How to address tensions in cosmology by modified gravity with 2 dof

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Tensions in Cosmology, Corfu, Sept. 9, 2022

[with A. Doll, K-i. Maeda, S. Mukohyama, M. Pookkillath]

[follow-up of enjoyable Shinji M's talk]

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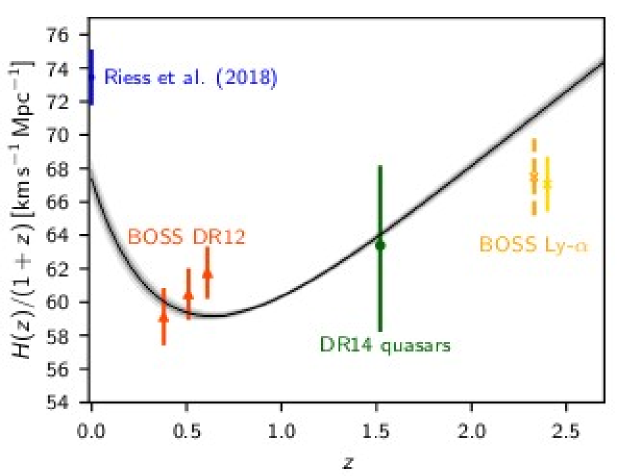
Experimental puzzle

- Different experiments give different values for today's value of the Hubble parameter in the context of ΛCDM



Planck coll., 2018

- The experiments & hypothesis are correct: so there exists H(z) which fits all the correct experiments and ΛCDM is ruled out
- There is no H(z) which can fit all the experiments: experiments or/& [e.g. hom./iso.] hypothesis are wrong

Which H(z)?

- In the context of a dyn. PF fluid component $\rho + P > 0 \wedge c_s^2 = \frac{\dot{p}}{\dot{\rho}} > 0$ as to avoid ghost or gradient instabilities
- Previous implies $\dot{H} < 0$
- Also need to keep control/screen extra degrees of freedom [solar system constraints, fifth force experiments, etc.]
- How to have an instability free arena on which to test H(z)?

A second cosmological tension?

- The universe might be too thin
- Early time & late time experiments seem to show tension in the parameter S_8
- $S_8 = \sigma_8 \left(\Omega_m / 0.3 \right)^{0.5} = 0.759^{+0.024}_{-0.021}$ From KiDS-1000, arXiv:2007.15633
- Here tension seems to be in the growth of perturbation
- Could both of cosmology's big puzzles share a single fix?

VCDM Lagrangian

[ADF, Doll, Mukohyama 2020 + Shinji M's talk]

- Introduce the Lagrangian (via Legendre transf)

$$\mathscr{L}_{tot}=\frac{M_P^2}{2}N\sqrt{\gamma}[K_{ij}K^{ij}-K^2+R-2V(\phi)]-\frac{3M_P^2}{4}N\sqrt{\gamma}\lambda^2-M_P^2 N\sqrt{\gamma}\lambda(K+\phi)-\sqrt{\gamma}\lambda_{gf}D^2\phi+\mathscr{L}_{SM}$$

- Here we have introduced 2 fcs $f_0=f_{,\phi},\ \ f_1=f_{,\psi}, but\ \ V\propto\tilde{\Lambda}/(f_1 f_0^{3/2})$
- Theory with only 2 dof in gravity [ϕ does not propagate]
- No need to screen extra dof! [e.g. GR static BHs solutions]
- Not equivalent to cuscuton theory

[ADF, Doll, Larruturou, Mukohyama, 2021

[ADF, Maeda, Mukohyama, Pookkillath 2022

Reconstructing V(ϕ)

- We have the choice of one free function $V(\phi(t))$
- 2nd Friedmann eq. $\frac{d\phi}{dN}=\frac{3}{2}\frac{\rho+P}{H M_P^2}$, $\rho=\sum_I \rho_I-\frac{3M_P^2 K}{a^2}$, $P=\sum_I P_I+\frac{M_P^2 K}{a^2}$, $N=\ln(a/a_0)$
- Given a H(N)>0 and ρ(N), P(N) and ρ+P>0, integrate to find N=N(ϕ), [where ρ, P is sum over all SM matter fields]
- From 1st Friedmann $V=\frac{\phi^2}{2}+\frac{\rho(N(\phi))}{M_P^2}$ reconstruct V
- Arena to attack H0-tension: only requirement H>0

