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Alleviating the σ8 tension via Soft Cosmology and Modified Gravity

Based on Saridakis (2021) and Tzerefos, Saridakis (in prep.)

National and Kapodistrian University of Athens & National Observatory of Athens
chtzeref@phys.uoa.gr

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Outline of the talk

- We shall examine the possibility of "soft" cosmology, namely small deviations from the usual cosmological framework due to the effective appearance of soft-matter properties in the Universe sectors.
- These properties could help alleviate issues of the standard cosmological paradigm, such as the growth tension.
- In this talk, preliminary results of how an $f(R)$ modified theory of gravity could naturally facilitate such properties for the dark Universe sectors will be presented.

Standard Cosmological Perturbation theory

Linear scalar metric perturbations (NG) :

$$ds^2 = a^2(\tau)\left[-(1+2\Psi)d\tau^2 + (1-2\Phi)\delta_{ij}dx^i dx^j\right]. \tag{1}$$

Stress-energy perturbations:

$$T_0^{0(l)} = -(\bar{\rho}^{(l)} + \delta\rho^{(l)})$$
$$T_i^{0(l)} = (\bar{\rho}^{(l)} + \bar{p}^{(l)})\upsilon_i^{(l)}\,,\ \upsilon_i^{(l)} \equiv a u_i^{(l)} \text{ and } \upsilon_i^{(l)} = -\upsilon_{,i}^{(l)}$$
$$T_j^{i(l)} = \bar{p}^{(l)}(\delta_j^i + \Pi_j^{i(l)}),$$

Conservation equations ($\nabla_\mu T_\nu^\mu = 0$):

$$\delta^{(l)\prime} = -3\mathcal{H}\left(c^{(l)2}_{eff} - w_{eff}^{(l)}\delta^{(l)}\right) - (1+w_{eff}^{(l)})k\upsilon^{(l)} + 3(1+w_{eff}^{(l)})\Phi' \tag{2}$$

$$\upsilon^{(l)\prime} = -\mathcal{H}\left[1 - 3w_{eff}^{(l)} - \frac{w_{eff}^{(l)\prime}}{\mathcal{H}(1+w_{eff}^{(l)})}\right]\upsilon^{(l)} + k\left[\Psi + \frac{(c_{eff}^{(l)})^2}{1+w_{eff}^{(l)}} - \frac{2\Pi^{(l)}w_{eff}^{(l)}}{3(1+w_{eff}^{(l)})}\right], \tag{3}$$

Standard Cosmological Perturbation theory

where $\delta^{(l)} \equiv \delta\rho^{(l)}/\bar{\rho}^{(l)}$ is the density contrast, $w_{eff}^{(l)} \equiv \bar{p}^{(l)}/\bar{\rho}^{(l)}$ is the parameter of state and $\left(c^{(l)}\right)_{eff}^{2} \equiv \delta P^{(l)}/\delta\rho^{(l)}$ is the effective sound speed square of the l-th species respectively.

The evolution of Φ and Ψ is governed by the perturbed Einstein equations, which in Fourier space are:

$$3\mathcal{H}(\Phi' + \mathcal{H}\Psi) + k^2\Phi = -4\pi G a^2\, \delta\rho^{tot} \tag{4}$$

$$k(\Phi' + \mathcal{H}\Psi) = 4\pi G a^2 \sum_l (\bar{\rho}^{(l)} + \bar{p}^{(l)})\upsilon^{(l)} \tag{5}$$

$$\Phi'' + \mathcal{H}(\Phi' + 2\Psi') + (\mathcal{H}^2 + 2\mathcal{H}')\Phi - k^2(\Phi - \Psi)/3 = 4\pi G a^2\, \delta p^{tot} \tag{6}$$

$$k^2(\Phi - \Psi) = 8\pi G a^2 \sum_l \bar{p}^{(l)}\Pi^{(l)} \tag{7}$$

Standard Cosmological Perturbation theory

By combining the (2) and (3) one can get the following general equation:

$$-\delta^{(I)\prime\prime}-\mathcal{H}\delta^{(I)\prime}\left(1-6w_{eff}^{(I)}\right)+\left(\frac{\delta P^{(I)}}{\bar{\rho}^{(I)}}-w_{eff}^{(I)}\delta^{(I)}\right)\left[3\mathcal{H}^2(3w_{eff}^{(I)}-1)-3\mathcal{H}'\right]$$
$$-3\mathcal{H}\left(\frac{\delta P^{(I)}}{\bar{\rho}^{(I)}}\right)'+3\mathcal{H}w_{eff}^{(I)\prime}\delta^{(I)}=$$
$$=-3(1+w_{eff}^{(I)})\left[\Phi''+\Phi'\mathcal{H}\left(1-3w_{eff}^{(I)}+\frac{w_{eff}^{(I)\prime}}{\mathcal{H}(1+w_{eff}^{(I)})}\right)\right]+$$
$$k^2\left[(1+w_{eff}^{(I)})\Psi+\frac{\delta P^{(I)}}{\bar{\rho}^{(I)}}-\frac{2\Pi^{(I)}w_{eff}^{(I)}}{3}\right] \quad (8)$$

Note: Equation (8) *doesn't depend on the underlying gravitational theory*, but rather only on the conservation of the energy-momentum tensor of the I-th sector.

