Assessing the hemispherical power asymmetry with gravitational waves

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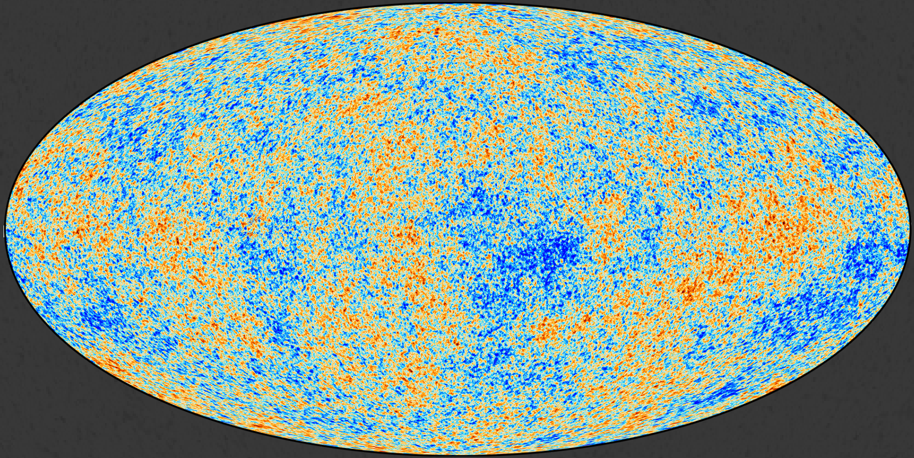
Based on:

Giacomo Galloni, Nicola Bartolo, Sabino Matarrese, Marina Migliaccio, Angelo Ricciardone and Nicola Vittorio, *Test of the statistical isotropy of the Universe using gravitational waves*, accepted JCAP