VCDM phenomenology

[ADF, Maeda, Pookkillath, Mukohyama (2022)]

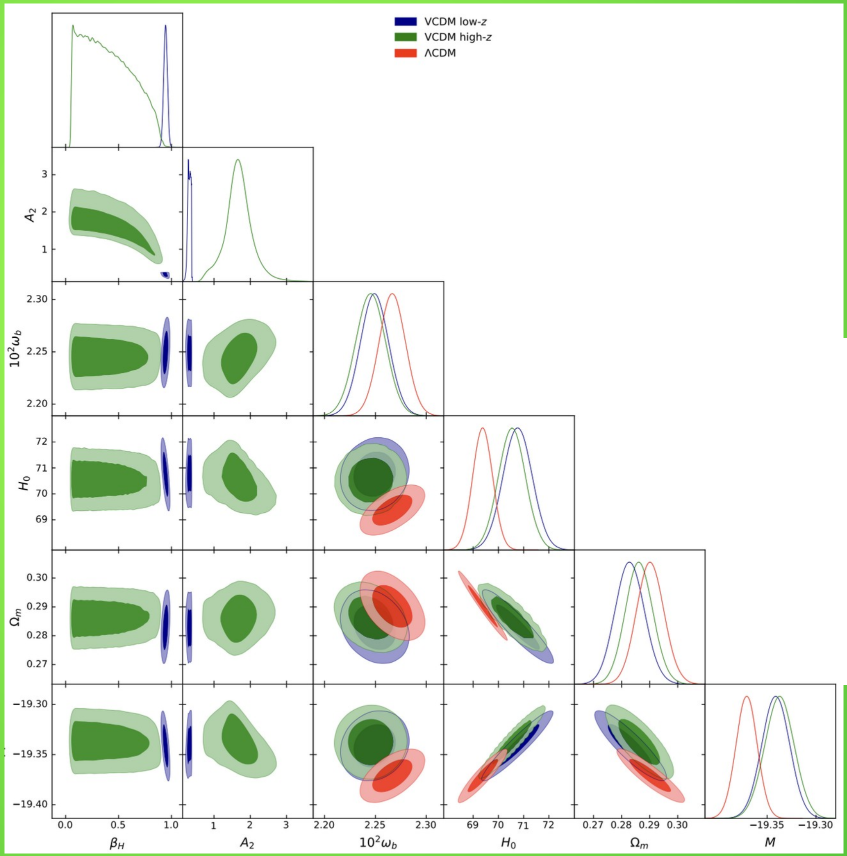
- Any positive H(z) can be given [no ghost or gradient inst]. H-arena: possible to address H0 tension
- “Wild„ possible modifications of H(z) [at any redshift]
- If experiments & hypothesis are correct there exists H(z) fitting all the data
- If such an H(z) exists (and H(z) > 0) then VCDM solves the H0-tension
- If not, namely there is no H(z) fitting the data, then experiments &/or hypothesis are wrong

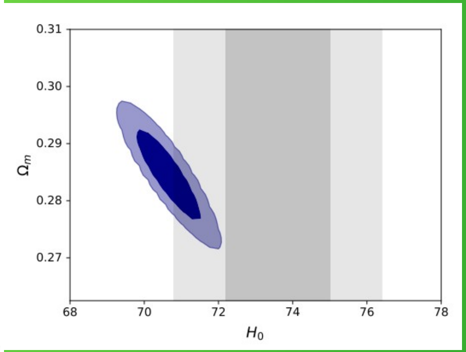
- Given $$\frac{H^2}{H_0^2} = \Omega_{\rm m0}(1+z)^3 + \Omega_{\rm r0}(1+z)^4 + (1-\beta_H^2)\frac{1+\tanh\left(\frac{A_2-z}{A_3}\right)}{1+\tanh\left(\frac{A_2}{A_3}\right)} + \beta_H^2\left(1-\frac{\Omega_{\rm m0}}{\beta_H^2}-\frac{\Omega_{\rm r0}}{\beta_H^2}\right)$$
- Leading to two effective Ccs (early & late times)

