Barrow Holographic Dark Energy and a Possible Reduction of the Hubble Tension

Mariusz P. Dąbrowski

Institute of Physics, University of Szczecin, Poland &
National Centre for Nuclear Research, Otwock, Poland &
Copernicus Center for Interdisciplinary Studies, Kraków, Poland

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Plan:

- 1. Barrow horizon entropy.
- 2. Barrow holographic dark energy
- 3. Results of statistical analysis against cosmological data
- 4. Conclusions.

References

- inspired by: J.D. Barrow, Phys. Lett. B 808, 135643 (2020) (ArXiv: 2004.09444).
- MPD and V. Salzano, Geometrical observational bounds on a fractal horizon holographic dark energy, Physical Review D 102, 064047 (2020) (3 Sept 2020), and then ArXiv:2009.08306 (16 Sept 2020) because it was banned from ArXiv on the original submission in June 2020 and put on Research Gate instead (https://www.researchgate.net/publication/342571909).
- MPD, Vincenzo Salzano and Tomasz Denkiewicz, (Sept. 2022) - work in progress (new cosmological set of data applied).
- + many papers which followed (M. Saridakis, ArXiv: 2005.04115, ArXiv: 2006.01105, Anagnostopoulos et al., ArXiv: 2005.10302, Yusufi et al. 2110.07258 etc. - apologies for those who have not been mentioned in this talk)

1. Barrow horizon entropy

- Inspiration from the COVID-19 pseudofractal geometrical structure
- consider the core sphere of the horizon with the attached number N of heavily packed smaller spheres, to each of which, step by step, a number N of some smaller spheres are latched on and so on, forming a fractal - **"sphereflake"**: https://www.youtube.com/watch?v=pPb5NKEYCD8;

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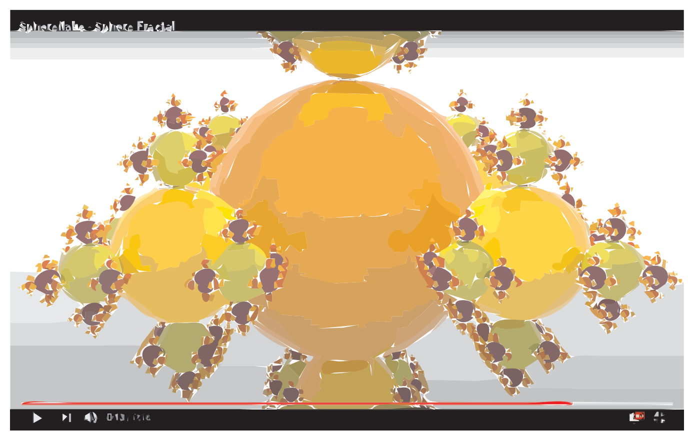
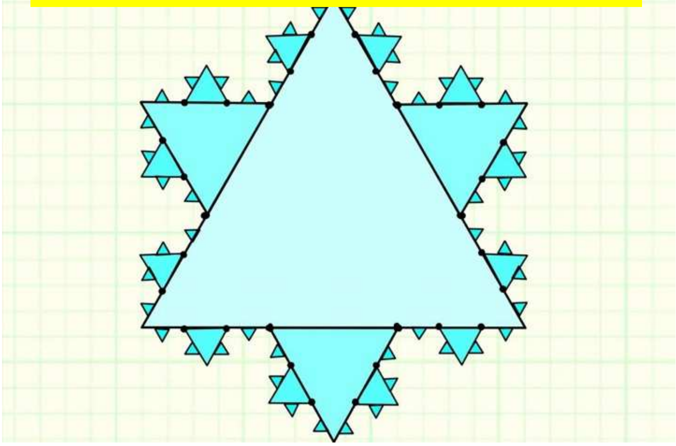


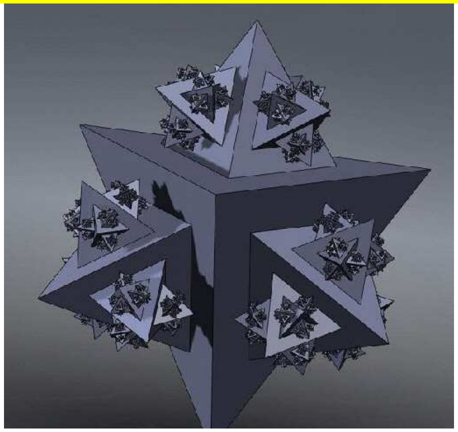
Barrow horizon entropy ctd.

