Hemispherical asymmetry of primordial power spectra

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Hemispherical power asymmetry

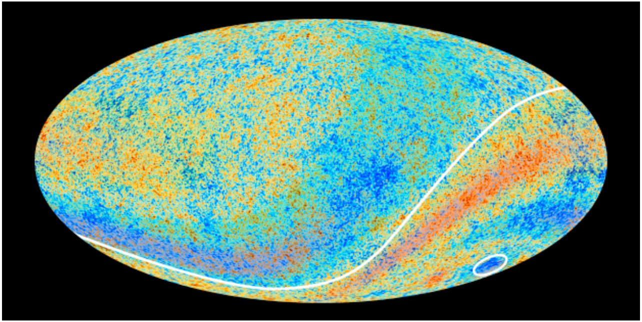


Figure: HPA breaks the isotropy of primordial fluctuations and CMB appears to be asymmetric with slightly higher temperatures in the north and slightly lower temperatures in the south.

- HPA is observed to be significant at low multipoles $\ell \sim 2 - 64$ or large angular scales or $k \lesssim 0.0045\mathrm{Mpc}^{-1}$.
- HPA can be parameterized as

$$\mathcal{P}_{\mathcal{R}}\left(k,\, \hat{\boldsymbol{n}}\right) \simeq \mathcal{P}_{\mathcal{R}\ iso}(k)\left(1 + 2A(k)\hat{\mathbf{p}}\cdot\hat{\mathbf{n}}\right)\ ,$$

which implies

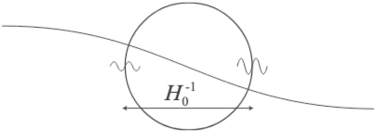
$$A(k) = \frac{\mathcal{P}_{\mathcal{R}}\left(k,\, \hat{\mathbf{n}}\right) - \mathcal{P}_{\mathcal{R}}\left(k,\, -\hat{\mathbf{n}}\right)}{4\mathcal{P}_{\mathcal{R}\ iso}}\,.$$

$\hat{\boldsymbol{p}}$ is the direction of maximal symmetry and $\hat{\mathbf{n}} = \frac{\boldsymbol{x}}{x_{\mathrm{ls}}}$ is the line of sight from earth and $x_{\mathrm{ls}} = 14,000\mathrm{Mpc}^{-1}$ is the co-moving distance to the surface of last scattering.

- The constraint on HPA is $|A| = 0.066 \pm 0.021\ (3.3\sigma)$ for $\ell < 64$ and the Planck data reports the existence of asymmetry even up to $\ell \sim 600$ (Planck 2013, 2015 and 2019 and Y. Akrami et al (2014)).
- From the latest Planck data 2019 HPA suspected to be present even in the 3-point correlations.

Attempts to explain HPA

- HPA is most often seen as a signature of Non-Gaussianity at large scales.
- Most explanations for HPA invoked existence of an **additional** scalar field whose amplitude is modulated on super-horizon scales at the onset of inflation. See A. L. Erickcek et al 2008



- Another explanation for HPA came from introducing asymmetric space-dependent initial conditions for the quantum fluctuations A. Ashoorioon and T. Koivisto (2015)

- A best theoretical explanation is which introduces less new parameters and explain more data points.
- HPA poses a significant challenge to the foundations of inflationary cosmology (Y. Akrami's talk yesterday). Question is:

> Can we explain HPA within the context of standard single-field slow-roll inflation? Can the answer lies somewhere in the our understanding of inflationary "quantum fluctuations"?

Standard formulation of inflationary quantum fluctuations

- Finding background solutions corresponding to quasi-de Sitter.
- We perturb metric and matter degrees of freedom around the given background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad \phi = \bar{\phi}(t) + \delta\phi \,.$$

- We quantize the effective gravitational degrees of freedom

$$\delta \hat{G}_{\mu\nu} = \delta \hat{T}_{\mu\nu}$$

- Through inflationary quantum fluctuations we witness the linearized "quantum gravity" J. Martin (2004).