Experiments	ΛCDM	VCDM low−z ($\Delta\chi^2$)	VCDM high−z ($\Delta\chi^2$)
Planck_highl_TTTEEE	2354.01	2349.56 (4.45)	2347.03 (6.98)
Planck_lowl_EE	397.37	395.92 (1.45)	395.83 (1.54)
Planck_lowl_TT	22.16	22.89 (−0.73)	23.25 (−1.09)
Pantheon	1027.28	1031.64 (−4.36)	1027.31 (−0.03)
bao_boss_dr12	4.79	5.38 (−0.59)	9.27 (−4.48)
bao_smallz_2014	3.14	5.31 (−2.17)	4.58 (−1.44)
absolute_M	11.47	6.57 (4.9)	6.85 (4.62)
H_0 (SH0ES)	8.54	3.31 (5.23)	4.34 (4.2)
H_0 (H0LiCOW)	4.69	1.88 (2.81)	2.43 (2.26)
H_0 (MEGAMASER)	2.25	1.04 (1.21)	1.29 (0.96)
Total	3835.71	3823.50 (12.21)	3822.19 (12.51)

Table I: Comparison of effective χ^2 between VCDM and ΛCDM for individual data sets.

	VCDM low−z	VCDM high−z	ΛCDM
Parameters	95% limits	95% limits	95% limits
β_H	$0.947^{+0.031}_{-0.037}$	≤ 0.80	−
A_2	$0.295^{+0.086}_{-0.052}$	$1.82^{+0.69}_{-0.93}$	−
$10^2\omega_{\rm b}$	$2.254^{+0.022}_{-0.032}$	$2.240^{+0.034}_{-0.024}$	$2.270^{+0.023}_{-0.029}$
$\tau_{\rm reio}$	$0.054^{+0.018}_{-0.013}$	$0.053^{+0.017}_{-0.014}$	$0.061^{+0.015}_{-0.017}$
n_s	$0.9677^{+0.0771}_{-0.0769}$	$0.9664^{+0.008}_{-0.009}$	$0.9736^{+0.0066}_{-0.0076}$
$10^{10}A_s$	$3.043^{+0.036}_{-0.028}$	$3.044^{+0.032}_{-0.032}$	$3.052^{+0.031}_{-0.036}$
H_0	$70.83^{+1.07}_{-1.13}$	$70.49^{+1.11}_{-1.09}$	$69.40^{+0.76}_{-0.8}$
$\Omega_{\rm m}$	$0.282^{+0.011}_{-0.009}$	$0.2865^{+0.0096}_{-0.0097}$	$0.2899^{+0.0101}_{-0.0092}$
M	$-19.34^{+0.03}_{-0.03}$	$-19.34^{+0.03}_{-0.03}$	$-19.37^{+0.02}_{-0.02}$





[ADF, Pookkillath, Mukohyama (2021)]

Addressing H_0 & S_8 (I): VCCDM

- Extend VCDM construction for dark matter
- Introduce standard matter fields with standard actions
- Dark sector consisting of CC and dust in the new frame
- Doing so DM breaks 4d diffeo
- SM doesn't

Lagrangian of VCCDM

[ADF, Mukohyama 2021 + Shinji M's talk]

- If DM is a pressureless dust

$$\mathscr{L}_{tot} = \frac{M_P^2}{2} N \sqrt{\gamma}\left[K_{ij}K^{ij} - K^2 + R - 2V(\phi)\right] - \frac{3M_P^2}{4} N\sqrt{\gamma}\lambda^2 - M_P^2 N\sqrt{\gamma}\lambda(K+\phi) - \sqrt{\gamma}\lambda^i_{gf}\partial_i\phi$$
$$+\mathscr{L}_{SM}[g_{\mu\nu}, \chi_{SM}] + \mathscr{L}_{DM}[g^{eff}_{\mu\nu}, \chi_{DM}]$$