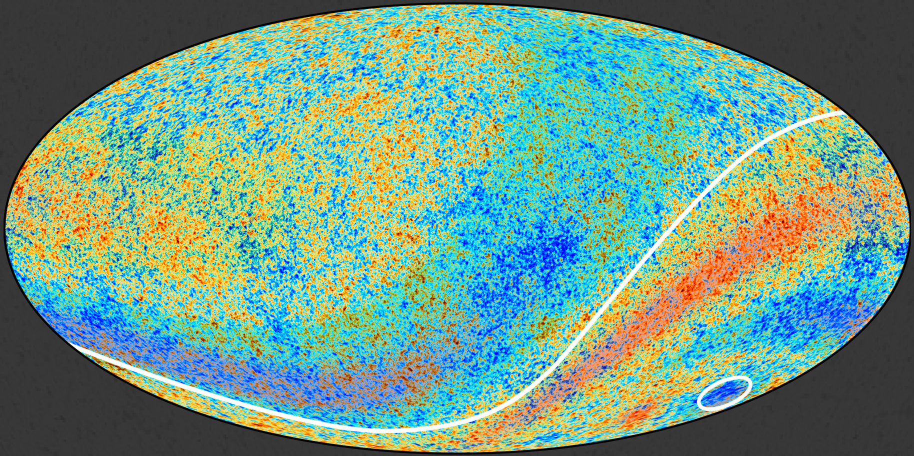
2202.12858

Cosmic Microwave Background



Credits: Planck's collaboration

Anomalies



Credits: Planck's collaboration

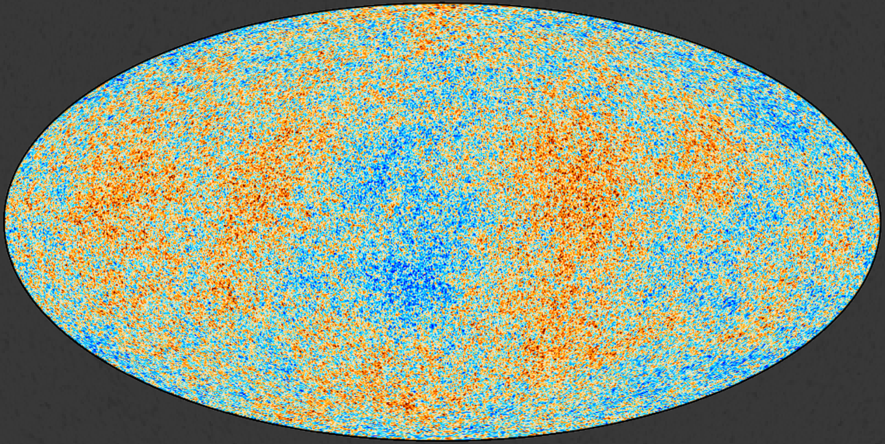
Tackling the Anomaly

Why isn't everyone aware of this?

The significance is about 2σ.

Phenomenological approach VS Null-hypothesis

Cosmological GW Background



Bartolo et al. - 1912.09433

Angular power spectra

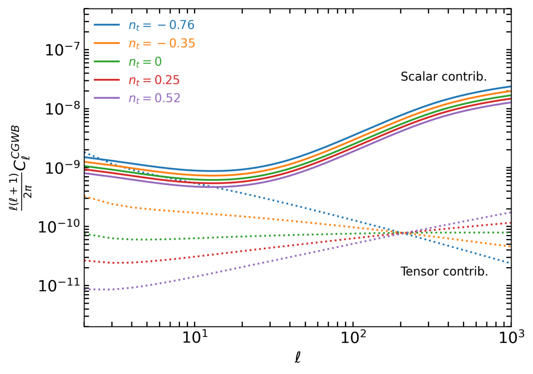


Figure: Angular power spectrum of CGWB (GG et al. - 2202.12858)

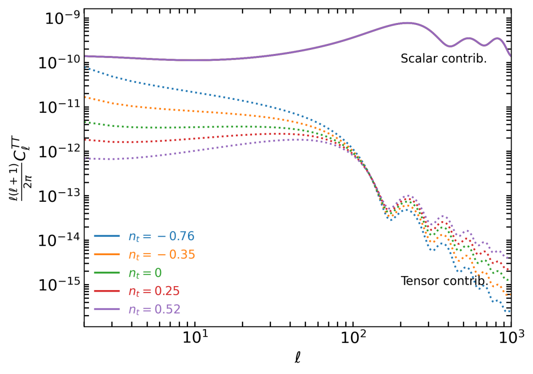


Figure: Angular power spectrum of CMB TT (GG et al. - 2202.12858)

For updated constraints on r and n_t, GG et al. - 2208.00188

Cross-correlation power spectrum

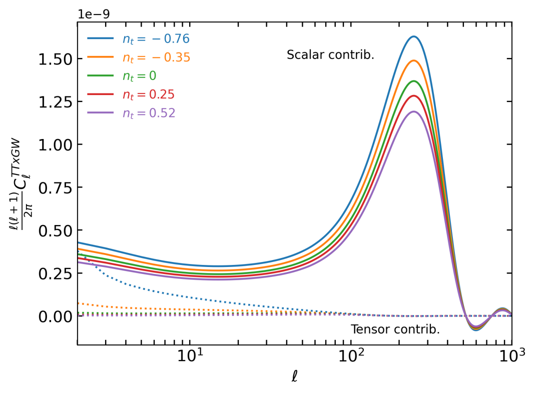


Figure: Cross-spectrum between CGWB and CMB temperature (GG et al. - 2202.12858)

Ricciardone et al. - 2106.02591

Modulated Gravitational Potentials

Gravitational potentials $\Psi, \Phi \propto$

$$\zeta(\vec{x}) = g(\vec{x})[1 + h(\vec{x})]$$

with

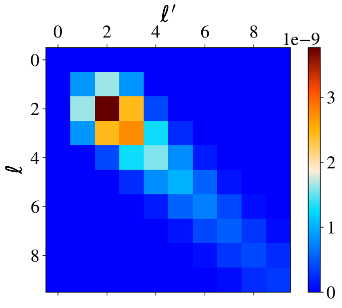
$$h(\vec{x}) \propto \omega \frac{\sin \vec{k}_0 \cdot \vec{x}}{k_0 D},$$

$$k_0 \ll k$$

Dvorkin et al. - 0711.2321

This results in

Spherical harmonic coefficients have non-diagonal m-dependent covariance in $\ell - \ell'$



