The Hubble Constant Tension and its evolution New perspectives on Gamma-Ray Burst cosmology

Dr. Maria Giovanna Dainotti

NAOJ, DIVISION OF SCIENCE, Tokyo, Japan
SOKENDAI UNIVERSITY, Kanagawa, Japan,
SPACE SCIENCE INSTITUTE, Boulder, Colorado, USA

In collaboration with: Biagio De Simone, Tiziano Schiavone, Giovanni Montani, Enrico Rinaldi, Gaetano Lambiase, Malgorzata Bogdan, Sahil Ugale, Salvatore Capozziello, Aleksander Lenart, Giada Bargiacchi, Via Nielson, Giuseppe Sarracino, Shigehiro Nagataki, Oleg Gnedin

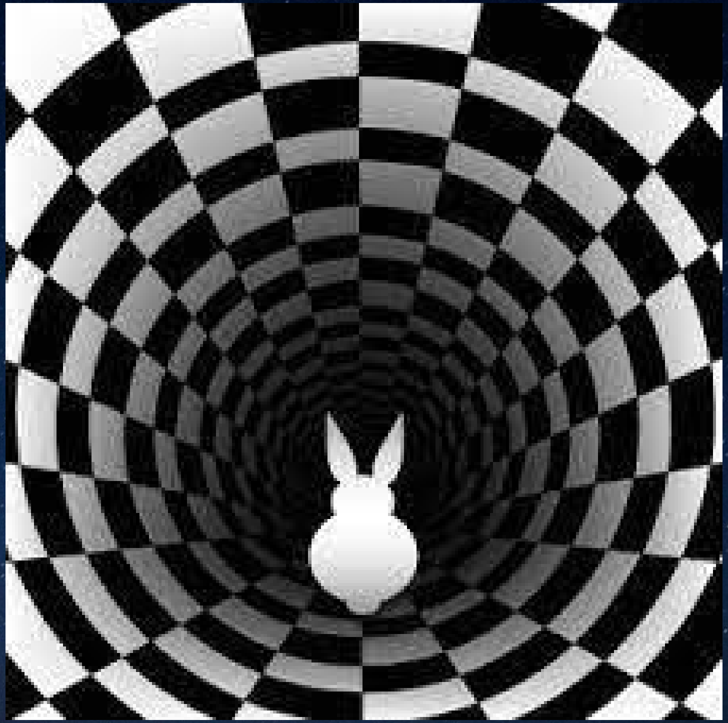
SPACE SCIENCE INSTITUTE

National Astronomical Observatory of Japan

Tensions in Cosmology, 9th September 2021

National University SOKENDAI The Graduate University for Advanced Studies

Going down the rabbit hole of the Hubble constant tension



The Hubble constant

THE HUBBLE CONSTANT H_0 IS A PARAMETER THAT DESCRIBES THE RATE OF EXPANSION OF THE UNIVERSE

$$H_0 \stackrel{\text{def}}{=} \frac{R'(t_0)}{R(t_0)}$$

$R(t_0)$ SCALE FACTOR OBTAINED FROM THE METRIC AND COMPUTED IN THE PRESENT t_0

FOR SMALL REDSHIFT VALUES (FOR SMALL COSMOLOGICAL DISTANCES) H_0 CAN BE USED IN THE HUBBLE'S LAW

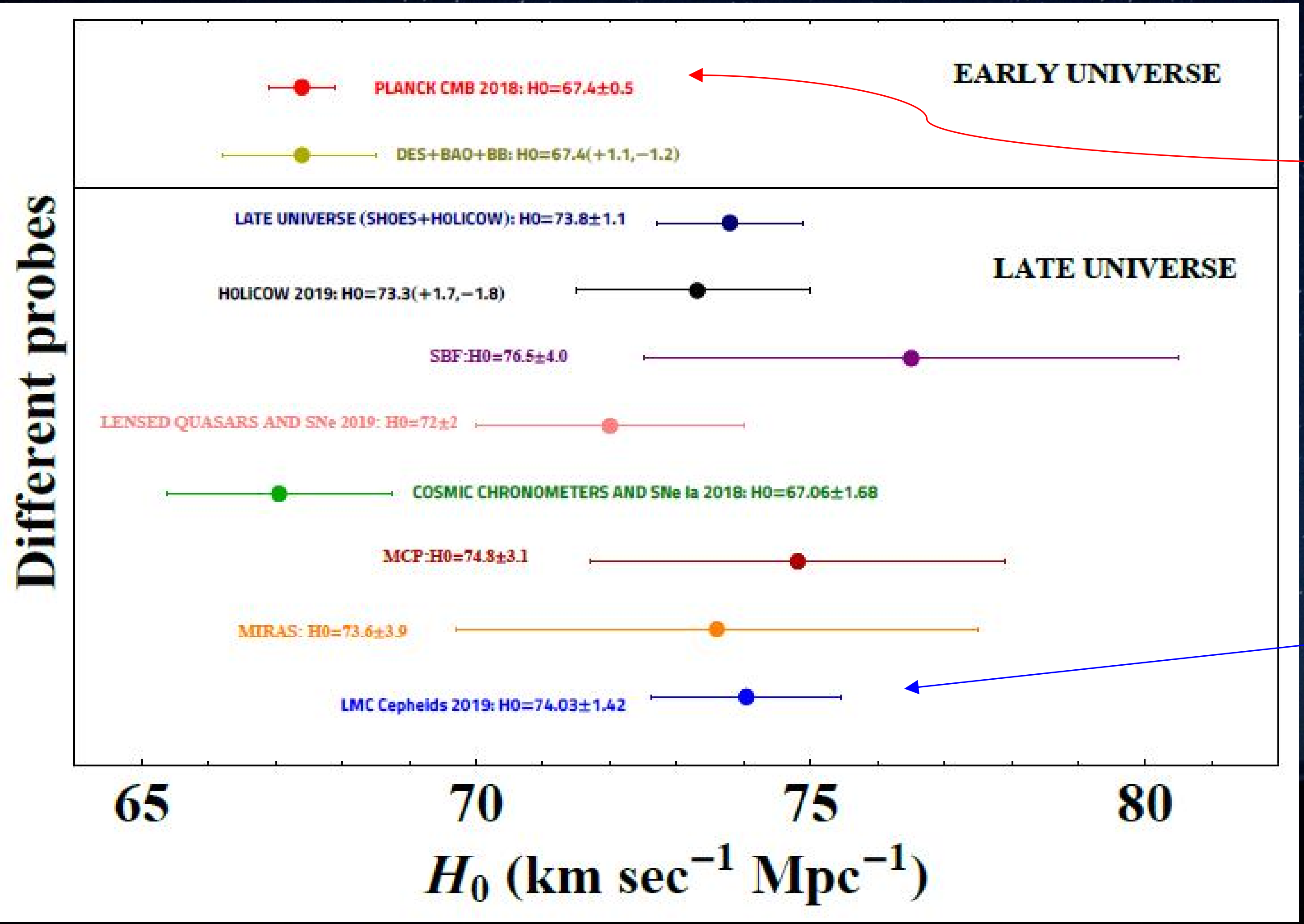
$$v = H_0 * D$$

VELOCITY OF THE ESCAPING GALAXY (v)

DISTANCE OF THE ESCAPING GALAXY (D)

The H_0 tension

M. G. Dainotti et. al 2021, ApJ ,912, 150



«H_0 TENSION»:

the discrepancy in 4.4 - 6σ between the local value of the Hubble constant H_0 based on Supernovae Ia (SNe Ia) and Cepheids and the value of H_0 referred to the Cosmic Microwave Background (CMB)

Supernovae Ia

SNe Ia are among the best standard candles so far discovered, Riess et al. 1998, A J, 116, 3, 1009 (given their almost uniform peak brightness)

Their distance moduli, $\mu_{obs}^{(SN)}$ can be expressed through the Tripp formula (Tripp 1998) with the addition of correcting terms (Scolnic et al. 2018):

$$\mu_{obs}^{(SN)} = m_B - M + \alpha x_1 - \beta c + \Delta M + \Delta B$$

where

m_B is the apparent magnitude of the SN Ia in B-band

x_1 is the stretch factor, c is the color correction

M is the fiducial absolute magnitude of a SN Ia with zero values of x_1 and c

ΔM is the host galaxy mass correction term

ΔB is the bias correction

SNe Ia absolute magnitude and H_0

Di Valentino et al. 2020, JCAP 2007, 045

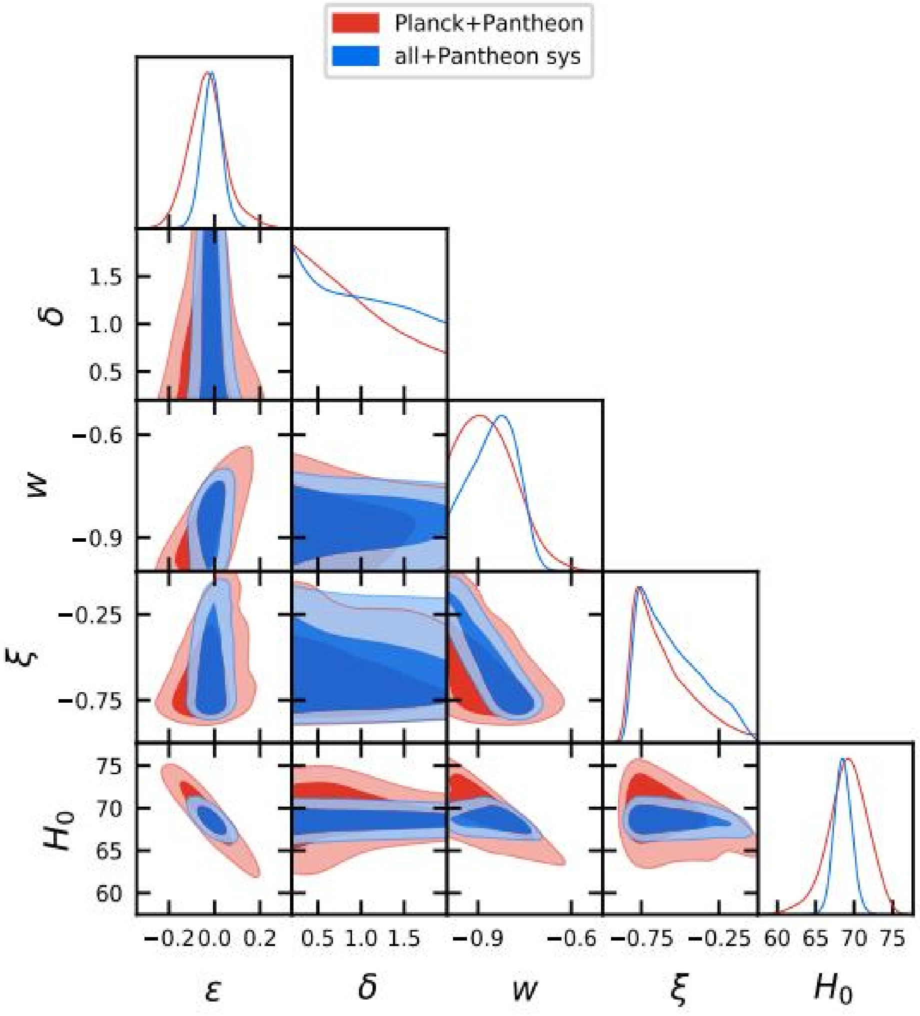


Figure 11. As in Fig. 3, for the parameters of the coupled quintessence ξqCDM cosmology described in Sec. 4.3, with the qualitatively similar JLA results not shown here.

