Hubble tension and quantum gravity effects

Antonino Marcianò

Fudan University

based on

M. Lulli, A. Marciano & X. Shan, arXiv:2112.01490v2

M. Lulli, A. Marciano & L. Visinelli, Work in progress…

M. Lulli, A. Marciano & R. Pasechnik, Work in progress…

Tensions in Cosmology — September 07-12, Corfu

Main message

Looking for a solution of the Hubble tension

that involves a geometric RG flow

Running of the Hubble parameter in a stochastic thermal time

Strategy: from an empty Universe to a Universe filled with matter

First principle discussion and running of the cosmological constant

M. Lulli, A. Marciano & X. Shan, arXiv:2112.01490v2

Analysis for the Hubble parameter in the realistic case

M. Lulli, A. Marciano & L. Visinelli, Work in progress…

Plan of the seminar

Symmetries and the path integral approach

The Ricci flow from the RG perspective

The Ricci flow and the Stochastic Quantization

Thermal time and conformal transformation

The Hamiltonian version of the Ricci flow and its interpretation

QG macroscopic effect: a way out from Hubble tension?

Can the Ricci flow matter for matter?

Gravitational back-reaction to YM fields

Topological features of vacua

Searching over 100 years…

Summing over quantum histories

Imposition of the constraints at the quantum level vs fluctuations

The fate of the symmetries

Spontaneous vs dynamical symmetry breaking

Frisch, Rumpf…

Emergence of conformal symmetry in the UV

Finite gravity vs Convergence to zero of all coupling constants

Stelle, Toumbolis, Modesto…

Searching over 100 years…

Conformal anomaly and dimensional transmutation

Emergence of scale and new degrees of freedom

Renormalization group flow

Searching for a non-trivial fixed point and control of UV behaviour

Ricci flow

Developing a geometric intuition on the RG flow

$$S = \alpha' \int_{\mathcal{M}} d^2\sigma \sqrt{h} h^{ab}(\sigma) g_{ij}(X) \frac{\partial X^i}{\partial \sigma^a} \frac{\partial X^j}{\partial \sigma^b}$$

The case of the non-linear sigma models

$$\frac{\partial g_{ij}}{\partial \lambda} = -\alpha' R_{ij} - \frac{\alpha'^2}{2} R_{iklm} R_j{}^{klm} + \cdots$$

$$\frac{\partial}{\partial \lambda} g_{\mu\nu} = -\, 2 R_{\mu\nu}$$

Hamilton

Complementing with randomness

Ricci flow and the variational principle

$$\frac{\partial}{\partial\lambda}g_{\mu\nu} = -2R_{\mu\nu}$$
$$= -2\left[R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right] - g_{\mu\nu}R$$
$$= -2\frac{\delta S_G}{\delta g^{\mu\nu}} - g_{\mu\nu}R.$$

substitute with noise

Langevin equation and random noise

$$\frac{\partial}{\partial\lambda}\phi_A(x^\mu,\lambda) = -\frac{\delta S[\phi]}{\delta\phi_A} + \eta(x^\mu,\lambda)$$

Parisi & Wu

Stochastic quantization

Describe approach to equilibrium

$$\frac{\partial}{\partial \lambda}\phi_A\left(x^\mu,\lambda\right) = -\frac{\delta S\left[\phi\right]}{\delta \phi_A} + \eta_A\left(x^\mu,\lambda\right)$$