Standard Cosmological Perturbation theory

- In the framework of Λ_{CDM} cosmology, for the study of structure formation it is standard to consider only adiabatic scalar perturbations ($c_{eff}^2 \approx w$), neglect the radiation contribution and take the sub-horizon limit $k >> \mathcal{H}$, so (8) yields:

$$\delta_m'' + \mathcal{H}\delta_m' = -4\pi G \delta_m \bar{\rho}_m, \qquad (9)$$

  which is the usual *growth equation*.

- There is a rather strong assumption that is implied in usual perturbation theory analysis, namely that the sectors which constitute the Universe are *simple*

Introduction to "Soft" Cosmology

In condensed matter physics, it is well known that there is a large variety of “soft” matter forms, which are characterized by complexity, simultaneous co-existence of phases, entropy dominance, extreme sensitivity, viscoelasticity, etc, properties that arise effectively at *intermediate* scales due to *scale-dependent* effective interactions that are not present at the fundamental scales.

Opportunity: These properties can help alleviate the aforementioned cosmological tensions.

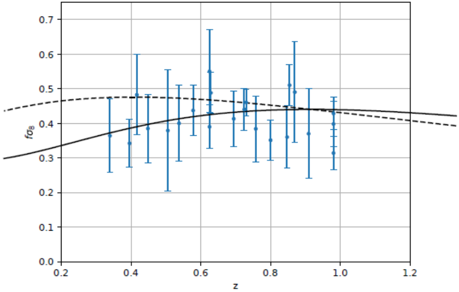
Why investigate "Soft" Cosmology?

As an illustration, we can introduce the effective “softness parameter” s_{de} for the dark energy sector. At cosmological, large scales (ls) dark energy has the usual EoS, namely w_{de-ls}, at intermediate scales (is) we have

$$w_{de-is} = s_{de} \cdot w_{de-ls}, \tag{10}$$

and standard cosmology is recovered for $s_{de} = 1$. The background evolution remains unaffected. Then, borrowing from E. Saridakis, (2021) ,(2105.08646) :

Improved growth behaviour



The $f\sigma_8$ as a function of z. The dashed curve is for ΛCDM. The solid curve is for soft dark energy with $s_{de} = 1.1$, i.e. with $w_{de-ls} = -1$ and $w_{de-is} = -1.1$, and $c_{\text{eff}}^{(de)} = 0.1$, while dark matter is standard (i.e. not soft) with $w_{dm} = 0$.

Soft properties from $f(R)$ gravity

Aim: The purpose of the following work is to explicitly demonstrate that such properties can naturally arise within the framework of $f(R)$ gravity.

We are going to accomplish that by reformulating the $f(R)$ field equations into the GR ones plus an effective "curvature" fluid, as for instance is done in: Arjona et al, (2018), (1811.02469).

$f(R)$ gravity action and field equations

The characteristic action of $f(R)$ is the following:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,f(R) + \int d^4x\sqrt{-g}\mathcal{L}_m, \qquad (11)$$

where $f(R)$ is a general function of the Ricci scalar R, and $T^{(m)}_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta\mathcal{L}_m}{\delta g^{\mu\nu}}$. Varying the action (11) with respect to $g_{\mu\nu}$ yields the following field equation:

$$FR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)F = 8\pi G T^{(m)}_{\mu\nu}, \qquad (12)$$

where we set $F \equiv df(R)/dR$. The aforementioned reformulation can proceed as follows:

Reformulation as an effective fluid

$$G^{\mu}_{\nu} = 8\pi G \left(T^{(m)\mu}_{\nu} + T^{(eff)\,\mu}_{\nu} \right) \equiv T^{(tot)\,\mu}_{\nu} \text{ with} \tag{13}$$

$$T^{(eff)\,\mu}_{\nu} \equiv (1-F)R^{\mu}_{\nu} + \frac{1}{2}\delta^{\mu}_{\nu}(f-R) - (\delta^{\mu}_{\nu}\Box - \nabla^{\mu}\nabla_{\nu})F \tag{14}$$