The H_0 and the M fiducial absolute magnitude for SNe Ia are degenerate (Tripp 1998)

$$\Delta m_{evo}(z) = \varepsilon\, z^{\delta}$$

$\Delta m_{evo}(z)$ is the redshift dependence of the intrinsic absolute magnitude M of the SNe Ia

Di Valentino et al. 2020, JCAP07, 045
the ξqCDM model shows a dependence on the redshift. However, here several parameters are constrained simoultaneously.

To see more clearly a one dimension parametrization, we varied only H_0 and in bins within the Pantheon sample

Bins division of the Supernovae Ia sample

WE DIVIDE THE PANTHEON SAMPLE IN DIFFERENT BINS: 3 BINS, 4 BINS, 20 BINS, AND 40 BINS ORDERED IN REDSHIFT

we check if evolution is present by deriving H0 from each of the redshift bin. We found such an evolution

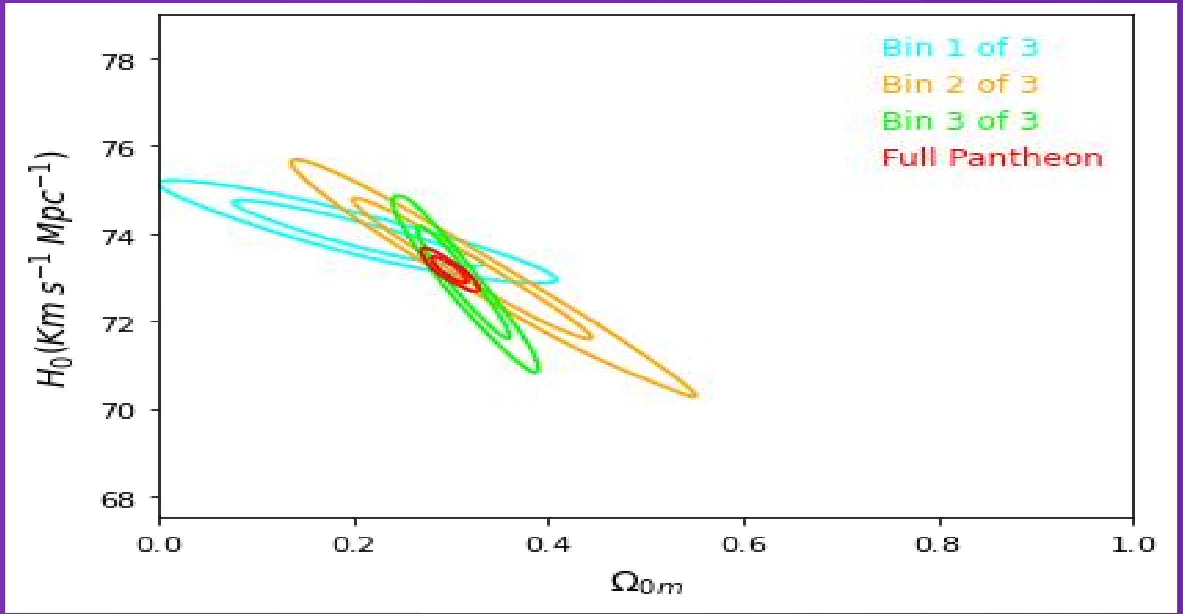
α = evolution parameter

$$g(z) = \frac{\widetilde{H}_0}{(1+z)^{\alpha}}$$

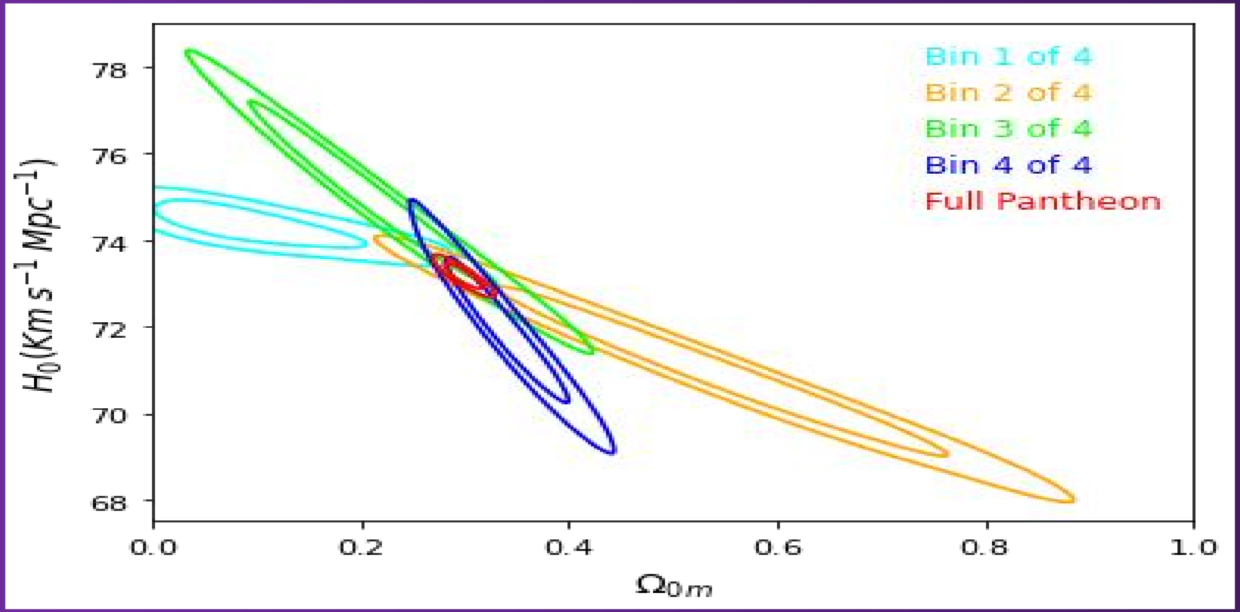
$$\widetilde{H}_0 = H_0(z=0)$$

The criteria for choosing bins number

3 BINS (AROUND 350 SNe PER BIN)



4 BINS (262 SNe PER BIN)



«Contours closed» for $0 < \Omega_{0m} < 1$, $(60 < H_0 < 80\,)km/(\sec Mpc)$

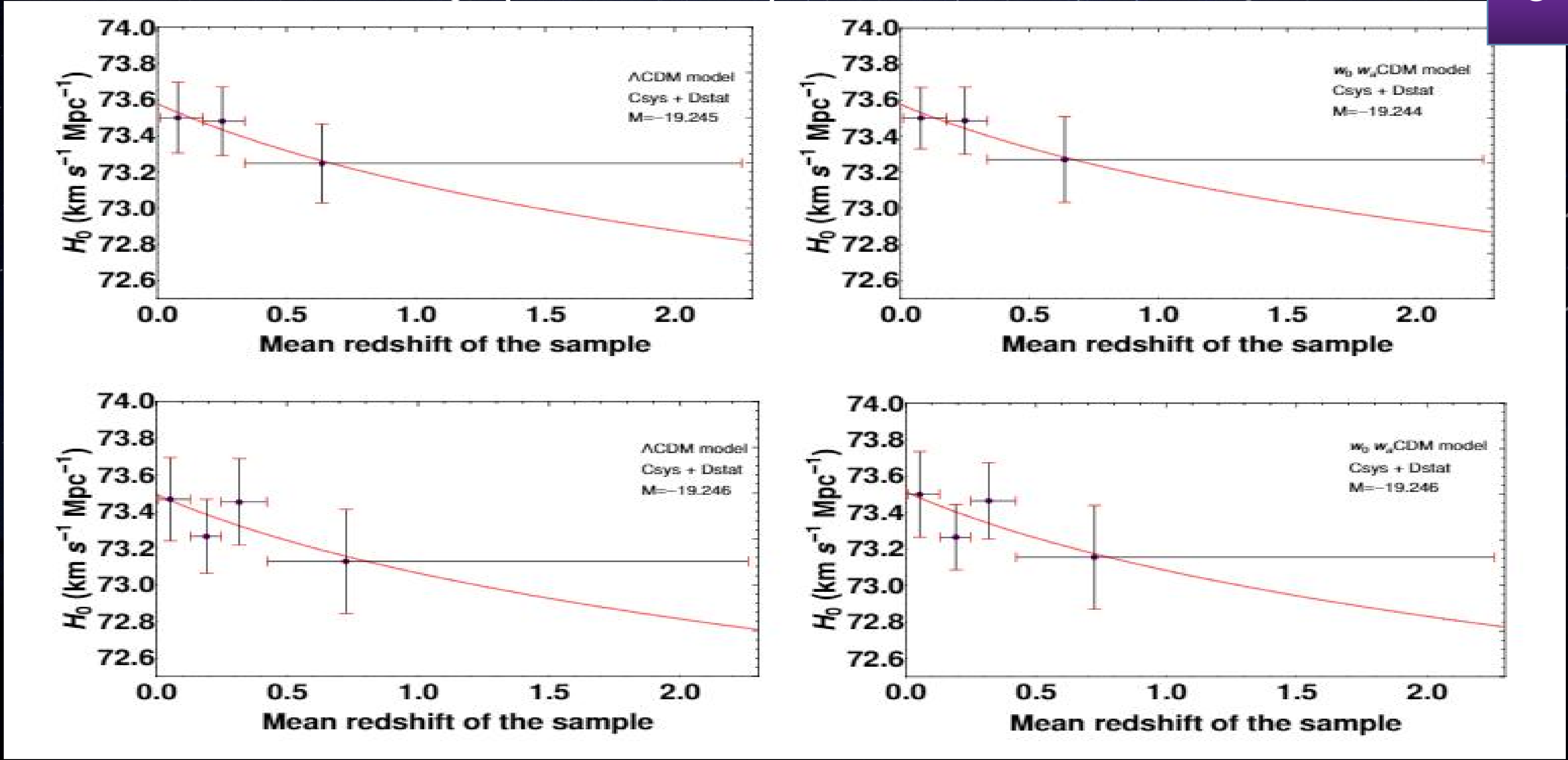
Values compatible in 2σ with the total Pantheon case

The first bin has an «open contour» in $0 < \Omega_{0m} < 1$, $(60 < H_0 < 80\,)km/(\sec Mpc)$and only the second bin is not compatible in 2σ with both the parameters for the total Pantheon case

(20, 40 bins cases have been added to test the independence on bins division)

