H2020 MSCA COFUND
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Towards realistic constraints on alternative theories of gravity

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Workshop on Tensions in Cosmology

Which framework shall I use? The two ways ...

Covariant theory approach

[Horndeski (1974)]
[Gubitosi, *et al.* (2013)]
[EB, Sawicki (2014)]

$$S=\int \mathrm{d}^4x\,\sqrt{-g}\left[\sum_{i=2}^{5}\frac{1}{8\pi G_{\mathrm{N}}}\mathcal{L}_i(g_{\mu\nu},\phi)+\mathcal{L}_{\mathrm{m}}(g_{\mu\nu},\psi_M)\right]$$

$$\begin{aligned}
\mathcal{L}_2 &= G_2(\phi,\,X)\\
\mathcal{L}_3 &= -\,G_3(\phi,\,X)\Box\phi\\
\mathcal{L}_4 &= G_4(\phi,\,X)R+G_{4X}(\phi,\,X)\left[(\Box\phi)^2-\phi_{;\mu\nu}\phi^{;\mu\nu}\right]\\
\mathcal{L}_5 &= G_5(\phi,\,X)G_{\mu\nu}\phi^{;\mu\nu}-\frac{1}{6}G_{5X}(\phi,\,X)\left[(\Box\phi)^3+2\phi_{;\mu}{}^{\nu}\phi_{;\nu}{}^{\alpha}\phi_{;\alpha}{}^{\mu}-3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi\right]
\end{aligned}$$

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$$\mathcal{L}_5 = G_5(\phi,\, X)G_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}G_{5X}(\phi,\, X)\left[(\Box\phi)^3 + 2\phi_{;\mu}{}^{\nu}\phi_{;\nu}{}^{\alpha}\phi_{;\alpha}{}^{\mu} - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi\right]$$

Effective theory approach

Covariant Theory		Effective Description (linear)		Predictions
$S[g_{\mu\nu},\phi]$ +initial cond.	→	Expansion $w(\tau)$	→	$H(z)$
$G_i(X,\phi)\big\|_{i=2,3,4,5}$		Kineticity $\alpha_K(\tau)$		C_ℓ^{ij}
$\phi'(\tau_0),\, \phi(\tau_0)$		Braiding $\alpha_B(\tau)$		$P(k,z)$
		M_p running $\alpha_M(\tau)$		...
		Tensor speed $\alpha_T(\tau)$		

Effective theory approach ⟶ (from Effective Description to Predictions)

Covariant theory approach ⟶ (from Covariant Theory to Predictions)

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$$G_2(\phi, X) \quad G_3(\phi, X) \quad G_4(\phi, X) \quad G_5(\phi, X) \quad F_4(\phi, X) \quad F_5(\phi, X)$$
$$\phi_i \quad \dot{\phi}_i \quad H_0 \quad \Omega_{\rm m0}$$

↓

$$\alpha_{\rm K}(t) \quad \alpha_{\rm B}(t) \quad \alpha_{\rm M}(t) \quad \alpha_{\rm T}(t) \quad \alpha_{\rm H}(t) \quad H(t) \quad \Omega_{\rm m0}$$

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- arbitrary metric and background

$$\alpha_{\mathrm{K}}(t) \quad \alpha_{\mathrm{B}}(t) \quad \alpha_{\mathrm{M}}(t) \quad \alpha_{\mathrm{T}}(t) \quad \alpha_{\mathrm{H}}(t) \quad H(t) \quad \Omega_{\mathrm{m0}}$$

- finite order in perturbations theory on FRW (this is linear)

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- need some understanding to choose a model

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- fully consistent and non-trivial dynamics of the background

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- decouple background from perturbations, map to ST not clear

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- arbitrary metric and background
- need some understanding to choose a model
- fully consistent and non-trivial dynamics of the background
- each theory has to be solved from the beginning (probability of numerical noise)

$$\alpha_{\mathrm{K}}(t) \quad \alpha_{\mathrm{B}}(t) \quad \alpha_{\mathrm{M}}(t) \quad \alpha_{\mathrm{T}}(t) \quad \alpha_{\mathrm{H}}(t) \quad H(t) \quad \Omega_{\mathrm{m0}}$$

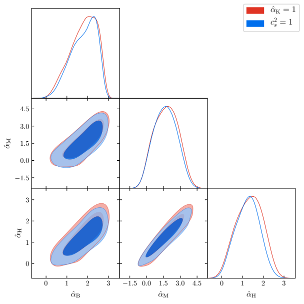
- finite order in perturbations theory on FRW (this is linear)
- easy to model
- decouple background from perturbations, map to ST not clear
- easy to jump from one theory to the other

The linear universe

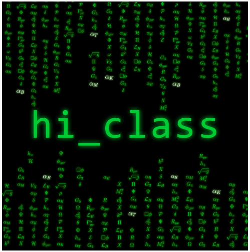
[Zumalacàrregui, EB, *et al.* (2017)]
[EB, Sawicki,Zumalacàrregui (2020)]

Guiding principles

- level of detail and control available to standard cosmology
- better a (controlled) error than a wrong solution



Traykova, *et al.* (2020)



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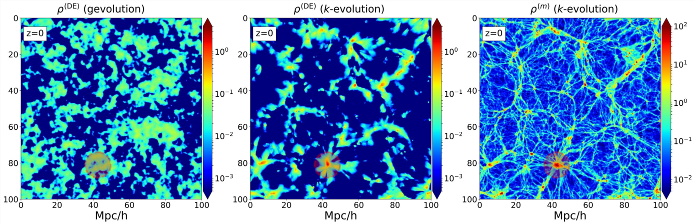
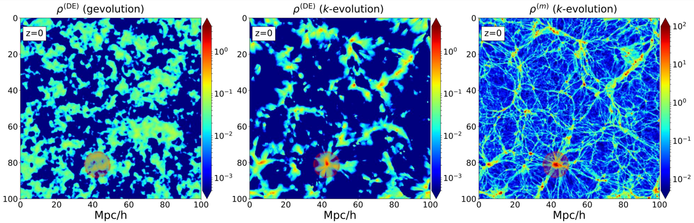


Hassani, *et al.* (2019)

The status

- N-body simulations for specific models, e.g. nDGP, $f(R)$, cubic Galileons, IDE
- fast methods more general, but many parameters
- we need emulators

The non-linear universe



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The status

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The dream

A unified framework to compress the number of non-linear parameters and describe broad classes of models.

Summary

- choose carefully how you want to describe gravity
- linear order with `hi_class` we are ready
- non-linearities: still a lot to do, but there are efforts (it's tough)

Thank you!