- For instance DM action via a scalar field

$$N_{eff} = \frac{N}{f_1}, N^i_{eff} = N^i, \quad \gamma^{eff}_{ij} = \frac{\gamma_{ij}}{f_0}, \qquad \mathscr{L}_{DM} = -\frac{1}{2}\sqrt{-g^{eff}}\,\rho\left(g^{\mu\nu}_{eff}\partial_\mu\sigma\,\partial_\nu\sigma + 1\right)$$

General framework

[ADF, S. Mukohyama, 2021]

- Standard matter coupled with ADM metric

$$g_{\mu\nu} dx^\mu dx^\nu = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

- 3D scalar ϕ and two functions $f_1(\phi), f_2(\phi)$
- Introduce effective metric felt by DM

$$g^{eff}_{\mu\nu} dx^\mu dx^\nu = -\frac{N^2}{f_1^2} dt^2 + \frac{\gamma_{ij}}{f_0}(dx^i + N^i dt)(dx^j + N^j dt)$$

Similar to coupled DE: Wetterich (1995); Amendola (2000); Damour Gibbons, Gundlach (1990); Fuzfa, Alimi (2006); Amendola, Tsujikawa (2020)

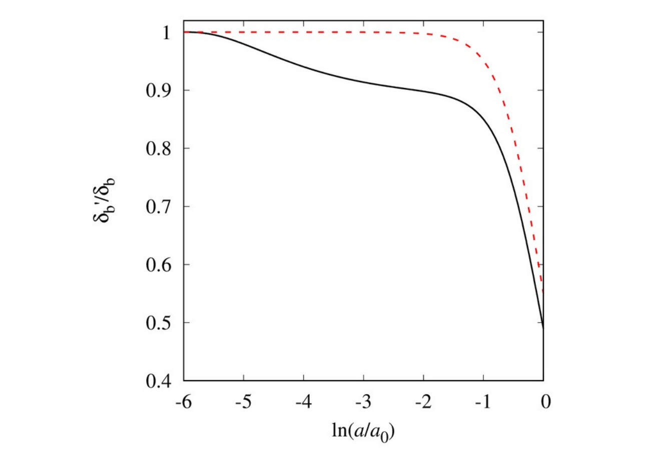
Reconstructions of H & Geff

- We have the choice of two free functions $f_0(\phi(t)), f_1(\phi(t))$
- Consider fixing the functions in terms of cosmological observables: H & Geff

$$\delta_b'' + \frac{(Ha)'}{Ha}\delta_b' - \frac{4\pi G_N}{H^2}(\rho_c\delta_c + \rho_b\delta_b) = 0,$$

$$\delta_c'' + \frac{(Ha f_1/f_0)'}{Ha f_1/f_0}\delta_c' - \frac{4\pi G_{eff}}{H^2}(\rho_c\delta_c + \rho_b\delta_b) = 0,$$

$$\frac{G_{eff}}{G_N} = \frac{f_0}{f_1^2}$$



VCCDM prospectives

- Baryon-baryon gravitational interactions unchanged
- Breaking of equivalence principle for DM
- Similar to phenomenology of DE-DM interactions
- But without extra dof
- Possible studies linear & non-linear (N-body simulations)

Addressing H_0 & S_8 (2): (ext)MTMG

[ADF, S. Mukohyama, 2016], [ADF, S. Mukohyama, M. Pookkillath, 2022]

- Maybe massive graviton responsible for the tensions?
- 2 physical dof only = massive gravitational waves
- FLRW is unstable for dRGT: no stable FLRW cosmology

  de Rham, Gabadadze, Tolley 2010 ADF, Gumrukcuoglu, Mukohyama, 2012
- no BD ghost, no Higuchi ghost, no nonlinear ghost, if:
- 1. Fix local Lorentz to realize ADM vielbein in dRGT
- 2. Switch to Hamiltonian
- 3. Add 2 additional constraints

Cosmology of MTMG (self acc branch)

- Background constraint $\left(c_3+2c_2X+c_1X^2\right)\left(\dot{X}+NHX-MH\right)=0, \quad X=\tilde{a}/a$

$$X=\frac{-c_2\pm\sqrt{c_2^2-c_1c_3}}{c_1},$$

$$3M_P^2H^2=\frac{m^2M_P^2}{2}\left(c_4+3c_3X+3c_2X^2+c_1X^3\right)+\rho$$

- Λ_{eff} from graviton mass term (even when $c_4 = 0$)
- Scalar/vector equal to ΛCDM
- Time-dependent mass for the gravity waves