Credits: GG et al. - 2202.12858

Null-hypothesis

Assume no modulation and ΛCDM+tensors

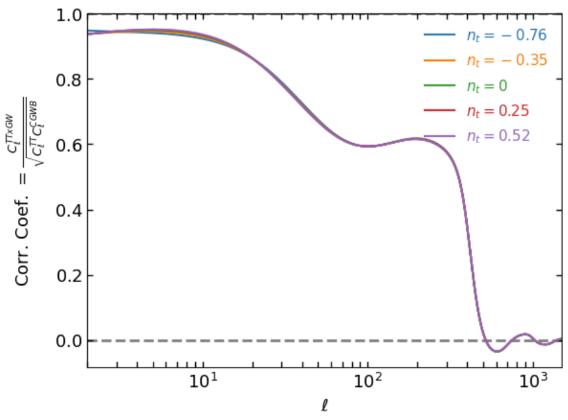
$\Downarrow$

N realizations of CMB TT $+$ N realizations of CGWB

$\Downarrow$

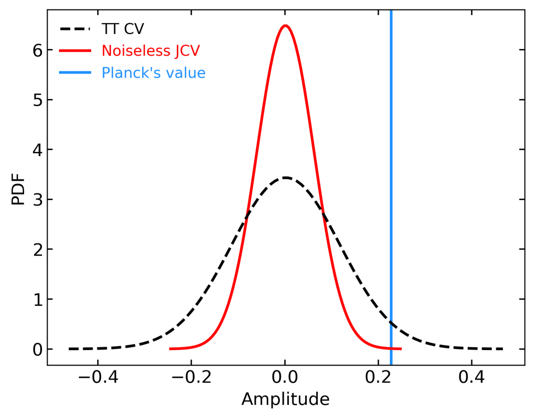
Estimate the amplitude either with CMB alone or with CMB+CGWB

