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Well-Tempered Teleparallel Horndeski Cosmology

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Cosmolgoical Constant Problem

- Cosmological constant Λ offers a scenario to describe a net vacuum energy that is responsible for the accelerated expansion of the universe.
- Vacuum energy density ρ_Λ:

Observational	Planck Scale	Proton-Neutron
$\leq (10^{-12}\ \text{GeV})^4$	$\approx (10^{18}\ \text{GeV})^4$	$\approx 10^{-38}\ \text{GeV}^4$

- A positive Λ value:
  - Account for quantum fluctuations
  - Corresponds to an older universe

Teleparallel Gravity

General Relativity

- Curvatureful
- Metric $g_{\mu\nu}$ is the fundamental dynamical object
- Levi-Civita connection $\mathring{\Gamma}^{\lambda}_{\mu\nu}$
- Riemann curvature tensor $\mathring{R}^{\lambda}{}_{\mu\rho\nu}$

Teleparallel Gravity

- Torsionful
- Tetrad $e^{A}{}_{\mu}$ is the fundamental dynamical object
- Weitzenbock connection $\Gamma^{\lambda}_{\mu\nu}$
- Torsion tensor $T^{\lambda}{}_{\mu\nu}$

$$\underbrace{\mathring{R}}_{\text{Ricci Scalar}} = -\underbrace{T}_{\text{Torsion Scalar}} + \underbrace{B}_{\text{Boundary Term}} \tag{1}$$

Action

$$\mathcal{S}_{\text{BDLS}} = \overbrace{\sum_{i=2}^{5} \int d^4x\, e\, \mathcal{L}_i}^{\text{Standard Horndeski}} + \overbrace{\int d^4x\, e\, \mathcal{L}_{\text{Tele}}}^{\text{Teleparallel}} + \overbrace{\int d^4x\, e\, \mathcal{L}_{\text{m}}}^{\text{Matter}}, \tag{2}$$

where

$$\mathcal{L}_2 := G_2(\varphi, X), \qquad \mathcal{L}_3 := -G_3(\varphi, X)\mathring{\Box}\varphi,$$

$$\mathcal{L}_4 := G_4(\varphi, X)(-T + B) + G_{4,X}(\varphi, X)[(\mathring{\Box}\varphi)^2 - \varphi_{;\mu\nu}\,\varphi^{;\mu\nu}],$$

$$\mathcal{L}_5 := G_5(\varphi, X)\mathring{G}_{\mu,\nu}\varphi^{;\mu\nu} - \frac{1}{6}G_{5,X}(\varphi, X)[(\mathring{\Box}\varphi)^3 + 2\varphi_{;\mu}{}^{\nu}\,\varphi_{;\nu}{}^{\alpha}\,\varphi_{;\alpha}{}^{\nu} - 3\varphi_{;\mu\nu}\,\varphi^{;\mu\nu}\,\mathring{\Box}\varphi],$$

and

$$\mathcal{L}_{\text{Tele}} := G_{\text{Tele}}(\underbrace{\varphi}_{\text{scalar}}, \underbrace{X}_{\text{kinetic}}, \underbrace{T, T_{\text{ax}}, T_{\text{vec}}}_{\text{irreducible parts of } T^{\lambda}{}_{\mu\nu}}, \underbrace{I_2}_{\text{linear } T^{\lambda}{}_{\mu\nu} \text{ and } \varphi_{;\mu}}, \underbrace{J_1, J_3, J_5, J_6, J_8, J_{10}}_{\text{quadratic } T^{\lambda}{}_{\mu\nu} \text{ and } \varphi_{;\mu}}),$$

Late-Time Cosmology

Homogeneous and isotropic background described by the FLRW metric:

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 (dx^2 + dy^2 + dz^2)\,, \tag{3}$$

and the tetrad is expressed in terms of the lapse function $N(t)$ and scale factor $a(t)$:

$$e^A{}_\mu = \mathrm{diag}(N(t), a(t), a(t), a(t)) \quad \text{and} \quad e = \det(e^A{}_\mu) = N(t)a(t)^3\,, \tag{4}$$

from which one can construct

- Hamiltonian equation: varying with respect to $N(t)$
- Hubble equation: varying with respect to $a(t)$
- Scalar equation: varying with respect to $\varphi(t)$

For simplification:

$$G_2(\varphi, X) = V(\varphi, X)\,, \quad G_3(\varphi, X) = G(\varphi, X)\,, \quad G_4(\varphi, X) = \frac{1}{16\pi G} + \frac{A(\varphi, X)}{2}\,,$$
$$G_5(\varphi, X) = 0\,, \quad G_{\text{Tele}}(\varphi, X, \ldots) = \mathcal{G}(\varphi, X, T, I_2)\,.$$

- $T_{\text{ax}} = J_1 = J_3 = J_5 = J_6 = J_8 = J_{10} = 0$ and $T_{vec} = -\frac{3}{2}T$ at background.
- Observations GW170817 (by A-LIGO, Virgo collaboration and GRB surveys) showed that gravitational wave speed is almost equivalent to speed of light in homogeneous universe, therefore $A(\varphi, X) \to A(\varphi)$ since mixing of the form $g^{\mu\nu}\varphi_{;\mu}\varphi_{;\nu}$ alters graviton speed. **This has been challenged, and X dependency is kept within a teleparallel framework.**

Well-Tempering

- Dynamically cancel out large cosmological constant and replace it with a much lower de Sitter state.
- Takes into account that different energy fields come into play as the Universe evolves.
- Quantum radiative corrections accounted for through shift symmetry.
- Ensures that a viable cosmic history is obtained with radiation and matter dominated periods.

Conditions

$$\text{Hubble Equation} \qquad \dot{H} = \ddot{\varphi}\mathcal{Z}(\varphi, \dot{\varphi}, H) + \mathcal{Y}(\varphi, \dot{\varphi}, H)\,, \tag{5}$$

$$\text{Scalar Equation} \qquad 0 = \ddot{\varphi}\mathcal{D}(\varphi, \dot{\varphi}, H) + \mathcal{C}(\varphi, \dot{\varphi}, H, \dot{H})\,. \tag{6}$$

For a de Sitter vacuum: $P_\Lambda = -\rho_\Lambda\,,$ and $H(t) = h\,.$

$$\text{Degeneracy Equation:} \qquad \mathcal{Y}\mathcal{D} - \mathcal{C}\mathcal{Z} = 0\,, \tag{7}$$

$$\text{Consistency Conditions:} \qquad \mathcal{Z} \neq 0 \quad \text{and} \quad \mathcal{D} \neq 0\,, \tag{8}$$

$$\varphi(t) \text{ in Hamiltonian Equations:} \qquad 3h^2 = \rho_\Lambda + F(\varphi(t), \dot{(\varphi)})\,. \tag{9}$$

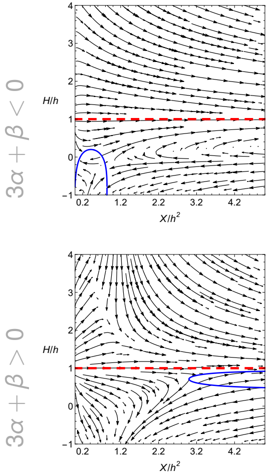
Examples

	Degeneracy	Consistency	Dynamical Hamiltonian
$\mathcal{G}(I_2)$	☒	☒	☐
$\mathcal{G}(\varphi, I_2)$	☒	☒	☒
$A(\varphi) + \mathcal{G}(I_2)$	☒	☒	☒
$A(X) + \mathcal{G}(\varphi, X, I_2)$	☒	☒	☒
$V(\varphi, X) + G(X) + A(X) + \mathcal{G}(X, I_2)$	☒	☒	☒
$V(X) + G(X) + A(X) + \mathcal{G}(X, I_2)$	☒	☒	☐