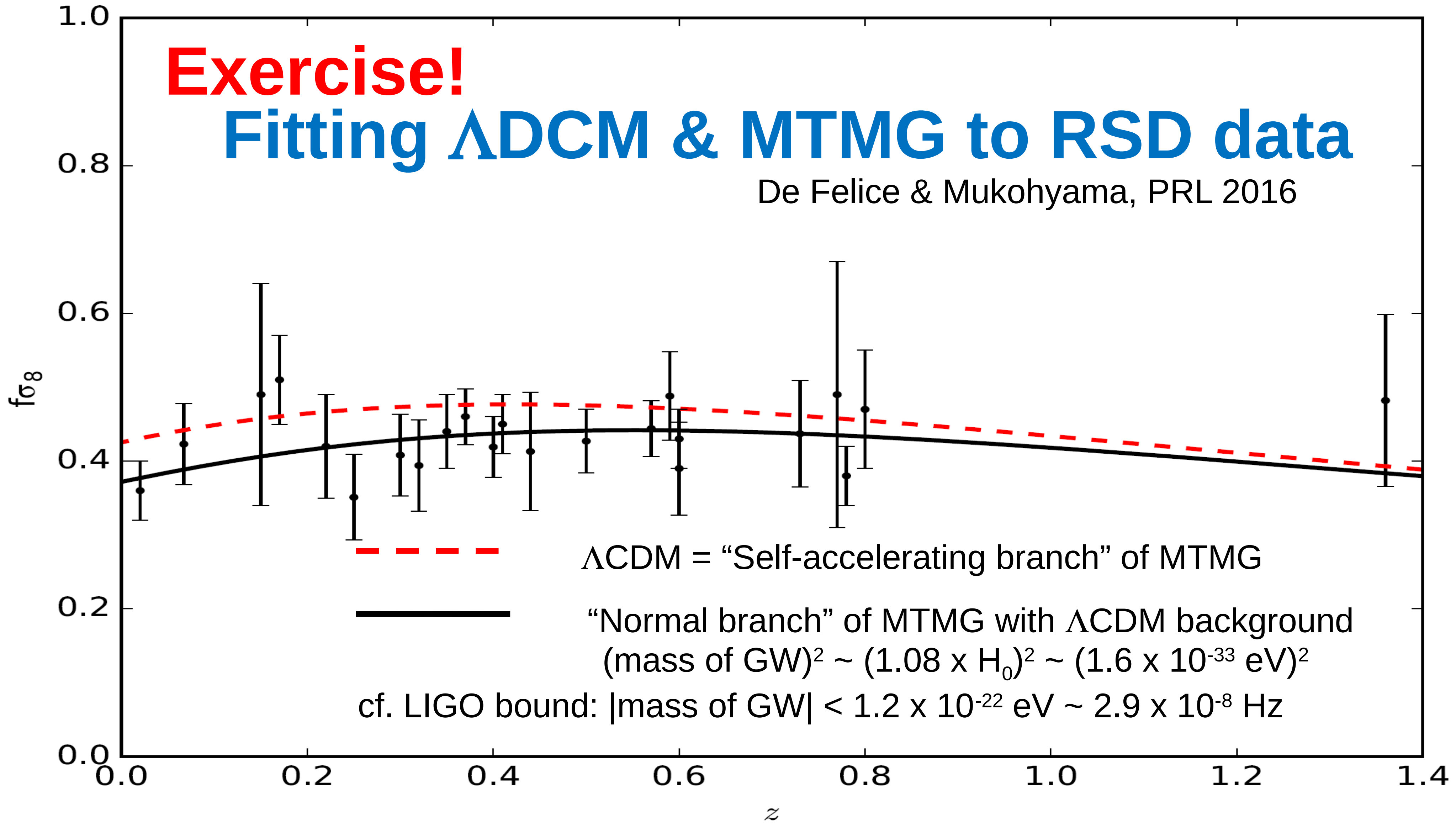
Cosmology of MTMG (normal Branch)