Constrained realizations of the CGWB



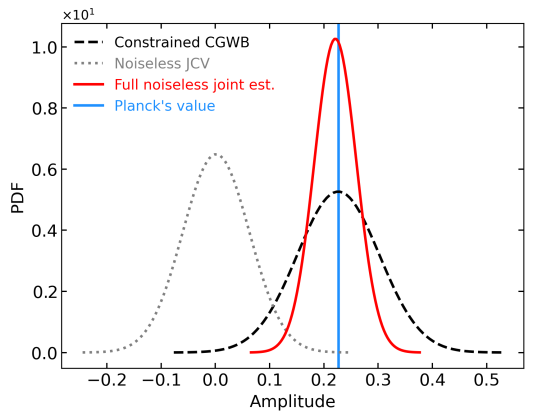
Credits: GG et al. - 2202.12858

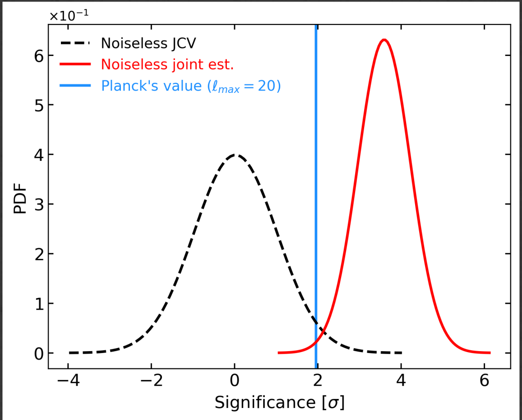
Cosmic Variance Distribution



$$\ell_{max} = 20, \; n_t = 0$$

Assessing the Significance





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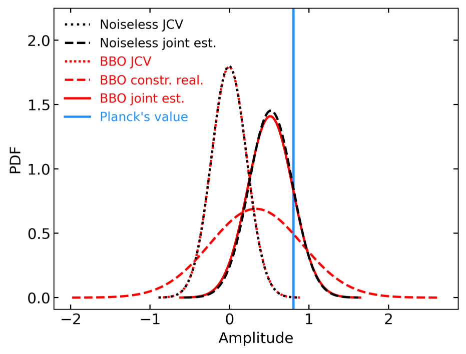
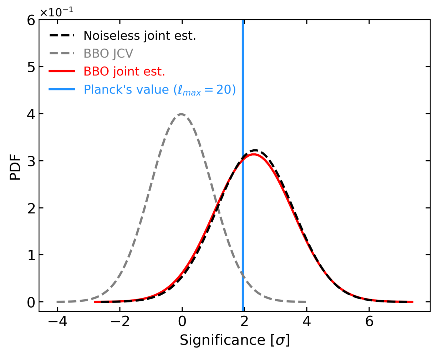
Assessing the Significance





$$\ell_{max} = 6,\ n_t = 0.52$$

For updated constraints on r and n_t, GG et al. - 2208.00188

Conclusions

2202.12858

- Analytical expressions for the variance of the spherical harmonics coefficients
- Statistical tools to study the significance
- GWs have the potential of being crucial to assess the significance of the hemispherical power asymmetry

Future Perspectives

- What do other observables have to say about this?
- What is the underlying physical model?
- What about other indications of a departure from statistical isotropy? Could they be related to this dipole modulation model?

Thank you for your attention!

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Backup

Boltzmann Equations

$$\mathscr{L}[f(x^\mu, p^\mu)] = \mathscr{C}[f(x^\mu, p^\mu)] + \mathscr{I}[f(x^\mu, p^\mu)]$$

$$\Downarrow$$

$$f_{gw}(\eta, \vec{x}, \vec{q}) = \bar{f}_{gw}(q) - q\frac{\partial \bar{f}_{gw}}{\partial q}\Gamma(\eta, \vec{x}, \vec{q})$$

$$\Downarrow$$

$$\frac{\partial \Gamma}{\partial \eta} + n^i \frac{\partial \Gamma}{\partial x^i} = \frac{d\Psi}{d\eta} - \frac{d\Phi}{dx^i} n^i - \frac{1}{2}\frac{d\chi_{jk}}{d\eta} n^j n^k$$

Boltzmann Equations

$$\frac{\partial \Gamma}{\partial \eta} + n^i \frac{\partial \Gamma}{\partial x^i} = \frac{d\Psi}{d\eta} - \frac{d\Phi}{dx^i} n^i - \frac{1}{2} \frac{d\chi_{jk}}{d\eta} n^j n^k$$

$$\Downarrow$$

$$\Gamma(\eta, \vec{k}, q, \hat{n}) \longleftrightarrow \Gamma_{\ell m}$$

$$\Downarrow$$

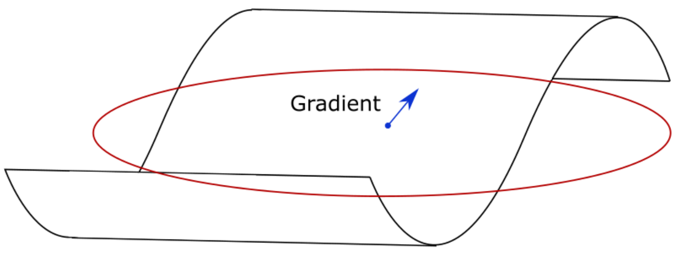
$$\langle \Gamma_{\ell m} \Gamma^*_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell^{CGWB}$$