M. G. Dainotti et al 2021 ApJ 912 150

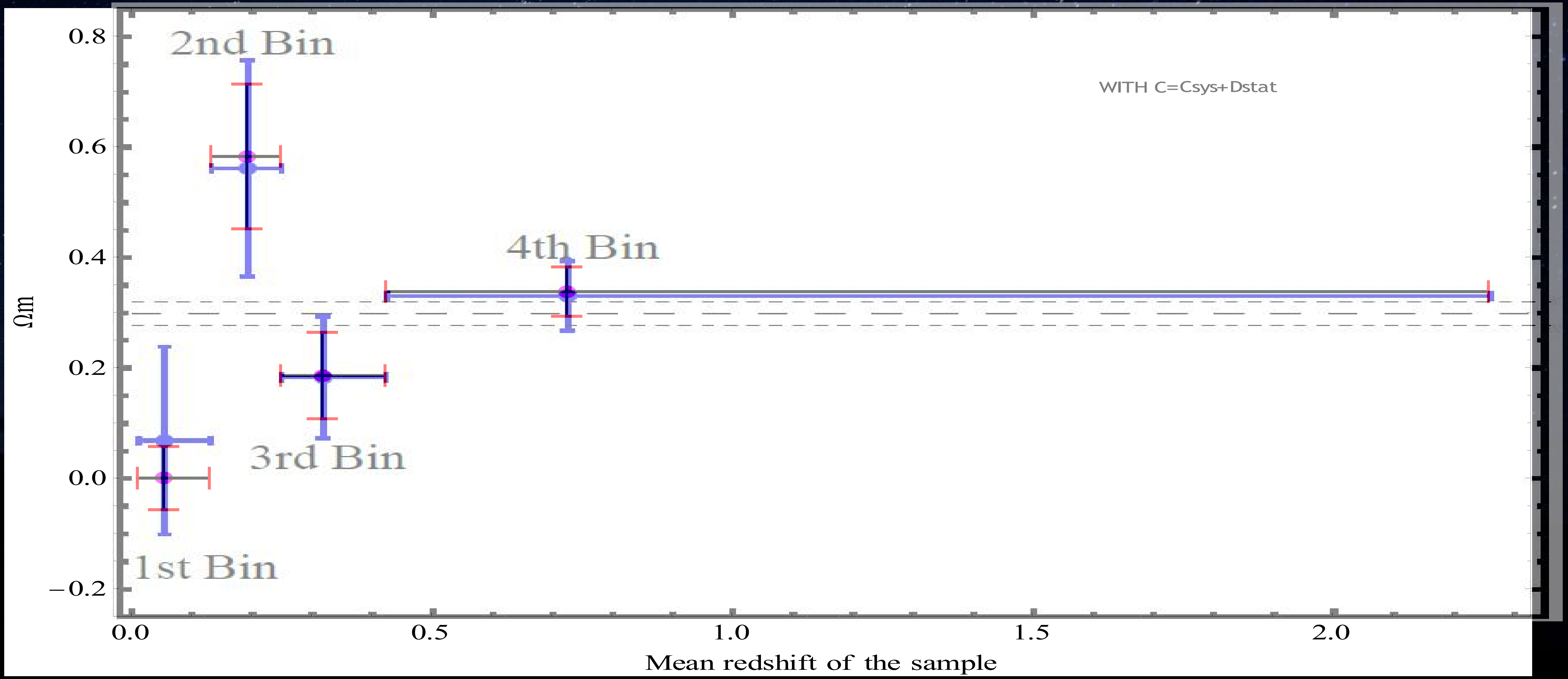
Results for H_0 (3, 4 bins) *M. G. Dainotti et al 2021 ApJ,912, 150*



THE ANALYSIS IS PERFORMED ON A 1-D PARAMETER SPACE FOR THE MCMC (WE VARY ONLY H0)

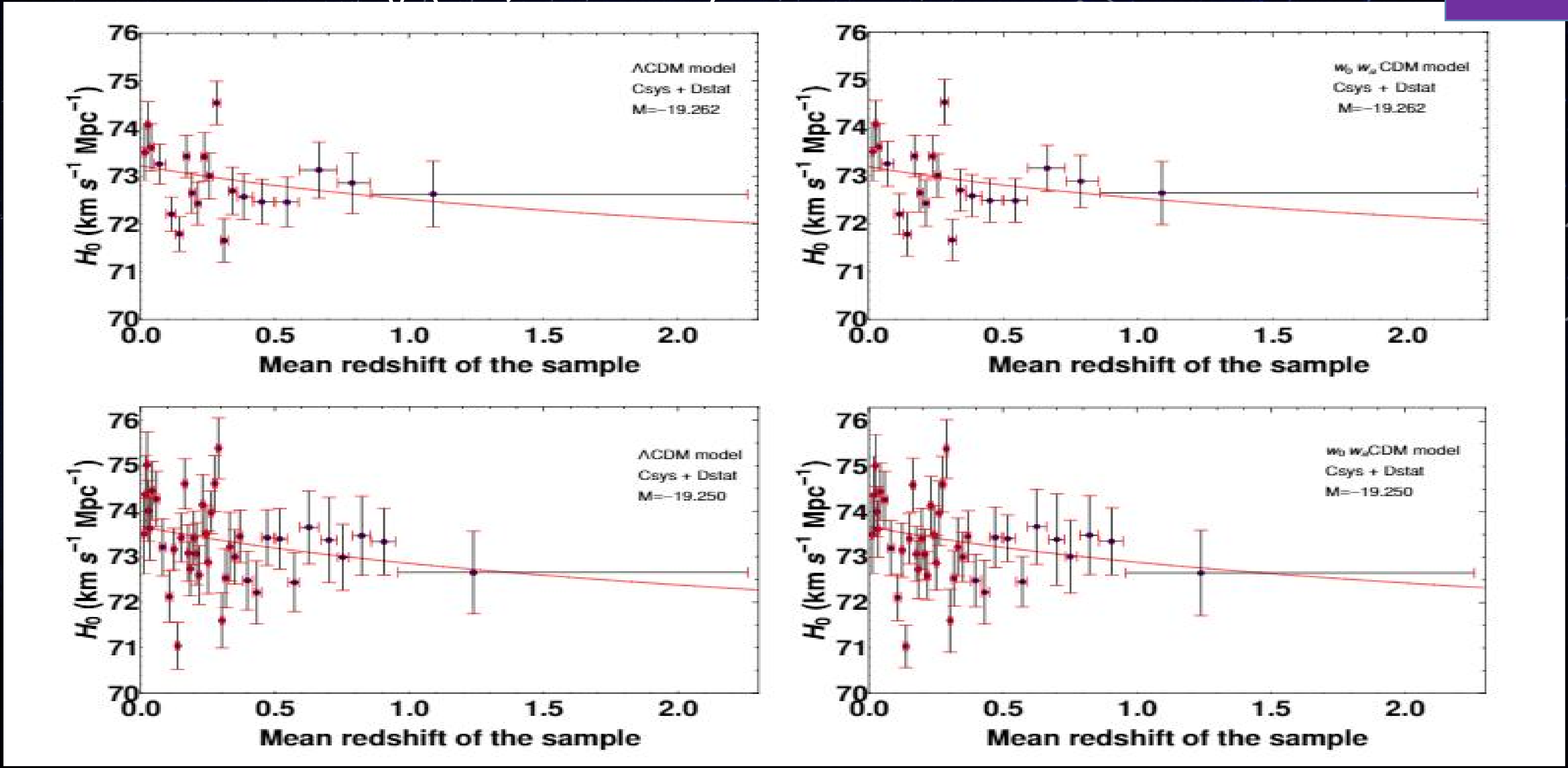
COMPARISON WITH KAZANTZIDIS AND PERIVOLAROPOULOS (2020)

Our 4 bins with C=Csys+Dstat submatrices (in black and red points) superimposed and compared with the results in L. Kazantzidis and Perivolaropoulos, 2020, Phys. Rev. D 102, 023520 (in blue)



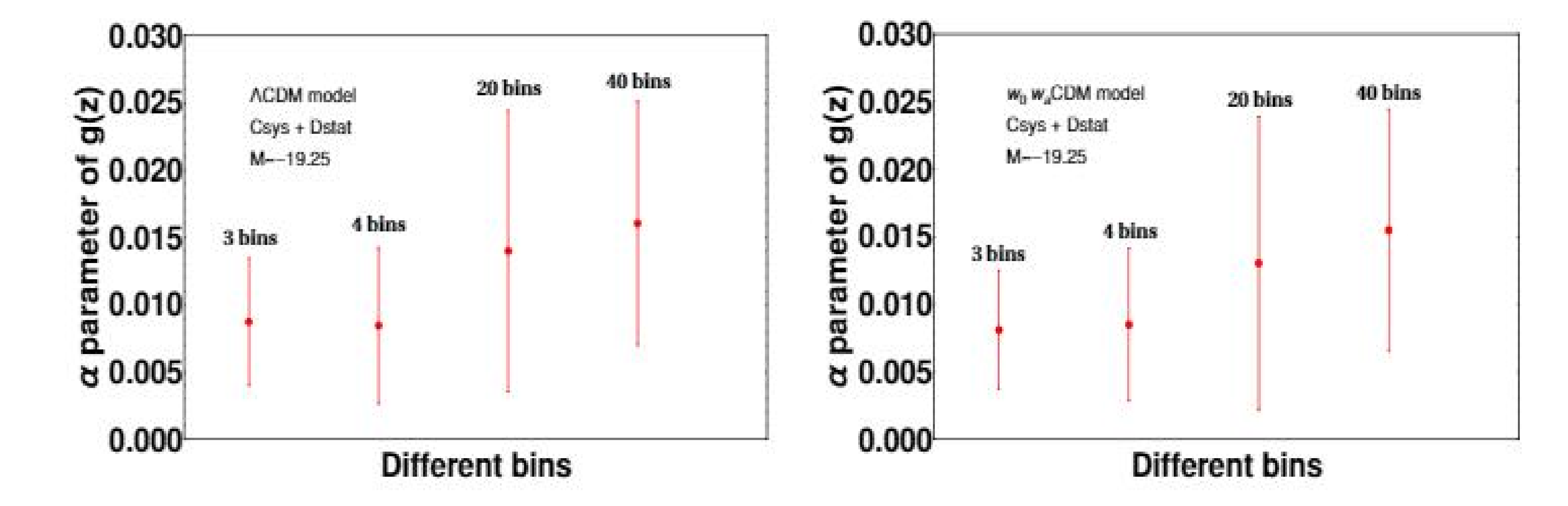
Results for H_0 (20, 40 bins)

M. G. Dainotti et al 2021, ApJ, 912, 150



The analysis is performed on a 1-D parameter space for the MCMC (we vary only H_0)

Results for ΛCDM model (α)

