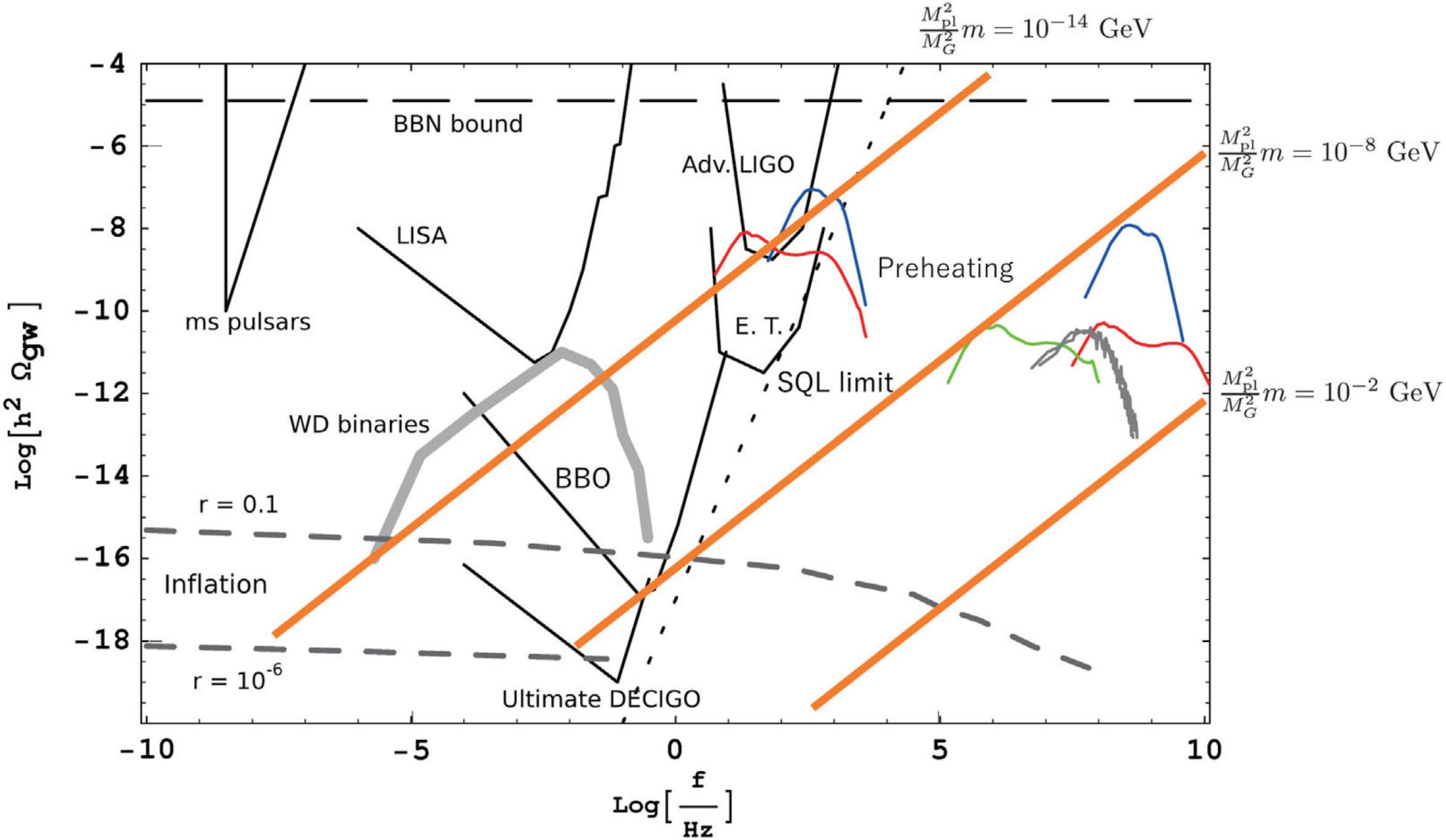
[2]*Center for Gravitational Physics, Yukawa Institute for Theoretical Physics, Kyoto University, 606-8502 Kyoto, Japan*

[3]*Kavli Institute for the Physics and Mathematics of the Universe (WPI), UTIAS, The University of Tokyo, Kashiwa, Chiba 277-8583, Japan*