Additive noise associated

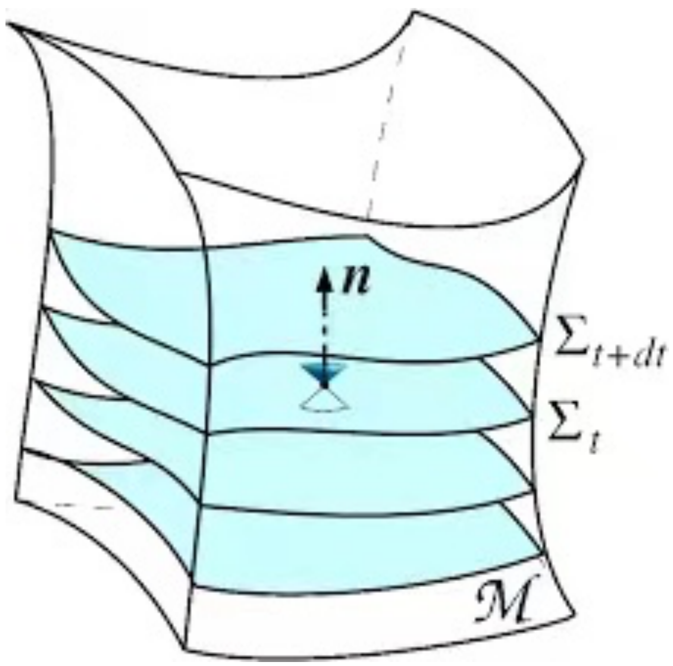
$$\langle \eta_A\left(x,\lambda\right)\eta_B\left(x',\lambda'\right)\rangle \quad = \quad \alpha_\eta \delta_{AB}\delta\left(x-x'\right)\delta\left(\lambda-\lambda'\right)$$

Compute gauge invariant quantities without fixing the gauge

Thermal time and conformal transformation

$$\varepsilon^2(\lambda) = \exp\left[2\int_{\tau_0}^{\tau} \mathrm{d}\bar{\tau}\varphi(\bar{\tau},\lambda)\right]$$

The norm of the metric is not constant



$$\lambda \to \tau$$

From having introduced a projective term in the connection

$$\frac{\mathrm{d}g_{\mu\nu}(\tau,\lambda)}{\mathrm{d}\tau} = -2\varphi(\tau,\lambda)g_{\mu\nu}(\tau,\lambda)$$

Projective connection

$$\bar{\Gamma}^{\gamma}_{\alpha\beta} = \Gamma^{\gamma}_{\alpha\beta} + \mathcal{C}^{\gamma}_{\alpha\beta}$$

$$\mathcal{C}^{\gamma}_{\alpha\beta} = \lambda_1 \delta^{\gamma}_{\alpha} u_\beta + \lambda_2 u_\alpha \delta^{\gamma}_{\beta} + \lambda_3 w_{\alpha\beta} u^\gamma + \lambda_4 u_\alpha u_\beta u^\gamma$$

$$\sqrt{-g}\bar{R} = \sqrt{-g}R + \sqrt{-g}g^{\beta\delta}\left(\mathcal{C}^{\mu}_{\beta\delta}\mathcal{C}^{\alpha}_{\mu\alpha} - \mathcal{C}^{\mu}_{\beta\alpha}\mathcal{C}^{\alpha}_{\mu\delta}\right)$$

Cosmological term induced in the action

$$\sqrt{-g}g^{\beta\delta}\left(\mathcal{C}^{\mu}_{\beta\delta}\mathcal{C}^{\alpha}_{\mu\alpha} - \mathcal{C}^{\mu}_{\beta\alpha}\mathcal{C}^{\alpha}_{\mu\delta}\right)$$
$$= \sqrt{-g}\left[\left(\lambda_2^2 + \lambda_3^2\right)(D-1)\, u_\mu u^\mu\right]$$

Breakdown of the conformal symmetry

$$h_{\mu\nu} = h^{\perp}_{\mu\nu} + \partial_\mu a^{\perp}_\nu + \partial_\nu a^{\perp}_\mu + \left(\partial_\mu \partial_\nu - \frac{1}{4}\eta_{\mu\nu}\Box\right) a + \frac{1}{4}\eta_{\mu\nu}\varphi$$

$$\Phi = \varphi - \bar{\Box} a$$

Einstein-Hilbert expanded on dS or AdS backgrounds

$$\mathcal{S}^{(2)}_{\mathrm{EH}} = \frac{c^3}{16\pi G}\int d^4x\sqrt{-\bar{g}}\left[\frac{1}{4}h^{\mu\nu}_{\perp}\left(\bar{\Box} - \frac{\bar{R}}{6}\right)h^{\perp}_{\mu\nu} - \frac{3}{32}\Phi\left(\bar{\Box} + \frac{\bar{R}}{3}\right)\Phi\right]$$

Residual gauge transformation: conformal Killing vector and disappearance of the ghost

$$h^{\perp}_{\mu\nu} \to h^{\perp}_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu\,, \qquad \Phi \to \Phi + 2\nabla^\mu k_\mu$$