Classical vs quantum mechanical time

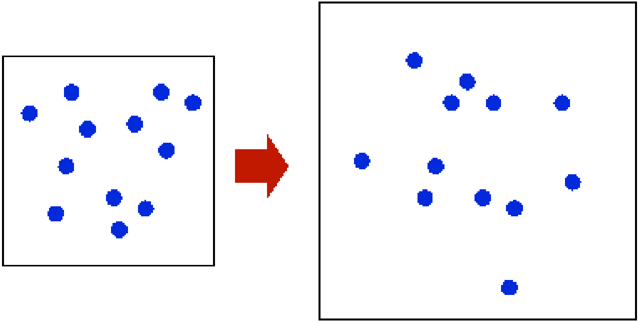


Figure: Pariticles in an expanding Universe (picture taken from https://galileospendulum.org).

Does the quantum fluctuations necessarily follow the classical or thermodynamical arrow of time?

Time reversal in quantum theory

- Time reversal operation in quantum theory can be implemented by replacing $i \to -i$ since time is an anti-unitary operator.
- What we call positive energy and negative energy is associated with our convention about arrow of time.
- Schrödinger equations $i\frac{d\psi}{dt} = H\psi$ and $-i\frac{d\psi}{dt} = H\psi$. Nature's laws are perfectly symmetric under time inversion.
- Quantum theory in the context of gravity is highly non-trivial and one must understand better the notion of time. See J. F. Donoghue and G. Menezes 2019, 2020, 2021

Open questions

Understanding inflationary quantum fluctuations require a robust formulation of QFT in curved space-time.

- Standard QFT: Particles that propagate forward and backward in time (anti- particle). What happens to particles and anti-particle states in a curved spacetime? What is time reversal operation in curved spacetime?
- What happens to $(\mathcal{C})\mathcal{PT}$ in curved spacetime?
- In GR time is a coordinate and in quantum theory time is a parameter. What is the consistent way to quantize gravitational degrees of freedom?
- (Quantum) gravity is special and surprising: Wheeler-De Witt equation is timeless: $\mathcal{H}\Psi = 0$. The problem of time in quantum cosmology (C. Keifer, 2nd ed. 2007.)

Formulating new set of rules for quantization (in the context of inflation)

- Separate completely classical and quantum mechanical notion of time
- Expansion of Universe: Shrinking Horizon $r_H = |\frac{1}{aH}|$ when $|H| \approx \mathrm{const}$ (classical arrow of time).
- Quantum theory requires understanding of discrete symmetries. Notion of observers, regions of spacetime, Penrose diagrams are classical concepts J. F. Donogue, G. Menezes 2021. and they must be important only we after we quantize fields respecting discrete symmetries.
- In a quantum theory an arrow of time only emerges only after we specify initial and final states otherwise quantum theory is time symmetric (J. Hartle 2013).

Doubling the number of quantum states: $\mathcal{PT}$ transformations in gravitational context

- Respecting discrete symmetries $\mathcal{PT}$ we propose quantum fields are always created as $\mathcal{PT}$ pairs.

We represent the total vacuum as direct sum of two different vacua related by $\mathcal{PT}$ transformations.

$$|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_{\mathrm{I}} \oplus |0\rangle_{\mathrm{II}}\right).$$

In the vacuum $|0\rangle_{\mathrm{I}}$ we create quantum fields at the position $\mathbf{x}$ that evolve forward in time and in the vacuum $|0\rangle_{\mathrm{II}}$ we create quantum fields at $-\mathbf{x}$ which evolve backward in time.

Quantization in de Sitter spacetime

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2 \quad = \frac{1}{H^2\tau^2}\left(-d\tau^2 + d\mathbf{x}^2\right) .$$

where $d\tau = \frac{dt}{a}$,

$$a(t) = e^{Ht}, \quad H^2 = \left(\frac{1}{a}\frac{da}{dt}\right)^2 = \mathrm{const}\,.$$

The metric is $\mathcal{PT}$ symmetric.

$$t : -\infty \to \infty, H > 0 \implies \tau < 0, \quad t : \infty \to -\infty, H < 0 \implies \tau > 0.$$

Expanding Universe means $\tau : \pm\infty \to 0$.