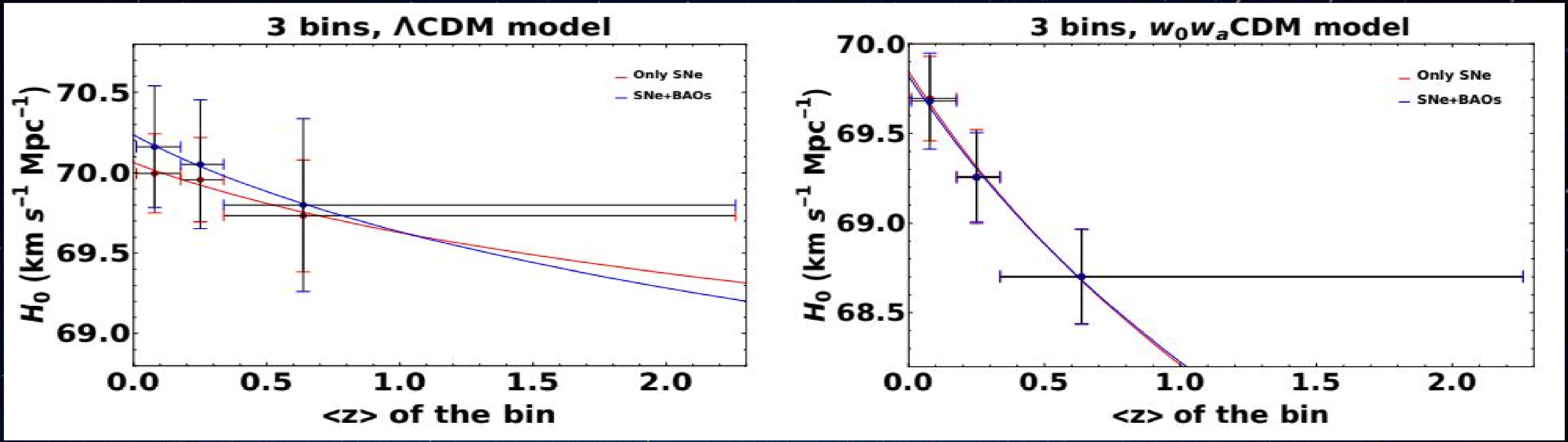


COMPATIBLE IN 1 σ WITH THE PLANCK CMB VALUES

Flat ΛCDM Model, Fixed Ω_{0m}, with Full Covariance Submatrices $\mathcal{C}$							
Bins	$\tilde{H}_0$ (km s^{-1} Mpc^{-1})	α	$\frac{\alpha}{\sigma_\alpha}$	M	$H_0\,(z=11.09)$ (km s^{-1} Mpc^{-1})	$H_0\,(z=1100)$ (km s^{-1} Mpc^{-1})	% Tension Reduction
3	73.577 ± 0.106	0.009 ± 0.004	2.0	-19.245 ± 0.006	72.000 ± 0.805	69.219 ± 2.159	54%
4	73.493 ± 0.144	0.008 ± 0.006	1.5	-19.246 ± 0.008	71.962 ± 1.049	69.271 ± 2.815	66%
20	73.222 ± 0.262	0.014 ± 0.010	1.3	-19.262 ± 0.014	70.712 ± 1.851	66.386 ± 4.843	68%
40	73.669 ± 0.223	0.016 ± 0.009	1.8	-19.250 ± 0.021	70.778 ± 1.609	65.830 ± 4.170	57%

What if we add BAO and vary H_0 and another parameter contemporaneously?

Dainotti et al 2022, Galaxies, 10, 1, 24



WE VARY H_0 , Ω_{0m} FOR THE ΛCDM MODEL AND H_0 , w_a FOR THE w_0w_a CDM MODEL DIVIDING THE PANTHEON SAMPLE IN 3 BINS.

Flat ΛCDM model, without BAOs, varying H_0 and Ω_{0m}			
Bins	$\mathcal{H}_0$	η	$\frac{\eta}{\sigma_\eta}$
3	70.093 ± 0.102	0.009 ± 0.004	2.0
Flat ΛCDM model, including BAOs, varying H_0 and Ω_{0m}			
Bins	$\mathcal{H}_0$	η	$\frac{\eta}{\sigma_\eta}$
3	70.084 ± 0.148	0.008 ± 0.006	1.2

Flat w_0w_aCDM model, without BAOs, varying H_0 and w_a			
Bins	$\mathcal{H}_0$	η	$\frac{\eta}{\sigma_\eta}$
3	69.847 ± 0.119	0.034 ± 0.006	5.7
Flat w_0w_aCDM model, including BAOs, varying H_0 and w_a			
Bins	$\mathcal{H}_0$	η	$\frac{\eta}{\sigma_\eta}$
3	69.821 ± 0.126	0.033 ± 0.005	5.8

Discussion of the results I

SNe Ia ANALYSIS: POSSIBLE ASTROPHYSICAL EFFECTS

There is a redshift evolution intrinsic to H_0 IF
these results are not due to residual evolutionary effects on color, stretch, mass correction, or statistical fluctuations or hidden biases.
- Nicholas et al. 2021 sshows that the stretch factor has a drift with the redshift and this may explain our results.

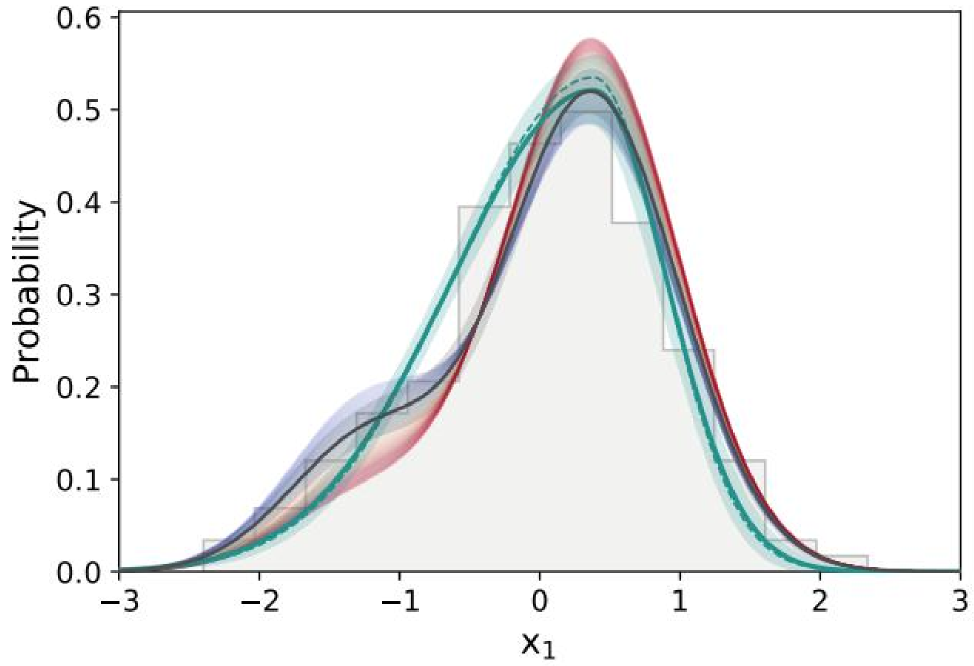


Fig. 8. Distribution of the PS1 SN Ia *SALT2.4* stretch (x_1) after the fiducial redshift limit cut (gray histogram). This distribution is supposed to be a random draw from the underlying stretch distribution. The green lines show the BBC model of this underlying distribution (asymmetric Gaussian). The full line (band) is our best fit (its error); the dashed line shows the Scolnic et al. (2018) result. The black line (band) shows our best-fit base modeling (its error, see Table 2) that includes redshift drift. For illustration, we show (colored from blue to red with increasing redshifts) the evolution of the underlying stretch distribution as a function of redshift for the redshift range covered by PS1 data.

ALTERNATIVE SCENARIOS CAN BE INVOKED:

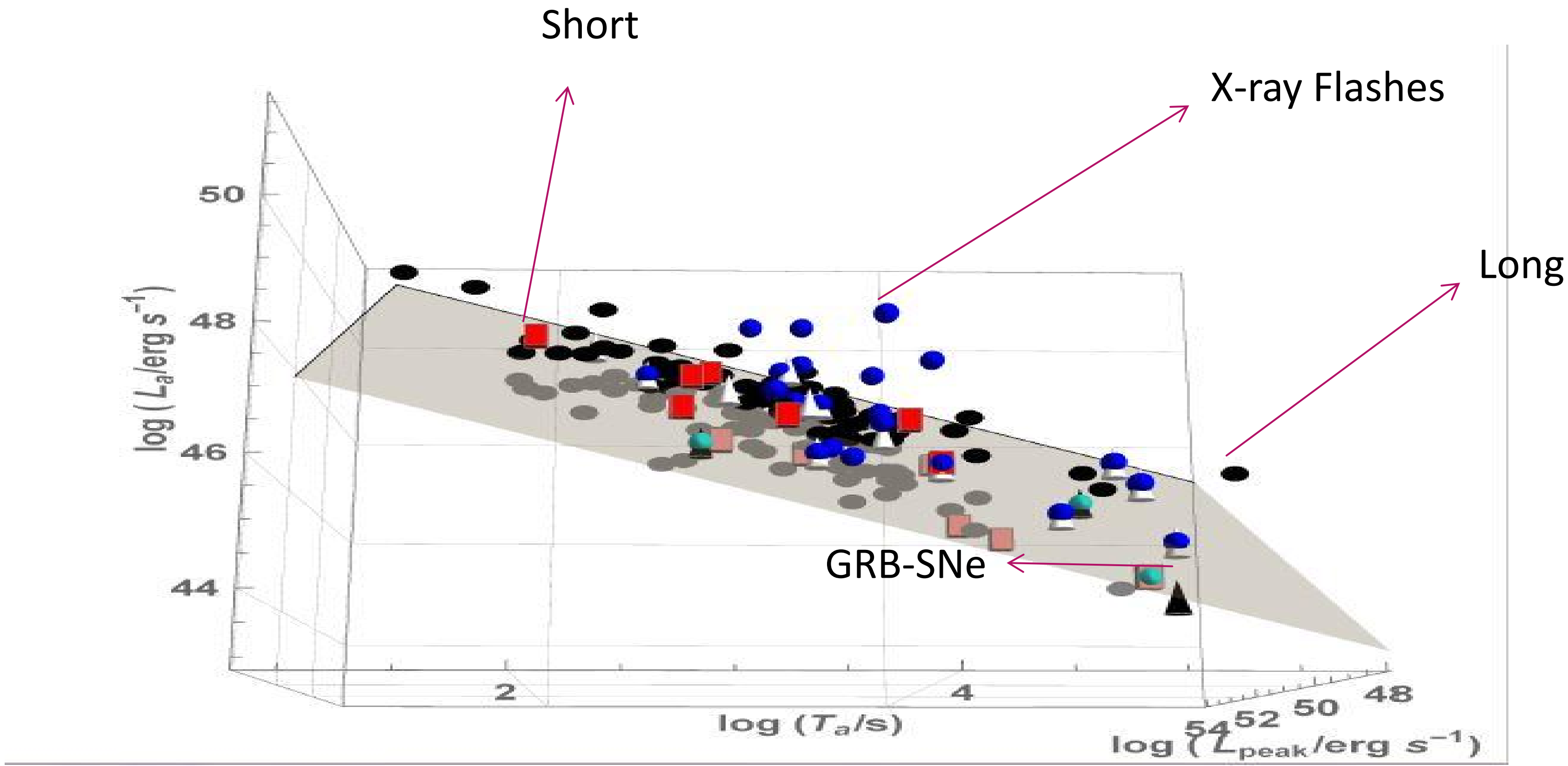
modified gravity theories,
$G = G(z)$ -> in modified theories there is a variation of G constant (ex. $f(R)$ THEORIES)

GRB COSMOLOGY VIA THE GRB FUNDAMENTAL PLANE DAINOTTI RELATION

Press release by NASA:
https://swift.gsfc.nasa.gov/news/2016/grbs_std_candles.html
Mention in Scientific American, Stanford highlight of 2016, INAF Blog
UNAM gaceta, and many online newspapers took the news.