Spectral dimension & Heat flow equation

$$\partial_s K(x,y;s) + \Delta_x K(x,y;s) = 0$$

$$d_s \equiv -2\frac{\partial \mathrm{Tr} K}{\partial \log s}$$

Schwinger time as the Wick rotation of the thermal time

$$D_F(x,y) = \int_0^\infty ds\, e^{-s\epsilon}\, e^{-\imath s m^2} K_\eta(x,y;-\imath s)$$

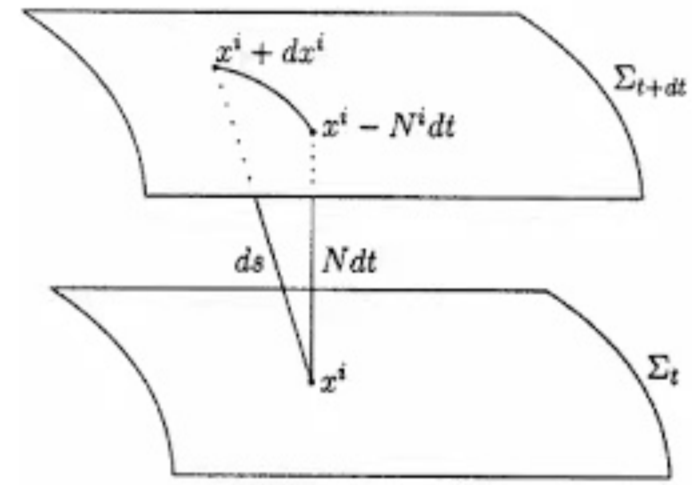
$$\frac{dg_{\mu\nu}}{ds} = -2R_{\mu\nu}$$

Ricci flow precisely as a heat equation of either Riemannian or pseudo-Riemannian space

ADM decomposition

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$
$$= -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$n_\mu = (-N, 0)\,, \qquad n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N}\right)$$



$$K_{ij} = -\nabla_{(j}n_{i)} =$$
$$= \frac{1}{2N}\left(-\partial_t h_{ij} + {}^{(3)}\nabla_{(i}N_{j)} + {}^{(3)}\nabla_{(j}N_{i)}\right)$$

$$\bar{\nabla}_{(\alpha}n_{\beta)} = \nabla_{(\alpha}n_{\beta)} - \mathcal{C}^{\gamma}_{(\alpha\beta)}n_\gamma$$
$$= \nabla_{(\alpha}n_{\beta)} - (\lambda_1 + \lambda_2)\, n_\alpha n_\beta$$
$$+ \varepsilon(\lambda)\left[\lambda_3 w_{\alpha\beta} + \lambda_4 n_\alpha n_\beta\right]$$

Fokker-Planck and cosmological constant

Langevin equation with complex additive noise

$$\frac{\partial g_{\mu\nu}}{\partial s} = \imath \mathcal{G}_{\alpha\beta\mu\nu} \frac{\delta S}{\delta g_{\alpha\beta}} + g_{\mu\nu}\eta \quad \longrightarrow \quad \eta = \sigma_{\tilde{\eta}} \tilde{\eta}$$

Related Fokker-Planck within the Ito differential calculus

$$\frac{\partial p}{\partial s} = -\frac{\delta}{\delta g_{\mu\nu}} \left[\mathcal{G}_{\alpha\beta\mu\nu} \frac{\delta S}{\delta g_{\alpha\beta}} p \right] + \frac{\delta^2}{\delta g_{\mu\nu} \delta g_{\rho\sigma}} \left[g_{\mu\nu}^2 \, p \right]$$

$$p \simeq \frac{D}{g_{\mu\nu}^2} \exp \left[2 \int^{g_{\mu\nu}} \mathcal{D} g_{\alpha\beta} \frac{\mathcal{G}_{\rho\sigma\alpha\beta} \frac{\delta S}{\delta g_{\rho\sigma}}}{\Lambda_0 g_{\alpha\beta}^2} \right] \longrightarrow \mathcal{G}_{\rho\sigma\mu\nu} \frac{\delta S}{\delta g_{\rho\sigma}} - \imath \Lambda_0 g_{\mu\nu} = 0$$

$$\sigma_{\tilde{\eta}} = \sqrt{\Lambda_0} \to e^{-\imath \frac{\pi}{4}} \sqrt{\Lambda_0}$$