As we explained earlier, via the bianchi identities this tensor is conserved $\nabla_{\mu} T^{(eff)\,\mu}_{\nu} = 0$. Thanks to this construction, for the FLRW metric we can get the Friedmann equations with:

$$\bar{\rho}_{eff} \equiv -T^{(eff)\,0}_{0} = \frac{1}{8\pi G a^2}\left(3\mathcal{H}^2 - \frac{1}{2}\alpha^2 f + 3F\mathcal{H}' - 3\mathcal{H}F'\right) \tag{15}$$

$$\bar{p}_{eff} \equiv \frac{T^{(eff)\,i}_{i}}{3} = \frac{1}{8\pi G a^2}\left(-2\mathcal{H}' - \mathcal{H}^2 + \frac{1}{2}a^2 f - F\mathcal{H}' - 2F\mathcal{H}^2 + F'' + \mathcal{H}F'\right) \tag{16}$$

Perturbations in $f(R)$

After performing the same (scalar) perturbations as in the previous section, the set of perturbed field equations is the same as in GR with:

$$\delta\rho_{eff} \equiv -\delta T_0^{(eff)\,0} = -\frac{1}{8\pi G a^2}\Big\{(1-F)\big[-6\mathcal{H}'\Psi + k^2\Psi - 3\mathcal{H}(\Phi'+\Psi') - 3\Phi''\big]$$
$$-3\mathcal{H}'\delta F + a^2\delta f/2 - k^2\Psi + 2k^2\Phi + 6(\mathcal{H}'+\mathcal{H}^2)\Psi + 3\Phi'' + 3\mathcal{H}(\Psi'+3\Phi')$$
$$+k^2\delta F + 3\mathcal{H}\delta F' - 3F'(\Phi'+2\mathcal{H}\Psi)\Big\}, \tag{17}$$

$$\delta p_{eff} \equiv \frac{\delta T_i^{(eff)\,i}}{3} = \frac{1}{8\pi G a^2}\Big\{(1-F)\big[-k^2\Phi - \Phi'' - 3\mathcal{H}(5\Phi'+\Psi')$$
$$-(2\mathcal{H}'+4\mathcal{H}^2)\Psi - k^2(\Phi-\Psi)/3\big] - (\mathcal{H}'+2\mathcal{H}^2)\delta F + a^2\delta f/2 +$$
$$+3\Phi'' + k^2(2\Phi-\Psi) + 3\mathcal{H}(\Psi'+3\Phi') + 6(\mathcal{H}'+\mathcal{H})\Psi +$$
$$+\delta F'' + 2k^2\delta F/3 + \mathcal{H}\delta F' - F'(2\Phi'+2\mathcal{H}\Psi+\Psi') - 3\Psi F''\Big\}$$
$$\tag{18}$$

Perturbations in $f(R)$

$$(\bar{\rho}_{eff} + \bar{p}_{eff})\upsilon^{eff}_{,i} \equiv -\delta T^{(eff)\,0}_{i} = \frac{1}{8\pi G}\Big[2(1-F)(\Phi' + \mathcal{H}\Psi)_{,i} +$$
$$+ \delta F'_{,i} + F'\Psi_{,i} - \mathcal{H}\delta F_{,i}\Big] \text{ and}$$
$$\Pi^{eff}_{ij}\bar{p}_{eff} \equiv \delta T^{(eff)\,i}_{j} = \frac{1}{8\pi G a^2}\Big[(1-F)(\Phi-\Psi)_{,ij} + \delta F_{,ij}\Big],\ i \neq j \qquad (19)$$

We now aim to reveal the "soft" properties of this fluid - we can't just use the simple expression $w_{eff} = \bar{p}_{eff}/\bar{\rho}_{eff}$ from equations (15) and (16), since they only hold on large scales.

Instead, we shall infer w_{eff} by constructing an equation like (8) via algebraic manipulations of the field equation (4) with $\delta\rho_{tot} = \delta\rho_m + \delta\rho_{eff}$ where $\delta\rho_{eff}$ is given by (17).

Revealing soft properties

- First, we construct the quantities $\delta_{eff} \equiv \delta\rho_{eff}/\bar{\rho}_{eff}$, δ'_{eff} and δ''_{eff}.
- Next, we add to both sides of the equation (4) every perturbation term in (8) that doesn't have a w_{eff} factor, which means the terms $\delta''_{eff} + \mathcal{H}\delta'_{eff} + 3\frac{\delta p_{eff}}{\bar{\rho}}(\mathcal{H}' + \mathcal{H}^2) + 3\mathcal{H}\left(\frac{\delta p_{eff}}{\bar{\rho}}\right)'$.
- After using the equations (15) and (17), we get a very large number of terms. Luckily, we are only interested in the terms which contain a factor of $k^2\Psi$, since our aim is to identify those terms with w_{eff} using the corresponding term of equation (8)