- Background constraint $(c_3+2c_2X+c_1X^2)(\dot{X}+NHX-MH)=0, \quad X=\tilde{a}/a$

$$H=XH_f, \quad H_f=M^{-1}\dot{\tilde{a}}/\tilde{a},$$

$$3M_P^2H^2=\frac{m^2M_P^2}{2}(c_4+3c_3X+3c_2X^2+c_1X^3)+\rho$$

- Dark component without extra dof
- Scalar part recovers GR in UV (L << 1/m) but not GR when L >> 1/m
- Non-zero mass for the gravity waves



Exercise!
Fitting ΛDCM & MTMG to RSD data
De Felice & Mukohyama, PRL 2016
ΛCDM = "Self-accelerating branch" of MTMG
"Normal branch" of MTMG with ΛCDM background
(mass of GW)^2 ~ (1.08 x H_0)^2 ~ (1.6 x 10^{-33} eV)^2
cf. LIGO bound: |mass of GW| < 1.2 x 10^{-22} eV ~ 2.9 x 10^{-8} Hz
$f\sigma_8$
z
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

ISW-galaxy correlation

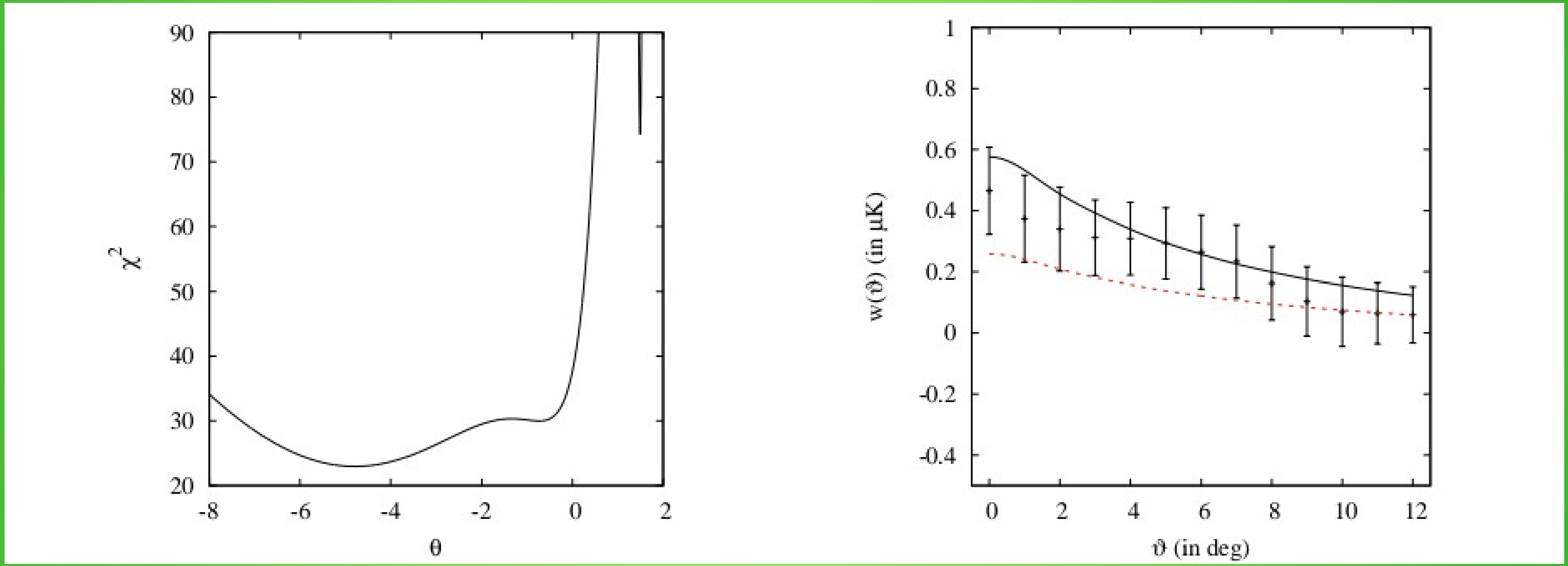
[N. Bolis, ADF, S. Mukohyama, ‘18]