Quantization in de Sitter space-time

- For a state evolving forward in time $\tau < 0$ and for a state that is evolving backward in time $\tau > 0$.
- Let us take a massless field in de Sitter space. We split the field operator into two parts

$$\hat{\phi}(\tau, \mathbf{x}) = \frac{1}{\sqrt{2}}\hat{\varphi}_{\mathrm{I}}(\tau, \mathbf{x}) \oplus \frac{1}{\sqrt{2}}\hat{\varphi}_{\mathrm{II}}(-\tau, -\mathbf{x}) ,$$

corresponding to two vacua

$$a_{\mathbf{k}}|0\rangle_{\mathrm{I}} = 0, \quad b_{\mathbf{k}}|0\rangle_{\mathrm{II}} = 0, \quad [\hat{\varphi}_{\mathrm{I}}(\tau, \mathbf{x}), \hat{\varphi}_{\mathrm{II}}(-\tau, -\mathbf{x})] = 0 .$$

Quantum mechanically $\hat{\varphi}_{\mathrm{I}}(\tau, \mathbf{x})|0\rangle_{\mathrm{I}}$ is the postive energy state that propagate forward in time at $\mathbf{x}$ while $\hat{\varphi}_{\mathrm{II}}(-\tau, -\mathbf{x})|0\rangle_{\mathrm{II}}$ is the postive energy state that propagate backward in time at -$\mathbf{x}$.

$\mathcal{PT}$ symmetry in dS spacetime

Since dS spacetime is perfectly $\mathcal{PT}$ symmetric we have that the quantum fields $\hat{\varphi}_{\rm I}\left(\tau,\, \mathbf{x}\right)|0\rangle_{\rm I}$ and $\hat{\varphi}_{\rm II}\left(-\tau,\, -\mathbf{x}\right)|0\rangle_{\rm II}$ behave identically, which can be seen from the fact that their equal time correlations are the same

$$\begin{aligned} &\frac{1}{a^2}{}_{\rm I}\langle 0|\hat{\varphi}_{\rm I}\left(\tau,\, \mathbf{x}\right)\hat{\varphi}_{\rm I}\left(\tau,\, \mathbf{x}'\right)|0\rangle_{\rm I} = \\ &\frac{1}{a^2}{}_{\rm II}\langle 0|\hat{\varphi}_{\rm II}\left(-\tau,\, -\mathbf{x}\right)\hat{\varphi}_{\rm II}\left(-\tau,\, -\mathbf{x}'\right)|0\rangle_{\rm II} = \frac{H^2}{4\pi^2 k^3}\,. \end{aligned} \tag{1}$$

Quantization in quasi-de Sitter space-time: Single field inflation case

- In the inflationary space-time we quantize the Mukhanov-Sasaki variables $v = 2a\frac{\dot{\phi}}{H}\zeta$ and $u_{ij} = \frac{a}{2}h_{ij}$.
- Inflationary space-time is not $\mathcal{PT}$ symmetric like dS. Expectation is $\mathcal{PT}$ symmetry must be spontaneously broken at the quantum level.
- $$\hat{v}(\tau, \mathbf{x}) = \frac{1}{\sqrt{2}}\hat{v}_{\rm I}(\tau, \mathbf{x}) \oplus \frac{1}{\sqrt{2}}\hat{v}_{\rm II}(-\tau, -\mathbf{x}) \,.$$
  corresponding to $|0\rangle_{\rm qdS} = |0\rangle_{\rm qdS_I} \oplus |0\rangle_{\rm qdS_{II}}$.
- $\hat{v}_{\rm II}(-\tau, -\mathbf{x})$ is the fluctuation that goes backward in time. Logically, if the fluctuation propagates forward in time in a slow-roll background, the fluctuation that goes backward in time experience space-time as a "slow-climb". Therefore "quantum mechanically" we solve for $v_{\rm II, k}(\tau)$ following the time reversal operation

$$t \to -t \implies H \to -H, \quad \epsilon \to -\epsilon, \quad \eta \to -\eta \,.$$