Dainotti, Postnikov, Hernandez, Ostrowski 2016, ApJL, 825L, 20

the 3D Lpeak-Lx-Ta correlation **is intrinsic** and it has a reduced scatter, σ int of 54% for a gold sample.



GRB cosmology: *Dainotti M.G. et al. 2022, MNRAS, 514, 1828*

GRBs have proven to be standardizable candles and cover the universe up to redshift 9.4 (far beyond the SNe Ia)

We used the **Dainotti fundamental plane relation (or 3D relation, Dainotti et al. 2016,2017)** that correlates the peak luminosity of the GRB L_{peak}, the plateau end luminosity L_a, and the rest-frame plateau end time T^*_a in the X-rays.

$$\log_{10} L_{\rm a} = a \cdot \log_{10} T^*_{\rm a} + b \cdot \log_{10} L_{\rm peak} + c$$

Fundamental plane relation

$$\log_{10}(d_{\rm L}) = \frac{a \log_{10} T^*_{\rm a}}{2(1-b)} + \frac{b \cdot (\log_{10} F_{\rm peak} + \log_{10} K_{\rm peak})}{2(1-b)} + \frac{(b-1)\log_{10}(4\pi) + c}{2(1-b)} - \frac{\log_{10} F_{\rm a} + \log_{10} K_{\rm a}}{2(1-b)}$$

Luminosity distance

variables definitions $a_1 = a/(2(1-b))$; $b_1 = b/(2(1-b))$; $c_1 = ((b-1)\log_{10}(4\pi) + C)/(2(1-b))$; $d_1 = -1/(2(1-b))$; $F_{\rm peak,cor} = F_{\rm peak} \cdot K_{\rm peak}$; and $F_{\rm a,cor} = F_{\rm a} \cdot K_{\rm a}$, we obtain:

re-writing the parameters

$$\mu_{\rm obs,\,GRB} = 5 \cdot (a_1 \log_{10}(T^*_{\rm a}) + b_1 \log_{10}(F_{\rm peak,cor}) + c_1 + d_1 \log_{10}(F_{\rm a,cor})) + 25$$

observed distance moduli

$$\mu_{\rm theory} = 5 \cdot \log_{10} d_{\rm L}(z, H_0, \Omega_{\rm M}) + 25$$

theoretical distance moduli

$$\mathcal{L}_{\rm GRB} = \sum_i \left(ln\left(\frac{1}{\sqrt{2\pi}\sigma_{\mu,i}}\right) - \frac{1}{2}\left(\frac{\mu_{\rm th,GRB,\it i} - \mu_{\rm obs,GRB,\it i}}{\sigma_{\mu,i}}\right)^2 \right)$$

THE LIKELIHOOD

Simulating the GRBs

THROUGH THE APPLICATION OF
1) MACHINE LEARNING (ML) METHODS
2) GRBs LIGHT CURVE RECONSTRUCTION (LR)

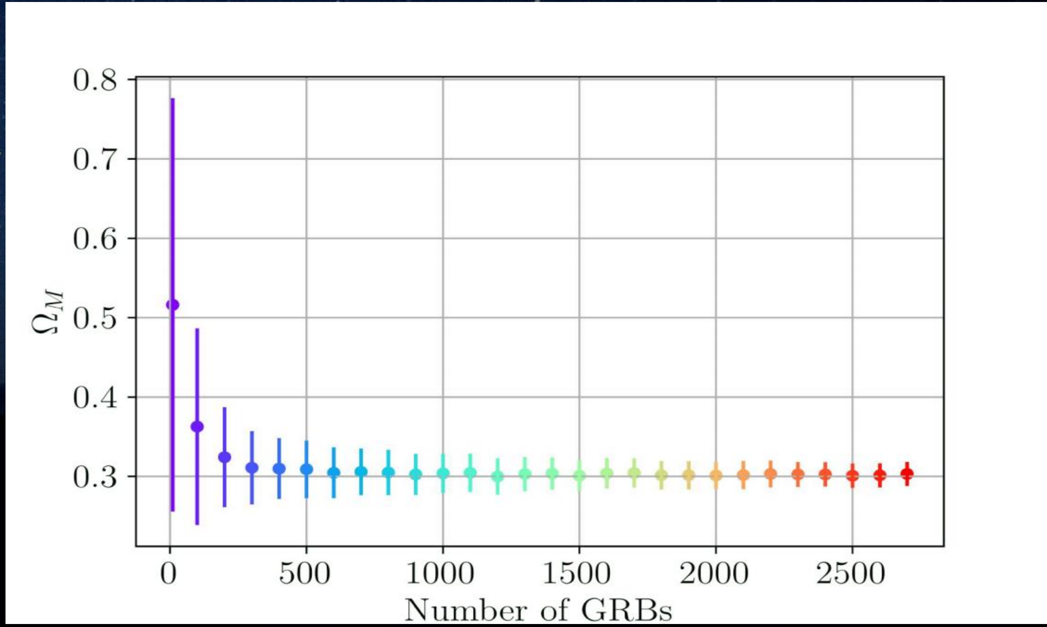
WITH THE CURRENT DATA OF GRBs WITH PLATEAU EMISSION, WITH THE OPTICAL SAMPLE, ML, LR

FOR THE ESTIMATION OF Ω_{0m} THROUGH THE GRBs WE EXPECT TO REACH A PRECISION COMPATIBLE WITH THE ONES OF

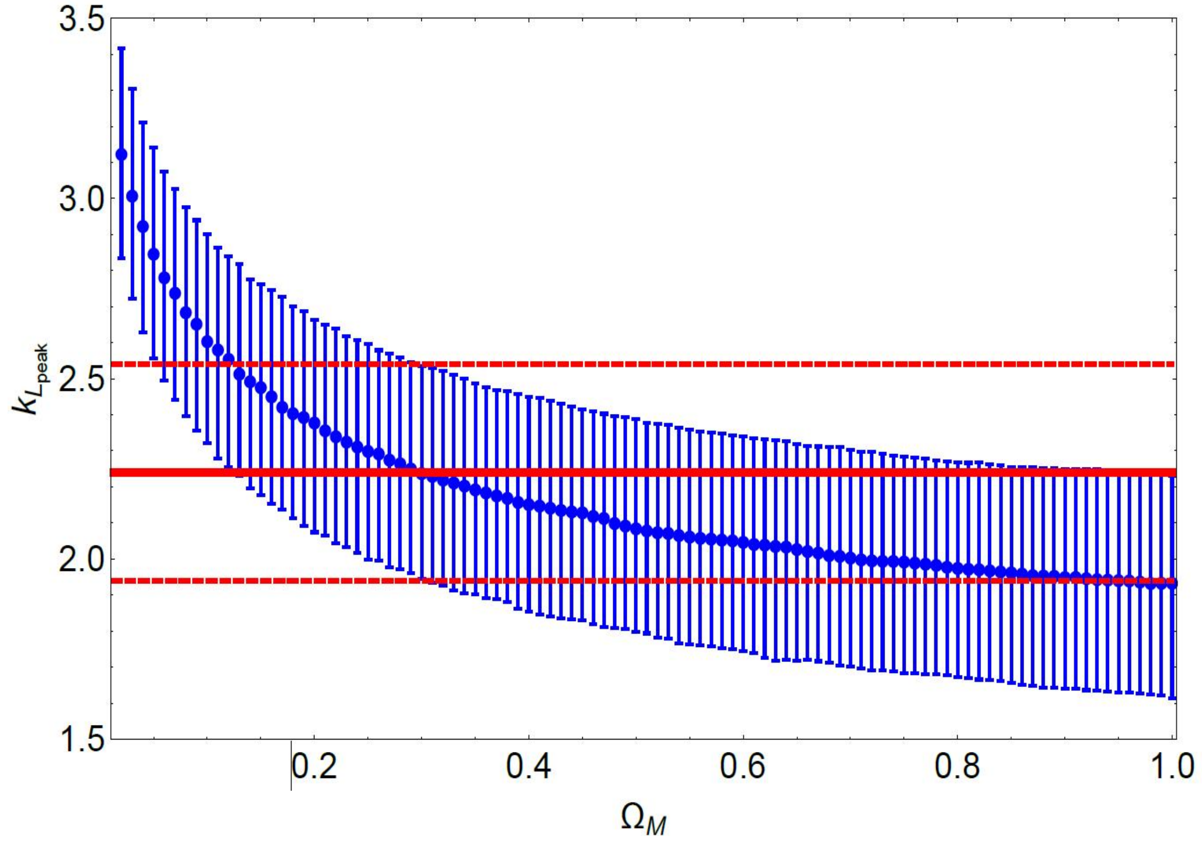
CONLEY ET AL. 2011 -> EVEN NOW $\Omega_m = 0.214^{+0.072}_{-0.097}$

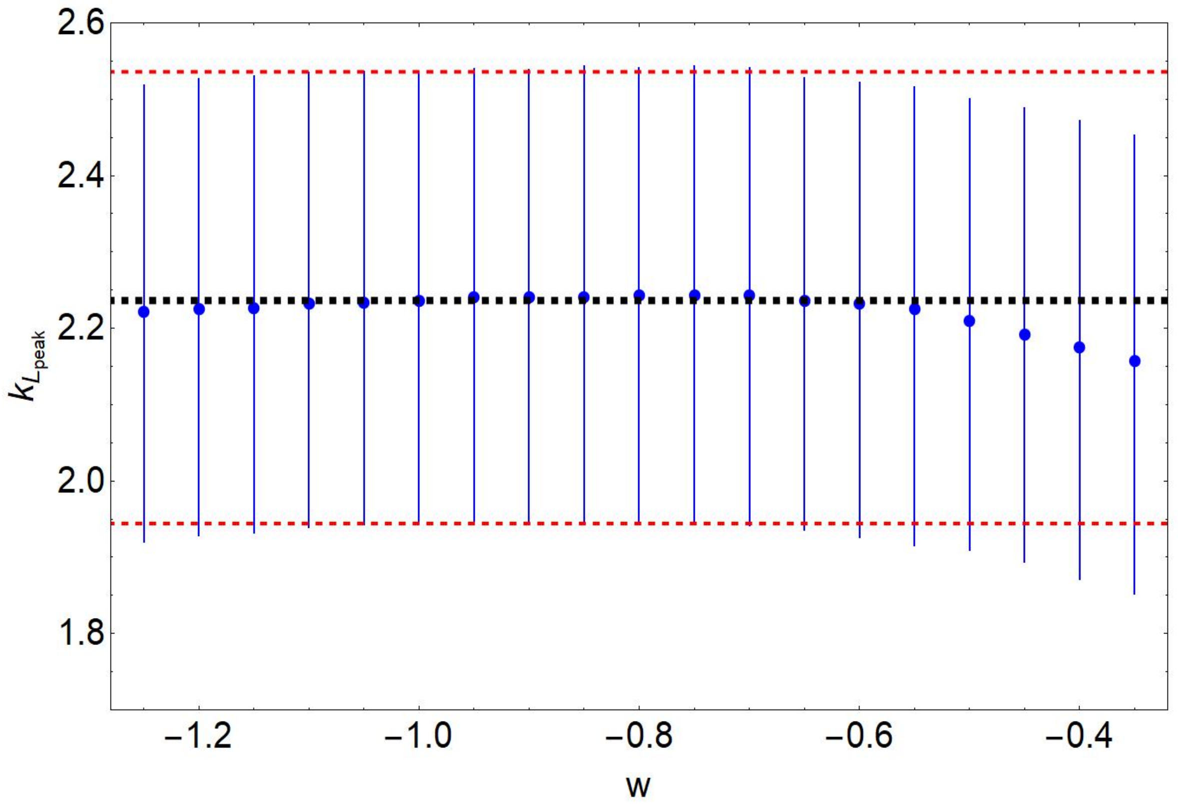
BETOULE ET AL. 2014 -> BY 2030 Ω_m: 0.295 ± 0.034