- This is a kind of analogue of known fractals: Koch snowflake, Sierpiński gasket, Menger sponge

Koch snowflake



3-D Koch snowflake



- geometrical series with the recurrence formula for the radius r_{n+1} of the $(n+1)$-th sphere is $r_{n+1} = \lambda r_n$, where r_n is the radius of the n-th sphere, and $\lambda < 1$;

Barrow horizon entropy ctd.

- take infinite number of steps ($n \to \infty$) to count up the effective surface area of all the spheres:

$$A_{eff} = \sum_{n=0}^{\infty} N^n 4\pi(\lambda^n r)^2 = \frac{4\pi r^2}{1 - N\lambda^2} = 4\pi r_{eff}^2; \quad r_{eff} = \frac{r}{\sqrt{1 - N\lambda^2}} \equiv r^{1+\Delta/2}. \tag{1}$$

where $0 \leq \Delta \leq 1$, provided that $N\lambda^2 < 1$.

- effective surface area $A_{eff} \propto r_{eff}^2 \geq$ a sphere surface area $A \propto r^2$ (so $r \propto A^{1/2}$) of radius r;
- in the extreme (most intricate surface) case of $\Delta = 1$, it would act **as if it was a volume**;

Barrow horizon entropy ctd.

- leading to a change of Bekenstein entropy (making it larger), according to the formula

$$S_{eff} \propto A_{eff} \propto r_{eff}^2 \propto r^{2+\Delta} \propto A^{1+\frac{\Delta}{2}}, \qquad (2)$$

where r can be either the black hole Schwarzschild radius r_s or the cosmological horizon length L.

- The exact formula in terms of the Planck area $A_{Pl} = 4l_{pl}^2$ was given by Barrow (2020)

$$S_{eff} = k_B \left(\frac{A}{A_{Pl}} \right)^{1+\frac{\Delta}{2}}, \qquad (3)$$

where k_B is the Boltzmann constant.

Barrow horizon entropy ctd.

- More accurately, the Bekenstein entropy generalizes into (here ”B” stands for ”Barrow”)

$$S_B = k_B \pi^{1+\frac{\Delta}{2}} \left(2\frac{M}{m_{pl}} \right)^{2+\Delta}, \quad (4)$$

- and the generalized Hawking temperature reads

$$T_B = \frac{c^2}{4\pi^{1+\frac{\Delta}{2}} k_B \left(1+\frac{\Delta}{2}\right)} \frac{m_{pl}^{2+\Delta}}{(2M)^{1+\Delta}}, \quad (5)$$

  where M is a black hole mass, k_B is the Boltzmann constant, c is the speed of light, and $m_{pl}^2 = \hbar c/G$ is the Planck mass.

- In the limit $\Delta \to 0$ they reduce to standard Bekenstein and Hawking formulas for the black holes:

$$S_{Bek} = \frac{4\pi G k_B M^2}{\hbar c} \quad \text{and} \quad T_{Haw} = \frac{\hbar c^3}{8\pi G k_B M}. \quad (6)$$

Barrow entropy vs Tsallis entropy ctd.