Now we classically rescale the fields back to the original variables using background quantities. This implies ${}_{\rm I}\langle 0|\zeta_{1\mathbf{k}}\zeta_{1\mathbf{k}'}|0\rangle_{\rm I} = \left(\frac{1}{2a^2\epsilon}\right)\Big|_{\rm classical}\, {}_{\rm I}\langle 0|v_{\rm I\mathbf{k}}v_{I\mathbf{k}'}|0\rangle_{\rm I}$.

$$
\begin{aligned}
P_{\zeta 1} &\approx \frac{H^2}{8\pi^2\epsilon}\left(1+\left(\frac{k}{aH}\right)^2\right) \\
&\quad + \frac{H^2}{8\pi\epsilon}\left(\epsilon+\frac{\eta}{2}\right)\left(\frac{k}{aH}\right)^3 H^{(1)}_{3/2}\left(\frac{k}{aH}\right)\frac{\partial H^{(1)}_{\nu_s}\left(\frac{k}{aH}\right)}{\partial\nu_s}\Big|_{\nu_s=3/2} \\
P_{\zeta 2} &\approx \frac{H^2}{8\pi^2\epsilon}\left(1+\left(\frac{k}{aH}\right)^2\right) \\
&\quad - \frac{H^2}{8\pi\epsilon}\left(\epsilon+\frac{\eta}{2}\right)\left(\frac{k}{aH}\right)^3 H^{(1)}_{3/2}\left(\frac{k}{aH}\right)\frac{\partial H^{(1)}_{\nu_s}\left(\frac{k}{aH}\right)}{\partial\nu_s}\Big|_{\nu_s=3/2}.
\end{aligned}
$$

The total power spectrum is $P_\zeta = \frac{1}{2}P_{\zeta 1} + \frac{1}{2}P_{\zeta 2} = \frac{H^2}{8\pi^2\epsilon}\Big|_{k=aH}$ The spectral index remains the same in the two hemispheres compatible with observations

$$
\frac{d\ln P_{\zeta 1}}{d\ln k} \approx \frac{d\ln P_{\zeta 2}}{d\ln k} \approx n_s - 1 \approx -2\epsilon - \eta\,. \tag{2}
$$

Hemispherical asymmetry of scalar power spectra

$$A(k) = \frac{P_{\zeta 1} - P_{\zeta 2}}{4P_\zeta}$$

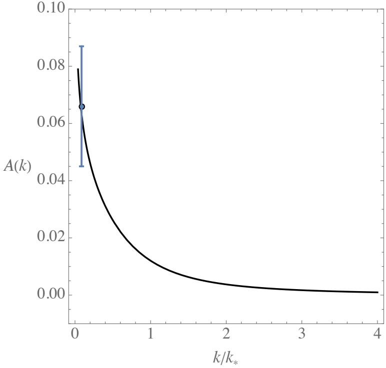


Figure: Here $k_* = a_* H_* = 0.05\mathrm{Mpc}^{-1}$ and we are within $|A| = 0.066 \pm 0.021$ for $k < 10^{-1} k_*$.

Hemispherical asymmetry for tensor power spectra

Similarly, double vacuum scheme of quantization predicts the power asymmetry of the tensor-power spectrum

$$T(k) = \frac{P_{h1} - P_{h2}}{4P_h}$$

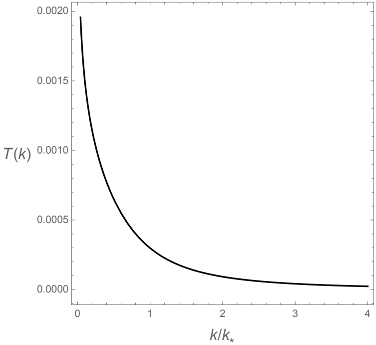


Figure: This plot is obtained for the case of Starobinsky and Higgs inflation.

Conclusions

- Based on several open theoretical questions about QFT in curved space-time and the surprising anomalies we proposed a new vacua structure for inflationary quantum fluctuations.
- Our scheme of quantization naturally produces HPA for both scalar and tensor power spectra. If detected we greatly learn about nature of QFT in curved space-time

Thank you very much for your attention.