SCOLNIC ET AL. 2018 -> BY 2042 0.309 ± 0.007



Dependence of the k parameter on w and Ω_M





Conclusions

SNe Ia + BAOs ANALYSIS

> THE OBSERVED DECREASING TREND OF H0 IN THE PANTHEON SAMPLE CAN BE EXPLAINED THROUGH HIDDEN ASTROPHYSICAL BIASES OR, IF THIS IS NOT THE CASE, THROUGH MODIFIED GRAVITY SCENARIOS (E.G. THE f(R) THEORIES)

> SUCH A TREND IS STILL VISIBLE EVEN EXPANDING THE DIMENSIONS OF THE PARAMETERS SPACE UP TO 2

> THE BAOs CONTRIBUTION CONFIRMED THE OBSERVED TREND WITH SNe Ia

GRBs ANALYSIS

> THE SIMULATION OF GRBs WITH PLATEAU EMISSION FOLLOWING THE FUNDAMENTAL PLANE RELATION SHOWED HOW IN THE NEXT YEARS THE GRBs WILL REACH A PRECISION FOR THE ESTIMATION OF THE COSMOLOGICAL MATTER DENSITY PARAMETERS SIMILAR TO THE ONE WE HAVE TODAY WITH SNe Ia

FUTURE PERSPECTIVES: *NEW SNe Ia DATA (PANTHEON+, SCOLNIC ET AL. 2022)*

Thank you for your attention!

IF THERE ARE
ANY QUESTIONS,
PLEASE FEEL
FREE TO ASK

Have a look at our papers:
https://arxiv.org/abs/2103.02117

If you want to join us:

maria.dainotti@nao.ac.jp

Have a look at our papers:
https://arxiv.org/pdf/2201.09848.pdf

1 – SOME CONSIDERATIONS

<u>In this case, the parameter space has been enlarged up to 2-dimensions.</u>

1) In order to have a reliable statistical representation of the Pantheon sample, we focus our analysis on the case of 3 bins, ignoring the subsequent divisions of the Pantheon sample.

2) In the current analysis, it is important to consider the following constraint in the $w_0 w_a CDM$ case,

$w(z) > -1$ where $w(z) = w_0 + w_a * \frac{z}{1+z}$ is the CPL parametrizatio n

otherwise the analysis would describe a universe which is not expanding (contradicting the main cosmological observations).

The starting value of the MCMC minimization

FOR EACH BIN OF SNe Ia, A χ^2 TEST IS PERFORMED IN ORDER TO FIND THE BEST VALUE FOR H_0

$$\mu_{obs}^{(SN)} = m_B - M + \alpha x_1 - \beta c + \Delta M + \Delta B$$

$$\mu_{th}^{(SN)}(z, H_0, \ldots) = 5 * \log_{10}\left(\frac{d_L(z.H_0, \ldots)}{10pc}\right) + 25$$

$$\chi^2 = \sum_i \frac{(\mu_{obs}^i - \mu_{th}^i)^2}{{\varepsilon_{\mu obs}^i}^2}$$

THE CANONICAL χ^2 DEFINITION; HERE THE MODEL H IS REPRESENTED BY $\mu_{th}^{(SN)}$ WITH PARAMETER H_0 (1-D ANALYSIS)

$$\chi_{SN}^2 = \Delta\mu^T C^{-1} \Delta\mu$$

THIS IS THE GENERALIZATION WITH THE COVARIANCE MATRIX C, WHICH INCLUDES STATISTICAL UNCERTAINTIES (DIAGONAL PART) AND SYSTEMATIC CONTRIBUTIONS (OFF-DIAGONAL)

$$\Delta\mu = \mu_{obs}^{(SN)} - \mu_{th}^{(SN)}$$

A CUSTOMIZED CODE WAS WRITTEN TO EXTRACT THE SUBMATRICES FOR THE GIVEN SUBVECTORS OF REDSHIFT ORDERED SUPERNOVAE

The systematics of SNe Ia

IT WAS SUGGESTED BY SULLIVAN ET AL. 2010 THAT THE HOST GALAXY MASS CONTRIBUTION COULD BE INSERTED IN THE $\mu_{obs}^{(SN)}$ FORMULA AS A THIRD CORRECTING PARAMETER

THIS CORRECTION IS PERFORMED IN SCOLNIC ET AL. 2018 -> THE SIZE OF THESE SISTEMATIC EFFECTS IS ON THE 1%

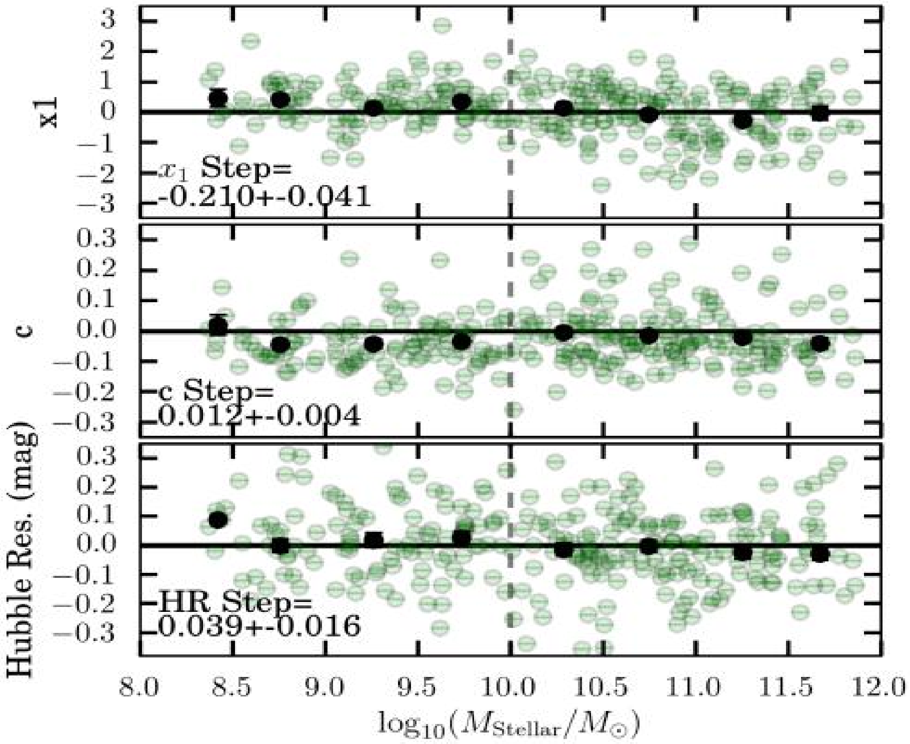


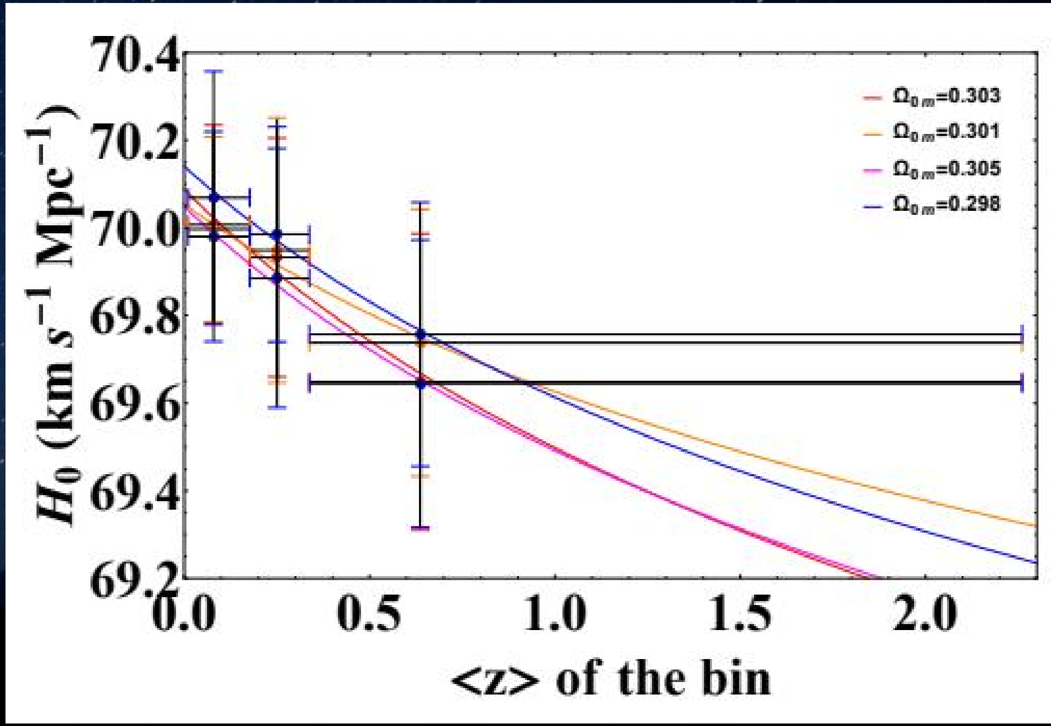
Figure 9. Correlations in the data between color, stretch and Hubble Residuals with host galaxy mass. A vertical line is shown at a host galaxy mass equal to $\log_{10}(M_{Stellar}/M_\odot) = 10$. Steps are expressed as parameters for the higher mass group minus the lower mass group.