Hamiltonian analysis of the Ricci flow

$$\eta_{\mu\nu} = \eta \, g_{\mu\nu}$$

Multiplicative choice of the noise source entails additivity in the Hamiltonian ADM picture

$$\frac{\partial N}{\partial \lambda} = -\frac{N}{2}\left[\frac{\mathcal{H}}{\sqrt{h}} + \eta\right]$$

"00"

$$\frac{\partial N^k}{\partial \lambda} = \frac{N\mathcal{H}^k}{\sqrt{h}}$$

"0i"

$$\frac{\partial h_{ij}}{\partial \lambda} = \frac{1}{N}\mathcal{L}_m\left[\mathcal{H}, h_{ij}\right] + \left[\mathcal{H}, \left[\mathcal{H}, h_{ij}\right]\right] + \frac{h_{ij}\mathcal{H}}{2\sqrt{h}} - h_{ij}\eta$$

"ij"

Physical interpretation

Thermal time and time de-parametrization

Measurement problem and collapse of the wavefunction

"00"

Navier-Stokes at equilibrium

$$r_c^{3/2}\partial^k T_{ki} = \partial_t v_i - \zeta\partial^2 v_i + \partial_i P + v^k\partial_k v_i = 0$$

Turbulence away from equilibrium

$$r_c^{3/2}\partial_k T^{ki} = \frac{1}{N}\frac{\partial N^i}{\partial\lambda}$$

"0i"

$$\frac{\partial\nu}{\partial\lambda} = -4\pi e^{-\nu} r^2 \left[2\left(\frac{\partial^2\nu}{\partial r^2} + \frac{1}{r}\frac{\partial\nu}{\partial r}\right) + \frac{1}{2}\left(\frac{\partial\nu}{\partial r}\right)^2 + \frac{2}{r^2}\left(1 - e^{-\nu}\right)\right] + \eta$$

Kardar-Parisi-Zhang Equation

"ij"

Ricci RG flow of Λ

$$ds^2 = -N^2 dt^2 + a^2(t)\left[\frac{(dr)^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

FLRW background

$$S = 6\int d^4x N a^3 R + \int d^4x N a^3 (D-1)\lambda_2^2 \epsilon(\lambda) \qquad R = 6\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}^2}{a^2}\right) + \frac{k}{a^2}\right)$$

$$\frac{\partial a}{\partial s} = -\frac{2\imath}{N^2}\left(a\dot{H} + 3aH^2 + \varepsilon N^2 \lambda_2^2\right) + a\eta\,,$$

$$\frac{\partial N}{\partial s} = -2\imath\left(\frac{3}{2N}(\dot{H} + H^2) + \frac{1}{16}N\left(\Lambda_0 + 8\lambda_2^2\right)\right) - N\eta\,,$$

$$\frac{\partial \lambda_2}{\partial s} = \imath\left(-2\varepsilon - \imath\eta\right)\lambda_2\,,$$

Ricci flow equations

Hubble tension: a macroscopic QG effect?

$$\langle \lambda_2^k(s) \rangle = \exp\left[\left(i\,(-2\varepsilon) + \frac{\Lambda_0}{2}\right) s\right] \langle \lambda_2^k(0) \rangle$$