CMB vs CGWB

$$\Gamma_{\ell m,S} = 4\pi(-i)^\ell \int \frac{d^3k}{(2\pi)^3} Y^*_{\ell m}(\hat{k}) \int_{\eta_{\text{in}}}^{\eta_0} d\eta \Bigg\{ \delta(\eta - \eta_{in}) \left[\Gamma_I(\eta, \vec{k}, q) + \Phi(\eta, \vec{k}) \right] + \frac{\partial \left[\Psi(\eta, \vec{k}) + \Phi(\eta, \vec{k}) \right]}{\partial \eta} \Bigg\} j_\ell[k(\eta_0 - \eta)]$$

$$\Theta_{\ell m,S} = 4\pi(-i)^\ell \int \frac{d^3k}{(2\pi)^3} Y^*_{\ell m}(\hat{k}) \int_{\eta_{\text{in}}}^{\eta_0} d\eta \Bigg\{ g(\eta) \left(\Theta_0(\eta, \vec{k}) + \Phi(\eta, \vec{k}) \right) + e^{-\tau(\eta)} \frac{\partial \left[\Psi(\eta, \vec{k}) + \Phi(\eta, \vec{k}) \right]}{\partial \eta} \Bigg\} j_\ell[k(\eta_0 - \eta)]$$

Boltzmann Equations for Modulated Fields



$$\left\langle \zeta(\vec{k})\zeta^*\left(\vec{k}'\right)\right\rangle = (2\pi)^3\delta\left(\vec{k}-\vec{k}'\right)P_g(k) + \left[P_g(k)+P_g\left(k'\right)\right]h\left(\vec{k}-\vec{k}'\right) + \int \frac{d^3\widetilde{k}}{(2\pi)^3}P_g(\widetilde{k})h(\vec{k}-\vec{\widetilde{k}})h^*\left(\vec{k}'-\vec{\widetilde{k}}\right)$$

Zeroth Order Term in the Modulating Field

$$\left\langle \Gamma_{\ell m,S} \Gamma^{*}_{\ell' m',S} \right\rangle^{(0)} = \delta_{\ell \ell'} \delta_{m m'} \mathcal{C}_{\ell}^{(0)}$$

$$\mathcal{C}_{\ell}^{(0)} \equiv 4\pi \int \frac{dk}{k} \frac{k^3}{2\pi^2} P_g(k) T_{\ell}^2$$

First Order Term in the Modulating Field

$$\left\langle \Gamma_{\ell m,S}\Gamma^{*}_{\ell' m',S}\right\rangle^{(1)} = \omega\delta_{mm'}\left[R^{1,\ell}_{\ell' m}\mathcal{C}^{(1)}_{\ell} + R^{1,\ell'}_{\ell m}\mathcal{C}^{(1)}_{\ell'}\right]$$

$$\mathcal{C}^{(1)}_{\ell} \equiv 4\pi\int\frac{dk}{k}\frac{k^3}{2\pi^2}P_g(k)T_\ell \times T^{\star}_{\ell}$$

Second Order Term in the Modulating Field

$$\left\langle \Gamma_{\ell m,S} \Gamma^*_{\ell' m',S} \right\rangle^{(2)} = \omega^2 \delta_{mm'} \sum_j R^{1,j}_{\ell m} R^{1,j}_{\ell' m} \mathcal{C}^{(2)}_j$$

$$\mathcal{C}^{(2)}_\ell \equiv 4\pi \int \frac{dk}{k} \frac{k^3}{2\pi^2} P_g(k) \left(T^\star_\ell\right)^2$$

Details on calculations

$$ds^2 = a^2(\eta)\left[-(1+2\Phi)d\eta^2 + \left[(1-2\Psi)\,\delta_{ij} + \chi_{ij}\right]dx^i dx^j\right]$$

$$\delta_{GW}(\eta_0, \vec{x}, q, \hat{n}) \equiv \left[4 - \frac{\partial \ln \bar{\Omega}_{GW}}{\partial \ln q}\right]\Gamma(\eta_0, \vec{x}, q, \hat{n})$$