- Though purely geometrical in its motivation - Barrow entropy has a fully supported thermodynamically companion - Tsallis entropy (J. Stat. Phys. 52, 479 (1988); book of 2009 etc.) which reads

$$S_{eff} = k_B \left(\frac{A}{A_{Pl}} \right)^{\delta}, \tag{7}$$

where δ is the real parameter. One easily notices that

$$\delta = 1 + \frac{\Delta}{2} \tag{8}$$

and so the range of Barrow entropy in Tsallis thermodynamics is

$$1 < \delta < \frac{3}{2}. \tag{9}$$

The derivations related to Tsallis entropy in this range are also valid for Barrow entropy (e.g. A. Mamon, 2007.0159, Y. Liu, 2201.00657).

Tsallis nonextensive thermodynamical entropy

- Tsallis entropy reads (Tsallis 2009)

$$S_T = S_q = -\sum_i [p(i)]^q \ln_q p(i), \tag{10}$$

where $p(i)$ is the probability distribution defined on a set of microstates Ω, $q \in \mathcal{R}$ is the nonextensivity parameter.

- The q-logarithmic function $\ln_q p$ is defined as

$$\ln_q p = \frac{p^{1-q} - 1}{1 - q}, \tag{11}$$

such that, in the limit, $q \to 1$, Tsallis entropy (10) *reduces to Gibbs-Shannon* entropy

$$S_G = -\sum_i p(i) \ln p(i). \tag{12}$$

Tsallis nonextensive thermodynamical entropy

- Tsallis entropy (10) satisfies a *nonadditive* composition rule.
- However, via “formal logarithm" approach (Biro & Van 2011), one can write a corresponding additive entropy in terms of S_q such that

$$S_R = \frac{k}{1-q}\left[\ln\left(1+\frac{1-q}{k}S_T\right)\right], \tag{13}$$

which is the Rényi entropy (Rényi 1959)

$$S_R = k\frac{\ln\sum_i p^q(i)}{1-q}. \tag{14}$$

The Bekenstein entropy is also nonextensive and nonadditive. Often it is put as Tsallis entropy to make it additive via logarithmic formula. For $\delta = 3/2$ (Tsallis) and $\Delta = 1$ (Barrow) we have the entropy scaling with volume and so it is an extensive and additive quantity.

2. Barrow holographic dark energy

Holographic dark energy is given by $\rho_H \propto S_{eff} L^{-4}$ (Wang 2016) with the effective Bekenstein entropy $S_{eff} \propto A_{eff} \propto L^{2+\Delta}$, where L is the horizon length. One can express Barrow holographic dark energy (BH) as (Saridakis 2020):

$$\rho_{BH} = \frac{3\,C^2}{8\pi G} L^{(\Delta-2)}\,, \tag{15}$$

where C is the *holographic parameter* with dimensions of $[\mathsf{T}]^{-1}[\mathsf{L}]^{1-\Delta/2}$. **Note that ΛCDM ($\Delta = 2$, $\rho_{BH} =$ const.) is excluded in Barrow holography**.

We may identify the length L with the future event horizon, later called BH1 (or Hubble horizon $r_H = c/H$ - but since there is no analytic solution for H, it takes too much time to run chains, possible BH2 case, not considered here):

$$L \equiv a \int_t^{\infty} \frac{dt'}{a} = a \int_a^{\infty} \frac{da'}{H(a')a'^2}\,, \tag{16}$$

where a is the scale factor.

Barrow holographic dark energy ctd.

The cosmological equation is simply

$$H^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_r + \rho_{BH}\right) , \tag{17}$$

where the suffices m and r refer respectively to matter and radiation.
Standard continuity equation for matter and radiation is still valid, i.e.

$$\dot{\rho}_{m,r} + 3H\left(\rho_{m,r} + \frac{p_{m,r}}{c^2}\right) = 0 , \tag{18}$$

where the pressure $p_i = w_i\rho_i$. We can rewrite (6) as

$$1 = \Omega_m(a) + \Omega_r(a) + \Omega_H(a) , \tag{19}$$

introducing the dimensionless density parameters $\Omega_i(a)$, defined as

$$\Omega_{m,r}(a) = \frac{H_0^2}{H^2(a)}\Omega_{m,r}a^{-3(1+w_{m,r})} , \quad \Omega_{BH}(a) = \frac{C^2}{H^2(a)}L^{(\Delta-2)} . \tag{20}$$

Barrow holographic dark energy ctd.

