Generalizing the Friedmann Model in Light of Cosmological Tensions

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in collaboration with Theo Anton

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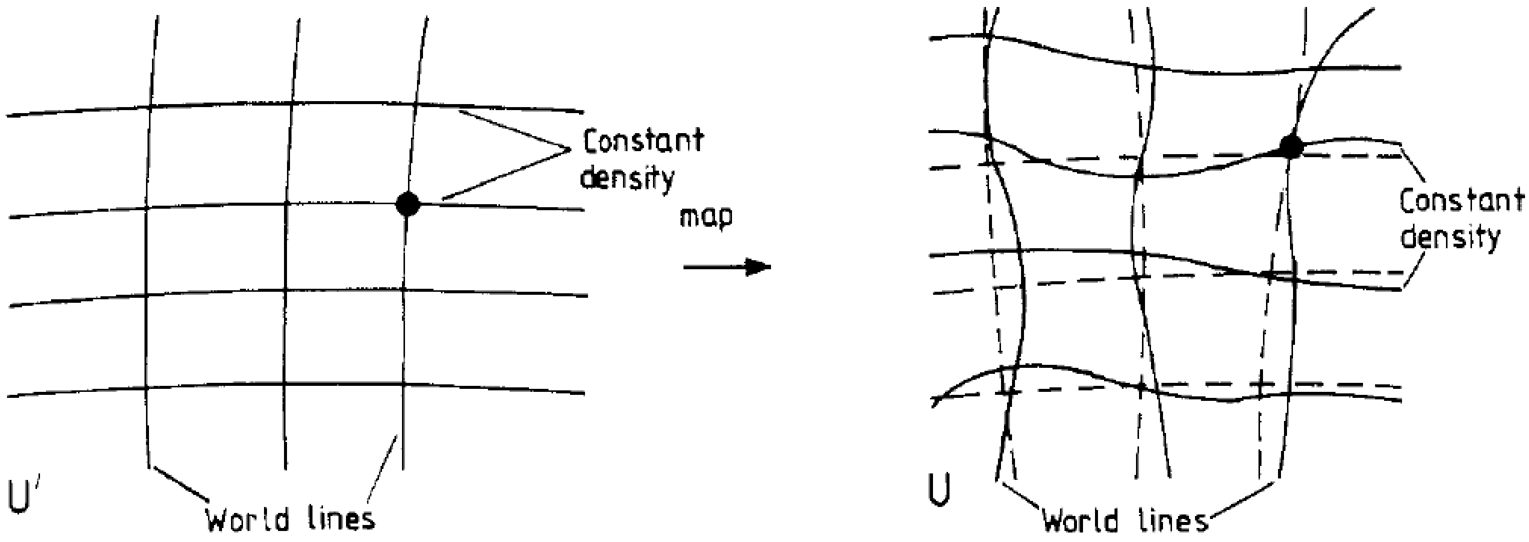

[Ellis & Stoeger CQG 4, 1697 (1987)]

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   - *à la* Gasperini *et al.*

Buchert's approach

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- Define an 'average' scale factor: $a_{\mathcal{D}}(t) \equiv \left(\frac{\int_{\mathcal{D}} d^3X \sqrt{^{(3)}g(t, X^i)}}{\int_{\mathcal{D}} d^3X \sqrt{^{(3)}g(t_0, X^i)}} \right)^{\frac{1}{3}}$

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where: $\langle\psi\rangle_{\mathcal{D}}(t) \equiv \frac{\int_{\mathcal{D}} d^3X \sqrt{^{(3)}g(t, X^i)}\,\psi(t, X^i)}{\int_{\mathcal{D}} d^3X \sqrt{^{(3)}g(t, X^i)}}$, $\qquad \mathcal{Q}_{\mathcal{D}} \equiv \frac{2}{3}\left(\langle\Theta^2\rangle_{\mathcal{D}} - \langle\Theta\rangle_{\mathcal{D}}^2\right) - 2\langle\sigma^2\rangle_{\mathcal{D}}$.

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most current modelling suggests these effects are small

Anisotropic observations

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  see e.g. MacCallum, arXiv:2001.11387

Locally rotational symmetry

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(the spatially homogeneous LRS solutions)

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$$\frac{2}{9}\langle\Theta\rangle^2 - \frac{1}{2}\langle\phi\rangle^2 + \frac{1}{3}\langle\Theta\rangle\langle\Sigma\rangle - \langle\Sigma\rangle^2 - \frac{2}{3}(\langle\mu\rangle + \Lambda) - \frac{1}{2}\langle\Pi\rangle - \langle\mathcal{E}\rangle + 2\langle\xi\rangle^2 = \mathcal{Q}_1$$

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$$\langle\Sigma\rangle^{\cdot}+\frac{2}{3}\langle\Theta\rangle\langle\Sigma\rangle+\frac{1}{2}\langle\Sigma\rangle^2+\langle\mathcal{E}\rangle-\frac{1}{2}\langle\Pi\rangle-\frac{1}{3}\left(2\langle\mathcal{A}\rangle-\langle\phi\rangle\right)\langle\mathcal{A}\rangle=\mathcal{Q}_3$$

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where, e.g.,

$$\mathcal{Q}_3=\frac{1}{3}\mathrm{Cov}\left(\Theta,\Sigma\right)+\frac{2}{3}\mathrm{Var}\mathcal{A}-\frac{1}{3}\mathrm{Cov}\left(\phi,\mathcal{A}\right)+\frac{2}{3}\langle m^a D_a\mathcal{A}\rangle-\frac{1}{2}\mathrm{Var}\Sigma-\frac{1}{3}\left\langle M^{ab}D_a\mathcal{A}_b\right\rangle$$
$$-\frac{1}{3}\langle\Sigma_a\Sigma^a\rangle+\frac{1}{3}\langle\mathcal{A}_a\mathcal{A}^a\rangle+\frac{1}{3}\left\langle\Sigma_{ab}\Sigma^{ab}\right\rangle+2\langle\alpha_a\Sigma^a\rangle-\frac{2}{3}\langle a_a\mathcal{A}^a\rangle\,.$$

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publication to appear!

Thank you