$$\left\langle \Gamma_{\ell m, S}\Gamma^*_{\ell' m', S}\right\rangle = (4\pi)^2(-i)^{\ell-\ell'}\int \frac{d^3k}{(2\pi)^3}\mathrm{e}^{i\vec{k}\cdot\vec{x}_0}\int \frac{d^3k'}{(2\pi)^3}\mathrm{e}^{-i\vec{k}'\cdot\vec{x}_0}\left\langle \zeta(\vec{k})\zeta^*\left(\vec{k}'\right)\right\rangle$$
$$\times\, Y^*_{\ell m}(\hat{k})Y_{\ell' m'}\left(\hat{k}'\right)T^S_\ell\left(k, \eta_0, \eta_{\rm in}\right)T^S_{\ell'}\left(k', \eta_0, \eta_{\rm in}\right)$$

Details on calculations

$$h(\vec{k}) = \frac{\omega}{2i}\sqrt{\frac{3}{4\pi}\frac{(2\pi)^3}{k_0 D}}\left[\delta\left(\vec{k}-\vec{k}_0\right)-\delta\left(\vec{k}+\vec{k}_0\right)\right]$$

$$R^{\ell_1,\ell_2}_{\ell m} \equiv (-1)^m\sqrt{\frac{(2\ell+1)(2\ell_1+1)(2\ell_2+1)}{4\pi}} \times \begin{pmatrix} \ell_1 & \ell_2 & \ell \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \ell_1 & \ell_2 & \ell \\ 0 & m & -m \end{pmatrix}$$

Estimator of the Modulating Amplitude

$$\langle X^*_{\ell m} Y_{\ell+1,m} \rangle = \omega f^{XY}_{\ell} R^{1\ell}_{\ell+1,m}$$

$$\Downarrow$$

$$\widehat{\omega}^{XY}_{\ell m} = \frac{X^*_{\ell m} Y_{\ell+1,m}}{f^{XY}_{\ell} R^{1\ell}_{\ell+1,m}}$$

$$\Downarrow$$

$$\widehat{\omega} = \sum_{XY} \sum_{\ell m} A^{XY}_{\ell m} \widehat{\omega}^{XY}_{\ell m}$$

Dvorkin et al. - 0711.2321

Instrumental noise

schNell provides the noise level N_ℓ

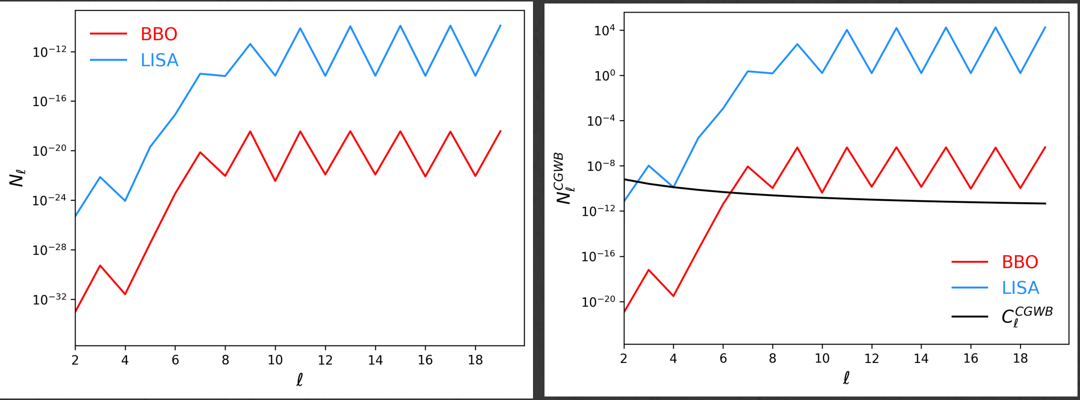
$\Downarrow$

$$N_\ell^{CGWB} = N_\ell / \bar{\Omega}_{GW}^2$$

$\Downarrow$

$$\bar{\Omega}_{GW}(f) = \frac{r A_s}{24 z_{eq}} \left(\frac{f}{f_{pivot}} \right)^{n_t}$$

Instrumental noise



$$r = 0.06,\ n_t = 0.52$$