Thermal time oriented as the proper time implies mild increase of Λ

Cosmological measurements $67.4 \pm 1.4 \quad (\mathrm{km/s})/\mathrm{Mpc}$

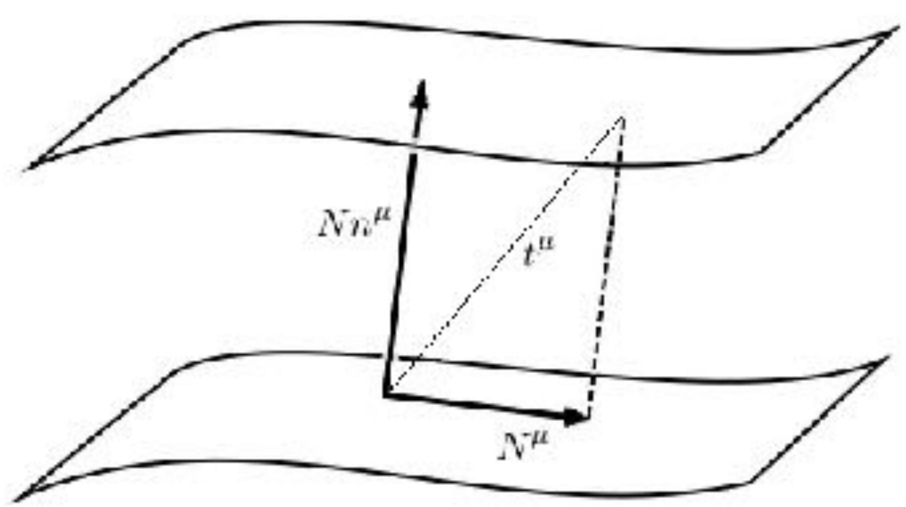
Astronomical measurements $74.03 \pm 1.42 \quad (\mathrm{km/s})/\mathrm{Mpc}$

Can the Ricci flow matter for matter?

$$R^T_{\mu\nu} = \frac{8\pi G}{c^4}\left[T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right]$$

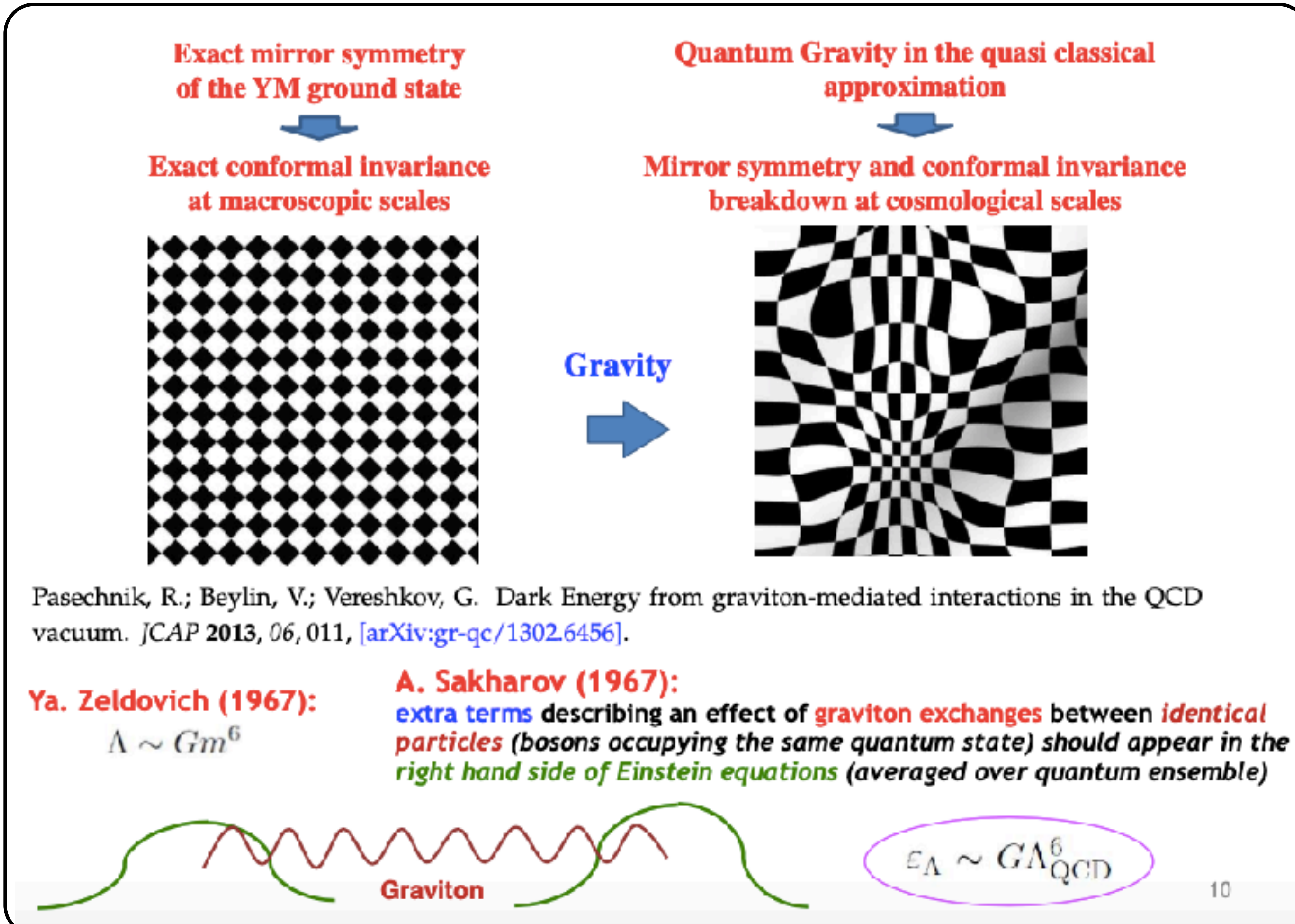
$$\begin{aligned}\frac{\partial}{\partial s}g_{\mu\nu} &= -2\left[R_{\mu\nu} - R^T_{\mu\nu}\right]\\ &= -2\left[R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{8\pi G}{c^4}T_{\mu\nu}\right] - g_{\mu\nu}\left(R - T\right)\end{aligned}$$