Combining above equations one can express the Hubble parameter as

$$H(a) = H_0 \sqrt{\frac{\Omega_m a^{-3} + \Omega_r a^{-4}}{1 - \Omega_{BH}(a)}} \,. \tag{21}$$

Final relation for the Barrow Holographic dark energy reads (prime is derivative with respect to a):

$$\begin{aligned} a\Omega'_{BH}(a) \quad &= \quad \Omega_{BH}(a)\left(1 - \Omega_{BH}(a)\right) \times \\ \times \quad & \left[\left(1 + \frac{\Delta}{2}\right)\mathcal{F}_r(a) + (1+\Delta)\,\mathcal{F}_m(a)\right. \\ + \quad & \left.(1 - \Omega_{BH}(a))^{\frac{\Delta}{2(\Delta-2)}}\,\Omega_{BH}(a)^{\frac{1}{2-\Delta}}\,\mathcal{Q}(a)\right] \,, \end{aligned} \tag{22}$$

Barrow holographic dark energy ctd.

with

$$
\begin{aligned}
\mathcal{F}_r(a) &= \frac{2\Omega_r a^{-4}}{\Omega_m a^{-3} + \Omega_r a^{-4}}\,, \\
\mathcal{F}_m(a) &= \frac{\Omega_m a^{-3}}{\Omega_m a^{-3} + \Omega_r a^{-4}}\,, \\
\mathcal{Q}(a) &= 2\left(1 - \frac{\Delta}{2}\right)\left(H_0\sqrt{\Omega_m a^{-3} + \Omega_r a^{-4}}\right)^{\frac{\Delta}{2-\Delta}} C^{\frac{2}{\Delta-2}}\,.
\end{aligned}
\tag{23}
$$

3. Results of statistical analysis against cosmological data

In MPD, Salzano (2020) we applied:

- Type Ia Supernovae (SNeIa) from the Pantheon sample;
- Cosmic Chronometers (CC);
- the gravitational lensing data from COSMOGRAIL's Wellspring project (H0LiCOW);
- the "Mayflower" sample of Gamma Ray Bursts (GRBs);
- Baryon Acoustic Oscillations (BAO) from several surveys;
- latest *Planck* 2018 release for Cosmic Microwave Background radiation (CMB).

Considered 2 cases:

- "full data", where we join both early- (CMB + BAO data from SDSS) and late-time observations (SNeIa, CC, H0LiCOW, GRBs + BAO data from WiggleZ);
- "late-time" data set, which includes only late-time data.

Results of statistical analysis (2020)

Here we report 1σ confidence intervals for each parameter.

	ΛCDM		BH	
	late	full	late	full
Ω_m	$0.293^{+0.016}_{-0.016}$	$0.319^{+0.005}_{-0.005}$	$0.290^{+0.019}_{-0.019}$	$0.314^{+0.006}_{-0.006}$
Ω_b	–	$0.0494^{+0.0004}_{-0.0004}$	–	$0.049^{+0.001}_{-0.001}$
h	$0.713^{+0.013}_{-0.013}$	$0.673^{+0.003}_{-0.003}$	$0.715^{+0.014}_{-0.013}$	$0.676^{+0.007}_{-0.007}$
Δ	–	–	> 0.63	> 0.84
C	–	–	$3.93^{+1.77}_{-1.88}$	$4.66^{+0.87}_{-1.07}$