- Consider Bardeen potential Ψ, Φ
- Define ISW field $\psi_{ISW} = \Psi + \Phi$
- Calculate for both branches the triple integral

$$C_l^{GI} = \frac{2}{\pi D_0^2} \int_{k_m}^{k_M} dk\, k^2 P(k) \int_{N_0}^{N_i} dN_1\, j_l(k\chi_1)\, \psi_{ISW}' \int_{N_0}^{N_i} dN_2\, e^{-N_2} \phi(N_2)\, b_s D(N_2)\, j_l(k\chi_2)$$

- Perform integral for small l without taking subhorizon limit approx.
- Compare with data (SDSS, 2dmass)

Bounds from ISW-galaxy cross correlations



- Existence of Schwarzschild BH in self acc branch

ADF, Larrouturou, Mukohyama, Oliosi 2019

Constraints from multiple data sets

[ADF, Pookkillath, Mukohyama 2021]

- Geff takes the form $$\bar{G}_{\rm eff} = \frac{2}{3} G_N \left[\frac{3}{2-\theta Y} - \frac{9\theta Y \Omega_m}{2(\theta Y - 2)^2}\right]$$
- Here $\theta = \mu_0^2 / H_0^2, \ \ Y = H_0^2 / H^2$
- Strong bounds on graviton mass
- Negative values allowed(!?)
- $G_{\rm eff}$ might blow up(!?)

	Planck	Planck+BAO+Pantheon	All joint analysis
$10^2\omega_b$	$2.242^{+0.031}_{-0.030}$	$2.242^{+0.027}_{-0.027}$	$2.247^{+0.027}_{-0.027}$
$\omega_{\rm cdm}$	$0.1197^{+0.0028}_{-0.0028}$	$0.1195^{+0.0020}_{-0.0020}$	$0.1189^{+0.0019}_{-0.0019}$
$100\theta_s$	$1.04194^{+0.00059}_{-0.00058}$	$1.04194^{+0.00056}_{-0.00056}$	$1.04198^{+0.00057}_{-0.00056}$
$\ln 10^{10} A_s$	$3.044^{+0.032}_{-0.032}$	$3.045^{+0.033}_{-0.032}$	$3.037^{+0.031}_{-0.031}$
n_s	$0.9671^{+0.0090}_{-0.0088}$	$0.9674^{+0.0077}_{-0.0076}$	$0.9683^{+0.0074}_{-0.0075}$
$\tau_{\rm reio}$	$0.055^{+0.016}_{-0.015}$	$0.055^{+0.016}_{-0.015}$	$0.052^{+0.015}_{-0.015}$
A_1	$0.57^{+1.1}_{-0.57}$	$0.63^{+0.73}_{-0.63}$	$0.71^{+0.43}_{-0.71}$
A_2	$6.2^{+8.4}_{-7.0}$	$6.4^{+8.5}_{-6.4}$	$3.9^{+11}_{-3.9}$
$\bar{c}_1$	$0.0^{+9.2}_{-9.2}$	$0.2^{+9.0}_{-9.2}$	$-0.1^{+8.3}_{-8.5}$
$\bar{c}_2$	$0.1^{+8.4}_{-8.4}$	$0.0^{+8.5}_{-8.3}$	$-0.4^{+6.8}_{-7.1}$
$\bar{c}_3$	$1.2^{+8.1}_{-8.1}$	$1.1^{+8.2}_{-8.0}$	$0.9^{+6.6}_{-6.5}$
Ω_m	$0.318^{+0.17}_{-0.068}$	$0.306^{+0.012}_{-0.012}$	$0.302^{+0.011}_{-0.011}$
H_0	67^{+8}_{-10}	$68.11^{+0.92}_{-0.92}$	$68.37^{+0.87}_{-0.93}$
σ_8	$0.816^{+0.089}_{-0.15}$	$0.822^{+0.021}_{-0.018}$	$0.816^{+0.016}_{-0.017}$
S_8	$0.832^{+0.040}_{-0.040}$	$0.830^{+0.028}_{-0.027}$	$0.819^{+0.023}_{-0.024}$
Δ	$-0.4^{+2.7}_{-4.2}$	$-0.4^{+2.5}_{-4.1}$	$-0.1^{+1.3}_{-1.5}$
θ_0	$0.18^{+0.64}_{-0.40}$	$0.16^{+0.27}_{-0.28}$	$0.12^{+0.21}_{-0.22}$
$\bar{c}_4$	3^{+10}_{-10}	3^{+11}_{-11}	$3.2^{+5.9}_{-6.9}$

Table II: Constraints at 95% CL on the primary and derived parameters of dynamical MTMG.