2 – Hu-Sawicki model

Testing the Hu & Sawicki (2007) model with $n = 1$

$$S_g = -\frac{1}{2\chi}\int d^4x\sqrt{-g}f(R)$$

$$f(R) = R + F(R) = R - m^2\frac{c_1\left(R/m^2\right)^n}{c_2\left(R/m^2\right)^n + 1}$$



In the case of $F_{R0} = -10^{-7}$ (value of the field at the present time)

3 - The code in action (1/3)

```
def my_likeSNE(H0):

    O_m=0.298
    M=-19.24988619
    H0_s=H0/(3.085677581491367*10**19)
    c_cm=scipy.constants.c*100
    d_par=np.array([])

    def integrand(r,O_m):
        return 1/(((O_m*(1+r)**3)+1-O_m))**(1/2)

    for i in z:
        I=quad(integrand, 0, i, args=(O_m))
        d_par=np.append(d_par,I[0])

    d=np.array(c_cm*(1+zhel)*d_par/H0_s)
    d_megaparsec=d*(3.2408*10**(-25))

    logdl_th=np.log10(d_megaparsec)

    muthSNE=5*logdl_th+25


    mu=mb-M
    Deltamu=mu-muthSNE


    return(-np.sum(np.matmul(Deltamu,np.matmul(Cinverse,Deltamu))))
```

MINIMIZING THE NEGATIVE LIKELIHOOD IS LIKE MAXIMIZING THE LIKELIHOOD

```
guess=73.5

def log_prior(array):
    H0=array
    if 60<H0<80:
        return 0.0
    return -np.inf

def log_probability(array):
    H0=array
    lp = log_prior(array)
    if not np.isfinite(lp):
        return +np.inf
    return -(lp + my_likeSNE(H0))

"""
def neg_like(array):
    O_m,H0=array
    return -1*my_likeSNE(O_m,H0)
"""

soln=minimize(log_probability, guess, method="SLSQP")
print(soln.x)

H0=float(soln.x)
```

GUESS VALUE FOR MINIMIZING

MINIMIZING VALUE AS START FOR MCMC

3 - The code in action (2/3)

```
info = {"likelihood": {"agostini": my_likeSNE}}

from collections import OrderedDict as odict

info["params"] = odict([["H0", {"prior": {"min":60, "max":80},"ref":H0,"proposal": 0.01}]])


info["sampler"] = {
    "mcmc": {"burn_in": 300, "max_samples": 10000000, "Rminus1_stop": 0.1, "Rminus1_cl_stop": 0.2, "learn_proposal": True}}

from cobaya.run import run

updated_info, products = run(info)

%matplotlib inline
from getdist.mcsamples import MCSamplesFromCobaya
import getdist.plots as gdplt

gdsamples = MCSamplesFromCobaya(updated_info, products["sample"],ignore_rows=0.3)
gdplot = gdplt.getSubplotPlotter(width_inch=5)
gdplot.triangle_plot(gdsamples, ["H0"], filled=True)



mean = gdsamples.getMeans()[:1]
sigma = np.sqrt(np.array(gdsamples.getVars()[:1]))



print("Mean:")
print(mean)
print("Sigma:")
print(sigma)
```

PRIORS

$n = 300$ (BURN-IN)

$R - 1$ CONDITIONS

PLOTTING OPTIONS

The power of Bayesian approaches

GIVEN THE HYPOTHESIS AS «H» AND THE DATA OBSERVED AS «A», THE BAYES THEOREM STATES

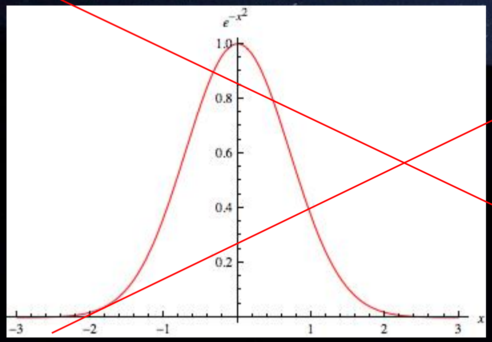
$$P(H|A) = \frac{P(H) * P(A|H)}{P(A)}$$

Posterior probability of hyphotesis H, given data A

Prior probability of hypothesis

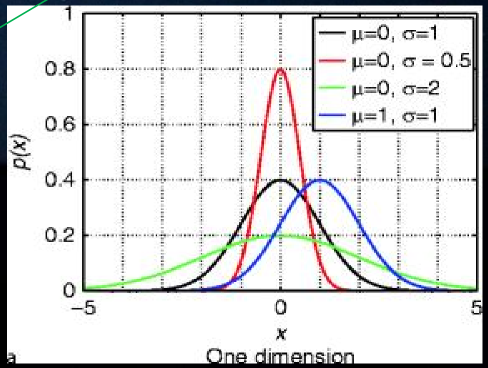
Likelihood function: probability of obtaining A given the H (to be maximized)

Prior probability of data (normalization)



Probability of having a given data set

Give n a data set

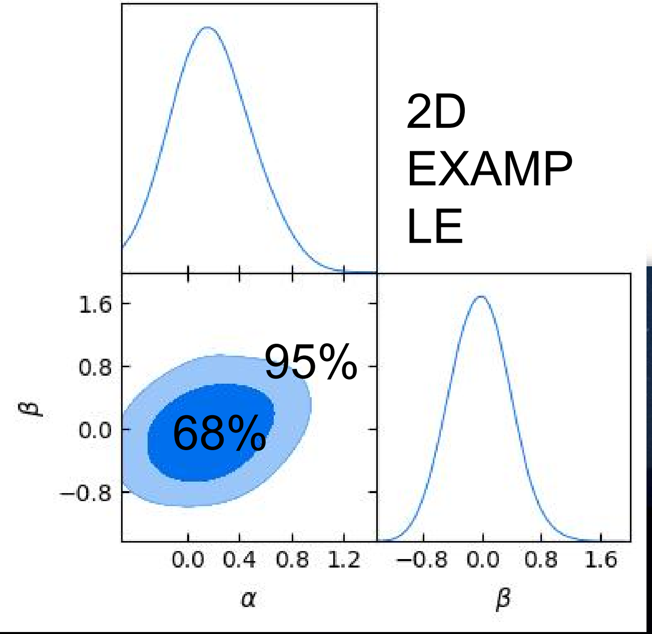


Probability that this data set comes from a given distribution

3 - The code in action (3/3)

```
[74.37113291]
[agostini] Initialised external likelihood.
[mcmc] Covariance matrix not present. We will start learning the covariance of the proposal earlier: R-1 = 30 (was 2).
[mcmc] Initial point:
[mcmc]
 weight  minuslogpost         H0  minuslogprior  minuslogprior__0       chi2  chi2__agostini
    1.0      32.20881  74.371133       2.995732          2.995732  58.426156       58.426156
[mcmc] Sampling! (NB: nothing will be printed until 300 burn-in samples have been obtained)
[mcmc] Finished burn-in phase: discarded 300 accepted steps.
[mcmc] Checkpoint: 40 samples accepted.
[mcmc] Ready to check convergence and learn a new proposal covmat
[mcmc] Convergence of means: R-1 = 0.727792 after 40 accepted steps
[mcmc] Updated covariance matri
[mcmc] Checkpoint: 80 samples a
[mcmc] Ready to check convergen
[mcmc] Convergence of means: R-
[mcmc] Updated covariance matri
[mcmc] Checkpoint: 120 samples
[mcmc] Ready to check convergen
[mcmc] Convergence of means: R-
```

MINIMIZING




```
[mcmc] Convergence of means: R-1 = 0.072575 after 320
[mcmc] Convergence of bounds: R-1 = 0.356695 after 320
[mcmc] Updated covariance matrix of proposal pdf.
[mcmc] Checkpoint: 360 samples accepted.
[mcmc] Ready to check convergence and learn a new prop
[mcmc] Convergence of means: R-1 = 0.064717 after 360
[mcmc] Convergence of bounds: R-1 = 0.291676 after 360
[mcmc] Updated covariance matrix of proposal pdf.
[mcmc] Checkpoint: 400 samples accepted.
[mcmc] Ready to check convergence and learn a new prop
[mcmc] Convergence of means: R-1 = 0.053685 after 400
[mcmc] Convergence of bounds: R-1 = 0.146532 after 400
[mcmc] The run has converged!
[mcmc] Sampling complete after 400 accepted steps.
[root] *WARNING* outlier fraction 0.1
Mean:
[74.37492303]
Sigma:
[0.82045577]
```

We thank A. Lenart and G. Sarracino for the support on the
cosmological computations

Monte Carlo Markov-Chain (1/3)

POPULAR METHOD TO OBTAIN INFORMATION ABOUT POSTERIOR DISTRIBUTIONS

MONTE CARLO: estimate the properties of a distribution studying extracted random samples

MARKOV-CHAIN: the chain of creation of the random samples.

Markov property: the step x_i depends on the step x_{i-1} but not on the step x_{i-2}

CONSIDERING H = THE COSMOLOGICAL MODEL FOR OBSERVED DATA

$$\mathrm{P}(H|A) \sim P(H) * P(A|H)$$

- Starting from a guess prior value for the $\mathrm{P}(H)$, a series of posterior values $\mathrm{P}(H|A)$ is obtained to check the average and 1-sigma for the posterior distribution
- After the first guess prior value, adding a small perturbation to $\mathrm{P}(H)$, a proposal step is created
- If accepted, the proposal becomes the new value from which the proposal is drawn, otherwise another proposal is created

Lewis, A., 2013, Phys. Rev. D87, 103529

Monte Carlo Markov-Chain (2/3)

LET'S CONSIDER y AS SLOW VARIABLE AND x AS FAST VARIABLE

↓

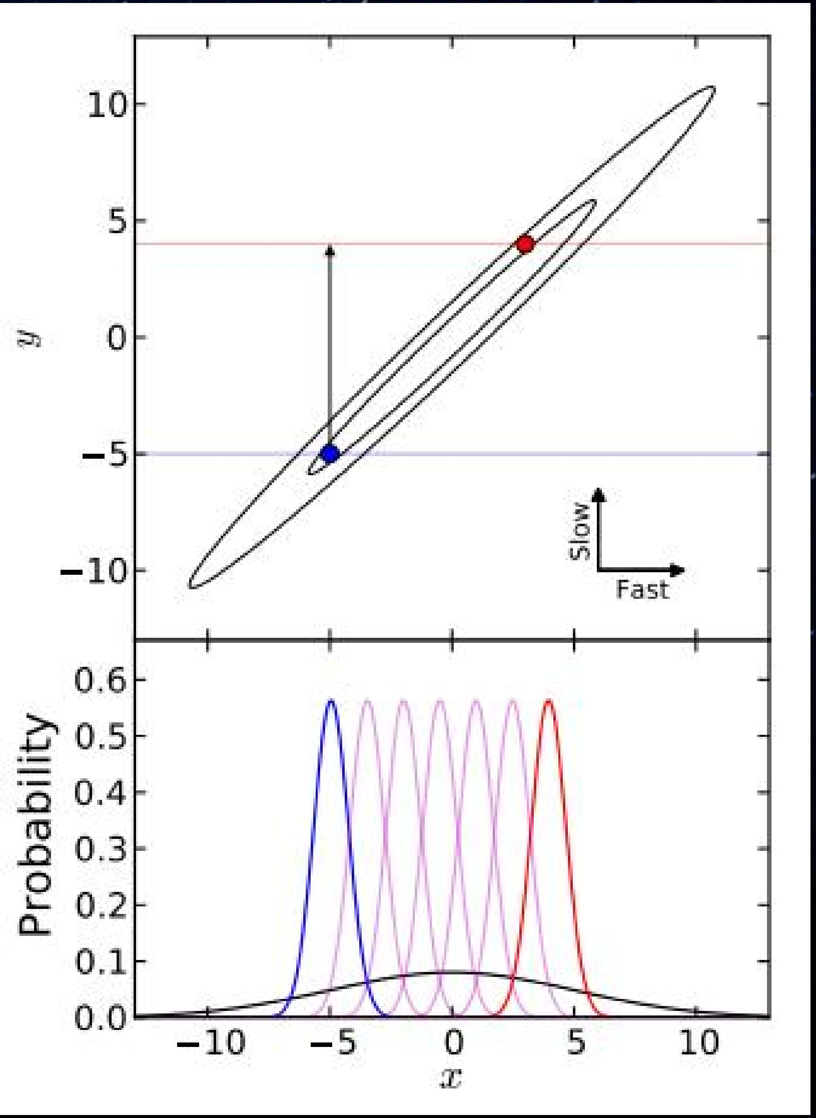
FROM THE SLOW y VALUE A NEW y' VALUE IS PROPOSED