New samples (2022), previous data in blue, also dynamical tests (RSD):

	geo-late	geo-full	geo-late+dyn	geo-full+dyn
Pantheon SNeIa	✓	✓	✓	✓
Cosmic Chronometers	✓	✓	✓	✓
GRBs	✓	✓	✓	✓
CMB	–	✓	–	✓
SDSS-IV DR16 ELG	–	✓(BAO)	✓(RSD)	✓(BAO+RSD)
SDSS-IV DR16 LRG	–	✓(BAO)	✓(RSD)	✓(BAO+RSD)
SDSS-IV DR16 LRG+Void	–	✓(BAO)	✓(RSD)	✓(BAO+RSD)
SDSS-IV DR16 Lyman α	–	✓(BAO)	–	✓(BAO)
SDSS-IV DR16 QSO (BAO)	–	✓(BAO)	✓(RSD)	✓(BAO+RSD)
SDSS-IV DR14 QSO (BAO)	–	✓(BAO)	✓(RSD)	✓(BAO+RSD)
WiggleZ	✓(BAO)	✓(BAO)	✓(BAO+RSD)	✓(BAO+RSD)
2dFGRS	–	–	✓(RSD)	✓(RSD)
6dFGS	–	–	✓(RSD)	✓(RSD)
6dFGS Voids	–	–	✓(RSD)	✓(RSD)
FASTSOUND	–	–	✓(RSD)	✓(RSD)
GAMA	–	–	✓(RSD)	✓(RSD)
BOSS-WiggleZ	–	–	✓(RSD)	✓(RSD)
BOSS LOWZ	–	–	✓(RSD)	✓(RSD)
SDSS-IV DR15 LGR-SMALL	–	–	✓(RSD)	✓(RSD)
SDSS DR7 MGS	–	–	✓(RSD)	✓(RSD)
VIPERS Voids	–	–	✓(RSD)	✓(RSD)
VIPERS	–	–	✓(RSD)	✓(RSD)
VIPERS+GGL	–	–	✓(RSD)	✓(RSD)

New samples (2022) with references:

	geo-late	geo-full	geo-late+dyn	geo-full+dyn	ref.
Pantheon SNeIa	✓	✓	✓	✓	Pan-STARRS1:2017jku
Cosmic Chronometers	✓	✓	✓	✓	Jiao:2022aep
GRBs	✓	✓	✓	✓	Liu:2014vda
CMB	–	✓	–	✓	Zhai:2019nad
SDSS-IV DR16 ELG	–	✓(BAO)	✓(RSD)	✓(BAO+RSD)	Tamone:2020qrl,deMattia:2020fkb
SDSS-IV DR16 LRG	–	✓(BAO)	✓(RSD)	✓(BAO+RSD)	BOSS:2016wmc,Gil-Marin:2020bct,Bautista:2020ahg
SDSS-IV DR16 LRG+Void	–	✓(BAO)	✓(RSD)	✓(BAO+RSD)	Nadathur:2020vld
SDSS-IV DR16 Lyman α	–	✓(BAO)	–	✓(BAO)	duMasdesBourboux:2020pck
SDSS-IV DR16 QSO (BAO)	–	✓(BAO)	✓(RSD)	✓(BAO+RSD)	Hou:2020rse,Neveux:2020voa
SDSS-IV DR14 QSO (BAO)	–	✓(BAO)	✓(RSD)	✓(BAO+RSD)	Zhao:2018gvb
WiggleZ	✓(BAO)	✓(BAO)	✓(BAO+RSD)	✓(BAO+RSD)	Blake:2012pj
2dFGRS	–	–	✓(RSD)	✓(RSD)	Song:2008qt
6dFGS	–	–	✓(RSD)	✓(RSD)	Achitouv:2016mbn
6dFGS Voids	–	–	✓(RSD)	✓(RSD)	Achitouv:2016mbn
FASTSOUND	–	–	✓(RSD)	✓(RSD)	Okumura:2015lvp
GAMA	–	–	✓(RSD)	✓(RSD)	Blake:2013nif
BOSS-WiggleZ	–	–	✓(RSD)	✓(RSD)	Marin:2015ula
BOSS LOWZ	–	–	✓(RSD)	✓(RSD)	Lange:2021zre
SDSS-IV DR15 LGR-SMALL	–	–	✓(RSD)	✓(RSD)	Chapman:2021hqe
SDSS DR7 MGS	–	–	✓(RSD)	✓(RSD)	Howlett:2014opa
VIPERS Voids	–	–	✓(RSD)	✓(RSD)	Hawken:2016qcy
VIPERS	–	–	✓(RSD)	✓(RSD)	Mohammad:2018mdy
VIPERS+GGL	–	–	✓(RSD)	✓(RSD)	Jullo:2019lgq

Results of statistical analysis (new data).