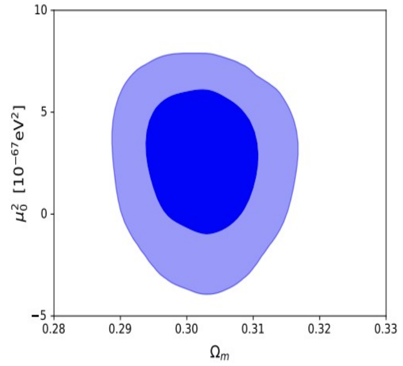


Figure 4: Constraints on μ_0^2 for the joint analysis.

Extended version of MTMG

[ADF, S. Mukohyama, M. Pookkillath, 2022]

- Can we extend MTMG so that $\mu > 0\,,\ \ G_{eff}/G_N > 0$?
- At any redshifts?
- But still allowing $H \neq H_{\Lambda CDM}\,,\ \ 0 < \frac{G_{eff}}{G_N} < 1$
- If implemented bullet-proof massive graviton theory
- Interesting phenomenology

Extended MTMG

[ADF, Pookkillath, Mukohyama 2022]

- Introduce the theory with $\mu>0\,,\quad 0<G_{eff}/G_N\leq 1$ for any dynamics

$$
\begin{aligned}
\mathcal{L} = {} & \frac{M_{\rm P}^2}{2}\sqrt{\gamma}N\left[\gamma^{ij}\gamma^{kd}(K_{ik}K_{jd}-K_{ij}K_{kd})+R\right] \\
& -\frac{1}{2}m^2M_{\rm P}^2\sqrt{\gamma}NF_1([\mathfrak{K}],[\mathfrak{K}^2],[\mathfrak{K}^3])-\frac{1}{2}m^2M_{\rm P}^2\sqrt{\tilde{\gamma}}MF_2([\mathcal{K}],[\mathcal{K}^2],[\mathcal{K}^3]) \\
& +\frac{m^4M_{\rm P}^2\lambda^2M^2}{64N}\sqrt{\gamma}\gamma_{ik}\gamma_{jd}(2\Theta^{ij}\Theta^{kd}-\Theta^{ik}\Theta^{jd}) \\
& +\lambda\sqrt{\gamma}\left[\mathcal{C}_\zeta-\frac{1}{4}m^2M_{\rm P}^2M\,K_{ij}\Theta^{ij}\right]+\sqrt{\gamma}\,(D_j\lambda^i)\mathcal{C}^j{}_i\,.
\end{aligned}
$$

with

$$
F_1 = c_4+\left(2[\mathfrak{K}][\mathfrak{K}^2]-\frac{10}{9}[\mathfrak{K}]^3\right)\xi^2\,,
$$

$$
F_2 = \left(2[\mathcal{K}][\mathcal{K}^2]-\frac{10}{9}[\mathcal{K}]^3\right)\zeta_1^2+\left(2[\mathcal{K}][\mathcal{K}^3]-\frac{4}{9}[\mathcal{K}]^4\right)\zeta_2^2+\left(2[\mathcal{K}^2][\mathcal{K}^3]-\frac{2[\mathcal{K}]^5}{15}\right)\zeta_3^2+\left([\mathcal{K}^3]^2-\frac{[\mathcal{K}]^6}{45}\right)\zeta_4^2
$$

- Phenomenology?
- Can a graviton mass fix the tensions in cosmology?

Conclusions

- Implement minimal theories to attack late time puzzles
- Minimal theories allow for wild modifications of observables H & Geff
- VCDM [as to solve H0-tension]
- VCCDM [as to solve H0 and S8 tensions]
- ExtMTMG [as to attack H0 & S8 tensions via a massive graviton]
- Each having different (interesting) phenomenology
- Let us wait for more data to come
- Detailed understanding of these theories is necessary