↓

A SERIES OF POISSON DISTRIBUTIONS $P_i(x)$ INTERPOLATE BETWEEN $\mathrm{P}(x|y)$ AND $\mathrm{P}(x|y')$ VALUE WHEN A NEW y' VALUE IS PROPOSED

↓

THE STEP $(x, y) \rightarrow (x', y')$ IS ACCEPTED WITH A GIVEN PROBABILITY



Lewis, A., 2013, Phys. Rev. D87, 103529

Monte Carlo Markov-Chain (3/3)

THE METHOD THAT HERE IT'S USED IS THE D'AGOSTINI METHOD (G. D'AGOSTINI, 2005 FOR REVIEW)

- THE ERRORS ON THE DIFFERENT PARAMETERS OF THE MODEL H ARE COMPARABLE
- IT'S NOT EASY TO SAY WHICH OF THE PARAMETERS IS THE INDEPENDENT AND WHICH IS THE DEPENDENT

<u>THE MONTE CARLO MARKOV-CHAIN STARTS FROM THE VALUE THAT MAXIMIZES THE LIKELIHOOD FUNCTION</u>

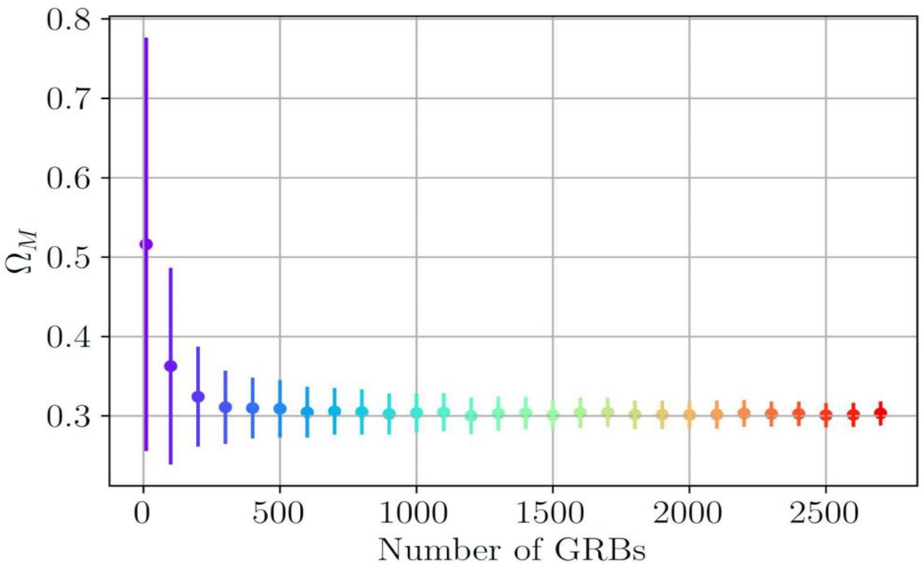
A SEQUENCE OF BURN-IN SAMPLES IS CONSIDERED:
THE BURN-IN IS A COMMON PRACTICE IN MCMC COMPUTATIONS. THE FIRST n STEPS OF THE CHAIN ARE THROWN AWAY (NOT USED AS PROPOSALS), with $n \sim 100$. THIS ALLOWS THE CHAIN TO ENTER THE REGION WHERE THE STATES OF THE MARKOV CHAIN ARE MORE REPRESENTATIVE OF THE PARAMETER SAMPLE

TO CHECK CONVERGENCE, THE GELMAN RUBIN-STATISTICS $R - 1$ IS APPLIED (GELMAN, RUBIN, 1992)

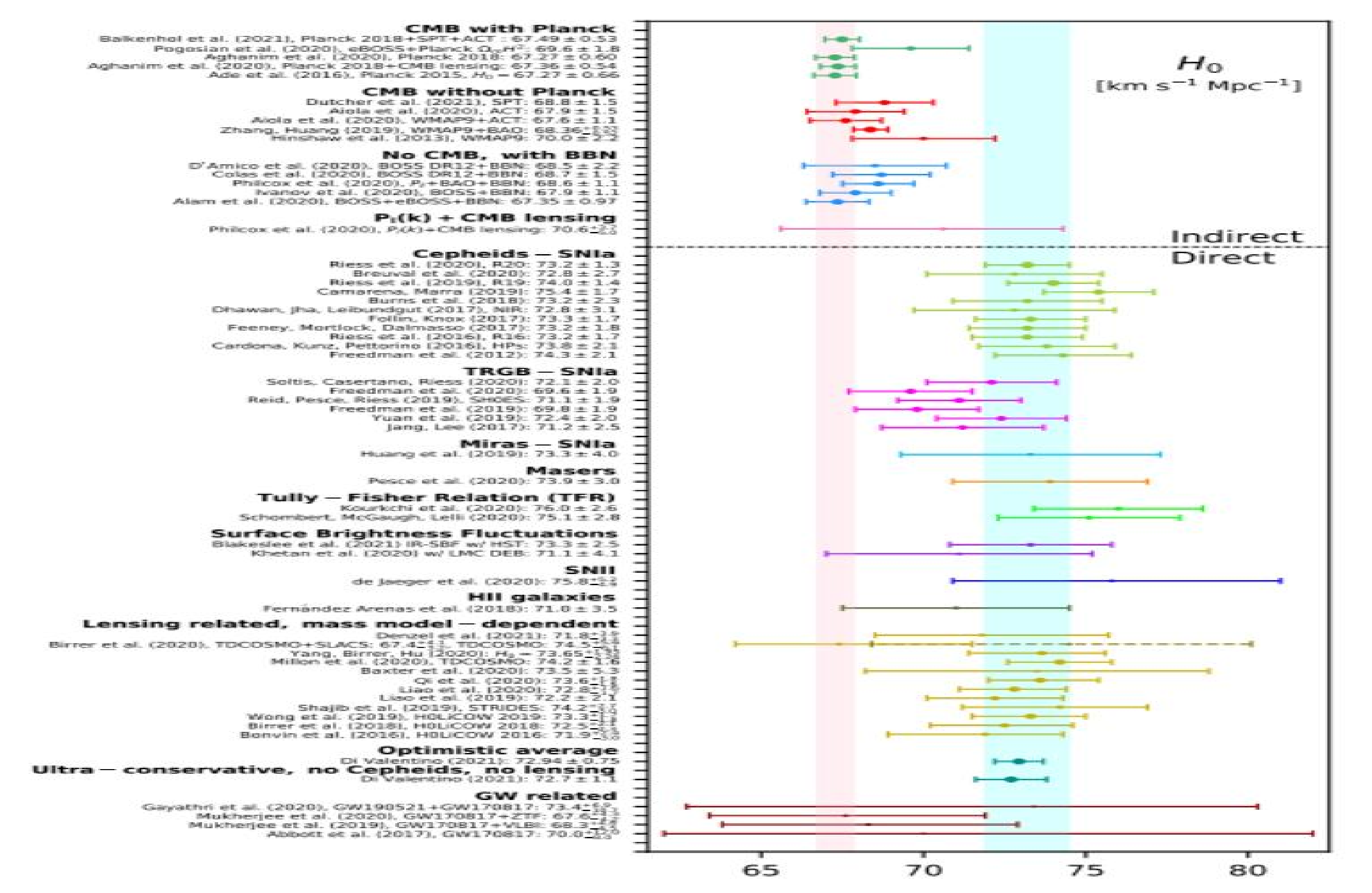
- Simulating additional GRBs:

how many GRBs are needed as standalone probes to achieve a comparable precision on Ω_M to the one obtained by SNe Ia only? In which years will these numbers be reached?

  - Same error measurements as SNe Ia in 2011:
    - with 142 simulated optical GRBs with errorbars halved → reached in 2038
    - with a doubled sample (future machine learning approaches for LC reconstruction and estimates of GRB redhifts) with errorbars halved → already reached now
  - Same error measurements as SNe Ia in 2014:
    - with 284 simulated optical GRBs with errorbars halved → reached in 2047
    - with a doubled sample and errorbars halved → reached in 2026
  - Same error measurements as current SNe Ia:
    - With 390 (doubled) simulated optical GRBs with errorbars halved → reached in 2054



OPTICAL | Simulation Results for the Full Optical Base with Halved Errors



Di Valentino et al. 2021, CQG, 38, 153001

THE CONTRIBUTIONS TO THE χ^2 GIVEN BY THE BAOs IS ADDED TO THE CONTRIBUTION OF SNe

$$D_V(z) = \left[\frac{czd_L^2(z)}{(1+z)^2H(z)}\right]^{1/3}, \qquad d_z(z) = \frac{r_s(z_d)}{D_V(z)}.$$

$$r_s \approx \frac{55.154 \cdot e^{-72.3(\omega_\nu+0.0006)^2}}{\omega_{0m}^{0.25351}\omega_b^{0.12807}} \text{ Mpc}$$

SOUND HORIZON SCALE

where $\omega_i = \Omega_i \cdot h^2$, and $i = m, \nu, b$ represent matter, neutrino and baryons

$$\Delta d = d_z^{obs}(z_i) - d_z^{theo}(z_i)$$

M is the covariance matrix for the BAO $d_z^{obs}(z_i)$ values.

TOTAL χ^2

$$\chi^2 = \frac{1}{2}\chi^2_{SN} + \frac{1}{2}\chi^2_{BAO}$$

$$\chi^2_{BAO} = \Delta d^T \cdot M^{-1} \cdot \Delta d$$

Data set and methodology:

- Subsample of 222 GRBs with redshift measurements and LC plateaus from all 1064 GRBs of Swift-XRT
- GRB standardization with 3D fundamental plane relation and 3D optical Dainotti correlation
- Correction for redshift evolutionary effects with EP method
- No circularity problem

Results:

- Using EP method: smallest intrinsic scatter on X-ray 3D fundamental plane in the literature (44.4% reduction)
- Flat ΛCDM + combining GRBs with SNe Ia:
  - $\Omega_M = 0.299 \pm 0.009$ with and without correcting GRBs for selection biases and redshift evolution
- 3D optical Dainotti correlation as efficient as the X-ray sample in determining Ω_M