"geo" - geometrical, "late" - late time data, "dyn" - dynamical, BH1 - event hor.

	Ω_m	Ω_b	h	$\sigma_{8,0}$	$S_{8,0}$	Δ	C	$\log \mathcal{B}^i_j$
LCDM (geo-late)	$0.303^{+0.018}_{-0.017}$	–	$0.689^{+0.017}_{-0.018}$	–	–	–	–	*0*
LCDM (geo-full)	$0.332^{+0.007}_{-0.007}$	$0.0504^{+0.0007}_{-0.0007}$	$0.663^{+0.005}_{-0.005}$	–	–	–	–	*0*
LCDM (geo-late+dyn)	$0.301^{+0.017}_{-0.016}$	–	$0.690^{+0.018}_{-0.017}$	$0.771^{+0.019}_{-0.020}$	$0.772^{+0.025}_{-0.024}$	–	–	*0*
LCDM (geo-full+dyn)	$0.326^{+0.006}_{-0.006}$	$0.0499^{+0.0006}_{-0.0006}$	$0.668^{+0.004}_{-0.004}$	$0.780^{+0.017}_{-0.017}$	$0.813^{+0.020}_{-0.019}$	–	–	*0*
BH1 (geo-late)	$0.303^{+0.022}_{-0.020}$	–	$0.689^{+0.018}_{-0.018}$	–	–	> 0.59	$3.57^{+1.83}_{-1.82}$	$-0.41^{+0.02}_{-0.02}$
BH1 (geo-full)	$0.263^{+0.006}_{-0.006}$	$0.0417^{+0.0002}_{-0.0002}$	$0.727^{+0.004}_{-0.004}$	–	–	> 0.79	$3.92^{+0.94}_{-1.48}$	$-12.25^{+0.03}_{-0.03}$
BH1 (geo-late+dyn)	$0.291^{+0.018}_{-0.018}$	–	$0.690^{+0.017}_{-0.017}$	$0.787^{+0.026}_{-0.023}$	$0.776^{+0.025}_{-0.024}$	> 0.61	$3.87^{+1.96}_{-1.74}$	$-1.81^{+0.03}_{-0.03}$
BH1 (geo-full+dyn)	$0.317^{+0.008}_{-0.007}$	$0.049^{+0.001}_{-0.001}$	$0.673^{+0.007}_{-0.008}$	$0.777^{+0.018}_{-0.017}$	$0.799^{+0.023}_{-0.022}$	> 0.79	$4.40^{+1.02}_{-1.29}$	$-1.85^{+0.04}_{-0.04}$

- In one case (BH1, geo-full) the value of h is larger (SNa value). Also, the S_8 tension alleviated in all cases. All BH1 models statistically disfavoured.

4. Conclusions

- **All data tests** lead to the conclusion that the Barrow fractal index Δ is **bound from below** ($\Delta > 0.59$) which means that cosmological horizon **should be of the fractal nature** (peaking towards $\Delta \to 1$).
- The ’’standard” limit of a non-fractal horizon, i.e. $\Delta = 0$, is excluded by the data.
- ΛCDM ($\Delta = 2$) is also excluded in Barrow holography (though admitted in Tsallis holography $\delta = 1 + \frac{\Delta}{2} = 2$)!
- Hubble tension can be **resolved** in just one case of BH1 (geo-full) of new data.
- S_8 tension can be **alleviated**.
- The Bayes factor $\mathcal{B}^i_j$, given Jeffreys’ scale $\ln \mathcal{B}^i_j$, the **new data disfavours Barrow entropy models** w.r.t. ΛCDM.