Changes of topologies through defects are induced by singularities in the Ricci flow



Changes of topologies in the manifolds —> change of topologies of the ground state structure

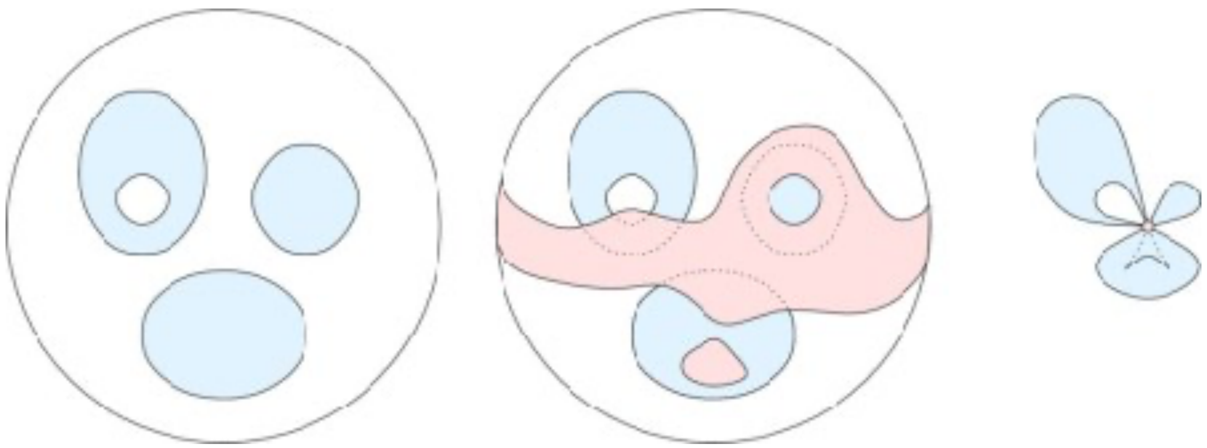
Gravitational back-reaction to YM



Credit to R. Pasechnik

Topological features of vacua

The Ricci flow allows for topology changes from equilibrium



Geometrical interpretation of ground-states

Topological charges label ground-states structures and are related to the characterization of the matter content —e.g. Atiyah-Singer Index theorem

Stochastic dynamics and the Ricci RG flow

Holography in 4+1D and dynamics in the stochastic time parameter

The Ricci flow amounts to a conformal transformation of the 3D-hypersurfaces

The Langevin equation and the probability distributions for manifolds with Lorentzian signature and complex structure

Manifolds with Lorentzian signature enable to fully take into account dynamics of out-of-equilibrium systems and relaxations features

Chromo-magnetic vortices and turbulences can be addressed as a by-product of the Ricci flow driven relaxation processes

Outlooks

RG flow for matter fields with gravitational back-reaction

Binary systems and growth of instabilities

Inflationary scenario from conformal symmetry breaking

Gravitational collapse of the wave-functions and its dynamics

Emergent gravity and topological phase

Thank you! **Ευχαριστώ!**

Grazie! **谢谢!**