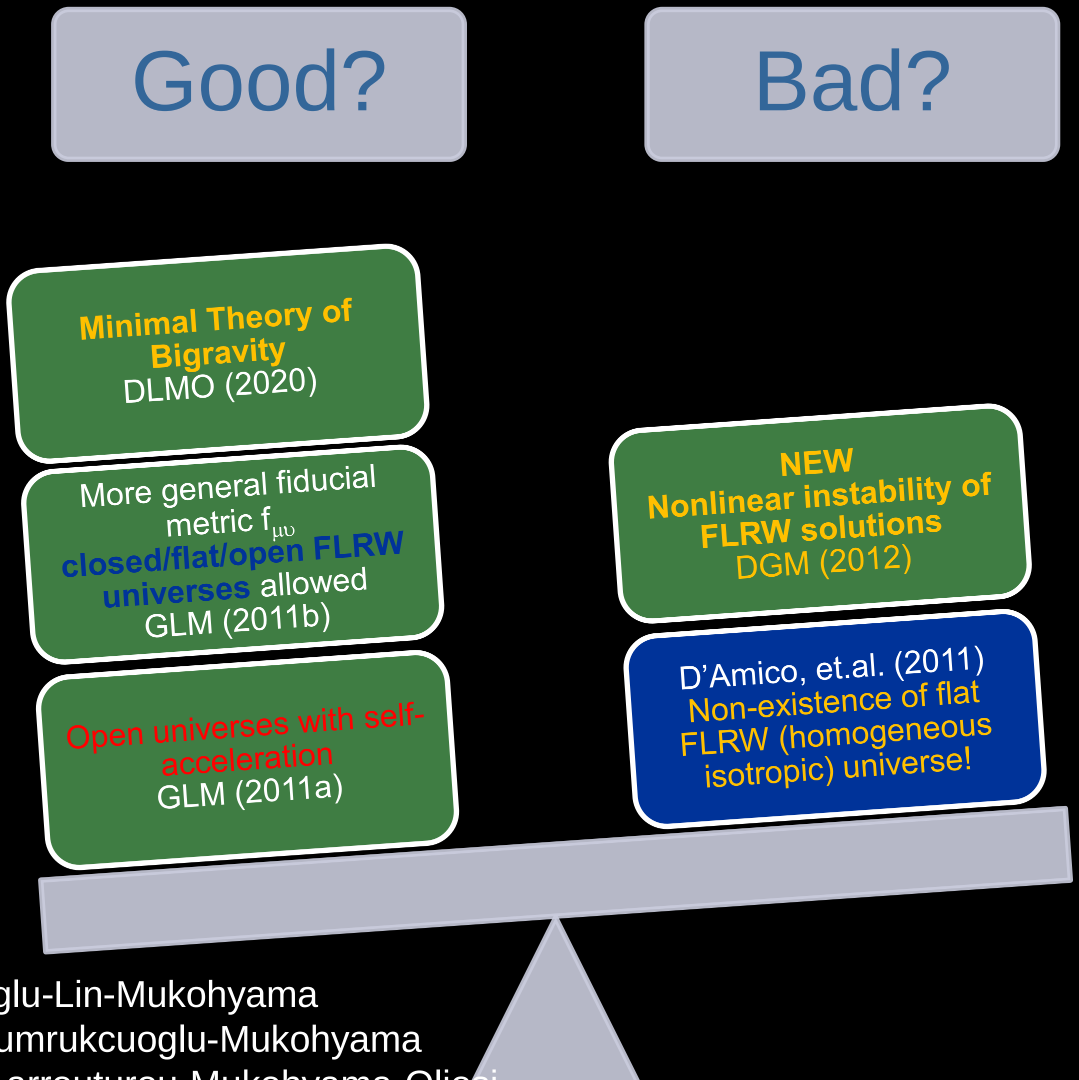
(Received 2 May 2016; published 1 July 2016)

We consider the possibility that the massive graviton is a viable candidate for dark matter in the context of bimetric gravity. We first derive the energy-momentum tensor of the massive graviton and show that it indeed behaves as that of dark matter fluid. We then discuss a production mechanism and the present abundance of massive gravitons as dark matter. Since the metric to which ordinary matter fields couple is a linear combination of the two mass eigenstates of bigravity, production of massive gravitons, i.e., the dark matter particles, is inevitably accompanied by generation of massless gravitons, i.e., the gravitational waves. Therefore, in this scenario some information about dark matter in our Universe is encoded in gravitational waves. For instance, if LIGO detects gravitational waves generated by the preheating after inflation, then the massive graviton with the mass of ~0.01 GeV is a candidate for dark matter.



$\frac{M_{\rm pl}^2}{M_G^2} m = 10^{-14}$ GeV
$\frac{M_{\rm pl}^2}{M_G^2} m = 10^{-8}$ GeV
$\frac{M_{\rm pl}^2}{M_G^2} m = 10^{-2}$ GeV
BBN bound
Adv. LIGO
LISA
Preheating
ms pulsars
E. T.
SQL limit
WD binaries
BBO
r = 0.1
Inflation
r = 10^{-6}
Ultimate DECIGO
$\mathrm{Log}\left[h^2\ \Omega_{gw}\right]$
$\mathrm{Log}\left[\frac{f}{\mathrm{Hz}}\right]$
-4
-6
-8
-10
-12
-14
-16
-18
-10
-5
0
5
10

Cosmological solutions in nonlinear massive gravity



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