Revealing soft properties

Therefore, if we only consider the aforementioned terms and perform some factorisations, we finally obtain the following equation:

$$\delta''_{eff}+\mathcal{H}\delta'_{eff}+3\frac{\delta p_{eff}}{\bar{\rho}}(\mathcal{H}'+\mathcal{H}^2)+3\mathcal{H}\left(\frac{\delta p_{eff}}{\bar{\rho}}\right)' = k^2\frac{\Psi}{\rho_{eff}}\Bigg\{\mathcal{H}^2+\mathcal{H}'+\frac{\rho_{eff}}{2}+2A^2$$

$$+k^2\left\{2\frac{F_{,R}}{a^2}\left[-4\mathcal{H}^2-4\mathcal{H}'-\frac{\rho_{eff}}{2}-2A^2+6\mathcal{H}A-B\right]-2\frac{F'_{,R}}{a^2}(5\mathcal{H}+2A)+2\frac{F''_{,R}}{a^2}\right\}+$$

$$+F\left(3\mathcal{H}^2-2\mathcal{H}'+2B+2\mathcal{H}A\right)-F'2\mathcal{H}+$$

$$+2\frac{F_{,R}}{a^2}\left\{12\mathcal{H}^4+3(\mathcal{H}')^2+21\mathcal{H}^2\mathcal{H}'+9\mathcal{H}\mathcal{H}''-3\mathcal{H}'''+\frac{\rho_{eff}}{2}\left(3\mathcal{H}'+6\mathcal{H}^2\right)\right\}$$

$$+2\frac{F'_{,R}}{a^2}\left[15\mathcal{H}^3-3\mathcal{H}\mathcal{H}'-3\mathcal{H}''-\frac{3}{2}\mathcal{H}\rho_{eff}+A\left(12\mathcal{H}^2-6\mathcal{H}A\right)-3\mathcal{H}B\right]$$

$$-18\frac{F''_{,R}}{a^2}\mathcal{H}A\Bigg\}+\text{other terms}$$

$$\equiv(1+w_{eff})k^2\Psi+\text{other terms}, \quad (20)$$

Revealing soft properties

where $A \equiv \rho'_{eff}/\rho_{eff}$, $B \equiv \rho''_{eff}/\rho_{eff}$ and ρ_{eff} and its derivatives are given by equation (15). Our final result can be written as:

$$
\begin{aligned}
w_{eff} = -1 + \frac{1}{\rho_{eff}} \Bigg\{ & \mathcal{H}^2 + \mathcal{H}' + \frac{\rho_{eff}}{2} + 2A^2 \\
& + k^2 \left\{ 2\frac{F_{,R}}{a^2} \left[-4\mathcal{H}^2 - 4\mathcal{H}' - \frac{\rho_{eff}}{2} - 2A^2 + 6\mathcal{H}A - B \right] \right. \\
& - 2\frac{F'_{,R}}{a^2}\left(5\mathcal{H} + 2A\right) + 2\frac{F''_{,R}}{a^2} + +F\left(3\mathcal{H}^2 - 2\mathcal{H}' + 2B + 2\mathcal{H}A\right) - F'2\mathcal{H} + \\
& 2\frac{F_{,R}}{a^2}\left\{ 12\mathcal{H}^4 + 3(\mathcal{H}')^2 + 21\mathcal{H}^2\mathcal{H}' + 9\mathcal{H}\mathcal{H}'' - 3\mathcal{H}''' + \frac{\rho_{eff}}{2}\left(3\mathcal{H}' + 6\mathcal{H}^2\right) \right\} + \\
& + 2\frac{F'_{,R}}{a^2}\left[15\mathcal{H}^3 - 3\mathcal{H}\mathcal{H}' - 3\mathcal{H}'' - \frac{3}{2}\mathcal{H}\rho_{eff} + A\left(12\mathcal{H}^2 - 6\mathcal{H}A\right) - 3\mathcal{H}B \right] \\
& \left. - 18\frac{F''_{,R}}{a^2}\mathcal{H}A \right\} \qquad (21)
\end{aligned}
$$

Conclusions

- We briefly examined the possibility of “soft" cosmology, namely small deviations from the usual framework due to the effective appearance of soft properties in the Universe sectors.
- We demonstrated that modified theories of gravity (MG) and $f(R)$ in particular can facilitate such a possibility.
- A potential refinement of these results could provide a more quantitatively useful bridge between parameters of MG and these phenomenological "soft" parameters.
- In order to incorporate complexity and estimate the scale-dependent behavior of the EoS’s from first principles we should revise and extend the cosmological perturbation theory.