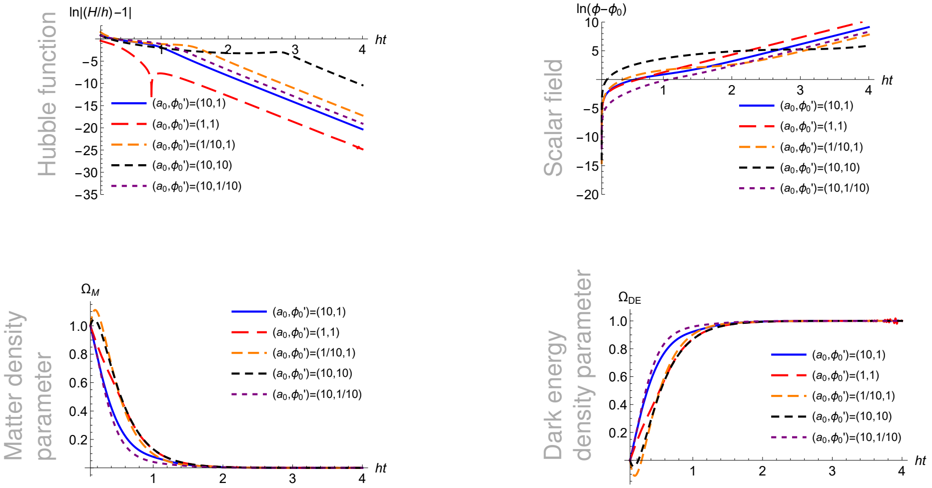
Dynamical Stability of Well Tempered Vacuum

For the example $A(X) + V(X) + G(X) + \mathcal{G}(\varphi, X, T, I_2)$:

- The differences in the portrait depends on the coefficient of A (α) and the coefficient of $\mathcal{G}$ (β) as a linear combination.
- The size energy density of the vacuum $\rho_\Lambda = 3h^2\lambda$ vanishes in the system i.e. it holds regardless of vacuum size.
- Red-dashed line represents the well-tempered vacuum.
- Blue-solid line represents the critical curve for which a particular function is undefined.

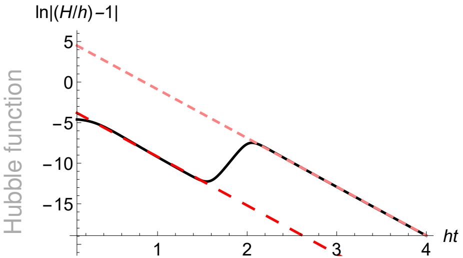


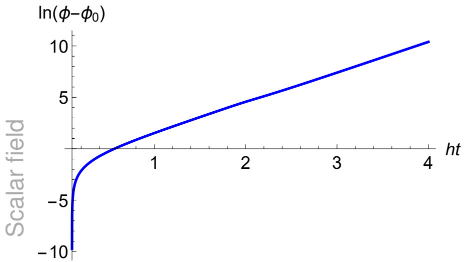
Compatibility with Matter Era



Stability through Phase Transition

For a disturbance to represent the phase transition





- Red-long-dashed line depicts the exponential solution before the disturbance.
- Pink-short-dashed line depicts the same solution after the disturbance.
- The scalar field is seen to continue to propagate despite the disturbance.

Conclusion

- Well-tempering offers a model to dynamically cancel the vacuum energy to end up with a net value attributed to dark energy.
- An approach to tackle the cosmological constant problem.
- Teleparallel analog of Horndeski theory offers a plethora of cosmological models to obtain a well-tempering model due to the conformal pontetial A and the teleparallel potential $\mathcal{G}$.
- All results correspond to the their Horndeski limit counterparts.

Future Work

- Include $G_5(\varphi, X)$ terms.
- Soundness through checking for ghost and Laplacian instabilities to verify the stability of the de Sitter asymptote.
- Constraining well-tempering with cosmological data.
- Exploring well-tempering in BDLS at the strong gravity regime.

References

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